Is Hintikka's Independence-Friendly Logic a revolutionary non-classical first-order language?

Abstract

This paper investigates Hintikka's Game-Theoretical Semantics (GTS), starting from its application to classical First-Order Logic (FOL), in preparation to its further transposition into Hintikka's Independence-Friendly Logic (IFL). I shall argue that the literal transferring of the clauses of GTS into IFL is, first, unjustified; and second, unnatural. Finally, I will suggest that a natural semantics for IFL is well-behaved, does not violate the central classical theorems which Hintikka claims it violates, and is thus only an extension of FOL with more expressive power, just like any other extension of FOL with non-classical (e.g. generalized) quantifiers.

Hintikka (mostly in his 1996 book, among many other places) has vigorously claimed that Independence-Friendly Logic (IFL) together with Game-Theoretical Semantics (GTS) constitute the really fundamental first-order language, and should as such replace classical First-Order Logic (FOL) in general practice —even for the purpose of teaching basic logic in the universities. Among the various advantages of IFL-GTS over FOL he points out, the most striking ones are related to the invalidity of central theorems of classical logic, especially Gödel's completeness and Tarski's undefinability of truth. Moreover, IFL-GTS is three-valued and, according to Sandu and Hintikka (2001) (in spite of Hodges 1997), is not compositional in a suitable sense. Now we ask: Does IFL really deserve the status of revolutionary non-classical logic?

1 Game-Theoretical Semantics for First-Order Logic

GTS introduces games which are played only with complex sentences [1], by two players, the Verifier (*aka* Eloise) and the Falsifier (*aka* Abelard). As one would

[1] We shall concentrate on quantifiers and negation and omit conjunction and disjunction throughout the paper, since these latter connectives are treated in GTS much the same way as quantifiers.

expect, games are played with respect to some given structure of interpretation. Therefore, we use the general notation $G(\varphi, \mathbf{M})$ to mean a game played with the sentence φ in structure $\mathbf{M}$. The individual domain of $\mathbf{M}$ is written M.

Quantifiers $(\exists)$ and $(\forall)$ are then viewed as moves in the game associated with choices by Eloise and Abelard respectively. The motivation behind the picture is the following. If we have a sentence of the kind $\varphi = \exists x R x$, to prove that φ is true in $\mathbf{M}$ is the same thing as finding some individual $\bar{a} \in M$ of name a, such that Ra is true in $\mathbf{M}$, i.e. $(\bar{a}) \in R^{\mathbf{M}}$. This is a natural move for Eloise, since she has the power of verifying (or not) φ in just one move: it suffices for her to give such a suitable a, and the game is over. Similarly for Abelard and the universal $(\forall)$. Thus we give Eloise the right of playing under the presence of an existential quantifier, whereas the universal quantifier indicates that choice is attributed to Abelard.

The well-known overall mechanism of games $G(\varphi, \mathbf{M})$ for the other logical constants is given by the clauses below. For all sentences of FOL, we may reach through (a finite number of) moves by Abelard and Eloise an atomic sentence which is directly evaluable in the relevant structure. In other words, Abelard and Eloise's choices eliminate quantifiers and connectives of the initial sentence. All these observations can be informally stated through the following set of inductive rules given by Hintikka (1996, p. 25) for games $G(S)$ in some given structure $\mathbf{M}$:

(R. $\exists$) $G(\exists x S x)$ begins with the choice by the verifier of a member of M. If the name of this individual is b, the rest of the game is as in $G(S b)$.

(R. $\forall$) $G(\forall x S x)$ likewise, except that falsifier makes the choice.

(R. $\neg$) $G(\neg S)$ is like $G(S)$, except that the roles of the two players (as defined by these rules) are interchanged.

(R. At) If A is a true atomic sentence (or identity), the verifier wins $G(A)$ and the falsifier loses it. If A is a false atomic sentence (or identity), vice versa.

Next we may ask: if Eloise (Abelard) verifies (falsifies) some φ in a given $\mathbf{M}$, does it mean that φ is true (false) in $\mathbf{M}$? The general answer is "No". For φ to be true (false), we must actually require that the choices by Eloise (Abelard) be such that Abelard (Eloise) has *no possible* good choice for him (her). In other words, φ is true (false) in $\mathbf{M}$ if and only if Eloise (Abelard) has some sequence of choices such that she (he) wins every play of the game regardless of how Abelard (Eloise) plays. A sequence of choices of that kind is called a *winning strategy*. Hintikka glues all these observations as follows (*ibid.*, p. 26):

(R. T) S is true in $\mathbf{M}$ if and only if there exists a winning strategy for the verifier in the game $G(S)$ when played in $\mathbf{M}$.

(R. F) S is false in $\mathbf{M}$ if and only if there exists a winning strategy for the falsifier in the game $G(S)$ when played in $\mathbf{M}$.

1.1 From Games to Quantification

Let us mind our step. What exactly are Eloise and Abelard willing to verify and falsify with respect to some quantified sentence φ?

Consider the atomic formula $R(x, y)$. Since quantifiers are interpreted as determining which player has the move in the game, the formula $\forall x \exists y R(x, y)$ is at bottom related to a game $G(R, \mathbf{M})$ over the matrix $R(x, y)$ with two (ordered) choice-points, the first for x which is taken care of by Abelard, the second for y whose choice is carried through by Eloise. Generally, quantifier games in GTS are not really played over quantified sentences, but rather over their matrices according to the choice-points determined by the quantifiers. In other words, the function of quantifiers is to settle the turns and the order of the choices related to a game over the matrix. In our example, the sequence $\langle \forall x, \exists y \rangle$ determines that the game $G(R, \mathbf{M})$ is to be played with the choice of x being ascribed to Abelard, and that of y to Eloise [2].

From this point of view, the mechanism of $G(R(x_A^1, y_E^2), \mathbf{M})$ is described as follows: Abelard plays first, and picks an element of M whose name is a_1, and the game goes on as $G(R[a_1], \mathbf{M})$ —with the obvious notation. Then Eloise picks an element of M whose name is a_2, and the games goes on as $G(R[a_1, a_2], \mathbf{M})$. Since $R[a_1, a_2]$ is an atomic sentence, we say that Abelard wins $G(R, \mathbf{M})$ if $(a_1, a_2) \notin R^{\mathbf{M}}$; and Eloise wins the game if $(a_1, a_2) \in R^{\mathbf{M}}$.

Players are not playing for just winning, but rather for winning *every possible play* of the game. Not every sequence $(a_i, a_j) \in M^2$ is a possible outcome of the game, because players will never make choices which may lead to his/her opponent's victory. If $(a_i, a_j) \notin R^{\mathbf{M}}$ for some i, j, then Eloise will never choose a_j if Abelard chooses a_i, therefore (a_i, a_j) is not a possible outcome of $G(R(x_A^1, y_E^2), \mathbf{M})$.

To sum up, we may now say that Abelard's final goal is to falsify *the matrix* $R(x, y)$ of $\forall x \exists y R(x, y)$, for all possible outcomes of $G(R(x_A^1, y_E^2), \mathbf{M})$. Similarly, Eloise's challenge is to verify *the matrix* $R(x, y)$ of $\forall x \exists y R(x, y)$, for all possible outcomes of $G(R(x_A^1, y_E^2), \mathbf{M})$. But this in turn is to say the following: Abelard's goal is to provide some constant a for x such that for all elements possibly chosen for y, it is the case that $(a, y) \notin R^{\mathbf{M}}$. Eloise's goal is to provide, for all elements

[2] This further suggests that GTS-style games can be defined also for atomic sentences like $R(a, b)$, with $\bar{a}, \bar{b} \in M$, the particular feature being that the "choices" (which we had better call "random draws") are no longer supplied by active players (who are now merely bettors, one betting for truth and the other for falsity), but by a kind of lottery machine. I investigate this line in another working paper.

possibly chosen for x, some constant b such that $(x, b) \in R^{\mathbf{M}}$. As a consequence, each player has a quantified sentence to prove in the game: Abelard wants to show

$$\varphi = \exists x \forall y\, \neg R(x, y); \tag{1}$$

whereas Eloise wants to show

$$\varphi' = \forall x \exists y\, R(x, y). \tag{2}$$

As we see, the game over $R(x, y)$ in which we give the first choice to variable x and player Abelard, and the choice of y to Eloise, is associated with two quantified conditions, namely (1) and (2), each one giving the respective goal of each player. If we give different variables to different players in different orders, the conditions will vary accordingly.

Thus as it stands, the game $G(R(x^1_A, y^2_E), \mathbf{M})$ gives truth conditions *for two different quantified sentences*, again (1) and (2). Specifically, we have:

$$\begin{aligned} &\varphi \textit{ is True in } \mathbf{M} \text{ iff Abelard has a winning strategy in } G(R(x^1_A, y^2_E), \mathbf{M})\,; \\ &\varphi' \textit{ is True in } \mathbf{M} \text{ iff Eloise has a winning strategy in } G(R(x^1_A, y^2_E), \mathbf{M}). \end{aligned} \tag{3}$$

We are starting here from the definition of the semantic game in order to come upon the quantified conditions. The conclusion so far is that the game $G(R(x^1_A, y^2_E), \mathbf{M})$ gives only truth (but no falsity) conditions for two different quantified sentences, one for each player. GTS is essentially the same game as we have just described, the only difference being a notational one: instead of writing $G(\forall x \exists y R(x, y), \mathbf{M})$, we put $G(R(x^1_A, y^2_E), \mathbf{M})$. However, this latter notation says in addition that φ' is related to *the same game* as φ.

But if so, GTS games do *not in principle* give any falsity conditions on quantified sentences. Abelard is the falsifier, but what he is intrinsically falsifying is the matrix of the quantified sentence, not the sentence itself. In order for us to securely state that he is falsifying the whole sentence, we need to be sure that —to take our particular case— the first clause of (3) is in fact equivalent to the negation of the second, without which the statement that Abelard is the falsifier of the quantified sentence turns out to be an *ad hoc* affirmation.

Fortunately, the equivalence does hold. On the other hand, however, it is an equivalence which goes beyond the game-theoretical perspective, and which is crucially grounded on the standard semantics of FOL. The equivalence is, of course: $\neg\varphi' \leftrightarrow \varphi$. Under this equivalence, we can define the falsity of φ' as the truth of φ (the same for the falsity of φ), which leads directly to Hintikka's clauses. Therefore, the justification of Hintikka's game-theoretical falsity condition draws crucially *on a peculiar fact of* the standard semantics of FOL, not on any intrinsic feature of GTS games.

As long as we are placed within FOL (with its standard semantics), there is thus no conflict involved (the following has the obvious notations):

Theorem. *For every $\varphi = Q_1 x_1 \dots Q_n x_n R(x_1, \dots, x_n)$ of* FOL*, φ is true (false) in* $\mathbf{M}$ *if and only if Eloise (Abelard) has a winning strategy in $G(\varphi, \mathbf{M})$, if and only if Eloise (Abelard) has a winning strategy in $G(R(x_{1_{Q_1}}, \dots, x_{n_{Q_n}}), \mathbf{M})$.*

I have not enough space here, but the full proof is a straightforward consequence of the observations in the above discussion.

2 Games for Independence-Friendly Logic

Independence-Friendly Logic (IFL), first presented by Hintikka and Sandu (1989) following previous work by Hintikka, introduces a kind of language featuring independence phenomena among quantifiers. As we have seen, quantifiers in FOL were linearly ordered, i.e. they were automatically dependent on every quantifier preceding them in the formula, and independent on those succeeding them. Starting from that, Hintikka and Sandu introduced a language extending FOL through a new slash symbol which stands between some quantified variable Qv, with $Q = \{\forall, \exists\}$, and a set of variables W, so that Qv/W counts as a well-formed string in IFL. The standard Qv of FOL is then the special case where $W = \varnothing$ [3]. The intended meaning of Qv/W is to the effect that Q is not in the semantic scope of the quantifiers bounding the variables of W, even though it is in their syntactic scope.

Now the question was: How should we conceive a natural semantics for IFL?

GTS suited so beautifully the standard semantics of FOL, that one seemed to take for granted that, in the absence of a standard semantics for IFL, *the* semantics of IFL would be most naturally given by the same GTS. Hintikka's more or less tacit motivation for that was roughly the following: compositional semantics like the standard Tarski-style semantics for FOL seemed unavailable for IFL, and since GTS was not compositional (on the issue of compositionality, cf. Hintikka 1996, Hodges 1997, Sandu and Hintikka 2001), it could conveniently fill in the gap: GTS was an alternative semantics for FOL equivalent to the Tarski-style semantics; now this latter is unavailable for IFL, but GTS is still here to save our semantical lives.

Indeed, since GTS describes games, we can still make sense of the slash operator by simply stating that it imposes a restriction on players' strategies. The game is the same as before, with every quantifier being associated with a choice-point; but now the strategy of the "slashed player" associated with Qv/W becomes restricted in the following way: his or her choice must be made independent on every choice

[3] We usually require that $v \notin W$, and that every $v' \in W$ be bounded by some quantifier preceding Q in the formula.

associated with the "slashed variables" of W —in the case where $W = \emptyset$, we obtain the standard unrestricted strategies.

This was then just too natural not to answer the above stated question. *The* semantics of IFL is simply GTS, with no amendment.

There is no justification at all, in *no single piece* of the literature, for the literal transposition of GTS into IFL. Of course, the insinuating movement sketched in the last paragraph leading from GTS for FOL to GTS for IFL can by no means stand for a sufficiently convincing argument; at best, it suggests a highly attractive option. But then, the answer to the question should be: GTS is *a* possible semantics for IFL. To begin with, there is no such thing as *the* semantics for some language, for every semantical system is a particular among other possibilities of interpreting the syntax of that language. Moreover, the fact that GTS is a possible semantics for IFL does not mean by itself and without any further argumentation, that GTS is *the most suitable* semantics for IFL. However, the two aspects are presupposed throughout the literature on the topic (actually, the second is presupposed in such a thorough way that the first is simply overlooked). In every exposition of IFL you will find a variation of the following theme: That is how GTS works; it is arguably equivalent to the standard semantics of FOL; here is the language of IFL, which extends FOL; GTS is the semantics of IFL.

The question I want to ask is: *Why* is that so? On which grounds are we to justify the literal transposition of GTS into IFL? It is too fundamental a question for us to simply skip it. And however astonishing it may seem now, it *was* skipped from the very beginnings of IFL.

2.1 Game-Theoretical Semantics for Independence-Friendly Logic

As we have just seen, GTS carries over to IFL without any modification. Pursuing our rephrasing of GTS in terms of choice-point games to the case of IFL, we obtain games like the two choice-point game $G(R(x^1_A, y^2_E), \mathbf{M})$ of the last section. The difference is that now the conditions imposed on the players for always winning the game may be distinct, according to the amount of information about the previous moves of the game they possess at the moment of their respective choices.

Suppose for example that we want to impose on Eloise the condition that her choice be made without knowing which element was picked by Abelard. Both strategies, Eloise's and Abelard's, must now take into account this new fact, just as they both took into account the fact that in the game played in FOL, Eloise's choice was made with her acceding to the information about the choice previously made by Abelard.

Thus Abelard's goal is now to provide some a for x such that for all elements possibly chosen for y *independently on the choice of* x, we have $(a, y) \notin R^{\mathbf{M}}$; and

Eloise's is to choose some constant b *independently on the choice of* x, such that for all elements possibly chosen for x, we have $(x, b) \in R^{\mathbf{M}}$. This can now be formally stated in the language of IFL, as we did for FOL through (1) and (2). Abelard now wants to show —we write "$/v$" instead of "$/\{v\}$" for clearness of notation—:

$$\varphi_{/} = \exists x(\forall y/x)\ \neg R(x, y); \tag{4}$$

while Eloise wants to show:

$$\varphi'_{/} = \forall x(\exists y/x)\ R(x, y). \tag{5}$$

Therefore, the condition imposed on Eloise's strategies is not only to the effect that these latter must be winning for her as it was the case before, but that they be *uniformly* winning with respect to possible choices by Abelard. Following Hodges (1997), we will say that slash operators trigger *uniformly winning* conditions on the players.

As before, we obtain the following truth conditions for $\varphi_{/}$ and $\varphi'_{/}$:

$$\begin{aligned} &\varphi_{/} \textit{ is True in } \mathbf{M} \text{ iff Abelard has a uniform winning strategy in } G(R(x^1_A, y^2_E), \mathbf{M})\ ; \\ &\varphi'_{/} \textit{ is True in } \mathbf{M} \text{ iff Eloise has a uniform winning strategy in } G(R(x^1_A, y^2_E), \mathbf{M}). \end{aligned} \tag{6}$$

Now the same kind of move is needed in order for us to obtain Hintikka's GTS clauses for truth and falsity of, say, $\varphi'_{/}$. We need to turn $\varphi_{/}$'s truth condition into a falsity condition for $\varphi'_{/}$, so that we can say that $\varphi'_{/}$ is true in $\mathbf{M}$ if and only if Eloise has a uniform winning strategy in $G(R(x^1_A, y^2_E), \mathbf{M})$; and that φ' is false in $\mathbf{M}$ if and only if Abelard has a uniform winning strategy in $G(R(x^1_A, y^2_E), \mathbf{M})$.

But again, there is no intrinsic fact of GTS which allows us to conclude that $\neg\varphi'_{/} \leftrightarrow \varphi_{/}$. Worst, we have no longer the standard semantics of FOL on which we could possibly rely. Yet, Hintikka tacitly takes it for granted, without any real justification. Let us *not* take it for granted, and see whether Hintikka's move was, if not explicitly justified, eventually justifiable.

Since we cannot rely on the standard semantics of FOL any longer, let us ask ourselves what the source of the equivalence $\neg\varphi' \leftrightarrow \varphi$ is in FOL. The source is of course the semantical clauses for universal and existential quantifiers, and negation. Those clauses in turn derive from the intended meanings of the respective syntactical elements. If it is false that every man is mortal, then it is true that there exists a man such that he is not mortal, and conversely.

It seems then that in order to be sure that GTS is the most natural semantics for IFL, we have to look at the intended meanings of slashed quantifiers. Consider the expression "*there is some x chosen independently on W* [i.e. on the choices for the variables belonging to W] *such that* ... ", which is the intended meaning of $\exists x/W$. What would be the intended negation of $\exists x/W$? That is, what is the intended meaning of the expression "it is not the case that *there is some x independent on W such that* ... "?

It seems fairly clear that to give some element for x independently on previous choices for W is the same as providing some element which suits every possible choice for W. The negation of this is then: "*For all x, there is some W* [i.e. some set of choices for the variables of W] *such that* it is not the case that ... ", for this expression just denies the fact that there is some good choice of x independently on the choice of W. We then obtain equivalences between $\neg\exists x/W$, $\neg\exists x\forall W$, and $\forall x\exists W\neg$ (where $\exists W$ and $\forall W$ have the obvious meaning).

Hintikka argues, however, that $\forall x(\exists y/x)R(x,y)$ is not equivalent to $\exists y\forall xR(x,y)$. If he is right, this is to my view a strong argument against the fact that (6) gives falsity conditions to $\forall x(\exists y/x)R(x,y)$. If we do not have any evidence to that effect within FOL, and if it goes against the intended meaning of the slash operator, I do not see any further reason in ascribing to $\exists x(\forall y/x)\ \neg R(x,y)$ the falsity condition of $\forall x(\exists y/x)R(x,y)$. But then, since falsity in GTS is defined precisely in terms of Abelard having a (uniform) winning strategy in the underlying game, then it follows that GTS does not suit IFL.

The only way out would be to reject that the intended meaning of choosing x independently on y is to make a choice for x such that it suits every choice for y, that is, to find some x such that for all y etc. It seems to me it would be the hardest, not to say impossible, way back to GTS.

Of course, Hintikka did not fail to see the evident close relationship between $\forall x(\exists y/x)$ and $\exists y\forall x$. It seems to me that his failure was to take the one-way street in the wrong direction. In presupposing that GTS falsity conditions applied literally to IFL, he was able to reject the equivalence (he just did not need it) between $\forall x(\exists y/x)$ and $\exists y\forall x$ without much harm (though not without arguments [4]). However, it was precisely that literal transposition of GTS which was in need of justification, and if justification were to be found in the intended meanings of IFL, as it is always the case when one builds up a semantic system for a language, then it would turn out that GTS clause for falsity conditions is inadequate for IFL.

[4] As we know, the argument is that the two formulas have equivalent truth conditions, but not equivalent falsity conditions. But this depends on already defined falsity conditions, which are of course provided by GTS, bringing us back to our initial concerns. The point is that we may well have cases in which neither Abelard nor Eloise have a uniform winning strategy in the game, but as is made explicit by (6), this just means, without the unjustified presupposition of GTS, that two *different* formulas are not true in the underlying structure, and not that the *same* formula is neither true nor false in the structure.

Limitation of space prevents me of carrying through here a fully and explicitly development of an alternative "intended semantics" for IFL, which I believe is the most natural one. But let me draw some final opportune remarks and hint at some future directions.

First, if $\forall x(\exists y/x)$ is to be the same thing as $\exists y \forall x$, one could think at first glance that IFL is merely a complicated typography for FOL. This is not so. I am arguing that GTS is not the right semantics for IFL, but this latter is still a legitimate extension of FOL. In the particular cases —as the one we were dealing with so far— in which we have a simple independence pattern, IFL nails down to FOL. However, in more complex cases like the double independence pattern in $\forall x \forall y(\exists u/x)(\exists v/y)$, it is a well-known fact (which the reader can easily verify) that we cannot eliminate all slashes. Thus in general, not every formula of IFL is expressible in FOL.

GTS is for sure a possible interpretation of IFL, which in addition brings logic closer to some game-theoretical notions and some well-known games like the so-called *Matching Pennies* game. But the mere fact that GTS and IFL fit some aspects of game theory is not to say that they fit language. We cannot let the game metaphor take over. Hintikka's procedure was to use an adequate metaphor for the semantics of FOL, and then crudely apply the metaphor to IFL, as if metaphor were more important than semantics. As it happened, the metaphor did fit language in the case of FOL, but it did not in the case of IFL. This is not to say, though, that game theory is not useful to the understanding of language; but if it is, it is not the way GTS as applied to IFL pretended it was.

Notice further that the rule of quantifier negation in IFL according to GTS is just the same as that of standard FOL: we just invert the quantifier and move the negation symbol inwards the formula, while the slash operator and the slashed set are kept frozen (e.g. $\neg\exists v/W$ becomes $\forall v/W\neg$). Everything was affected by moving the negation into the formula, except for the slash and its slashed set. Who told us that the slash-restriction must be left untouched, and not be switched elsewhere in the formula [5]? We are changing the nature of the quantifiers! we should at least ask ourselves whether FOL quantifier rules really should be directly transposed into IFL. And perhaps they could. But I think they do not. The only underlying "motivation" for such a move, is to make metaphor take over semantics.

As a matter of fact, I think we can still keep the players metaphor in IFL. To do that, we must define game semantics effectively over quantified sentences, and not over their matrices. The line of reasoning is the following: what Abelard wants to falsify in a game over $\forall x(\exists y/x)R(x,y)$ is not the matrix $R(x,y)$ for all possible movements associated with Eloise's choice-point, but rather $\forall x(\exists y/x)R(x,y)$ *tout court*. Then his condition for winning the game is that he can provide some x

[5] As I believe it will in a correct semantics for IFL; see below.

such that $\neg(\exists y/x)R(x,y)$ holds. Following the intended meaning of the slash operator, this means that he must find some x such that it is not the case that Eloise can find some y independently on the choice of x, such that $R(x,y)$ holds in the relevant structure. But the fact that Eloise cannot find any y independent on x means that for all y, there exists some x such that $R(x,y)$ is false, or to state it more briefly, Abelard's condition is: $\forall y \exists x \neg R(x,y)$. But finally, this is quite different from Abelard's previous condition of uniformly winning the GTS game, i.e. $\exists x(\forall y/x)\neg R(x,y)$. The upshot is that the verifier and the falsifier of GTS games aim at verifying or falsifying the matrix of a quantified formula for every possible outcome of the choice-point game, whereas the right move is to simply verify or falsify the quantified formula. In FOL, the two methods are equivalent: to falsify $\forall x \exists y R(x,y)$ is to provide some x such that there exists no y such that $R(x,y)$ holds, that is, some x such that $\forall y \neg R(x,y)$ holds. But this is the same thing as providing some x such that the matrix $R(x,y)$ fails to hold in connection with every possible choice of y. However, in IFL the two methods give two different falsity conditions.

Actually, the deep negation of $\forall x(\exists x/y)R(x,y)$ would be explicitly given by $\forall y(\exists x/\varnothing)\neg R(x,y)$, which in our particular example boils down to a FOL formula. However, in the general case falsity conditions will give IFL formulas which are not expressible in FOL. For instance, the negation of $\forall x \forall y(\exists u/x)(\exists v/y)R(x,y)$ would be given by $\forall u \forall v(\exists x/v)(\exists y/u)\neg R(x,y)$ —slashes and slashed sets also move as negation gets into the formula. Moreover, I believe that a compositional semantics can be supplied managing a modification in syntactic notation. By way of a hint, if we note $(\forall^Y_X)v$ to mean that the variables in X are independent on v, and the variables of Y are dependent on v, then we can write the negation $\neg(\forall^Y_X)v$ as $(\exists^X_Y)v\neg$, where $(\exists^X_Y)v$ means that v is independent on Y and dependent on X, inverting thereby not only the quantifiers as in FOL and in GTS-IFL, *but also the dependence-independence patterns*. Again, a topic to be continued.

Under its natural semantics, IFL would arguably present a rather well-behaved two-valued [6] compositional semantics. And since the lack of these two properties was according to Hintikka what mostly provided IFL with all of its other revolutionary features, it seems that classical results like Tarski's undefinability of truth and semantical completeness are still hovering around IFL.

In my opinion, IFL with its natural semantics is just an extension of FOL with a new kind of quantifier, just as any extension of FOL with other non-classical (e.g. generalized) quantifiers non-expressible in FOL. Its correct game-theoretical interpretation is in turn a correct extension of the application of a metaphor.

[6] The reader may already verify that for the formulas given above, Abelard wins if and only if Eloise looses, for every game with respect to any structure.

References

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