UniLog 2010
Book of Abstracts

World Congress and School on Universal Logic
(3rd edition)

April 18-25, 2010
Monte Estoril, Portugal

Edited by

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UniLog 2010

Universal Logic

In the same way that universal algebra is a general theory of algebraic structures, universal logic is a general theory of logical structures. During the 20th century, numerous logics have been created and studied: intuitionistic logic, deontic logic, many-valued logic, relevant logic, linear logic, non-monotonic logic, etc. Universal logic is not a new logic, it is a way of unifying this multiplicity of logics by developing general tools and concepts that can be applied to all logics. One aim of universal logic is to determine the domain of validity of such and such metatheorem (e.g., the completeness theorem) and to give general formulations of metatheorems. This is very useful for applications and helps to make the distinction between what is really essential to a particular logic and what is not, and thus enables a better understanding of this particular logic. Universal logic can also be seen as a toolkit for producing a specific logic required for a given situation, e.g., a paraconsistent deontic temporal logic.

This is the third edition of a world event dedicated to universal logic, following the very successful inaugural editions held in Montreux, Switzerland in 2005, and in Xi'an, China in 2007. The format of the event has already become traditional, and consists of a combination of a congress and a school.

The Congress

The four day congress is a privileged forum for all those interested in logic and its universal aspects. It includes ten invited talks by prominent logicians, as well as more than 170 contributed talks, some of which are organized into special sessions, dedicated to selected topics and organized by specialists. As usual, there will also be a contest, this time to distinguish the best paper on *How to combine logics?*.

The congress is intended to be a major event in logic, providing a platform for future research guidelines. Such an event is of interest to all people dealing with logic in one way or another: pure logicians, mathematicians, computer scientists, AI researchers, linguists, psychologists, philosophers, etc.

The School

The four day school on universal logic precedes the school, and offers more than twenty tutorials by renowned specialists, presenting general techniques useful for a comprehensive study of logic. For PhD students, postdoctoral and young researchers interested in logic, artificial intelligence, mathematics, philosophy, linguistics and related fields, this is a unique opportunity to strengthen their background.

The school is intended to complement some very successful interdisciplinary summer schools which have been organized in Europe and the USA in recent years. The difference is that our school will be more focused on logic, there will be less students and a better interaction with the specialists.

It will all happen nearby Lisbon, Portugal, a place which was the departure point of many adventures. Join this adventure!

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The Congress

Invited Speakers

Hartry Field

Paracomplete Logics for Vagueness and the Paradoxes

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The talk is intended to introduce a subject that seems ripe for fruitful investigation, about which (as far as I'm aware) relatively little is now known.

By a paracomplete logic I mean, one without excluded middle. My focus will be on paracomplete logics that (unlike intuitionism) obey the deMorgan laws and double negation laws, and that (unlike strong Kleene logic) contain a reasonable conditional. Such logics are very natural, in connection with vagueness and also the semantic (and property-theoretic) paradoxes. Regarding the latter, they provide the only alternative to dialetheism for anyone who wants to adhere to "naive truth and satisfaction", e.g. the intersubstitutivity of True($< A >$) with A. (The idea is to assume classical mathematics in a classical fragment of the language, and conservatively extend it with a truth or satisfaction predicate not subject to excluded middle.) It is known that some such logics are consistent with naive truth and satisfaction, and that some otherwise natural-seeming ones (e.g. Lukasiewicz continuum-valued) aren't. But there has been little systematic exploration of such logics, and even less of the question of which ones are friendly to naive truth and satisfaction. This is also important to vagueness, because when a logic fails for truth there is usually a related worry about it as a logic for vagueness, for reasons I'll explain. In the talk I'll introduce a (rather obvious) algebraic framework for the study of such logics, mention a few basic results, but mostly focus on how much seems open.

Yuri Gurevich

Algebra and Logic: Pitfalls and Potential Benefits

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One danger in algebra and logic is over-abstraction. You rise to a rarified air with little substance and no good theorems. Another potential problem is that it is easy to formulate questions that are mathematically precise but uninteresting. In applications though the greater problem is under-abstraction. Whether you program or prove, it is all too easy to get bogged down with details so that you can't see the forest for the trees. How to get the level of abstraction right? That is where algebra and logic are indispensable, and that is the issue that we intend to dwell upon.

Gerhard Jäger

About the Suslin Operator in Applicative Theories

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In the seventies Feferman developed his so-called explicit mathematics as a natural formal framework for Bishop-style constructive mathematics. Explicit mathematics is strongly influenced by generalized recursion theory and soon turned out to be of independent proof-theoretic interest. Since the operations of explicit mathematics can be regarded as abstract computations, functionals of higher types can be added in a direct and perspicuous way.

A first important step in the proof-theoretic treatment of functionals of higher types in the framework of explicit mathematics was the analysis of the non-constructive minimum operator over a basic theory BON of operations and numbers. A further interesting type two functional is the Suslin operator which tests for well-foundedness of total binary relations.

In this talk I take up the proof-theoretic analysis of the Suslin operator in explicit mathematics due to Strahm and myself, but present a new and conceptionally preferable approach. This is joint work with Dieter Probst.

Marcus Kracht

Judgement

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The logical literature is filled with signs of judgement (typically the turnstile and all its graphical variants). Yet a discussion of their role in logic is typically absent. In this talk I want to rectify this imbalance by focussing on the nature judgement. I will show that certain logics can be motivated by the character of judgement alone. This provides a way to reconcile logical monism with logical pluralism. For we may maintain that there is just one objective logic while there are many subjective logics, each based on a different notion of judgement.

Hiroakira Ono

Regular embeddings of residuated lattices and infinite distributivity

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Recently there have been remarkable developments of the study of completions of residuated lattices, in particular of canonical extensions and MacNeille completions. Embeddings associated with MacNeille completions are always *regular*, which means that all existing infinite joins and meets are preserved, while embeddings associated with canonical extensions are never so. Meanwhile MacNeille completions do not always preserve distributivity. Here, we will consider completions of residuated lattices with regular embeddings which preserve (infinite) distributivity, since such completions would be quite useful in proving algebraic completeness of *distributive* substructural predicated logics. It will be shown that the join infinite distributivity (JID) will play a particularly important role.

Giovanni Sambin

The principle of reflection and basic logic

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In a dynamic perspective, every abstract concept can be seen as the interaction between two different levels: the word which labels the concept itself and the concrete instances to which it applies. This process is particularly well visible in the case of logical constants. Using a calculus of sequents as a framework, one can show that all connectives and quantifiers acting on propositions are the reflection of a meta-linguistic link acting on assertions. In other words, the inference rules of every logical constant are obtained uniformly as the solution of a "definitional equation" linking object and meta-language.

The minimal logical system satisfying this "principle of reflection" has been called basic logic (Sambin-Battilotti-Faggian, JSL 65, 2000). Formally, one can describe linear logic (without exponentials) as obtained from classical logic by adding control of the rules of weakening and contraction; then basic logic is obtained from linear logic by further adding control of contexts.

Basic logic is the common core of most logics, in the sense that linear, intuitionistic, paraconsistent, classical and other logics are obtained in a natural way by adding only some structural rules. This has also some technical advantages, as a uniform cut-elimination procedure.

Dana Scott

Mixing modality and probability

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For some time at many recent workshops, the author has lectured about a Boolean-valued model for higher-order logic (and set theory) based on the complete Boolean algebra of Lebesgue measurable subsets of the unit interval modulo sets of measure zero. This algebra not only carries a probability measure, but it also allows for a non-trivial S4-modality by using the proper subframe of open sets modulo zero sets. This provides rich ingredients for building many kinds of structures having non-standard random elements. The lecture will review the basics of this type of semantics and discuss several examples and their logical properties.

Jonathan Seldin

Logical Algebras as Formal Systems: H. B. Curry's Approach to Algebraic Logic

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Nowadays, the usual approach to algebras in mathematics, including algebras of logic, is to postulate a set of objects with operations and relations on them which satisfy certain postulates. With this approach, one uses the general principles of logic in writing proofs, and one assumes the general properties of sets from set theory. This was not the approach taken by H. B. Curry in [1] and [2], Chapter 4. He took algebras to be formal systems of a certain kind, and he did not assume either set theory or the '*rules of logic*. I have not seen this approach followed by anybody else. The purpose of this paper is to explain Curry's approach.

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Amílcar Sernadas

Parallel Composition of Logics

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The practical significance of the problem of combining logics is widely recognized, namely in knowledge representation (within artificial intelligence) and in formal specification and verification of algorithms and protocols (within software engineering and information security). In these fields, the need for working with several calculi at the same time is the rule rather than the exception. The topic is also of interest on purely theoretical grounds. For instance, one might be tempted to look at predicate temporal logic as resulting from the combination of first-order logic and propositional temporal logic. However, the approach will be significant only if general preservation results are available about the combination mechanism at hand, namely preservation of completeness. For these reasons, different forms of combining logics have been studied and several such transference results have been reported in the literature. To name just a few, fusion (of modal logics), temporalization and fibring are now well understood, although some interesting open problems remain concerning transference results. Fibring [1] is the most general form of combination and its recent graphic-theoretic account makes it applicable to a wide class of logics, including substructural and non truth-functional logics. Capitalizing on these latest developments on the semantics of fibring [2] and inspired by the notion of parallel composition of processes in its most basic form (interleaving), a novel form of combination of logics, applicable to a wide class of logics, is proposed together with a couple of transference results, and compared with other combination mechanisms,

showing how they can be recovered as special cases.

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Secret Speaker

???

Special Sessions

Abstract Algebraic Logic

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This discipline can be described as Algebraic Logic for the XXIst century. It gathers all mathematical studies of the process of algebraization of logic in its most abstract and general aspects. In particular it provides frameworks where statements such as "A logic satisfies (some form of) the interpolation theorem if and only if the class of its algebraic counterparts satisfies (some form of) amalgamation" become meaningful; then one may be able to prove them in total generality, or one may investigate their scope, or prove them after adding some restrictions, etc.

The term appeared for the first time in Volume II of Henkin-Monk- Tarski's "Cylindric Algebras", referring to the algebraization of first-order logics, but after the Workshop on Abstract Algebraic Logic (Barcelona, 1997) it has been adopted to denote all the ramifications in the studies of sentential-like logics that have flourished following Blok, Pigozzi and Czelakowski's pioneering works in the 1980's. Abstract Algebraic Logic has been considered as the natural evolution of the traditional works in Algebraic Logic in the style of Rasiowa, Sikorski, Wójcicki, etc., and integrates the theory of logical matrices into a more general framework.

The 2010 version of the Mathematics Subject Classification will incorporate Abstract Algebraic Logic as entry 03G27, which witnesses the well-delimited, qualitatively distinctive character of this discipline and its quantitative growth.

Topics that can fit this Special Session include, but are not limited to, the following ones:

- Studies of the Leibniz hierarchy, the Frege hierarchy and their refinements, and relations between them.

- Lattice-theoretic and category-theoretic approaches to representability and equivalence of logical systems.

- Use of algebraic tools to study aspects of the interplay between sentential logics and Gentzen systems, hypersequent systems and other kinds of calculi and logical formalisms

- Formulation of abstract versions of well-known algebraic procedures such as completions, representation theory and duality.

- Study of the algebraization process for logics where order, besides equality, is the main relation to be considered in the algebraic counterparts.

- Extensions to other frameworks motivated by applications to computer science, such as institutions, behavioural logics, secrecy logic, etc.

- Study of algebra-based semantics of first-order logics.

Accepted contributed talks

Implicative Bilattices

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Bilattices are algebraic structures introduced by Matt Ginsberg in the context of A.I. In recent years, Arieli and Avron introduced a logic defined from matrices called "logical bilattices". In a previous work we studied, from the perspective of Abstract Algebraic Logic, the implication less fragment of Arieli and Avron's logic. Here we complete this study considering the full system. We prove that this logic is strongly (but not regularly) algebraizable and define its equivalent algebraic semantics through an equational presentation. We call the algebras in this variety "implicative bilattices". We obtain several results on this class, in particular that it is a discriminator variety (hence arithmetical), generated by a single finite algebra, and characterize its members as certain bilattice products of two copies of a generalized Boolean algebra. We also characterize some subreducts of implicative bilattices that have a particular logical interest.

Behavioral algebraization of logics (parts I and II)

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The theory of Abstract Algebraic Logic (AAL) aims at drawing a strong bridge between logic and algebra. It can be seen as a generalization of the well known Lindenbaum-Tarski method. Although the enormous success of the theory we can point out some drawbacks. An evident one is the inability of the theory to deal with logics with a many-sorted language. Even if one restricts to the study of propositional based logics, there are some logics that simply fall out of the scope of this theory. One paradigmatic example is the case of the so-called non-truth-functional logics that lack of congruence of some of its connectives, a key ingredient in the algebraization process. The quest for a more general framework to the deal with these kinds of logics is the subject of our work. In this two-sessions talk we will present a generalization of AAL obtained by substituting the role of unsorted equational logic with (many-sorted) behavioral logic. The incorporation of behavioral reasoning in the algebraization process will allow to amenably deal with connectives that are not congruent, while the many sorted framework will allow to reflect the many sorted character of a given logic to its algebraic counterpart. In this first part of the talk we focus on syntactical issues, leaving the semantical issues for the second part of the talk. We illustrate theses ideas by exploring some examples, namely, paraconsistent logic C_1 of da Costa.

In the second part of the talk we focus on the semantical issues of the theory of behavioral algebraization of logics. This newly developed behavioral approach to the algebraization of logics extends the applicability of the methods of algebraic logic to a wider range of logical systems, namely encompassing many-sorted languages and non-truth-functionality. However, where a logician adopting the traditional approach to algebraic logic finds in the notion of a logical matrix the most natural semantic companion, a correspondingly suitable tool is still lacking in the behavioral setting. Herein, we analyze this question and set the ground towards adopting an algebraic formulation of valuation semantics as the natural generalization of logical matrices to the behavioral setting, by establishing some promising results. or illustration, we use again da Costa's paraconsistent logic C_1. This work was partly supported by FCT and EU FEDER, namely via the project KLog PTDC/MAT/68723/2006 of SQIG-IT. The second author acknowledges partial support of FCT under the postdoctoral grant SFRH/BPD/47245/2008.

A general approach to non-classical first-order logics

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The goal of this talk is to present a general theory of first-order non-classical logics. Our approach generalizes the tradition of Rasiowa's implicative logics, Gödel-Dummett first-order logic, and Hájek's first-order fuzzy logic, i.e. starting from a propositional non-classical logic we add

quantifiers in the same way as in first-order intuitionistic logic. The unifying idea of this treatment of first-order logics can be formulated simply as:

the truth value of a universally (resp. existentially) quantified formula is the infimum (resp. supremum) of all instances of that formula w.r.t. the existing matrix order.

To do so, one needs a good notion of order in the semantics which is typically obtained from a suitable implication in the syntax, thus our underlying propositional logics are the so called weakly p-implicational logics previously studied by the authors this differs from other approaches where the order is extralogical such as a recent paper by James Raftery). Given a propositional logic L we present a first-order Hilbert-style calculus extending it, and prove its completeness w.r.t. the class of all first-order structures based upon the matrix semantics of L and hence, it turns out to be the minimal first-order logic over L. Having this suitable minimal logic we can study several of its extensions in order to cope with important examples of variants of non-classical first-order logics in the literature. For instance, we find a uniform axiomatization for logics complete with respect to linearly ordered or witnessed semantics and characterize logics enjoying Skolemization for their prenex fragment.

Structural Theorems on Congruence – Modular Quasivarieties of Algebras

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The focus of the talk is on applying the methods worked out by Abstract Algebraic Logic (AAL) to the problem of finite axiomatizabilty of classes of algebras. AAL offers here convenient tools based on the notion of a commutator equation. This notion behaves pretty well in the context of relatively congruence-distributive (RCD) quasivarieties of algebras and yields an elegant proof of the theorem stating that every finitely generated RCD quasivariety is finitely based. For relatively congruence modular (RCM) quasivarieties the analogous problem is much harder. In the talk a number of observations concerning the structure of quasivarieties possessing the additive equational commutator is presented. The class of quasivarieties with the additive equational commutator encompasses RCM quasivarieties.

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Modular canonicity for bi-implicative algebras

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We will report on the canonicity of certain identities and inequalities in a signature consisting of constants $\top, \bot$ and implications $\rightarrow, \leftarrow$, which relates to the canonicity of logics associated with certain distinguished sub-quasivarieties of (bi-)implicative algebras, the best known of which are the varieties of (bi-)Hilbert algebras and (bi-)Tarski algebras. These results are instances of a research program connecting canonical extensions and Abstract Algebraic Logic (see [1]). Previous results of this kind were obtained in [2]. Our basic setting is the quasi-axiomatization of implicative algebras and expand it "symmetrically" with a subtraction operator and a bottom. Within this setting we analyze the independence and interdependence of certain axioms w.r.t canonicity, which yields a better, more modular understanding of the canonicity of Hilbert algebras.

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The Isomorphism Problem for modules over quantaloids (parts I and II)

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Given a structural consequence relation, the lattice of theories can be expanded to include the action of substitutions (or equivalently inverse substitutions). Blok and Pigozzi, in their monograph, showed that the expanded lattice of theories of an algebraizable sentential logic is isomorphic to the expanded lattice of theories of the corresponding algebraic consequence relation, and conversely any such isomorphism comes from an algebraizable sentential logic. The result was further extended from sentential logics to k-deductive systems. Blok and Jónsson, realizing that the action of the monoid of substitutions to the set of formulas plays a crucial role, developed a general framework, where one considers the action of an arbitrary monoid M on a set, yielding an M-set. Considering logics on (equivalently, closure operators over) M-sets, one can easily prove that every bidirectional syntactic translation between two such logics (a notion that specializes to algebraizability, if one of the two logics comes from a class of algebras) yields an isomorphism between the expanded lattices of theories. The converse (unlike in the case of sentential and k-deductive systems) is not always true, and determining when it holds is known as the Isomorphism Problem.

Blok and Jónsson gave a sufficient condition (existence of a basis) for the Isomorphism Problem and Gil-Férez provided a more general sufficient condition (existence of a variable), while a necessary and sufficient condition (a characterization of cyclic projective modules) was given by Galatos and Tsinakis. Their proof puts the problem in the correct level of abstraction, by extending the framework even further to modules (join-complete lattices) over complete residuated lattices (or quantales); this corresponds to passing to the action of the powerset of M to the powerset of the M-set. In particular, both the syntactic translation and the semantic isomorphism become morphisms at the same level, namely between modules.

Sentential logics and k-deductive systems are deductive systems where the syntactic objects involved all have a fixed "length," while this is not the case for sequent and hypersequent deductive systems. Although the latter are also examples of M-sets, the sufficient condition for the Isomorphism Problem of Blok and Jónsson does not apply. The Isomorphism Problem of sequent systems was addressed by Rebagliato and Verdú, Gil-Férez gave a sufficient condition (existence of a multi-variable) in the setting of M-sets, while the general solution (in the context of modules) follows from the work of Galatos and Tsinakis.

The Isomorphism Problem was also considered in the context of π-institutions by Voutsadakis, who provided a sufficient condition (the term condition). The context of π-institutions extends that of M-sets in a different direction than its extension to modules, and Voutsadakis condition covers extensions of "fixed length" deductive systems. A sufficient condition (the multi-term condition) for "variable length" π-institutions was provided by Gil-Férez. The problem of an exact solution of the Isomorphism Problem for π-institutions was open.

Our work provides, in particular, a solution to the Isomorphism Problem for π-institutions. We first provide a general categorical context that encompasses π-institutions and modules over quantales and we solve the Isomorphism Problem in its full generality.

More specifically, in the first of the two talks, we consider the category of modules over quantaloids. A quantaloid is an enriched category over the category of join-complete lattices. A one-object quantaloid is coextensive with a quantale, so our theory can be viewed as a categorical extension of the work of Galatos and Tsinakis. A module over a quantaloid is defined as an enriched functor from the quantaloid to the category of join-complete lattices; it is the natural generalization of a quantale module and of an M-set, where the action is identified with a homomorphism to the endomorphisms of the join-complete lattice or to the endomaps of the M-set, respectively.

We solve the Isomorphism Problem by characterizing the modules over quantaloids for which the theorem holds (every isomorphism is induced by syntactic translators), which end up being the projective objects in the category. The characterization for the cyclic projective modules is given by the existence of a generalized variable.

In the second part of the talk we expose the technical results on which the results of the first talk

are based. In particular, we describe the structure of the categories of modules over quantaloids. Monos are characterized thanks to the Yoneda Lemma, and by the Duality Principle that we provide, epis can also be characterized. Strong completeness (and cocompleteness) are shown, and also the Strong Amalgamation Property for modules over a quantaloid. Other properties as having enough projectives and injectives, or the (Epi,Mono)-structure of the categories of modules over quantaloids are explained, as well.

On Hilbertizable Gentzen systems associated with finite valued logics

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According to J.G. Raftery's [2] definition, a Gentzen relation is Hilbertizable if it is equivalent to some Hilbert relation. This is the strongest among several relations that have been considered in the literature between a Gentzen system and a Hilbert system. In this work we show that when we consider an m-dimensional sequent calculus associated with a finite algebra L (in the sense of M. Baaz et al. [1]) and its corresponding Gentzen system G_L, then G_L is Hilbertizable if and only if the elements of L can can be expressed by terms. This means that for every element l, there exists a term function $p_l(x)$ that always takes the value l when evaluated in L. This will be the case, for instance, if L is a finite MV-algebra.

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Equationally orderable quasivarieties and sequent calculi

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A quasivariety is equationally orderable if there is a finite set of equations in two variables that in every member of the quasivariety defines a partial order. We characterize the equationally orderable quasivarieties as the equivalent algebraic semantics of the sequent calculi with the binary cut rule which are algebraizable with the translation from equations to sequents performed by the map that sends an equation $t = t'$ to the pair of sequents $t \Rightarrow t', t' \Rightarrow t$. Sequents are taken as a pair of a finite sequence of formulas and a formula. We will discuss characterizations of these sequent calculi and the relations they may have with their external deductive systems.

Quasi-subtractive varieties

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Algebras in classical varieties have the property that there is an isomorphism between their lattice of congruences and the lattice of some "special" subsets: e.g. normal subgroups of groups, two-sided ideals of rings, filters (or ideals) of Boolean algebras. Why do such pleasant theorems occur? Abstract Algebraic Logic provides a rather satisfactory explanation along the following lines:

- In every member $\mathbf{A}$ of a τ-regular variety $\mathbb{V}$—namely, a variety that arises as the equivalent algebraic semantics of an algebraizable deductive system—there is a lattice isomorphism between the lattice of congruences of $\mathbf{A}$ and the lattice of deductive filters on $\mathbf{A}$ of the τ-assertional logic of $\mathbb{V}$ ([2]; [1]; [5,Section 4.4]; [4]).

- In the pointed case, if $\mathbb{V}$ is 1-protoregular in the sense of [3], then the deductive filters on $\mathbf{A} \in \mathbb{V}$ of the 1-assertional logic of $\mathbb{V}$ coincide with the $\mathbb{V}$-ideals of $\mathbf{A}$ in the sense of [6], which is even better due to the availability of a manageable concept of *ideal generation*. Since 1-subtractive varieties are 1-protoregular, this holds in particular for 1-subtractive varieties.

However, ideal-congruence isomorphism theorems abound in the literature that are not subsumed by this picture: for example, the correspondence between open filters and congruences in pseudointerior algebras, or between deductive filters and congruences in residuated lattices, or between ideals and MV congruences in quasi-MV algebras. The aim of the present talk is to appropriately generalise the concepts of subtractivity and τ-regularity in such a way as to obtain a general framework encompassing all the previously mentioned results.

A variety $\mathbb{V}$ whose type ν includes a nullary term 1 and a unary term $\square$ is called *quasi-subtractive* w.r.t. 1 and $\square$ iff there is a binary term $\rightarrow$ of type ν s.t. $\mathbb{V}$ satisfies the equations

$$\text{(i) } \square x \rightarrow x \approx 1 \qquad \text{(iii) } \square(x \rightarrow y) \approx x \rightarrow y$$
$$\text{(ii) } 1 \rightarrow x \approx \square x \qquad \text{(iv) } \square(x \rightarrow y) \rightarrow (\square x \rightarrow \square y) \approx 1$$

If $\square$ is the identity, this definition collapses onto the standard definition of subtractive variety. If it is not, we call $\mathbb{V}$ *properly* quasi-subtractive.

Examples include: subtractive varieties (and their nilpotent shifts), pseudointerior algebras, Boolean algebras with operators, residuated lattices, Nelson algebras, subresiduated lattices, basic algebras, quasi-MV algebras. Some of these varieties are not subtractive; some are but can be viewed as properly quasi-subtractive with a different choice of witness terms.

The most important tool for the investigation of quasi-subtractive varieties is the notion of *open filter*, which is to quasi-subtractive varieties as the concept of Gumm-Ursini ideal is to subtractive varieties. A $\mathbb{V}$-*open filter term* in the variables $\overrightarrow{x}$ is is an $n + m$-ary term $p(\overrightarrow{x}, \overrightarrow{y})$ of type ν s.t.

$$\{\square x_i \approx 1 : i \leq n\} \vdash_{Eq(\mathbb{V})} \square p(\overrightarrow{x}, \overrightarrow{y}) \approx 1.$$

A $\mathbb{V}$-*open filter* of $\mathbf{A} \in \mathbb{V}$ is a subset $F \subseteq A$ which is closed w.r.t. all $\mathbb{V}$-open filter terms p (i.e. whenever $a_1, \ldots, a_n \in F, b_1, \ldots, b_m \in A$, $p(\overrightarrow{a}, \overrightarrow{b}) \in F$) and such that for every $a \in A$, we have that $a \in F$ iff $\square a \in F$.

Finally, a variety $\mathbb{V}$ is called *weakly $(\square x, 1)$-regular* iff the $\{\square x \approx 1\}$-assertional logic $\mathcal{S}(\mathbb{V})$ of $\mathbb{V}$ is strongly and finitely algebraizable.

We have achieved several results, including:

1) A proof that every quasi-subtractive variety $\mathbb{V}$ is such that for all $\mathbf{A} \in \mathbb{V}$, every $\mathbb{V}$-open filter of $\mathbf{A}$ is a τ-class of some $\theta \in \mathrm{Con}(\mathbf{A})$ (for $\tau = \{\square x \approx 1\}$);

2) A proof that, if $\mathbb{V}$ is quasi-subtractive and $\mathbf{A} \in \mathbb{V}$, $\mathbb{V}$-open filters of $\mathbf{A}$ coincide with deductive filters on $\mathbf{A}$ of $\mathcal{S}(\mathbb{V})$;

3) A manageable description of generated $\mathbb{V}$-open filters;

4) A decomposition of a quasi-subtractive variety as a subdirect product of a subtractive and a "flat" variety (together with conditions under which the decomposition is direct);

5) An investigation of constructions generalising kernel contractions in residuated lattices;

6) A proof that, if $\mathbb{V}$ is quasi-subtractive and weakly $(\square x, 1)$-regular and $\mathbb{V}'$ is the equivalent algebraic semantics of $\mathcal{S}(\mathbb{V})$, then in any $\mathbf{A} \in \mathbb{V}$ there is a lattice isomorphism between the lattice of $\mathbb{V}$-$\mathbb{V}'$ congruences on $\mathbf{A}$ and the lattice of $\mathbb{V}$-open filters on $\mathbf{A}$.

In view of the examples mentioned above, we believe that the notion of quasi-subtractive variety could provide a common umbrella for the algebraic investigation of several families of logics, including substructural logics, modal logics, quantum logics, logics of constructive mathematics.

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Abstract Algebraic Logic approach to Algebraic Specification

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Standard abstract algebraic logic (AAL) cannot be straightforwardly applied to the theory of specification of abstract data types. Specification logic must be seen as a deductive system (i.e., as a substitution-invariant consequence relation on an appropriate set of formulas) and behavioral equivalence as some generalized notion of Leibniz congruence. The class of deductive systems has to be expanded in order to include multisorted as well as one-sorted systems. The notion of Leibniz congruence has to be considered in the context of the dichotomy of visible vs. hidden. In our approach ([6, 3]), the standard AAL theory of deductive systems is generalized to the hidden heterogeneous case. Data structures are viewed as sorted algebras endowed with a designated subset of the visible part of the algebra, called a filter, which represents the set of truth values. This new perspective helps to provide a better insight on the properties of the behavioral equivalence, the key concept in the behavioral algebraic specification theory ([2]).

In another direction, recently in [4, 5], the authors introduced an alternative approach to refinement of specifications in which signature morphisms are replaced by logic interpretations. Intuitively, an interpretation is a logic translation which preserves meaning. Originally defined in the area of algebraic logic, in particular as a tool for studying equivalent algebraic semantics ([1]), the notion has proved to be an effective tool to capture a number of transformations difficult to deal with in classical terms, such as data encapsulation and the decomposition of operations into atomic transactions.

Keywords: Behavioral Equivalence, Behavioral Specification, Refinement, Hidden Logic, Leibniz congruence, Interpretation.

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Finite axiomatizability theorems and sub-technology

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In 2002 Baker and Wang gave a very clear proof of Baker's theorem: Finitely generated congruence-distributive varieties are finitely axiomatizable. For this purpose they introduced *definable principal SUBcongruences*. In the talk we would like to notice that the sub-technology of Baker and Wang may be adapted to

- Quasivarieties: classes of algebras defined by quasi-identities, i.e. sentences of the form $(\forall \bar{x})[t_1(\bar{x}) = s_1(\bar{x}) \cdots t_n(\bar{x}) = s_n(\bar{x}) \to t(\bar{x}) = s(\bar{x})]$;

- Strict universal Horn classes: classes of models defined by sentences of the form $(\forall \bar{x})[\phi_1(\bar{x}) \cdots \phi_n(\bar{x}) \to \phi(\bar{x})]$, where $\phi(\bar{x}), \phi_1(\bar{x}), \ldots, \phi_n(\bar{x})$ are atomic formulas different from equations.

The last type of classes is especially interesting from abstract algebraic logic perspective. Indeed, each sentential logic corresponds to a strict universal Horn class of logical matrices (algebras endowed with one unary predicate). We obtained new proofs of

- Pigozzi's theorem: Finitely generated relative congruence-distributive quasivarieties are finitely axiomatizable;

- Pałasińska's theorem: Finitely generated filter-distributive protoalgebraic strict universal Horn classes are finitely axiomatizable.

To this end we introduced *definable relative principal SUBcongruences* and *definable principal SUBfilters*.

Algebras for Logics

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The use of algebra for the theory of reasoning was a fundamental turn in the development of logic. It was the first way to use mathematics to deal with logic, a fundamental step towards mathematical logic.

In this special session different algebraic structures and algebraic operators useful for the understanding of logic will be presented and discussed.

Accepted contributed talks

Completion and amalgamation of bounded distributive quasi lattices

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One of the basic motivations for studying the completion of a certain structure is due to the need of filling the gaps of the original one; a leading example could be turning a partial algebra into a total one. In the case of lattice ordered structures, the most relevant examples are represented by canonical extensions and Dedekind-MacNeille completions, see e.g. [2]. Nevertheless, once we move from lattice ordered structures to quasi ordered ones, the classical approach does not work. This observation motivated us in investigating a generalization of the classical filter-based approach to the case of *bounded distributive quasi lattices* (*bdq-lattices*), introduced by I. Chajda in [1]. The main problem in the completion of bdq-lattices lies in the fact that it may happen for elements x, y in a bdq-lattice **L** that $x \leq y$, $y \leq x$ but $x \neq y$. In this case, the usual notion of lattice filter is no longer useful to distinguish x and y. Thus, we use a particular system of congruences which allows to obtain, out of any bdq-lattice **L**, a quasi ordered space of functions $\langle E(\mathbf{L}), \tau, \preceq \rangle$, where τ is a topology on $E(\mathbf{L})$ admitting as a quotient the Priestley topology [5]. By this construction, we can embed the original bdq-lattice **L** into a (functionally) "complete" one. As an application of the previous results, we close the paper by proving, along the style of [4], the amalgamation

property for the class of bdq-lattices.

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Non-compatible Operations in Heyting Algebras

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Abstract: We study some non-compatible operations that can be defined using the min operator in the context of a Heyting algebra. One example is the minimum x-dense, that has been studied from a logical point of view in [2]. We are interested, for instance, in interdefinability and equationality. Regarding logical questions, we focus on different axiomatizations and we consider if the corresponding extensions of intuitionistic logic are conservative. Finally, we study the relationship with the successor (see [1]).

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Lattice of quasigroup formulas

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The family of power terms xy^n where n is a positive integer is inductively defined as follows: $xy^1 := xy$, $xy^n := (xy^{n-1})y$. With these terms the following family of formulas are associated: $\forall x, y \; xy^n = x$ where n is a positive integer. As a generalization of these formulas we propose the following torsion formula: $\forall x, y \; \exists n \; xy^n = x$. Using Steinitz numbers (for the original construction of these numbers see [6], a slightly reformulated construction is proposed in [3] and in [4]) we can introduce a family of new formulas as follows: $\exists n \; \forall x, y \; n|s, \; xy^n = x$ where s is a Steinitz number. As associated with the torsion formula we have the following family of formulas: $\forall x, y \; \exists n \; n|s, \; xy^n = x$ where s is a Steinitz number.

Formulas described above are named one-sided quasigroup formulas (groupoids satisfying at least one of them are one-sided quasigroups (see [2] and [4])). Evidently there are groupoids being one-sided quasigroups satisfying none of them. On the set of all formulas defining one-sided quasigroups we introduce a lattice structure and prove the following theorem:

Theorem One-sided quasigroup formulas form a lattice isomorphic to the lattice of closed Steinitz numbers with the divisibility relation.

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On some characterization of distributive lattice

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The smallest glued tolerance relation, so called skeleton tolerance, determines the decomposition of a finite distributive lattice D into its maximal boolean intervals, which themselves form a lattice. The lattice is said to be the skeleton of the distributive lattice D.

Every finite lattice is a skeleton of infinitely many finite distributive lattices. We try to characterize non-isomorphic distributive lattices with the same skeleton by some weighted graphs, which we will call the weighted double skeletons. They arise from the double skeleton of a lattice, which is the ordered set of all zeroes and units of blocks of its skeleton tolerance.

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Canonical inequalities on FL-algebras

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Canonicity of substructural logic, or more generally the canonicity of lattice expansions with non-smooth operations, has seen many contributions over the last decades. However, when we consider non-smooth operations, e.g FL-algebras, the canonicity results obtained from generalizing Jónsson-Tarski's methods run into the problem of the existence of two types of extensions (sigma-extensions and pi-extensions), which do not coincide in general. In this talk, we will show how to harness Ghilardi and Meloni's technique of "parallel calculation" (Ghilardi and Meloni, 1997) to obtain new canonicity results for substructural logic. The method will be presented in the light of the recent work (Dunn, Gehrke and Palmigiano, 2005).

Categorical Logic

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Categorical logic is a branch of mathematical logic that uses category theory as its principal mathematical tool and as mathematical foundation. This mathematical setting profoundly changes the conception of logic put forward by Frege and Russell in the beginning of 20th century both in its technical and philosophical aspects. On the technical side categorical logic inherits features from earlier constructive and algorithmic approaches to logic, in particular from realizability, lambda-calculus, intuitionistic logic and type theory. (In fact a typed intuitionistic logical calculus in the categorical setting appears to be the most natural system of logic while classical logic turns out to be a very special case that requires strong additional conditions.). This is one of the reasons why categorical logic is so successfully used in computer science. On its philosophical side categorical logic suggests a new notion of intrinsic logic that is analogous to the notion of intrinsic geometry that made a revolution in this mathematical discipline in 19th century. Frege and Russell after Aristotle conceived of logic as a system of universal rules of reasoning independent of any particular subject domain. Categorical logic not only diversifies the notion of logic by giving a space for different systems of logic, but also provides a mechanism of adjustment of a system of logic (i.e. a formal language) to a given domain of study and thought.

Some of the research in categorical logic sees a great dichotomy between "categorical proof theory" and "categorical model theory". Categorical proof theory is able to model different proofs of a given theorem, and compares these different proofs, using categorical concepts. Categorical model theory is an extension of traditional model theory, where models are categories. We see this meeting as encompassing both aspects of categorical logic.

Topics that fit this Special Session include, but are not limited to, the following:

- Relationships between logic and geometry in a topos-theoretic setting

- Categorical logic and Categorical foundations of mathematics

- Sketch theory; diagrammatic syntax

- Functorial semantics and Categorical Model theory

- Quantum logic categorically

- Extensions of categorical semantics to different kinds of logics, such as modal and substructural logics

- Comparison of different categorical frameworks

Invited Speaker

The role of the quotient completion for the foundations of constructive mathematics

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A key characteristic of the foundations for constructive mathematics is that they should enjoy a computational interpretation.

In joint work with G. Sambin [2] we argued that a foundation for constructive mathematics should have two levels: an intensional one acting as a programming language and an extensional one in which to develop mathematics. The link between the two levels should guarantee the extraction of programs from proofs.

Category theory offers a tool to characterize such a link in terms of quotient completion with respect to a suitable fibration developed in joint work with G. Rosolini.

Key examples of two-level foundations are available based on Martin-Lof's type theory and the minimalist type theory in [1] following [2].

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Accepted contributed talks

Fraïssé's construction from a topos-theoretic perspective

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We present a topos-theoretic interpretation of (a categorical generalization of) Fraïssé's construction in Model Theory, with applications to countably categorical theories.

The proof of our main theorem represents an instance of exploiting the interplay of syntactic, semantic and geometric ideas in the foundations of Topos Theory; specifically, the three concepts involved in the classical Fraïssé's construction (i.e. amalgamation and joint embedding properties, homogeneous structures, atomicity of the resulting theory) are seen to correspond precisely to three different ways (resp. of geometric, semantic and syntactic nature) of looking at the same classifying topos.

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Deduction rules are fractions

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A deduction rule is usually written as a "fraction" H/C. The aim of the talk is to actually define a deduction rule as a fraction in the categorical sense of Gabriel and Zisman. However, then it is rather written as C/H, with the hypotheses as denominator and the conclusion as numerator. This point of view relies on the definition of categorical entailments, which might be called "potential isomorphisms". In terms of logic, as long as models are concerned the entailments may be seen as isomorphisms, but for dealing with proofs it is essential to consider that the entailments are not invertible.

The link between logic and geometry in the mathematical pulsation between 3-ary and 2-ary laws

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The everyday mathematical structures could be presented by systems of 2-ary and 3-ary laws. In the process of axiomatization the 2-ary level and the 3-ary level interact each one with the other, through more or less good translations (via the calculus of 2-ary and 3-ary relations).

At first our intention is to put in light and to systematize the main cases of such translations (old and new examples), and to understand why very often the 3-ary level seem to be unnecessary, and what subtle geometrical facts are then left out. The point is the question of the lack of a natural origin O in a space (and so the question of the replacement of isomorphisms by isotopies). In this direction we will provide an embedding theorem for some particular 3-ary laws into systems of 2-ary laws.

Our second interest is in the logical aspect of this pulsation between 2 and 3, in the logical calculus of relations, and also particularly in the relationship between this question of the 3-ary laws and the question of borromean logic as modelized as the moving logic associated to the Klein's group of order 168. The point is the question of the lack of a natural true proposition T. In this direction we will provide an embedding theorem for some particular 2-valued moving logic into a 3-valued constant logic.

Downcasing Types

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When we represent a category C in a type system it becomes a 7-uple, whose first four components - class of objects, Hom, id, composition - are "structure"; the other three components are "properties", and only these last three involve equalities of morphisms.

We can define a projection that keeps the "structure" and drops the "properties" part; it takes a category and returns a "proto-category", and it also acts on functors, isos, adjunctions, proofs, etc, producing proto-functors, proto-proofs, and so on.

We say that this projection goes from the "real world" to the "syntactical world"; and that it takes a "real proof", P, of some categorical fact, and returns its "syntactical skeleton", P^-. This P^- is especially amenable to diagrammatic representations, because it has only the *constructions* from the original P — the diagram chasings have been dropped.

We will show how to "lift" the proto-proofs of the Yoneda Lemma and of some facts about monads and about hyperdoctrines from the syntactical world to the real world. Also, we will show how each arrow in our diagrams is a term in a precise diagrammatic language, and how these diagrams can be read out as definitions. The "downcased" diagrams for hyperdoctrines, in particular, look as diagrams about **Set** (the archetypical hyperdoctrine), yet they state the definition of an arbitrary hyperdoctrine, plus (proto-)theorems.

A fibrational semantics for System K

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The Curry-Howard isomorphism for intuitionistic modal logic S4 has been well established - there is both a well-established type theory and a well-established categorical semantics. In particular, fibrations can be used to model the distinction between modal and intuitionistic formulae.

For the weaker System K the situation is more complicated. There have been definitions of a suitable type theories and also categorical semantics, but a categorical semantics using fibrations in the same way as the one for intuitionistic S4 has not been given. We give such a categorical semantics and show that it also links in with the already existing type theories.

Categories and diagrammatic proofs

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Abstract: I describe a dynamical aspect of diagrammatic proofs in some elementary cases including the diagram chasing in the snake lemma and in the five lemma. I point to some features of geometrization of symbolic reasoning with categorical diagrams and analyze consequences of such a geometrization for proof proceeding.

Quantum Logics and Categories: Localism vs. Globalism

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Logic Diagrams

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The third World Congress on Universal Logic (April 22-25, 2010 at Lisbon, Portugal) will include, on April 25th, a special session to consider the role of diagrams in logic.

The use of diagrams in logic is old but unequal. The nineteenth century is often said to be the golden age of diagrammatic logic, thanks to the wide use of Euler and Venn diagrams, before a decline with the arrival of the Frege-Russell tradition of symbolization. One more obstacle that prevented the use of diagrams is the plurality of logical systems and the difficulty to deal with them. However, diagram studies have known a revival in recent years. It is thus legitimate to wonder what place diagrams hold in modern logical theory and practice.

The aim of this session is to discuss the logical status of diagrams. Topics include: What is a logic diagram? Is diagrammatic logic one more logic? Or are diagrams merely a notation that one can adapt to fit to different logics? Do diagrams fit to some logical systems better than to others? Is there still room for the use of diagrams in modern logic?

Invited Speaker

Reasoning in Diagrams: The Case of Begriffsschrift

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Frege designed his strange two-dimensional Begriffsschrift notation as a system of written signs within which to exhibit the contents of concepts, as those contents matter to inference, and to reason in mathematics. Frege furthermore claimed that reasoning from definitions in his language can be ampliative, a real extension of our knowledge, and in particular, that his Begriffsschrift proof of theorem 133 is ampliative (so synthetic in Kant's sense) despite being strictly deductive (or, as Kant would think of it, analytic). The proof is, in other words, constructive in something like Kant's sense, despite being strictly deductive. But if it is constructive then the notation within which the construction is made, that is, Frege's notation, must be functioning diagrammatically in something like Kant's sense (which encompasses not only Euclidean diagrams but also the symbolic language of arithmetic and algebra). I will explain what is required of such a notation, and show that Frege's notation can be read in just this way, as a mathematical language, a system of written signs, within which to reason from defined concepts in mathematics. So read, Frege's work, and the language he developed within which to do it, belongs not to the tradition of mathematical logic begun by Boole and mostly developed after Frege, but instead within the twenty-five hundred year long tradition of constructive paper-and-pencil mathematical reasoning that came before him.

Accepted contributed talks

Nonlinear Orthography or Nonlinear Thought?

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A couple of years ago Hintikka claimed that working out a system of abduction should be ranked among the most urgent tasks of the contemporary logic. According to Peirce if we are disregarding the intuition, the abduction is the only method of synthetic thought. It differs from deduction with that the later consists in mere unfolding the system of knowledge encapsulated in a given set of axioms. The way the axiomatic system is deductively unfolded is linear to the effect that if S^0 is a well determined set of premises there is a well determined set S^1 of its conclusions. The thus described situation changes as soon as one pays heed to the abductively evolved knowledge. Even if its starting point were a well determined set of presuppositions, its abductive upshot would be nothing but a fuzzy set of quasi-necessary consequences. (Incidentally, Zadech's characteristic of fuzzy set meets requirements of Peirce's synechist conception both of being as well as cognition.)

Giving a rough sketch or a mere skeleton of semantics for the knowledge abductively evolved is inflicting the lecture. The point of departure is Peirce's claim that there is not thought without external sign. In another words, the way we speak is the way we think, not otherwise. Taking it for granted, a concise comparison of Peirce's existential graphs and Frege's ideography is carried through. The comparison leads to the conclusion, that there is a split between what Frege proclaims and what he so to speak "prescribes". However, the thus found "inconsistency" in Frege's thought paves road to reconciling some of his views with a couple of bed-rock ideas of Peircean sketchy

theory of graphs. Finally with the concept of semilattices at hand we draw the aforesaid outline of semantics fora system of knowledge abductively conceived.

A Cognitive Approach to Diagrams in Logic

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The term *logocentric* was chosen to define the *dogma* of the standard view of mathematics, according to which proofs are syntactic objects consisting only of sentences arranged in a finite and inspectable way. By contrast, reasoning is a heterogeneous activity: it is necessary to expand the territory of logic by freeing it from a mode of representation only (Barwise and Etchemendy (1996), Shin (2004)). In this talk, I will argue that the antidote against the logocentric approach to diagrams consists neither in finding the right set of rules nor in assuming an *opticentric* view, but in considering that the most relevant aspect of diagrammatic reasoning is the way in which diagrams are manipulated to infer some new conclusion from them; this happens in continuous interaction with language. My view moves from a purely syntactic approach to a semantic and indeed pragmatic approach to problem solving.

Levels of Syntax for Euler Diagram Logics

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Euler diagrams have been used for centuries to convey logical information and have been formalized for this purpose. Related notations, such as Venn-II, Euler/Venn, spider diagrams, and constraint diagrams, have been developed and fully formalized as logics in their own right. Issues regarding their abstract (type) syntax and concrete (token) syntax have been discussed in the literature but there are more fine grained levels of syntax that have not, to the authors' knowledge, been given serious consideration. We discuss different levels of Euler diagram syntax. These include the drawn level, where the images of the diagrams live, for example, on a computer screen, a piece of paper, or a whiteboard; mathematical models of the drawn diagrams; the dual graph of an Euler diagram; and the abstract level of syntax that captures the semantic information present in the diagram and forgets all geometric and topological properties that have no impact on the semantics.

The Hardness of the Iconic Must: Can Peirce's Existential Graphs Assist Modal Epistemology?

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The current of development in 20th century logic bypassed Peirce's existential graphs, but recently much good work has been done by formal logicians excavating the graphs from Peirce's manuscripts, regularizing them and demonstrating the soundness and completeness of the alpha and beta systems (e.g. Roberts 1973, Hammer 1998, Shin 2002). However, given that Peirce himself considered the graphs to be his *chef d'oeuvre* in logic, and explored the distinction between icons, indices and symbols in detail within the context of a much larger theory of signs, much about the graphs arguably remains to be thought through from the perspective of philosophical logic. For instance, are the graphs always merely of heuristic value or can they convey an 'essential icon' (analogous to the now standardly accepted 'essential indexical')? This paper claims they can and do, and suggests important consequences follow from this for the epistemology of modality. It is boldly suggested that structural articulation, which is characteristic of icons alone, is the source of all necessity. In other words, recognizing a statement as necessarily true consists only in an unavoidable recognition that a structure has the particular structure that it in fact has. (What else could it consist in?)

Reasoning with Euler diagrams: a proof-theoretical approach

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This talk is concerned with a proof-theoretical investigation of Euler diagrammatic reasoning. We introduce a novel approach to the formalization of reasoning with Euler diagrams, in which diagrams are defined not in terms of regions as in the standard approach, but in terms of topological relations between diagrammatic objects. On this topological-relation-based approach, the unification rule, which plays a central role in Euler diagrammatic reasoning, can be formalized in a style of Gentzen's natural deduction. We prove the soundness and completeness theorems of our Euler diagrammatic inference system. We then investigate structure of diagrammatic proofs and prove a normalization theorem. Finally, we discuss some cognitive properties of Euler diagrammatic reasoning in our system, in comparison to linguistic reasoning and reasoning with Venn diagrams.

Diagrammatic Reasoning with Class Relationship Logic

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We discuss diagrammatic visualization and reasoning for a class relationship logic accomplished by extending Euler diagrams using higraphs. The considered class relationship logic is inspired by contemporary studies of logical relations in biomedical ontologies, and it appeals to the Closed World Assumption (CWA) unlike e.g. Description logic. The suggested diagrams provide inference principles inherent in the visual formalism. The considered logical forms are dealt with at the metalogic level by variables ranging over classes and relations. There are inference rules being formalized at the meta-level using definite clauses without compound terms (Datalog).

Sentential Modal Logic and the Gamma Graphs

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The system of Existential Graphs is a diagrammatic system of logic that was developed by Charles S. Peirce. This system has three main sections and Peirce named these three sections: Alpha, Beta, and Gamma. The Alpha section of EG is a diagrammatic account of sentential logic; the Beta section (which is an extension of Alpha) is a diagrammatic account of first order predicate logic with identity; and the Gamma section (which is an extension of Beta) gives a combined diagrammatic account of higher order predicate logic and modal logic. In my paper I give brief account of Alpha. Once this has been done, I extend Alpha to include some of the rules of Gamma (as presented in Peirce's 1903 version of Gamma), so that a system of sentential modal logic results. After investigating some of the properties of Gamma, I compare them to some of the properties of contemporary accounts of sentential modal logic.

Tableaux for the Gammas

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Tableaux methods are conveniently applied to existential graphs, because tableaux interpret the assertions in the endoporeutic fashion. Since graphs can be read in a number of logically equivalent ways, the number of tree rules is kept in a minimum (negative juxtapositions go to different branches, positive ones to the same branch). Already in 1885, Peirce had suggested tableaux for propositional logic. He never applied the idea to the EGs. But his semantic rules for EGs are equivalent to those of the game-theoretic semantics, and thus are naturally amenable also to semantic tableaux systems. The gamma part of EGs was Peirce's boutique of modal and higher-order logics, metagraphs, and many others. I will define semantic tableaux-type proofs for the modal gammas, and propose such transformation rules for the broken-cut gamma that yield better correspondences with modern characterisations of modal logical systems.

Diagrams in the Membership Game

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There is a natural way to represent sets by certain diagrams; it has grown customary in modern studies of ill-founded sets. We use this way to study a set-theoretic game, the *membership game*. This game, introduced probably by T.Forster in [1] (see also [2]) and originally related to NF-like set theories, is a perfect information game of two persons played on a given set along its membership: The players choose in turn an element of the set, an element of this element, etc.; a player wins if its adversary cannot make any following move, i.e. if he could choose the empty set. Sets that are *winning*, i.e. have a winning strategy for some of the players, form an ordinal hierarchy easily visualized by diagrams.

We show that all levels of the hierarchy are nonempty and the class of hereditarily winning sets is a full model of set theory containing all well-founded sets. Then we show that each of four possible relationships between the universe, the class of hereditarily winning sets, and the class of well-founded sets is consistent. For consistency results, we propose a new method to get models with ill-founded sets. Its main feature is that such models are stratified like the cumulative hierarchy and reflect certain formulae at own lowest layers. Thus to know properties of whole models it suffices to observe diagrams of their lowest layers, usually very simple objects. We apply this method to establish various fine results, some of which display a deep difference between odd- and even-winning sets.

Our results are proved in a weak set theory, ZF minus both choice and regularity axioms. Although they can be established without using of diagrams, diagrams make the constructions much more clear and easily observable. These results were announced in [3] and appeared with detailed proofs in [4].

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Multimodal Logics

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Contemporary modal logic, officially born in 1932, received a powerful impulse in the Sixties with the development of so-called relational semantics. After this important turn modal logic underwent a constant, and indeed impressive, progress passing through a specialized analysis of different concepts of necessity and possibility and giving rise to such branches as tense logic, epistemic logic, deontic logic, dynamic logic and so on.

The last step of this development has been provided by the growth of multimodal logics, i.e. of logics whose language contains more than one primitive modal operator and whose axioms define the logical properties of each one of them along with their interaction. Multimodal logic has already reached interesting results in the abstract analysis of the properties of multimodal systems. As a matter of fact, multimodal logic is not a new branch of modal logic but rather a new way to study modal notions by using a more general and deep approach, akin to the spirit of Universal Logic.

The aim of this session is to collect contributions to the field of multimodal logic. A wide number of subjects may be treated in this realm. Topics regarded as being of special interest are the following:

- Temporal logics

- Logics of physical modalities

- Epistemic-doxastic logics

- Multimodal analysis of conditionality

- Topological logics

- Multimodal systems of mathematical provability

- Multimodal systems with non-classical propositional basis

- Combinations of (multi)modal systems

- Incompleteness of multimodal systems

- Decision procedures for multimodal systems

- New semantics and proof methods for (multi)modal systems

- Multimodal quantified logics

- Modal treatments of quantification

- Abstract properties of multimodal systems

Accepted contributed talks

Description logics from a paraconsistent viewpoint

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Description logics (DLs) are used to represent knowledge that intend to express properties of structured inheritance networks. These systems can be interpreted as fragments of first-order logic and permit us to express particular restrictions as "Mary is a student", or relations between objects as "every student is a woman". However, DLs face difficulties in expressing existential restrictions as "Mary has a cat". Such difficulties are related to properties of negation in the first-order fragment which interprets DLs, as argued in [1]. We intend to present DLs in terms of paraconsistent systems, so offering a new way to cope with questions related to negation. This is a quite natural approach, considering that DLs are, in a certain sense, notational variants of multimodal logics, and that multimodal logics with weak negation can be seen in paraconsistent terms as the cathodic systems investigated in [2]. Cathodic systems are positive modal systems endowed with weak negation, extending the logics of formal inconsistency (see [3]). In this way, DLs based on paraconsistent logics tolerate contradictions (in the sense of [4], thus enhancing the revision process inherent in DLs.

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Polynomial ring calculus for S4, intuitionistic logic and multimodal logics

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Polynomial ring calculi (PRC) consist in translating logic sentences into polynomials over algebraically closed fields (usually finite fields) and into performing deductions by means of polynomial operations. Elements of the field represent truth-values and polynomial equations express truth-conditions (in a similar way as specifying conditions in valuation semantics). In this way PRC can be legitimately considered a semantics, as well as a tool for performing deductions by means of polynomial operations, establishing a proof method appropriated for automation.

PRC were introduced in [2], where the method is applied to the classical propositional calculus, many-valued logics and paraconsistent logics. In [1] a PRC for the modal logic S5 is defined, and it is conjectured that such PRC can be adapted to a large class of modal and multimodal logics. Our purpose here is to adapt the PRC defined in [1] to the modal logic S4, providing a new semantics and proof method not only for S4, but also for intuitionistic logic, using the well-known Gödel's translation (1933) of Heyting's intuitionistic logic into S4. We discuss some advantages of the PRC for modal and intuitionistic logics in contrast with the traditional Kripke semantics and tableau proof methods. We also show how PRC can be extended to multimodal logics and to the provability logic (called KGL in [3]). KGL has strong connections with Gödel's incompleteness theorems of 1931 and Löb's theorem of 1953. We conjecture that the celebrated fixed point theorem of KGL (D. de Jongh and G. Sambin, independently, in 1975) can even be proven by means of formal polynomials via PRC.

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A tense-modal logic with choices for agency and temporal distinctions

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Truth in BTC logic (Branching Time logic with Choices) is relative to a moment and to a particular choice (of a single agent) at that moment. BTC+ is an extension of BTC in which we can consider simultaneous choices of a set of agents. According to [1] choices are viewed as sets of histories (paths) and, according to [2] BTC and BTC+ semantics have both an Ockhamist and a Peircean dimension: a modal operator quantifies over choices and tense operators quantify within a given choice. As we are mainly interested in Computer Science applications, histories are assumed to be isomorphic to the set of natural numbers. The paper is mainly concerned with the expressive power of BTC+, and shows that CTL and a fragment of CTL* are expressed in the new logic.

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Specifying and reasoning with asynchrony: proof theory of DSL

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To specify and reason on distributed system with asynchronous communications, the use of a many-dimensional modal logic combining spatial and temporal operators is fostered. This combination has an adequate expressive power, but suffers from the lack of a complete proof system, direct standard proof techniques having failed, as well as those based on compositionality results.

The Labelled Sequent Calculi approach, introduced in [1], is proposed here to define a complete proof system for the logic. This calculus is a good candidate since both admissibility of the structural rules and completeness are obtained in a modular way: The same proof method works for all extensions and the addition of other modalities or frame properties requires only small local additions to the proofs. As a first step, the formalization of the spatial fragment in the labelled sequent calculus is addressed.

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On necessary and possible contradictions

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Despite several non-classical modal systems were proposed in the literature, the method for obtaining them is not too much divergent: most of them are the result of combining a propositional non-classical logic with a hierarchy of (multi)modal systems. On the other hand, the definition of non-classical primitive modal operator was investigated to a lesser extent.

Departing from paraconsistent logics (more precisely, from Logics of Formal Inconsistency - LFI's), and by analogy with the inconsistency operator, we propose here a new unary modality, reading as "necessary contradictory", and its dual, reading as "possible contradictory"'. From the semantical point of view, we consider paraconsistent Kripke structures such that "necessary contradictory" alpha holds in a state w whenever both alpha and not alpha hold in every state w such that wRw'. Analogously, "possible contradictory"' holds in w whenever there is some w' such that wRw' and both α and not α hold in w'.

We introduce a minimal normal system analogous to K with the new operators. After this, we analyze the new version of axioms D, T, B, 4 and 5 suitable to the new modalities. For instance, by considering paraconsistent Kripke structures where R is reflexive, we shown that the suitable version of axiom T is "necessary contradictory α implies α and not α"

The subtle discrepancies with the classical versions of such axioms can be explained by translating the propositional modal logic into a first-order LFI. This shows the important role that an appropriate non-classical Correspondence Theory would have for non-classical (multi)modal logics, mainly for systems with non-classical primitive modalities.

Finally, we shown how to combine these systems with the classical modalities necessary and possible and outline possible applications of such systems.

Contingency operators and multimodal logics

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Contingency operators and multimodal logics In Pizzi [2007] it is proved that both the deontic system KD and the minimal system K may be defined as fragments of two contingential systems, named Kô and Kôw respectively, whose language contains as primitives truth-functional operators, the non-contingency operator and a propositional constant . The paper aims at showing that

interesting bimodal systems – such as the logics of physical modalities – may be generated by extending contingential languages with suitable axiomatized propositional constants. It is then discussed the suggestion that systems with an arbitrary number of necessity operators may be defined by generalizing this strategy.

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Axiomatizing hybrid products of topologies

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The main aim of this paper is to propose a robust way to combine two topological hybrid logics. First, we introduce two kinds of: *nominals*: i (e.g. for a moment of time) and a (e.g. for a spatial point), and *satisfaction operators*: $@_i$ and $@_a$ to the bimodal logic of *products of topologies* [1] (note: 'product of topologies' is a different notion from product topology). Second, we give five interaction axioms and establish a general completeness result called *pure completeness* of *bi*-hybrid logic of products of topologies. Finally, we explain how to capture the dependence of one dimension (e.g. space) to the other (e.g. time) in a generalization of our setting. Our enrichment of the language was a suggested further direction in [1]. This work can also be regarded as a further extension of both hybrid logic of topological spaces [3] and bi-hybrid logic of *products of Kripke frames* [2].

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A structural semantics for multimodalities

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A setting of multiple-valued logics is set forth within the general framework of **QAS** (Question-Answer Semantics). Starting from the view that any formula is a structured object whose components are a function symbol and its argument, **QAS** is used in the following to reformulate two algebraic theories: Gottschalk's theory of quaternality, and Piaget's theory of reversibility (INRC Group).

We propose to extend the process to a semantics for multimodalities, where the arguments include at least one further function symbol. This will be done with either of Gottschalk's and Piaget's questions, the content of which is not the same. A structural semantics relies upon the use of two basic answer-values, namely: *affirmation* (1), and *denial* (0, or -1). This general setting gives rise to a family of logics, where logical pluralism lies in the plurality of questions **Q** to be asked among the variety of structured formulas.

Finally, the aim of our conceptual framework is to replace the basic notions of *truth* and *consequence* by those of *negation* and *opposition*; but some challenges are still to be overcome in order to confirm its relevance, including the reformulation of modal logics within **QAS**. It remains to see whether a *multiple*-valued family of systems could throw some new light upon the awkward relation between many-valued and modal systems: the latter challenge is reminiscent of Dugundji's negative theorem, according to which no finitely many-valued matrix can characterize the Lewis modal systems S1-S5.

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Interplay of Beliefs, Desires, and Actions

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In the bdi-stit logic of beliefs, desires, intentions, and actions, we combine normal modal operators as in Stit Theory with standard monotonic, regular operators for pro-attitudes and cognitive states. The emphasis of the talk is on postulating axioms describing a philosophically interesting interplay between beliefs, desires and actions of one and the same agent. On the other hand, we suggest an independence condition for neighbourhood semantics to ensure the independence of different agents concerning belief attitudes. Therefore, we relate properties of neighbourhood functions with the underlying Branching Time structure. The completeness of the bdi-stit logic is shown by expanding the canonical models of Xu for Stit Theory.

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Hybrid Language and Relation Algebra

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We shall take the development of modal logic and the theory of binary relations up to a meeting point at which they are combined (by means of an hybrid language and relation algebra (RA)) in a formalism which has full first order logic expressiveness. Our aim is discussing advantages and drawbacks of such formalism for both as a logical system equipolent to first order logic and as a formalism with which one can formalize set theory and work on foundational issues of mathematics.

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Whitehead's theory of space and time

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Whitehead is known not only as one of the authors of the famous book "Principia Mathematica". He also was an initiator of a new approach to the theory of space and time based on some simple spatio-temporal relations between material things taking as primitives [2]. Whitehead presented in [2] a quite detailed program how to build a mathematical formalization of the spatial part of his theory, known now as Region Based Theory of Space (RBTS) (see [1]). Unfortunately, the part of his theory dealing with time (called in [2] Epochal Theory of Time (ETT)) is developed only in an informal way, which makes quite difficult to extract a good mathematical theory of space and time. In our talk we will present an integrated mathematical formalization of a fragment of Whitehead's theory of space and time incorporating in it some modern versions of RBTS and some formal counterpart of ETT. The main result is a Stone-like representation theorem stating that any abstract system is representable in the class of standard models.

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Negation

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Dealing with a certain polarity of thought, negation is, perhaps, the most crucial among the logical connectives. It has been studied since antiquity and has been subject to thorough investigations in the development of philosophical logic, linguistics, artificial intelligence and logic programming. This development shows that bringing into play various types of negation may produce highly fruitful and promising results in many areas, such as paraconsistent logic, non-monotonic reasoning, the theory of data bases and logic programming.

The properties of negation - in combination with those of other logical operations and structural features of the deductibility relation - serve as gateways among logical systems. Moreover, a difference between various logical systems can often be reconstructed as a difference of certain features of negation operators used in these systems.

Notwithstanding the importance of negation, the immense literature on negation is full of disagreements concerning at least necessary conditions under which a unary connective ought to be regarded as a negation operation, the syntactical type to which a negation operator should belong , etc. We hope that this session will contribute to comparing different kinds of negation, developing a general theory of negation, and investigating the scope and validity of principles about negation.

Topics suitable for this Special Session include, but are not limited to, the following ones:

- proof-theoretical versus semantical treatments of negation

- negation, consistency, and inconsistency; interrelations between these notions

- negation and Galois connections; correspondence theory for negation

- negation in the light of modal logic

- negation in relevant and substructural logics

- constructive treatments of negation

- negation in belief revision

- negation in logic programming and non-monotonic reasoning

- negation in adaptive logics

- negation in paraconsistent logics

- negation in categorical grammar

- negation in concept analysis

Invited Speaker

Negation as a Primitive Duality in a Model for Quantum Computation

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We define the "random first order domain" of the set of outcomes of a measurement experiment on a quantum system. We consider a primitive negation of assertions originated by Girard's duality and extend it to random first order domains. So we obtain a representation of quantum states in dual couples, that allows to represent the NOT of quantum states. In this setting, we find a characterization of the eigenstates of the NOT matrix (the NOT gate in a quantum computer). As is well known, such eigenstates, including Bell's states, are very significant in quantum computation. In our representation, the eigenstates are fixed points in a cut elimination procedure exploiting the duality.

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Accepted contributed talks

All Natural Three-valued Paraconsistent Logics are Maximal

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Maximality is a desirable property of paraconsistent logics, motivated by the aspiration to tolerate inconsistencies, but at the same time retain from classical logic as much as possible. This notion of maximality, as well as other notions considered in the literature, is based on extending the set of theorems of a logic. In this paper, we use a strictly stronger notion of maximality: a paraconsistent logic L is maximally paraconsistent (in the strong sense) if every logic L' that properly extends L is not paraconsistent. We show that all the three-valued paraconsistent logics, satisfying some very natural conditions, are maximal in the strong sense. This includes well-known three-valued paraconsistent logics like Sette's logic P1, Priest's logic LP, the semi-relevant logic SRM3, the logics PAC and J3, as well as any extension of them obtained by enriching their languages with extra three-valued connectives.

Negations with a Contextual Meaning

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Some adaptive logics, including all inconsistency-adaptive logics, assign to negation a meaning which is contextual: the premise set determines which occurrences of negation have one meaning, say the classical one, and which the other, say a paraconsistent one. Making the meaning of negation contingent on the premise set results in sensible applications with respect to interpreting texts, but also with respect to the mathematical and empirical theories.

    The technicalities will be illustrated in terms of a specific inconsistency-adaptive logic. Several variant logics will be presented and their use and effect with respect to negation discussed. Some of these logics are not inconsistency-adaptive but realize an adaptation with respect to other properties of negation, for example its completeness.

    A different point concerns the way in which logical systems define the meaning of logical symbols and so determine theories from sets of non-logical axioms. Such questions are raised by adaptive logics in that, for example, the meaning assigned to all negations in *consistent* premise sets by inconsistency-adaptive logics is identical to the meaning assigned to them by Classical Logic.

Negation and the Fixed-point Property

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k-valued clone with constants is a set of k-valued functions that is closed with respect to composition and includes the projections and the constant functions. By definition, a clone F has the (Gupta-Belnap) fixed-point property if, and only if, every system of equations $x = f(x, y, ...)$, $y = g(x, y, ...), ...$ (for functions $f, g, ...$ in F) has a solution. Clones will be understood as the interpretation of propositional languages and systems of equations are meant to represent self-referential nets of sentences (for instance, the Liar paradox is represented as $x = "x$). By well-known fixed-point theorems, the three-valued clones with constants generated by Kleene logics (strong and weak) have the fixed-point property. In this talk we will characterize the fixed-point property for 2- and 3-valued clones as depending on whether the clones are able to express a generalized form of negation. We will also present some general results that specify classes of k-valued clones that have the fixed-point property. Finally those results will be applied to show that certain clones that generalize the weak Kleene operators and that include a pseudo-operator of strong negation and pathologicality operators (i.e. operators meaning –in the intended philosophical interpretation of the truth values- "p is a pathological sentence", "p is neither true nor false"...) have the fixed-point property.

Negation: Default is Explicit + Update

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Negation is still one of the controversial concepts underlying logic programming. While the negation-as-failure interpretation is operationally well-understood, the logical interpretation of negation is less clear. The two most prominent approaches—closed world assumption and completion—have helped us to understand the logic that underlies negation as failure. But, as Shepherdson remarks, "[t]heir disadvantage is that the logics involved are more complicated and less familiar

than classical logic so that they are not likely to help the naive programmer express his problem by means of a logic program, or to check the correctness of a program". We will show that default negation in (normal, generalized, and dynamic) logic programming can be understand as explicit negation in an update framework, as long as we consider stable model semantics. The updates are taken from dynamic logic programming. The technical result is not particularly complicated, but it has some interesting conceptual consequences. Default negation is a prime example in non-monotonic reasoning. Our result questions—on philosophical grounds—the status of default negation as a special, non-monotonic form of negation. Thus, default negation can be seen as explicit negation just involving an update aspect (which could also be considered temporally). Our analysis suggests that non-monotonic reasoning can be recast as classical reasoning incorporating forms of meta reasoning.

Some Modal Logics Seen as Subsystems of Classical Propositional Calculus

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One of the well-known and widely studied non-classical logics is the so-called Modal Logic, which is nowadays often regarded as *extension* of classical propositional calculus(CPC). However, there are other ways to formulate systems which have the "same" theorems as that of well-known modal logics. Namely, we can actually obtain such systems by first regulating CPC and second giving some appropriate definitions. This kind of results, recovering the original system Σ after restricting the system Σ, are not new. Indeed, Łukasiewicz showed that intuitionistic propositional calculus contains CPC and da Costa showed that systems of paraconsistent logics $C_n(1 \leq n < \omega)$ also contain CPC (cf. [3], [2]).

Now, the fact that modal logic S5 can be formulated by using "weak" negation instead of employing the classical negation and the necessity operator was first proved by Béziau and also by Waragai and Shidori independently(cf. [1], [6]). As for the problem of formulating other systems from S5, it remained open for a while until Marcos made an important contribution showing some formulations for systems such as K, T, KB, K5, B, etc (cf. [4]).

Based on these studies, the purpose of the present paper is twofold. Firstly, we extend and refine the results obtained by Marcos. This will include (1) formulations of some systems, including S4, which have been open after the work of Marcos and (2) an alternative economic formulation of systems treated by Marcos in [4]. Secondly, we emphasize that the systems developed by Béziau and Marcos can be seen as restriction of CPC. In order to make this viewpoint clear, a framework which is sketched roughly in [5] will be introduced and discussed in detail.

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Negation in Dual Intuitionistic Logic

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Weak Negations and Neighborhood Semantics

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In this talk, I explore the logic and semantics of weak negations: unary operators validating only some of the inferences usually associated with negation. In particular, I am concerned with operators that are downward entailing (operators N such that if A entails B, then NB entails NA); these seem to be particularly important for linguistic theory. As such, I focus on a base system with a single downward entailing operator and various strengthenings (requiring the operator to validate more inferences traditionally associated with negation).

In the course of this investigation, I adapt model-theoretic techniques from modal and substructural logic to provide a neighborhood semantics for these systems. The systems are shown to be sound and complete with respect to classes of frames, and inferences are shown to correspond to certain conditions on frames. Neighborhood semantics allows for the exploration of weaker negations than can be captured in a relational semantics like that pursued for negations by Dunn and others. Thus, the neighborhood approach allows for an expansion of Dunn and Zhou (2005)'s map of various negations and negation-like operators.

Negation as Test-Failure in DPL and Negation as Process Exclusion in Categorical Grammar

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We give a dynamic semantics for the Lambek Calculi, to which a dynamic negation is added. This dynamic negation is interpreted as procedural prohibition, or process exclusion. The resulting framework suggests connections with the analysis of negation as test–failure in Dynamic Predicate Logic (DPL). The aim of the present article is to explore this connection in detail.

Negation and Refutation in proof-theoretic semantics

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The intuitionistic informal account of negation, according to which a construction for the negation of A is a function mapping constructions of A onto constructions of the absurdity is somewhat unsatisfactory. This emerges in particular within the so-called proof-theoretic semantic framework, a development of intuitionism on the basis of Gentzen's work in proof-theory. In a truth-theoretic semantics, truth values are assigned to formulas: in the case of closed formulas (i.e. sentences) the semantics directly maps the sentence onto a truth value. In the case of open formulas, this happens indirectly, i.e. modulo an assignment of values to the open variables. A proof-theoretic semantics, on the other hand, takes proofs as the semantics values to be assigned to valid derivations in a deductive system. In a natural deduction setting, closed derivations are directly mapped onto proofs while open derivations are mapped onto them given an assignment of values for the open assumptions. When we come to negation, we have that a closed derivation of the negation of A is constituted by an open derivations having the absurdity as conclusion and A as only assumption, where the absurdity is what there is no proof of. The problem lays in the fact that the semantics is expected to map the open derivation onto a proof of the absurdity given an assignment of proofs of the assumption A. But if the negation of A is provable then of course there is no proof of A. The problem resembles the ones that led to the introduction of free logics: an analogous solution in this case would be a distinction between actual proofs and possible proofs. To avoid the ad hoc character of this strategy we consider an alternative one: the introduction of refutations alongside proofs as possible semantics values of derivations. We discuss a few alternative ways of doing this.

Nonmonotonic Functions Caused by Distributive Negatives

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A distributed system consisting of deductive processes with treatments of negatives conceives a nonmonotonic function over set variables (denoting true and false sets of atomic formulas). From model theory views, fixpoint and procedural semantics are well defined for logic programs with negation, as discussed in Pereira,P.M. et al. (1991), Shepherdson,J.C. (1998) and Yamasaki,S. et al. (2001), where the 3-valued logic is applied to the denotations of true and false sets. The procedural semantics is closely related to computing based on the first-order logic such that the fixpoint semantics is abstract enough to capture the computational behaviour of programs. So far varieties of negations are simultaneously treated for logic programming, where multi-negation induces a nonmonotonic function as in Yamasaki,S. (2006). In this talk, a general framework is defined for a nonmonotonic function, caused by a distributed system containing deductive derivations with negatives. The nonmonotonic function is not so easy to analyze, when we want to have a mathematical representation of the whole behaviour of the system in which the atomic formulas with and without negation are both transferred from one deductive process to others through communication. It is because the function cannot be in general monotonic even by means of the 3-valued logic. For a general theory, a fixpoint semantics can be considered as model theory, however, a pragmatic, procedural semantics may be reasonable with reference to model theory, in concretization of the framework applied to a program.

Non-Classical Mathematics

Organized by: **Libor Behounek (behounek@ff.cuni.cz)**
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The 20th century has witnessed several attempts to build parts of mathematics on grounds other than those provided by classical logic. The original intuitionist renderings of set theory, arithmetic, analysis, etc. were later accompanied by those based on relevant, paraconsistent, contraction-free, modal, and other non-classical logics. The subject studying such theories can be called non-classical mathematics, i.e., the study of any part of mathematics that is, or can in principle be, formalized in some logic other than classical. This special session at the 2010 UniLog World Congress is a follow-up to the conference on Non-Classical Mathematics that was held in Hejnice, Czech Republic, in June 2009.

The scope of interest of this special session contains, but is not limited to the following topics:

- Intuitionistic, constructive, and predicative mathematics: Heyting arithmetic, intuitionistic set theory, topos-theoretical foundations of mathematics, constructive or predicative set and type theories, pointfree topology, etc.

- Substructural and fuzzy mathematics: relevant arithmetic, contraction-free naive set theories, axiomatic fuzzy set theories, fuzzy arithmetic, etc.

- Inconsistent mathematics: calculi of infinitesimals, inconsistent set theories, etc.

- Modal mathematics: arithmetic or set theory with epistemic, alethic, or other modalities, modal comprehension principles, modal treatments of vague objects, modal structuralism, etc.

- Alternative classical mathematics: alternative foundational theories over classical logic, non-standard analysis, etc.

- Topics related to non-classical mathematics: metamathematics of non-classical or alternative mathematical theories, their relative interpretability, etc. Non-Classical Mathematics.

Invited Speaker

A New Approach to Predicative Set Theory

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The Poincare-Weyl-Feferman predicativist program for the foundations of mathematics seeks to establish certainty in mathematics without necessarily revolutionizing it (as the intuitionistic program does). The main goal of this paper is to suggest a new framework for this program by constructing an absolutely reliable predicative pure set theory PZF whose language is type-free, and (from a platonic point of view) the universe ZF is a model of it. Our basic idea is that the predicatively acceptable instances of the comprehension schema are those which determine the collections they define in an absolute way, independent of the extension of the *surrounding universe.* This idea is implemented using a syntactic safety relation between formulas and sets of variables. This relation is obtained as a common generalization of syntactic approximations of the notion of domain-independence used in database theory, and syntactic approximations of Godel's notion of absoluteness used in set theory. Two important features of our framework is that it makes an extensive use of abstraction terms, and its underlying language is that of ancestral logic (which is strictly stronger than first-order languages, but much weaker then full second-order languages). Another crucial feature is that it is possible to use it together with classical logic, but it equally makes sense to use it in combination with some non-classical logic, especially intuitionistic logic.

Accepted contributed talks

Features of Mathematical Theories in Contraction-Free Logics

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First-order contraction-free substructural logics are strong enough to support non-trivial mathematical theories. However, the absence of the law of contraction makes certain practices that are commonly used in classical mathematics meaningless. This talk summarizes the most important differences between classical and contraction-free mathematics and suggests several guidelines for developing contraction-free theories.

   In particular, in contraction-free mathematics, defined notions need be parameterized by multiplicities of subformulae of the defining formula; free combinations of subconditions replace conjunctive compound notions; non-contractive preconditions need occur as premises of theorems (with variable multiplicities) rather than in definitions; non-contractive subsets can only be viewed as an additional structure rather than universes for substructures; and instead of equivalences, theorems indicating mutual bounds for truth-values are regularly obtained in contraction-free mathematics.

   Some of these features were first observed in our paper "Features of mathematical theories in formal fuzzy logic" (LNAI 4529:523-532, 2007), but they actually apply to all contraction-free substructural mathematics. Further features, such as the splitting of classically equivalent notions into several variants, are common to all branches of non-classical mathematics.

Minimalistic Liberalism − a contradictory semi-formal set theory respecting classical logic, and with mathematical power beyond $PI(1,1) − CA(0)$

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We present the semantics and some salient partial axiomatic and inferential principles for a semi-formal set, or predication theoretic framework which we call Minimalistic Liberalism (ML). ML deals with paradoxes in a way which is akin to paraconsistent approaches, though differs in that classical logical principles are always theorems and their negations always fail to be theorems, even in the presence of what is called "liberal comprehension". E.g., if R is Russell's set, it will be a theorem of ML that R is in R, and it will be a theorem in ML that R is not in R, but it will not be a theorem of ML that R is in R and R is not in R. So ML is non-adjunctive. One focus of the presentation will be upon explaining the semantical framework, which relies upon an additional twist upon revisionary types of semantics in the tradition from Herzberger, Gupta and

Belnap, and to isolate some of the salient axiomatic and inferential principles which hold. It is a fact that ML is strong enough to interpret ACA. We show how we may make use of a fixed point construction going back to Cantini and Visser in order to show that a set of hereditarily non-paradoxical iterative sets in ML interprets the theory of finitely iterated inductive definitions $ID < \omega$ which has the same proof theoretic strength as $PI(1,1) - CA(0)$. So this is a lower limit for the kind of proof theoretic strength at stake.

Nonstandard Analysis in action: Zeno of Elea and his modern rivals revisited

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In my talk I consider The Achilles Paradox with regard to the structure of continuum. To compare Zeno's arguments with the modern solutions to the paradox I suppose that Zeno's space is that of Euclid. As a result, provided that $(F, +, ., 0, 1, <)$ is a real closed field, $F \times F \times F$ is a model of Zeno's space, so it does not have to be continuous (in Dedekind sense). In the classical resolution to the paradox (Ajdukiewicz, Grünbaum) it is supposed that Zeno's arguments can be represented in the arithmetic of reals and $R \times R \times R$ is the only model of Zeno's space. There is an implicit supposition both in Achilles paradox itself and its classical resolution: to overtake the Tortoise, Achilles must first reach the point where he catches up with the Tortoise. I show that Achilles can overtake the Tortoise and there may be no point where he catches up with the Tortoise. To this end an example is given: a continuous map defined on the set of hyperreals that does not has the intermediate value property. Next, I provide such a model (a hyperfinite time line that is a subset of hyperreal line) that Achilles overtakes the Tortoise, there is no point where he catches up with it, there is the last point where the Tortoise is ahead of Achilles, and the first point where Achilles is ahead of the Tortoise.

The Simple Consistency of Arithmetic, for Metacomplete Logics without a Quantified Form of Distribution

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We first prove the simple consistency of arithmetic by finitary methods, where the arithmetic is based on a logic MC of meaning containment. The essential difference between MC and classical logic is that it is conceptualized in terms of meanings rather than truth and falsity. This idea will permeate the axiomatization of the logic, and its quantificational and arithmetic extensions. The sentential logic MC is a weak relevant logic containing neither of the key classical principles: the Law of Excluded Middle (LEM) and the Disjunctive Syllogism (DS). It also does not include the sentential distribution properties. Further, the quantificational extension MCQ does not include the corresponding two distribution properties even in their rule forms. The reasons for dropping all these principles is that conjunction and disjunction are extensional whilst the entailment and the quantifiers are intensional, the latter possibly applying to a non-recursive property. We then set up the axiomatization of arithmetic capturing the spirit of Peano's axioms in the form of rules. This is because the entailment is an inappropriate relationship between statements involving distinct natural numbers. We do, however, take all the identity statements as classical, i.e. the LEM and the DS both apply to them. However, due to consistency, we can add the full DS admissibly. Since metavaluations are used for modelling purposes, this result will extend to metacomplete logics. Due to the recursiveness in the proof, we can add back in the existential distribution rule. Lastly, we will examine the development of arithmetic in Mendelson's *Introduction to Mathematical Logic* [1964] to compare what our system can and cannot do in relation to the classical Peano arithmetic.

Fuzzy Sets and Fuzzy Classes in Universes of Sets

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Traditionally, the concept of fuzzy sets is closely related to a fix set of interesting objects called the "universe of discourse". This restriction, however, seems to have several disadvantages. We can recognize a natural need to deal with fuzzy sets over different universes as in the case of fuzzy sets of "fresh apples" in the first basket and "fresh pears" in the second basket, where the sets of

all apples and pears in the baskets are the corresponding universes. Further, the presumption of a fix set as a universe for fuzzy sets has some fuzzy sets construction limitation. Practically, a fuzzy set theory cannot be introduced on a fix universe. An analogous disadvantage was also recognized by S. Gottwald and, therefore, he proposed a cumulative system of fuzzy sets [1].

In the presentation, we will introduce a universe of sets over which fuzzy sets are defined. The definition is based on the axioms of Grothendieck universe, i.e., on a set in which the whole set theory may be formed (see e.g. [2]), where an axiom ensuring the existence of fuzzy sets with membership degrees interpreted in a complete residuated lattice is added. Some of the examples and properties of the universe of sets will be demonstrated. Further, we will establish the concept of fuzzy set in a universe of sets and show several constructions of fuzzy objects and fuzzy relations that are well known in the fuzzy set theory. Finally, we will define the informal but very useful notion of fuzzy class in a universe of sets which generalizes the concept of fuzzy set. Some properties of fuzzy classes will be also presented.

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Intuitionistic Approach for Justification of a Set Theory

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The crisis in foundation of mathematics in the end of XIX - beginning of XX centuries initiated a number of axiomatic set theoretical systems during the first half of XX century. These systems were the result of different philosophical approaches (in view of second Godel's Theorem) which were aimed to overcome the above crisis. But the way out of this situation has never been found. In my report I will try to give a new approach to solve this problem using a basic axiomatic system of a set theory with intuitionistic logic. I will present many mathematical results which were obtained during last thirty years. We will survey more than thirty years development of the set theory with intuitionistic logic underlining main points and formulating unsolved problems and describe the basic system of intuitionistic set theory.

Many-valued (Fuzzy) Type Theories

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Mathematical fuzzy logic is a well established formal tool for modelling of human reasoning affected by the vagueness phenomenon and captured via degree theoretical approach. Besides various kinds of propositional and first-order calculi, also higher-order fuzzy logic calculi have been developed that are in analogy with classical logic called fuzzy type theories (FTT). These are generalization of classical type theory presented, e.g., in [1]. The generalization consists especially in replacement of the axiom stating *there are just two truth values* by a sequence of axioms characterizing structure of the algebra of truth values. The truth values form either an IMTL-algebra (a prelinear residuated lattices with double negation) or an EQ-algebra in which the main operation is a fuzzy equality equivalence) and, unlike residuated lattices, implication is a derived operation. The syntax of FTT is a generalization of the lambda-calculus constructed in a classical way, but differing from the classical one by definition of additional special connectives, and by logical axioms. The fundamental connective in FTT is that of a fuzzy equality interpreted by a reflexive, symmetric and weakly transitive binary fuzzy relation. This paper provides an overview of the main calculi of FTT.

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Relations between the classical differential calculus and da Costa's paraconsistent differential calculus

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Da Costa(2000) introduces a paraconsistent differential calculus, whose underlying logic and set theory are, respectively, his known paraconsistent predicate calculus $C_1^=$ and his paraconsistent set theory $CHU1$, introduced in 1986; two special algebraic structures are constructed, the hyper-ring A and the quasi-ring A^*, which extend the field of the real numbers and whose elements are called hyper-real numbers. From A^*, da Costa proposes the construction of a paraconsistent differential calculus, whose language is the language of $C_1^=$, extended to the language of $CHU1$, in which we deal with the elements of A^*. Carvalho (2004), by using the paraconsistent apparatus, studies and improves the calculus proposed by da Costa, obtaining extensions of several fundamental theorems of the classical differential calculus. In this paper, we introduce the concepts of paraconsistent super-structure and monomorphism between paraconsistent super-structures. By considering the paraconsistent super-structure R constructed over the set of standard real numbers, and mixing the structures A and A^* as basic sets for the construction of extensions of R, we prove a Transference Theorem, that "translates" the classical differential calculus into da Costa's paraconsistent calculus.

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Mathematical Pluralism

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There is a plurality of mathematical investigations. These cannot all be reduced to proofs within the framework of Zermelo-Fraenkel set theory, if only because some of them use non-classical logic (such as the various branches of intuitionist mathematics). How is one to understand this situation? In this paper, I suggest that one should see this plurality as analogous the plurality of games, any of which may be played. Various objections are considered and rejected, including the charge that the picture engenders a pernicious relativism.

Paraconsistent Logics

Organized by: Alexandre Costa-Leite (costaleite@unb.br)
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This session is in honor of Casey Neil McGinnis (USA) (*in memoriam*).

In this session the validity of the principle of non-contradiction will be discussed and various systems of paraconsistent logics will be presented investigating the nature of negation, the definition of paraconsistent logics and general aspects of paraconsistency.

Invited Speaker

The catuskoti

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In early Buddhist logic, it was standard to assume that for any state of affairs there were four possibilities: that it held, that it did not, both, or neither. This is the catuskoti (or tetralemma). Classical logicians have had a hard time making sense of this, but it makes perfectly good sense in the semantics of various paraconsistent logics, such as First Degree Entailment. Matters are more complicated for later Buddhist thinkers, such as Nagarjuna, who appear to suggest that none or these options, or more than one, may hold. The point of this talk to to examine the matter, including the formal logical machinery that may be appropriate.

Accepted contributed talks

Paraconsistentization: the general theory of paraconsistent logics

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This talk describes a method to convert any logic into a paraconsistent logic. This is a universal approach to the concept of *paraconsistency*. Instead of investigating particular systems of paraconsistent logics, we explore general and abstract properties of these systems inspired by the idea of universal logic.

Negation in Paraconsistent Logic

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Classically, the notion of consequence and inconsistency are interwoven. Could any such connection be found in case of paraconsistent consequence? This is the key motivator behind this study. In classical context, a formula and its negation together yield every formula. This property of classical consequence is known as the explosiveness condition. There is a general discord in accepting the idea that without any relevancy an arbitrary formula can follow from $\{a, \neg a\}$ or more generally, from an inconsistent set. Paraconsistent notion of consequence, by definition, negates this explosiveness condition. That is, in paraconsistent logics a set of formulae yielding a formula a and its negation a, does not necessarily yield every . So what should be the proper way of addressing such a set X which yields a pair of formulas a, a and does not yield at least one of , . Intuitively, it seems to be plausible to say, X behaves inconsistently if one considers a and behaves consistently if one considers . With this intuition, we looked for sets of axioms characterizing the notion of consequence as well as the notion of inconsistency in paraconsistent logic. While axiomatizing sets of axioms for the notion of consequence as well as the notion of inconsistency of paraconsistent logic by modifying already existing classical consequence axioms as minimum as possible, a strange connection of double negation has come out. In this paper we will propose this new notion viz. 'inconsistency with respect to formula' with a plausible axiomatization and explore the connection of double negation (which emerged from the proposed axioms) with paraconsistent logics in the existing literature.

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Aristotle and the Principle of Non-Contradiction: Where are the Proofs?

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If PNC is so strong a principle, why is it that Aristotle's justifications are so weak? Priest (2006) claims that the weak justifications illustrate the failure of PNC as a logical principle. My paper aims to show that Aristotle would have had no problem to agree with such a view. Lukasiewicz (1910) is the first to suggest a logical interpretation of Aristotle's PNC (cf. Metaphysics). Yet, it would have been more exact to restrict the interpretation to an ontological PNC and an epistemic PNC. The ontological PNC asserts that it is impossible for something to be both predicated and not predicated of a same thing at the same time and in the same respect. As for the epistemic PNC, it amounts to a belief in the ontological PNC, such that speakers believe that it is impossible for a predication of things to be both true and false at the same time and in the same respect. The necessary truth of the ontological PNC does not apply to the belief itself, since a belief may be false. Aristotle is also aware that a general proof of the epistemic PNC is circular: to prove that every belief in the ontological PNC is true requires that speakers already accept the epistemic PNC as true. Yet, it is possible to show that a belief rejecting the ontological PNC is false. Aristotle speaks of 'refutation' (elenchos), but adds that it is not a proper proof, as it cannot be generalised to every belief. Indeed, the refutation of a false belief involves a dialectical condition, namely a discussion between at least two speakers. Therefore, Aristotle is ready to assert that meaningful discourse requires an epistemic PNC; but he is quick to acknowledge that the justifications of the epistemic PNC are intrinsically weak, as they are nothing more than the individual refutations of false beliefs rejecting the ontological PNC.

Paraconsistent identities and modalities

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We analyze the concept of identity starting from a framework of paraconsistent logics with modalized satisfaction and truth, such as discussive logic by S. Jaśkowski and a new four-valued logic by J.-Y. Béziau (2005). In continuation, a logic of paraconsistent identity is proposed with a twofold (positive and negative) satisfaction relation based on a modally defined satisfaction of atomic formulas (Kovač, 2009). Within the logic proposed, the paraconsistent identity relation presupposes an equivalence relation at a world, $\approx_w$, with the classical identity relation as a subset. Members u_1 and u_2 of the universe are paraconsistently identical at w iff their respective $\approx$-counterparts at w are each other's $\approx$-counterparts at an accessible world w'. In that way, we can model the reasoning with identities in, for instance, discussive, social, and epistemic contexts, where one and the same thing often happens to be conceived in incompatible ways.

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Paraconsistent Intuitionistic Logic

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In standard intuitionistic logic INT, negation is proof theoretically characterized by the inference

rules reductio (RED) and ex falso quodlibet (EFQ). Because of EFQ, the logic INT is explosive, which means that anything follows from a contradiction. However, in view of the intuitive interpretation of intuitionistic negation, i.e. the possibility to derive a contradiction, it is hard to see why the logic INT should validate the inference rule EFQ. For, the construction of a contradiction certainly doesn't guarantee the construction of any formula whatsoever. An obvious solution to this problem consists in the overall rejection of the inference rule EFQ. The logic resulting from this move is the paraconsistent logic INTuN. However, the rejection of EFQ comes with a serious disadvantage, for most applications of the INT-derivable inference rule reductio ad absurdum (RAA) aren't valid either. In view of the intuitive interpretation of intuitionistic negation, most applications of RAA should be valid. In this paper, I will characterize the adaptive logic INTuNr. The latter adds to the logic INTuN all unproblematic applications of the inference rule EFQ (e.g. the applications that lead to unproblematic applications of RAA). In this way, the logic INTuNr doesn't only avoid explosion, but also captures the intuitive meaning of intuitionistic negation.

Paraconsistent logics adequate to consistency understood as the absence of the negation of any implicative theorem

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In [1] two senses of a so-called weak-consistency are defined: consistency as the absence of the negation of any theorem; and consistency as the absence of the argument of any negation theorem.

Let us now define a third sense of "weak Consistency", which is the concept of consistency the title of this paper refers to: consistency understood as the absence of the negation of any implicative theorem.

Definition 1 (Weak consistency in a third sense): Let S be a logic and T be a theory built upon S. Then, T is w3-inconsistent (weak inconsistent in a third sense) iff $\neg(A \rightarrow B) \in T$, $A \rightarrow B$ being a theorem of S (a theory is w3-consistent —weak consistent in a third sense— iff it is not w3-inconsistent).

The aim of this paper is to define a series of logics adequate to this sense of consistency in the ternary relational semantics with a set of designated points. These logics are said to be adequate to the concept of consistency in Definition 1 in the sense that the completeness proof can be carried out if consistency is understood as stated in this definition. If consistency were understood in the standard sense, the completeness proof would fail, at least in the present semantical context, i.e., the ternary relational semantics with a set of designated points. Now, let us define:

Definition 2 (w3-paraconsistency): A logic S is w3-paraconsistent iff the rule "If $\vDash A \rightarrow B$, then $\neg(A \rightarrow B) \vDash B$" is not a rule of S.

It will be proved that all logics in this paper are paraconsistent in the standard sense, but that none of them is w3-paraconsistent.

All logics are included in Lewis' S4, some of them include classical propositional logic, but none of them is relevant.

A Routley-Meyer type ternary relational semantics, negation being modelled with the Routley operator is provided for each one of these logics. Soundness and completeness theorems are proved. In some cases, strong —i.e., in respect of deducibility— soundness and completeness theorems are also proven.

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Paraconsistent classical logic

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Any system of relevant logic is known to be paraconsistent in a strict sense, that is to be both atomic and molecular paraconsistent. In my talk, I will show how paraconsistent 'hybrid logic' can be generated by 'spoiling' the system of the first degree relevant entailment (FDE).

Starting with Belnap's matrixes for negation, conjunction and disjunction I spoil the semantical logic with classical implication. The resulting logic, call it FDEP, is a hybrid of classical positive logic (TV+) with DeMorgan logic. An adequate axiomatization for this logic will be presented.

Paradoxes

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Between the end of the 19th century and the beginning of the 20th century, the foundations of logic and mathematics were affected by the discovery of a number of paradoxes, involving fundamental notions and basic methods of definition and inference, which were usually accepted as unproblematic. Since then paradoxes have acquired a new role in contemporary logic: basic notions of logic, as it is presently taught, and important metatheorems have reached their present shape at the end of a process which has been often triggered by various attempts to solve paradoxes.

The broad aim of this session is to compare and evaluate different logical systems able to solve paradoxes, which involve the notions of truth, set, operation, abstraction, vagueness. More precisely, the idea is to discuss most recent proposals which bring about new logical ideas and methods.

We encourage submissions of papers which use proof-theoretic as well model-theoretic methods, and deal with topics from the following list:

- Paradoxes from the viewpoint of (the whole spectrum of) substructural logics, fuzzy logics included;

- Theories of predication and truth;

- Paradoxes in type theories and theories of operations;

- Recent developments in the Neofregean approach;

- Theories of predication and truth;

- Epistemic paradoxes.

Papers dealing with the history of paradoxes are also welcome.

Invited Speaker

David Hilbert and the set-theoretic paradoxes

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In this talk we present the discussion of different set-theoretic paradoxes by David Hilbert in a series of lectures given at the Department of Mathematics at the University of Göttingen between 1905 and 1920. This discussion, preserved in unpublished lecture notes available at the library of the Mathematical Institute in Göttingen, shows that the paradoxes played an important role in Hilbert's elaboration of what is today called Hilbert's program.

Accepted contributed talks

A contradictory semi formal resolution respecting classical logic

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We undertake to show that there is a semi formal approach honouring classical logic which shares features with paraconsistent approaches and also retains virtually all naïve principles of truth and comprehension. The semantics and some salient partial axiomatic and inferential principles of the system, called Minimalistic Liberalism (ML), will be presented. ML differs from paraconsistent approaches in that classical logical principles are always theorems and their negations never are, even in the presence of paradoxical phenomena generated by the "liberal comprehension principle" implicitly defined by the theory; the latter happens by means of principles governing a truth operator T and by presupposing that it is, and never fails to be a theorem that s is an element of $\{x : F(x)\}$ iff $TF(s)$. E.g., if R is Russell's set, it will be a theorem of ML that R is in R, and it will as well be a theorem in ML that R is not in R. But it will not be a theorem of ML that R is in R and R is not in R. So ML is non-adjunctive; as like magnetic poles, contradictory theorems repel. One focus of the presentation will be upon explaining the semantical framework, which relies upon some additional twists upon revisionary types of semantics in the tradition from Herzberger, Gupta and Belnap, and to isolate some of the salient axiomatic and inferential principles which hold. Importantly, modus ponens is not generally valid. There are, however, a row of other inference rules which compensate for this. We will focus upon how paradoxes are resolved, and pay particular attention to the recalcitrant Curry-paradoxes, and how their resolution crucially hinges upon the fact that ML is, and must needs be a semi formal system.

An approach to paradoxes by Beppo Levi

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In a 1908 note of his [2], Beppo Levi argued that some of the most famous among the logical antinomies of mathematics, could be shown to be based on some fundamental misunderstanding of the meaning of the terms involved in their formulation. The aim of this talk is to introduce the audience to Levi's approach to the issue, and to discuss some consequences of it which might be useful to consider from the metamathematical viewpoint. Though this paper of Levi's carries no original methodology for dealing with the paradoxes, we will present reasons of a different sort for re-considering this contribution from a contemporary perspective: first of all, the similarities between Levi's proposed analysis of the logical antinomies and those provided elsewhere (as within the French group of so-called semi-predicativists, or even those given by Russell and Poincaré); relatedly, Levi's peculiarity of not making the choice between what are commonly considered as alternative views in the debate on the foundations of mathematics; finally, the relation with a slightly more popular contribution of Levi's in the metamathematical field, the one on Zermelo's axiom of choice 'naturality' as an assumption in the set–theoretic setting, and his proposed substitute for that assumption, his "approximation principle", which, although so far remained unclear and unexplored, has been recently undergone a process of reformulation (in [1]) that seems capable of making it intelligible.

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Coping With Falakros Paradox in the Meaning of Specials Linguistic Expressions

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A special but very important class of linguistic expressions is formed by the, so called evaluative linguistic expressions (e.g., small, medium, big, very weak, medium strong, extremely strong,

about 1 million, etc.). Logical analysis of their meaning reveals that the falakros (sorites) paradox is hidden in it. Moreover, the paradox extends even to real numbers ((very) small amount of water, an extremely strong pressure, etc.). We present a formal theory, the interpretation of which provides a model of their meaning and demonstrate that the theory copes well with the falakros paradox. The main assumption is that vagueness of the meaning of these expressions is a consequence of the indiscernibility relation between objects. Our main formal tool is the fuzzy type theory (FTT) introduced in [1]. We developed a formal theory T of the meaning of evaluative expressions.

Theorem 1: Let an evaluative linguistic expressions Sm be given (for example "very small", "extremely weak", etc.). Then in each context the following is provable in T: 1. $Sm(0)$ 2. There exists n such that surely not $Sm(n)$. 3. There does not exist x such that surely $Sm(n)$ and surely not $Sm(n+1)$. 4. For all n, if $Sm(n)$ then it is almost true that $Sm(n+1)$.

Theorem 2: In arbitrary context, there is no last surely small x and no first surely big x.

The proofs of these theorems are syntactical and so, they can have various kinds of interpretation. Full formalism, proofs and other details can be found in [2].

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Medieval obligations and paradoxes

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The medieval theory of obligations can be regarded as an attempt to codify in a highly regimented way the structure of a dialectical disputation. Consider a situation in which two disputants perform a series of actions, governed by rules. The Opponent is supposed to lay down an initial thesis and then, in case the Respondent admits it, to put forward further sentences with the aim of forcing the latter into a contradiction. The Respondent, on the other hand, should try to maintain consistency by conceding, denying or doubting the sentences proposed during the disputation according to certain criteria. Such criteria are precisely the object of a number of rules that constitute the theoretical bulk of medieval treatises on the topic.

One interesting feature of this literature is that, after setting up the stage at an abstract level, the focus shifts on the application of rules in the context of particular examples which are supposed to count as limit-cases devised to test the theory and see it at work. Now, such limit-cases often come in the form of paradoxes. Much in this game seems to depend on the Respondent's ability to detect the paradox in advance and choose either to reject it or, if it is possible, to work out a solution, i.e. accept the paradox as an initial thesis and find out a way to defend it.

I will examine a number of examples in order to clarify how the theory was actually employed in such contexts. The focus will be especially on epistemic paradoxes of the form: if p is the case, then you know that p is the case, and if not-p is the case, then you know that not-p is the case.

Paradoxes - propositionally

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A feature of natural discourse, not represented by a straightforward propositional model, is that every pronounced statement has, in addition to its actual content, also its unique identity (as witnessed, for instance, by cataphoric and anaphoric references.) The following discourse (*):

The next statement is not false.

The previous statement is false.

is purely propositional and, giving an explicit identifier to every statement, it can be represented by the two propositions (**):

x1 $\leftrightarrow$ not(not(x2))

x2 ↔ not(x1).

Simplicity of this representation notwithstanding, it gives a definite account of propositional paradoxes of so called "self-reference", determining uniquely the paradoxical or non-paradoxical status of every discourse. It does it without any use of higher-order logic or arithmetics and without recourse to any infinitary constructions. Still, it is not limited to finite, circular paradoxes and applies unmodified to the more recent, infinitary paradoxes without self-reference.

The model leads also to an interesting equivalence between consistent propositional theories and directed graphs possessing kernels (independent subset of nodes with an edge from every outside node to some node in the set). One central result guarantees the existence of a kernel in graphs without odd cycles, and this finds its counterpart in the consistency of our theories without "self-referential negation". (In (**) such a negation obtains, e.g.: x1 ↔ not(not(not(x1)))), and simplifies to the Liar: x1 ↔ not(x1).) Thus, besides illuminating the relation between paradoxes, consistency and circularity, as a more practical consequence, we can also envision transfer of the results and algorithms between kernel theory and the research on satisfiability.

Substructural Logics

Organized by: Francesco Paoli (paoli@unica.it)
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Substructural logics are usually described as logics that lack some members of the usual triple of structural rules: contraction, weakening, or exchange. From this description alone it is clear that substructural logics are intimately connected with sequent calculi. Indeed their origin is rooted in proof theory and Gentzen-style systems. Four broad families of logics immediately answer the description:

- relevant logics

- BCK logics, or logics without contraction

- linear logic and its extensions

- Lambek calculus

It was realized early on that substructural logics share a common algebraic characteristic. Namely, all the algebraic semantics for substructural logics are (embeddable in) residuated structures. Hence the slogan "Substructural logics are logics of residuated structures". This shift of focus brings forth a fruitful connection with more traditional areas of mathematical research, such as lattice-ordered groups, as well as encompassing two families of logics that the proof-theoretical description misses:

- fuzzy logics (where the sequent presentation is not obvious)

- intuitionistic and intermediate logics, including classical logic (where all structural rules are present)

Thus, a carefully stated proof-theoretical description of substructural logics could perhaps read: axiomatic extensions of any logic that, if presented as a sequent calculus, lacks zero or more structural rules. A phrase worthy of a logician, without a doubt.

We welcome submissions of papers on topics from (but possibly also outside of) the following list:

- Proof theory of substructural logics;

- Substructural logics from the viewpoint of abstract algebraic logic;

- Residuated lattices;

- Individual classes of residuated lattices (l-groups, MV algebras, etc.);

- Reducts and expansions of residuated lattices (BCK algebras, equivalential algebras, modal FL-algebras, etc.)

- Relationships between substructural logics and other non-classical logics (modal, paraconsistent, quantum logics, etc.);

- Applications of substructural logic

Invited Speaker

Projective algebras in Fregean varieties

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Projective algebras in Fregean varieties A variety of algebras with a distinguished constant 1 is called Fregean if it is 1-regular and congruence orderable. The subdirectly irreducible algebras in a Fregean variety can be characterized as those which have the largest non-unit element, traditionally denoted by *. Every congruence permutable Fregean variety has a binary term that turns each of its algebras into an equivalential algebra, where by an equivalential algebra we mean a grupoid that is a subreduct of a Heyting algebra with the naturally given equivalence operation. We show that for a congruence permutable Fregean variety V of finite type L the following conditions are equivalent: (1) every finitely generated algebra from V is projective; (2) A-* forms a subalgebra of A for every subdirectly irreducible algebra A from V; (3) V fulfils the identities: $t(1,\ldots,1) = 1$ and $t(x_1, \ldots, x_n)yy = t((x_1)yy, \ldots, (x_n)yy)$ for every t from L. The equivalence of (1) and (2) is also true under the weaker assumption that V is a substractive Fregean variety. The case where the language of V contains more than one constant is also discussed. In particular, we show that in the variety of equivalential algebras with 0, that gives the algebraic semantics for the equivalence-negation fragment of IPC, a finitely generated algebra is projective, iff 0 is not equal to 1.

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Accepted contributed talks

Substructural Logics are not Logics of Residuated Structures

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RMI_m is the logic which is obtained from the sequent calculus for R_m, the purely intensional (or "multiplicative") fragment of the relevant logic R, by viewing sequents as consisting of finite sets of formulas on both of their sides. This is equivalent to adding the converse of contraction to the usual sequential formulation of R_m. Now it is well known that RMI_m is a relevant logic, enjoying major properties like the variable-sharing property, cut-elimination, and a very natural version of the relevant deduction theorem. However, by enriching it with an "additive" conjunction, one gets an unconservative extension (with the full power of the intensional fragment of the semi-relevant system RM, for which the variable-sharing property fails). It follows that the substructural logic RMI_m is not the logic of any class of residuated structures. Despite this observation, RMI_m has a very useful and effective semantics. In this paper we construct an effective class S of finite-valued matrices, for which RMI_m is finitely strongly sound and complete. Moreover: all the matrices in S can be embedded in one effective infinite-valued matrix for which RMI_m is strongly sound and complete. We also show that the last result cannot be improved: there is no finite-valued matrix for which RMI_m is even weakly sound and complete.

An algebraic semantics for type similarity in Symmetric Categorial Grammar

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Starting with (unit-free) intuitionistic multiplicative linear logic, one descends the Lambek categorial hierarchy by gradually removing the structural rules of exchange and even associativity. The increase in structural discrimination so achieved is coupled with a decrease in the expressivity of type similarity: the reflexive, transitive and, crucially, the symmetric closure of derivability. Pentus and Moortgat found a counterexample to this pattern in the Lambek-Grishin calculus (LG), where associativity and commutativity remain underivable while at the same time being validated under type-similarity. Compared to the intuitionistic bias underlying the Lambek hierarchy, LG manifests an arrow-reversing duality by adding a co-residuated family of connectives (headed by the par), thus achieving full symmetry. In this talk, we revisit Pentus and Moortgat's results by showing type-similarity in LG sound and complete w.r.t. an algebraic semantics featuring two associative and commutative binary operations, related by linear distributivity. In addition, we show that this result extends to a recent suggestion of Moortgat to augment LG with families of (co-)Galois connected operators.

Structural rules and implication in a sequent calculus for quantum computation

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Basic logic is a core for sequent calculi of several substructural extensional logics, including intuitionistic, linear and quantum logics. It shows how to obtain such extensions by the addition of structural rules to the basic calculus, that is obtained translating metalinguistic links into logical connectives. The critical point is represented by the implication. In the framework of basic logic, we have recently introduced a paraconsistent and predicative interpretation of quantum parallelism in terms of sequents. It allows to consider logical implication as a causal link, in opposition to a semi-predicative associative link, formalizing a quantum link, which can have a meaning only in a paraconsistent setting, where the variable has the role of a random variable. Substitution of variables by closed terms is the structural rule which makes the quantum link collapse. The calculus exploits the limitation of contexts in the rules for quantifiers. One could argue a computational interpretation of "quantum contextuality", in terms of sequents.

Bunched logics displayed

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Display calculi, as formulated by Belnap in the 1980s, can be seen as natural successors of Gentzen's sequent calculi, suitable for the proof-theoretic analysis of substructural logics. They are characterised by the display property, which essentially states that proof judgements may be rearranged so that any chosen part appears alone on one side of the proof turnstile.

In this talk, we apply display calculus techniques to obtain a unified proof theory for *bunched logics*, which originate in O'Hearn and Pym's BI and can be seen as the result of freely combining a standard (additive) propositional logic with a (multiplicative) linear logic. The practical interest in bunched logics stems from their Tarskian "resource" interpretation of formulas, as used e.g. in the heap model of separation logic. However, the cut-free sequent calculus for BI does not extend naturally to its important variants such as Boolean BI. We show how cut-eliminating display calculi may be uniformly obtained for all the principal varieties of bunched logic, and incidentally provide an explanation as to why well-behaved sequent calculi seem very unlikely to exist for most of these varieties.

This talk is based upon a related paper by the speaker *A unified display proof theory for bunched logic. Submitted, 2009.*

Disjunction Property and Complexity of Substructural Logics

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In this talk we are going to present an algebraic method for proving PSPACE-hardness of a substructural logic which is less dependent on the sequent calculus. More precisely, we will prove by algebraic means that each substructural logic satisfying a stronger version of the disjunction property (SDP) is PSPACE-hard. This gives us a simpler method since it is usually easy to prove SDP from a cut-free sequent calculus. We demonstrate it by showing that the basic substructural logics (i.e., Full Lambek calculus and its extensions by the structural rules of exchange, contraction, left and right weakening) have SDP. Thus, as a corollary, we obtain PSPACE-hardness for these logics.

Admissible Rules for Substructural Logics

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The study of logical systems usually focuses on derivability: whether or not a rule, understood as a set of premises and a conclusion, belongs to the consequence relation of the system. Such rules may be thought of as providing an "internal" description of the system. However, an "external" perspective, describing properties of the system, can also be valuable. Following Lorenzen, a rule is said to be admissible in a system if the set of derivable structures is closed under the rule; that is, adding the rule to the system does not give any new derivable structures. In algebra, such rules correspond to quasi-equations holding in free algebras, while from a computer science perspective, admissibility is intimately related to equational unification.

For classical logic, derivability and admissibility coincide: the logic is structurally complete. However, for many non-classical logics, in particular, core modal, many-valued, intermediate, and substructural logics, this is no longer the case. In this work, we consider a selection of open problems for characterizing admissible rules. In particular, while the admissible rules of most fragments of intermediate logics have been characterized, we provide first bases for the implication-negation fragments. We also provide bases for the admissible rules of fragments of the relevant logic RM and identify some interesting research challenges in the area.

Extending the Gödel-McKinsey-Tarski Translation to Substructural Logics

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In 1933, Gödel suggested a way of translating formulas in the language of intuitionistic logic into the language of classical modal logic. He conjectured that for any intuitionistic formula φ, φ is provable in the intuitionistic logic LJ iff its translation $G(\varphi)$ is provable in the modal logic S4, making S4 a modal counterpart of LJ. This conjecture was later proven by McKinsey and Tarski, using algebraic methods. Then, Dummett and Lemmon extended this result to all superintuitionistic logics, showing that for any superintuitionistic logic L, there exists a smallest modal counterpart of L, among the normal modal logics over S4. Maksimova and Rybakov showed that there also is a greatest modal counterpart of L, and Blok proved a conjecture of Esakia's concerning an axiomatization for this greatest modal counterpart. In this talk, the sequent system S4FL for substructural modal logics (i.e., modal logics with all of the theorems of the substructural logic FL) will be introduced, corresponding to the classical modal logic S4. The Gödel-McKinsey-Tarski translation will then be extended to take formulas in substructural logics into formulas in substructural modal logics, and analogues of the aforementioned theorems will be considered for the class of substructural logics over FL with respect to the class of substructural modal logics over S4FL. In particular, we will show that there is a least element and a maximal element (with a specific axiomatization) among the modal counterparts of a given substructural logic.

A$_{\rightarrow}$, More Adorable

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In this paper we consider the implicational fragment of Abelian logic, $\mathbf{A}_\rightarrow$, as a formula logic and as a consequence relation. We show that although the abelian groups provide an algebraic semantics for the formula logic, they do not for the consequence relation. We then investigate both the formula logic and consequence relation with respect to algebraizability, in the sense of Blok and Pigozzi. We show that while the formula logic is algebraizable, the consequence relation is not. We then investigate various possible ways of extending $\mathbf{A}_\rightarrow$ to obtain an algebraizable consequence relation which has the Abelian groups as its equivalent algebraic semantics.

Permuting Nonassociating Lambek Calculus and Cognitive Grammars

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We show how the residuation structure that encodes the permuting, nonassociating Lambek calculus is a tempting base–structure for modelling the information flow in mono–agent deductive reasoning scenarios. The resulting model has a straightforward interpretation in terms of the data–base structure, or grammar, of the "cognitive language" of deductive reasoning.

Philosophical Aspects of Display Logic

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Many discussions of logical inferentialism take place against a backdrop of natural deduction systems. Against this, we argue that the proper setting for philosophical reflection on the meaning of connectives for the inferentialist is the consecution calculus. The arguments for this are based on the role of structure in natural deduction systems and in consecution calculuses. We present examples of elements of natural deduction that play a structural role, arguing that their influence on the meaning of the connectives is obscured in natural deduction systems. This role is made explicit in consecution calculuses, which present a clearer picture of the context of deducibility.

Once the inferentialist has shifted her attention to consecution calculuses, similar considerations seem to favor adopting the display logic generalization of consecution calculuses. This is because display logic makes the distinction between structure and non-structure sharper. There is a natural question of what constitutes harmony in this setting. We provide a novel characterization of harmony, suited to display logic, and close by presenting two applications of it.

Contest

How to combine logics?

When we have two logics we may want to put them together. For example on the one hand we have a temporal logic, on the other hand a deontic logic, how to produce then a temporal deontic logic? This is a very interesting question in the engineering of logic. People have been working in the subject since about 15 years. But there are still some fundamental problems not completely solved, such as the collapsing problem. These problems are connected to the very nature of what a logical system is. One may wonder if the intuitive definition of combination of logic as the smallest conservative extension of two given logics really works, and also if it is always possible to combine two logics.

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How to take part in the contest?

All participants of the school and the congress were welcome to take part in the contest by submitting short papers (10 to 15 pages) before January 15th, 2010. The papers selected for presentation at a special session during the congress, listed below, will be judged by a jury composed by three renowned specialists:

Markus Kracht (UCLA, USA)

Dana Scott (CMU & UC Berkeley, USA)

Cristina Sernadas (TU Lisbon, Portugal)

The winner will be awarded a special prize by *Birkhäuser Verlag*.

Simple type theory is well suited as framework for combining

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Simple type theory is well suited as framework for combining classical and non-classical logics. This claim is based on the observation that various prominent logics, including (quantified) multimodal logics and intuitionistic logics, can be elegantly embedded in simple type theory. Furthermore, simple type theory is sufficiently expressive to model combinations of embedded logics and it has a well understood semantics. Off-the-shelf reasoning systems for simple type theory exist that can be uniformly employed for reasoning within and about combinations of logics.

How to combine diagrammatic logics

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We claim that combining "things", whatever these things are, is made easier if these things can be seen as the objects of a category. We define the category of diagrammatic logics, so that categorical constructions can be used for combining diagrammatic logics. As an example, a combination of logics using an opfibration is presented, in order to study computational side-effects due to the evolution of the state during the execution of an imperative program.

Combining Logics from the Point of View of Universal Logic

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In number of papers concerning the issue of combining logics, in fact, one and the same type of combination is exploited. But there are another types of combination which as a rule lay beyond the scope of scholars. In the paper all these types are considered in categorical setting. The categorical constructions introduced allow to describe the nature and the structure of the general universe of possible combinations of logical systems. It is shown that categorically such universe turns out to be both a topos and a paraconsistent complement topos.

Contributed Talks

Information Overload and The Entailment Property in The Logic of Being Informed

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Informational reading of KTB (Brouwer's System), as proposed in Floridi 2006, formalizes the relation of "being informed". To be more precise, in that paper it is argued that there is an information logic (IL), different from epistemic logic (EL) and from doxastic logic (DL) (Hintikka, 1962), that formalizes the relation "a is informed that p" (holds the information). This sense of "being informed" is related to cognitive issues and to the logical analysis of an agent's possession of a belief or a piece of knowledge. In this paper we examine the process of combining a number of cognitively interpretable normal modal logics (NML) through the combination of axioms that satisfies IL. The motivation behind rebuilding Floridi's task is to analyse in detail two main philosophical consequences: Information overload and the dethroning of the $Kp \rightarrow Bp$ principle (Girle, 2000). We argue that these two consequences of IL pose a risk to theories about active externalism, such as "The Extended Mind" (Chalmers and Clark, 1998), bringing to light a weird kind of factual omniscience, with no logics that attribute such epistemic divinity to their agents. It is claimed that, to avoid this unexpected consequence, contents that were coupled to a cognitive system must have previously been consciously endorsed.

Implication and Causation: a realistic perspective

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According to Peirce (1958), regularities result from conditional final causes understood as crystallised habits. A conditional sentence *If A, then B* is composed of two clauses, the antecedent, or the if part, and the consequent, or the then part. We understand the conditional sentence *if A, then B* in terms of a final cause. The antecedent A will be understood in terms of an efficient cause. So, for example, the consequent "activate the production of histidine" will be nomologically determined, given the presence of the state of affairs "histidine less than X", by the final cause "if the quantity of histidine is less than X, then activate the production of histidine". The aim of this work is to defend the hypothesis according to which the logical form *If A, then B* is present in the mode of expression of a physico-chemical law, of a biological conditional and of a habit. But the nomological connection between the antecedent A and the consequent B is not the same, which leads us to relate different pairs of antecedents and consequents with different types of implication. We will argue that physico-chemical laws, which have strong nomological power, are compatible with strict implication (the antecedent implies the consequent in all possible states of affairs). Biological conditionals, which have moderate nomological power, are compatible with material implication (the antecedent may be true and the consequent false). Habits, which have weak nomological power, are compatible with relevant implication (the antecedent may be true and the consequent false, but we need – to avoid vacuously true consequents – to suppose that there is a causal connection between the antecedent and the consequent).

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Paraconsistent and paracomplete logics: or how to treat logical pluralism

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The aim of the talk is to be investigated some systems of paraconsistent and paracomplete logics and their role for the defense of Beall and Restall's interpretation of logical pluralism, namely that logic is both one and many [1:17]. This standpoint is strongly embraced with the view that the core notion for logic is logical consequence, i.e. the valid argument is one whose conclusion is true in every case in which all its premises are true [2:23]. The basic idea is that logical consequence is the stable kernel in the logical systems which entitle them to be called logic (this circumstance defines the uniformity of logic, it is a reason to be said that logic is one) and in the same time is a reason for their difference because the different logics present different explications of the mentioned cases. In this regard I will try to defend Restall and Beall's position that the specific cases, expressed by some paraconsistent and paracomplete logics, are the main characteristics which present some of the most essential and fruitful variety of logics. The reason is that they are connected with the precisification of consequence – the necessary requirement to form logic and are basic for the models – the sufficient condition to work this logic. I will illustrate the above circumstances regarding relevant logic and a couple of other paraconsistent and paracomplete systems. In the same time I will argue against the critiques towards the above view of logical pluralism.

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Logics of Reasonable Information Sources

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Extending the work of Belnap [1], we study structures which consist of a set of information sources providing information about formulas of classical logic and a processor collecting information from the sources and extending it using certain rules compliant with the truth tables of classical logic, which we call ESP structures. In continuation of our previous work [2], characterizing general ESP structures and the source-processor logics they generate, we now examine ESP structures with reasonable information sources which provide coherent information about formulas. We characterize the logic of a single reasonable source, and prove that the logics generated by ESP structures with reasonable sources coincide with the general source-processor logics described in [2]. However, we show that there are processor valuations in general ESP structures which cannot be obtained from any finite number of valuations defined by reasonable sources.

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Some Qualitative Adaptive Logics of Confirmation

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Due to its definition in terms of classical logic, Hempel's satisfaction criterion of confirmation is unable to handle both incomplete data sets and data sets containing irrelevant information. In order to adequately handle these problems, a logic of confirmation must be non-monotonic. As an alternative to the satisfaction criterion of confirmation, I will present an adaptive logic of induction (devised by Diderik Batens) of which the consequence relation also serves as a criterion of confirmation. A variant of this logic will allow us to confirm only those hypotheses of which there is a positive instance in our data set (the notion of positive instance can be traced back to the work of Nelson Goodman, and provides us with an adequate solution for Hempel's Raven Paradox). Each of these logics can then be further extended to an even stronger logic of confirmation by performing various minor operations on its respective definition. Thus we become a whole family of logics of confirmation, each of which naturally characterizes the dynamics of confirmation and provides us with new heuristic tools for further research.

Towards a universal logic of singularity

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There are many different logical resources to express different kinds of singularity: those that satisfy descriptions, those that are demonstrated, those that are rigidly designated. Inspired by the approach taken by Universal Logic, we can develop a general framework for all encounters with singular items of all sorts – a machinery that would enable us to express items in different patterns of singularity. In fact, singularities can be expressed by several kinds of logical resources, but there are arguably common elements to those resources. In particular, they work as some kind of glue between an expression and an item capable either to bear it or to be specified by it. These common elements to all kinds could start out with a break with all sorts of "any": whatever is singular contrasts with what is no more than an example – at least by being an example of too many things. In this work, I'll begin to explore what is common between the various forms of expressing what is singular.

About the point where Achilles catches up with the Tortoise

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1. There are many aspects of Zeno's paradoxes of motion, in my talk I will consider *Achilles* with regard to "the structure of continuum".
2. Reconstructing paradoxes some philosophers take into account verses referring to divisibility of space, movement and time (see [2]), some - those referring to the addition of magnitudes (see [1], [4]). I reconstruct paradoxes from the point of view of Euclid's *Elements* and focus on methods for determining points. I show that there is a difference between *Dichotomy* and *Achilles*: the former appeals to geometry (see *Elements*, I.10), the later - to the notion of speed.
3. In *Achilles* two sequences are defined: (A_n) - positions of Achilles, (T_n) - positions of the Tortoise (in short *Zeno's sequences*). The classical resolution of the paradox (see [1], [4]) consists in the proof of the existence of the limit point g, $lim_{n \to \infty} A_n = g = lim_{n \to \infty} T_n$, in which Achilles catches up with the Tortoise. To this end the axiom of continuity has to be used.
4. To compare Zeno's arguments with the modern solutions to the paradoxes I suppose that Zeno's space is that of Euclid, so it need not be continuous. To find a *neutral ground* for Zeno's

paradox and its classical resolution I represent both *Zeno's sequences* and the classical resolution limit-point on the line of hyperreals where the sequence of reals (A_n) is represented by the element $[(A_n)]$, the sequence (T_n) - by the element $[(T_n)]$, and the limit-point g - by the element $g^* = [(g, g, g, ...)]$, where $[(A_n)], [(T_n)], g^*$ are elements of the field of hyperreals (see [3]). Finally I show that in the very point g^* Achilles does not catches up with the Tortoise and relations obtain: $[(A_n)] < [(T_n)] < g^*$.

5. There is an implicit supposition both in the Achilles paradox and its classical resolution: to overtake the Tortoise, Achilles must first reach the point where he catches up with the Tortoise. There is also a supposition, dating back to Aristotle, that space, movement and time are continuous in the same way (*opposite* Bergson). In mathematics a continuity of a map and a continuity of its domain are defined in a different way, I make use of this difference. Provided that continuity of a map represents continuity of movement, and continuity of the domain of the map represents continuity of time and space, I show that Achilles can overtake the Tortoise and there may be no point where he catches up with the Tortoise. To show this, an example is given: a continuous map defined on the hyperreal line that does not has an intermediate value property.

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A Generalization of certain set-theoretical operations

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The starting point of the considerations are the upper and lower approximation operations introduced by Pawlak in his rough set theory (1982, 1991). These operations are determined by a binary *indiscernibility* relation (equivalence or tolerance relation) defined on a non-empty set called *universe*. They are unary set-theoretical operations defined on the power set of the universe. The upper approximation operations satisfy the conditions for the so-called unit operations introduced by Wybraniec-Skardowska (1989, 1992).

In the paper, some general properties of unit operations and their variations, as well as some of their topological and algebraic properties are given. The general properties of unit operations can be based on the relationships between unit operations and binary relations.

Certain important examples of unit operations are also some consequence operations (in Tarski's sense) serving the purpose of two-aspectual characterization of deductive systems with respect to acceptance and with respect to refutation (see Słupecki et al. 1971).

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On Gödel's completeness theorem

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In this contribution we will give a (constrained) constructive proof of Gödel's Completeness-Theorem. For this we will define a certain set of formulae and derive a model which means that we are taking a semantic point of view (in the tradition of Tarski). Additionally to our main goal we will gain a didactic method of introducing, using and, therefore, understanding the semantic point of view (terms, manner of speaking).

The cognitive path to universal Logic

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One path to universal logic (and aspects and components thereof) is to focus on logics themselves. We give specifically the metaphor of the universal logician working in a library whose volumes are logics, where he or she reads and processes the content in these books in various ways (merging them, etc.). The cognitive path is different. It is based on an attempt to figure out how human beings populated the library to begin with: How do we create logics in the first place? In words some may find a bit more helpful: What is "background logic," formally speaking, and how do human cognizers problem-solve, reason, and make decisions in and on the basis of background logic? An answer to this question would, we claim, enable significant progress in universal logic. We report on our research intended to answer this question, and on a working computational system - Slate - designed to assist humans reasoning in background logic.

The semantic hierarchies of First Order Gödel Logics - algebra versus Kripke frames

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The class of (first order) Gödel logics admits several layers of semantics. The most restrictive being semantics based on the Reals, the most permissive being Gödel (L-) algebras, with linear Kripke frames with constant domains between those two.

It has been shown that if one considers the generated logics as comparing factors, the subset of countable Kripke frames corresponds exactly to the real valued semantics, i.e., for each countable linear Kripke frame with constant domains there is a truth value set such that the respective logics coincide, and for each truth value set there is a countable linear Kripke frame with constant domains such that the respective logics coincide.

While both the study of logics based on a single truth value set or Kripke frame has been undertaken since about 20 years by various groups (e.g., [1,2]), in the algebraic case generally only logics defined by all or a large class of algebras has been studied.

The current work is targeted at pinpointing the sub-class of Gödel algebras that allow a similar representation result w.r.t. the set of linearly ordered Kripke frames with constant domains. Furthermore, we discuss the possibility to represent logics of single Gödel algebras as intersections of logics defined by Kripke frames.

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Referential opacity and epistemic logic

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Referential opacity is the failure of substitutivity of identity and in Quine's view of existential generalization as well. Quine thinks that no solution of opacity is available in belief or modal contexts. But epistemic logicians like Hintikka and Lenzen think that referential opacity can be solved since it is due not to absence of reference but to plurality of references: its solution is provided by stabilizing the reference in belief and knowledge contexts. However, stabilizing the reference needs in Hintikka's frame pragmatic methods of identification while Lenzen prefers using Kripke's frame and rigid designation. My aim in this paper is to analyze these solutions in order to show if and how they provide answers to Quine's criticisms. I will then compare between these options and answer these questions (among others): How can one identify individuals? Under what conditions may one use 'exportation'? How is the De Re / De Dicto distinction treated?. By answering these questions, I will show that Hintikka's approach keeps all its actuality since rigid designation is not always warranted and that we can preserve the De Re/De Dicto distinction, i.e. the singular/general distinction which is crucial in belief and knowledge contexts.

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On the Appropriateness of Taking RCS(R3) as Domain of Region – Why ROS(R3) is a good model of regions?

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This paper is a note on treating "ROS(R3)" as a model of regions. Ian Pratt-Hartmann proposes taking "ROS(R3)" as a good model of regions in[1]. First of all, there are two meanings of mereotopology defined by Ian Pratt-Hartmann: (1) we can see mereotopology as a model of formal language of Boolean algebra, and (2) we can see mereotopology as a model of first-order language which describing regions. I try to point out the difference here, since the existence of region and that of boundary are different. We can easily construct another Boolean algebraic structure (in this structure, region contains its boundary) which is isomorphic to mereotopology, however, we could also find that this structure will confront some problems as being a model of first-order language which describes regions. Second, I will explain the reason why Ian Pratt-Hartmann thinks that ROS(R3)(which is a mereotopology) is a good model of regions, moreover, I supply some proofs which Ian Pratt-Hartmann omits, such that readers interested in mereotopology can realize easily.

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Hypergraph as a semantic idiom

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A new semantic idiom is presented in the paper, one that uses hypergraphs as the semantic representatives of sentences of a propositional language. A standard hypergraph H on a base set X is a collection of hyperedges that are subsets of X. We trace a short history of this idiom, one that focuses on a special kind of hypergraph whose base set is the power set p(U) of the universe of a

model. Power-set hypergraphs serve as a unifying semantic idiom that combines various previously known semantics for the first degree entailment of the system E of entailment. In this idiom, an ordering relation between simple (that is, inclusion-disordered) hypergraphs models the entailment relation. Such an entailment relation preserves chromaticity of the hypergraphs. Hypergraph therefore affords a richer idiom for semantic investigation than that of truth and meaning in standard semantics. As the ordering relation between simple hypergraphs varies, the comparatively rich hypergraph-theoretic idiom also reveals other, hitherto unstudied, systems. Furthermore, we show that any distributive lattice can be represented as a hypergraph lattice, whose expressive power therefore can hardly be exaggerated. These hypergraph structures are instances of more abstract structures amenable to algebraic methods. We can adopt as base structures, abstract algebraic structures more general than the Boolean algebra of subsets, for example, distributive lattices of various kinds, and usefully generalize the notion of simple hypergraph itself, as an antichain on a lattice. Taking this lead, in the final portion of the paper we generalize hypergraphs to a more abstract setting. Here we show that the language of hypergraphs is quite powerful and give a characterization of boolean lattices in this language. An examination of the properties of lattice hypergraph not only generates familiar logic systems previously interpreted by more cumbrous semantic structures, but also affords new insights into connections between hypergraphs and lattices.

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Logic of Determination of Objects (LDO): How to articulate "extension" with "intension" and "objects" with "concepts"

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From a logical viewpoint, object is never defined, even by a negative definition. This paper is a theoretical contribution about object by the way of a new constructivist logical approach called Logic of Determination of Objects (LDO) founded on a basic operation, called determination. This new logics takes in account cognitive problems as heritage of properties by non typical occurrences

or by indeterminate atypical objects in opposition to prototypes that are typical completely determinate objects. We show how are defined and organized extensional classes, intensions, more and less determined objects, more or less typical representative of a concept, prototype, using a determination operation that constructs a class of indeterminate objects from an object representation of a concept called typical object.

The aim of the LDO is to provide conceptual and logical answers to the following questions and issues: What is a typical instance? What is an atypical instance? What is a prototypical instance? How does one define a typicality relation between instances without using a degree of membership? Articulate intension with extension? Define "family resemblances"? Manage exceptions? Manage prototypical properties? Manage multiple inheritances? To study these problems, we adopt the general framework of Combinatory Logic (CL) (Curry, Combinatory Logic, 1958) with types in which it becomes possible to define not only operators of predication (predicates), but also operators of determination of objects, and operators for building indeterminate objects from concepts, as well as explicitly articulating, for given concepts, the "structured intension of concepts" with the "structured extension of concepts" without taking into consideration that intension is defined only by duality with the extension.

The LDO is a non-classical logic of construction of objects. It contains a theory of typicality and a extended system of quantification.

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The undergeneration of permutation invariance as a criterion for logicality

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Permutation-invariance as a criterion for logicality is a much-discussed topic in the recent and not-so-recent literature. It has its enthusiastic defenders (Sher, Bonnay), but it has also met with considerable resistance. The most frequent charge of inadequacy is based on the claim that the criterion overgenerates, i.e. that it counts as logical notions that are arguably non-logical, in particular numerical notions. In fact, in his 1966 lecture 'What are logical notions?' Tarski had already observed that "it turns out that our logic [based on permutation invariance] is even less than a logic of extension, it is a logic of number, of numerical relations." (p.151) But what seems to be equally significant, and yet scarcely discussed, is the fact that the criterion also appears to undergenerate in light of the developments in logic of the last decades. At the time of Tarski's lecture, the logical systems being studied were still those which had been developed against the background of the logicist program, and as the purpose of logic according to the logicist program was to provide foundations to arithmetic, it was to be expected that these logical systems would be particularly sensitive to matters concerning cardinality and numbers. However, since then logic has developed in a variety of new directions, and its interface with computer science is particularly significant. Indeed, many of the notions and operators that are currently considered to be logical do not satisfy Tarski's permutation invariance criterion; this is the case for example of any non-S5 modal operator if interpreted on a Kripke-semantics (only the S5 modal operators satisfy the criterion). In my talk, I look into cases of undergeneration of the permutation-invariance criterion and discuss their significance for the matter of the true nature and scope of logic.

Leibniz: mathesis universalis, logic and science of forms

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There is a received talk about the Leibnizian Mathesis Universalis as a kind of general or universal

calculus. Our intention is not to contest or to reverse this interpretation, which has a long tradition imposed by the use, but rather to analyze more in detail the problems that are connected with the Leibniz's own conception about a Mathesis Universalis. Our objective here is not to get into the details of this Leibnizian conception, but to characterize the status that Leibniz gives to the Mathesis Universalis, and, if possible, to delimitate the reach of such discipline.

We will try to defend the idea that the Mathesis Universalis has a variable status. In some point of its intellectual development, Leibniz conceived it as a structural mathematical science not limited to quantity exclusively, therefore he included the science of forms (named too the science of similar and dissimilar) within its domain. Later, Leibniz retracted from that inclusion and limited the Mathesis Universalis to the domain of quantity. In this way, the Mathesis Universalis became practically identical to algebra, although it was an "extended" algebra, as we have said beforehand. On the other hand, the structural-qualitative aspect became the object of the sciences of forms, which turned into a subordinating science in relation to the Mathesis Universalis. The reasons for these variations are probably due to changes in points of view about the reach of the science of forms, the conception of the concept of number and the way of understanding the relation between arithmetic and geometry.

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Omitting Types Theorem Institutionally

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We present a generalization of forcing in institution-independent model theory which is used to prove an abstract omitting types theorem applicable to many first-order logics, which are, roughly speaking, logics whose sentences can be constructed from atomic formulae by means of boolean connectives and finitary first-order quantifiers. These include first-order logic (**FOL**), logic of order-sorted algebra (**OSA**), pre-order algebra (**POA**), partial algebras (**PA**)as well as their infinitary variants $\mathbf{FOL}_{\omega_1,\omega}$, $\mathbf{OSA}_{\omega_1,\omega}$, $\mathbf{POA}_{\omega_1,\omega}$, $\mathbf{PA}_{\omega_1,\omega}$.

Vuillemin between two Lukasiewicz's answers

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Hao Wang said once "There is more philosophical value in placing things in their right perspective than in solving specific problems?". Let us emphasize the phrase more philosophical value. It is taken for granted that the fundamental (philosophical) question "What is logic?" (the question Q for short) is a properly (i.e. well-posed) question. Is that so? Following Ajdukiewicz's pragmatic logic we introduce some modified concepts of a positive (negative) assumption of a given question. (Although the so called an assumption based conception of presupposition is no longer apt. We are aware well of that David I. Beaver?s belief.) We use the term assumption as more neutral than

the term presupposition; obviously, the latter is worked out with great care and nicety of detail not only in basic systems of erotetic logic. A question whose positive or negative assumption is not true is called an improperly posed question. It is our considered opinion that the question Q could be posed in the contexts of progressive structures based on Roman Suszko's diachronic logic in particular. For the sake of brevity we call them progressive contexts. It goes without saying that the non-reductionistic approach to questions is adopted. Two different (and both famous) Lukasiewicz's answers to the question Q are expanded within broader scope of philosophical systems classification by Jules Vuillemin. Such an interaction between the principal trend in universal logic (that leads from logical matrices to abstract logics) and philosophical studies of assumptions goes far beyond the very idea of practical turn in logic (in the Gabbay/Woods' sense).

Logic and Philosophy in the work of Kant and Hegel: The philosophical conditions for a general logic

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German Idealism, which dominated the philosophical scene in the 19th century, appears in the work of its two major figures, Kant and Hegel, as the endeavour to build and establish a general logic, to the determinations of which the classical metaphysical and ontological problems should be reduced. Although these attempts have scarcely contributed to the positive development of formal logic, it is possible to argue that they created the conditions that made possible the expansion of Frege's work's impact, thus allowing his essential logical achievement to also become a major philosophical event that played a critical role in the birth of contemporary philosophy. This event should be seen as the expression of the philosophical generality or universality that the new logical achievements were able to claim. From a detailed analysis of both Kant's Transcendental Logic and Hegel's Dialect, focused on the distinction between "conceptual determination" and "formal determination", this contribution will try to characterize this particular kind of universality that since the 19th century philosophy has expected from logic. Its aim is to propose a discussion on the conditions a logic should satisfy to be considered as general or universal from a philosophical point of view, as well as on the conditions that logic is, in return, allowed to impose on the philosophical discourse.

A Dialogical Semantics for Interrogative Consequence

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Interrogative consequence has been presented by (Hintikka et al. 1999), as an extension of classical consequence, in order to account for empirical reasoning as a variety of deductive reasoning with interrogative steps. Interrogative tableaux systems have also been used, and were given a game-theoretic semantics, reconstructing interrogative reasoning as a game against Nature. However, their puzzling proof theoretic aspects have never been thoroughly discussed, and the semantics offered is unsatisfactory on many aspects. We discuss the semantics offered by (Harris 1994), and show why it has to be improved. We then propose a dialogical reconstruction of interrogative logic, and we establish that proof-theoretically, the core system captures Anderson and Belnap's first degree entailment, recovering classical logic by ad hoc closure rules. We show that the dialogical semantics improves upon Harris' game-theoretic interpretation, and discuss the relation between this dialogic, classical game-theoretic semantics (for non-interrogative consequence) and dialogical semantics in general, with a particular attention to the interpretation of negation. We conclude by offering some comments on the modal interpretation of interrogative consequence—since the conclusion of an interrogative argument is, according to Hintikka, "known"—and its extension to cover nonmonotonic reasoning.

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On the applicability of mathematics

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The purpose of this talk is to show how the concepts of formal content and duality due to Granger [1,2] can help us to explain why mathematics fits so adequately to the description of empirical reality. We take quantum mechanics as a study of case to understand such a success of the applicability of mathematics. We try to give a different solution (from the well known ones) to the problem of the applicability of mathematics to empirical sciences. By those known solutions we mean the realist approaches derived from Plato, developed essentially by Frege [4], Steiner's [5] anthropocentric account and few other approaches (apud[4]) . We also try to account for another special feature of mathematics and that such a few philosophers have tried to understand; that's the role of the symbolic reasoning in helping scientists to make new discoveries, such as the ones we find in Dirac's work. We mean by that the prediction of anti-matter (see [3]). From a reasoning that seemed to be only a free symbol manipulation, Dirac was able to make these predictions that were confirmed in a short period of time; so we think that it's necessary to give an epistemological solution to this subject and that does not take into account the existence of mathematical objects in the sense the realists approaches believe they exist, i.e., in an analogous sense of the physical existence.

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Lambda calculus vs combinatory logic: quantitative approach

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In the beginning of the 20th century two new models of computations were introduced: lambda calculus and combinatory logic. It is a well known fact that they are equivalent in question of expression power. It turns out, however, that a typical lambda calculus term and a typical combinatory logic term are very unlike. Due to the quantitative approach towards those models we have shown that not only the structure, but also the semantics of typical terms differ radically. Using the notion of density we can define typical terms. By looking at them as at some special graph theoretical objects and applying combinatorial methods we can say how typical terms look like and on that basis claim some facts about their semantics (e.g. strong normalisation). Joint work with: Rene David, Jakub Kozik, Christophe Raffalli, Guillaume Theyssier, Marek Zaionc.

Gaining insight: towards a functional characterization of information in formal logics

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It is quite common for logicians to accept the idea that deductive inferences are non-informative. Yet, this idea clearly conflicts with our everyday reasoning experiences: most of us will acknowledge that some inferences bring along new insights and thus seem to contain some kind of 'information' (for example, it is quite hard to argue that Euclid's theorem contains no new information). In

this talk, we shall present various conceptual and technical aspects of our project aimed at developing a formal framework that allows classical and non-classical logics to deal with the notion information in an intuitive way. The formal core of the framework is based on a slightly modified version of the so-called *block semantics*, developed by Diderik Batens, cf. [1], and encompasses two new quantitative information measure functions that enable us to express how much information is present (at a certain line) in a formal logic proof. The functions are based on the formal notions of entropy and self-information as defined in standard information theory. More specifically, the first function is able to express a precise distance between the amount of information at a line of a proof and the total amount of information present in the premises. The second function is a *goal-directed variant* of the first function and allows us to formalize goal dependent accounts of information.

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Understandings of logic sublated by the dialectic

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Empirical, Formal and Speculative understandings of logic as identified by Hegel will be given an axiomatic interpretation. The domain of their values will be decided by a three-valued modal relation for the probability mass function. Keynes' and Popper's solution to the problem of unknown proportions will be challenged by an understanding of logic that puts the content of what the axioms mean for making rational decisions before their mere being. What is true for inferences will then work for the principle of a dialectic function, in contrast to the two proposed by Hume and the one proposed by David Lewis . In this way, it will be demonstrated that Hegel's understanding of logic is still more advanced than ones that fail to recognise they function within the dialectic.

Categoricity and Non-Classical Logics

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We explore Carnap's so-called categoricity problem. Smiley (1996) solved the problem for classical logic, and Rumfitt (1997) follows up with a solution for Strong Kleene. Using n-sided sequent calculus, we offer a proof-theoretic framework to handle the categoricity problem for a range of many-valued logics. In conclusion, we discuss some connections with proof-theoretic semantics and the semantic role of proof-conditions.

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An argument for universal logic: troubles relativizing truth-functions in Tarski's truth-schemata hierarchy

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Tarski avoids the liar paradox by relativizing truth and falsehood to particular languages and forbidding the predication to sentences in a language of truth or falsehood by any sentences belonging to the same language. The Tarski truth-schemata stratify an object language and indefinitely ascending hierarchy of meta-languages in which the truth or falsehood of sentences in a language can only be asserted or denied in a higher-order meta-language. However, Tarski's statement of the truth-schemata themselves involve general truth functions, and in particular the biconditional, defined in terms of truth conditions involving truth values standardly displayed in a truth table.

Consistently with his semantic program, all such truth values should also be relativized to particular languages for Tarski. The objection thus points toward the more interesting problem of Tarski's concept of the exact status of truth predications in a general logic of sentential connectives. Tarski's three-part solution to the circularity objection which he anticipates is discussed and refuted in detail. The upshot is to support a universal logic of propositional connectives whose truth values do not need to be relativized to a Tarskian hierarchy of linguistically relativized truth values.

4vbc is proved to be a group, ring, and module

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Four valued bit code (4vbc) consists of the elements $\{0,0\}$, $\{0,1\}$, $\{1,0\}$, and $\{1,1\}$. The motivation is to show that 4vbc is a vector space, or if not then to show what category 4vbc is. This paper proves that 4vbc is not a vector space but is an Abelian group, a ring, and a module.

Conditionals and contingentarianism

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Conditional statements are known to be problematic. It's not clear whether a conditional should be treated truth-functionally and, if so, whether it should be treated as a single proposition or as a relation between two propositions. Furthermore, treating conditionals truth-functionally results in some strange consequences: It seems counterintuitive that anything can follow from a false antecedent and that anything can lead to a necessarily true consequent. And, in general, it's not clear how to treat counterfactual conditionals. All of these problems are compounded when dealing with nested conditionals. I show that these and some other problems involving conditionals are due to the assumption that there is genuine contingency in the world. I argue that a rejection of contingentarianism in favor of necessitarianism results in the dissolution of all of these problems. I provide a necessitarian account of (so-called) conditional statements, according to which the only appropriate use of a conditional is when the truth value of the antecedent is unknown, and according to which (so-called) true conditional statements are more accurately expressed as nonconditional, universal or general propositions. Necessitarianism entails the collapse of modal logic (since the study of modal propositions will be the study of necessary propositions only (i.e. all propositions)). I argue that this consequence is both desirable and true.

Locally finite logics have the density

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Our aim is to give more general conditions on the existence of the density of truth. The notion of local finiteness (for propositional logics) turns out to be very helpful in this task. We do not restrict our attention to logical systems defined in the standard propositional language – such as classical or intuitionistic logic. We also take into consideration a family of modal logics which obey our criterium. We prove that the density of truth exists for a large class of locally finite (locally tabular) propositional logics. We are primarily interested in classical and intuitionistic logic and show that their implicational fragments have the same density. There are also given some locally finite logics without the density of truth.

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On Dynamic Logic for Phenomena Modeling

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This presentation provides a formal description of the Dynamic Logic for Phenomena modeling (DLP) along with the comparison of the goals of DLP and some other dynamic logics. DLP is presented with its syntactic, reasoning, and semantic parts in terms of the model theory. Computational complexity issues that motivate this work are presented using an example of polynomial models. Modeling of complex real-world phenomena such as the mind presents tremendous computational complexity challenges. DLP addresses these challenges in a non-traditional way. The main idea behind its success in applications is matching the levels of uncertainty of the problem/model and the levels of uncertainty of the evaluation criterion used to identify the model. When a model becomes more certain then the evaluation criterion is also adjusted dynamically to match the adjusted model. This process is called Dynamic Logic of Phenomena modeling, which mimics processes of the mind and natural evolution. There are two complimentary trends in modeling physical phenomena and logic. In the first one, it is adding more logic structures to classical mathematical techniques. In logic, it is *dynamification* of logic with various new non-classical logics about actions rather than about propositions as well as about making logic operations dynamic. To actually benefit each other these areas need to be close enough in specific tasks and goals. We use the comparison of goals to establish initial links of DLP with some dynamic logics and to facilitate further studies of linkages in both communities.

Completeness of S4 with respect to the measure algebra

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A Remark on the minimality of ω

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In this talk we will present the fact that the axioms of comprehension scheme and infinity in ZF do not exactly offer us the minimal collection of all natural numbers, as we usually expect or what we have learned from elementary set theory course. Since this is also true for any inductively defined set, inductive sets like the set of all terms (for a fixed first order language) or the quantifier-free part of a theory extension (constructed in [1] for a possible model of any given theory) are not as concrete as we think (though no paradox is generated and no harm is really made in the sense of formalism).

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Universality as generality: the case of Ernst Schroder's pasigraphy

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According to Ernst Schröder's original idea of formal algebra, a formal theory is a set of basic symbols with certain combination rules. For these operations and relations certain postulates hold, from which theorems are derived. In this conception, the structural side of formal algebra is implicit. A formal theory is merely schematic, including the extreme case of schemata consisting exclusively of "blanks". The applications of it to different domains can lead to formal systems with more meaningful symbols and less blanks. This general applicability of formal algebra suggested an idea of universality. It was implicit in Schröder's *Handbook of algebra and arithmetic* (1873) and underlies his Lectures on the Algebra of Logic. In volume III of this work (1895), he extended this program, adopting the algebra of relatives of Peirce. Now, in his paper "On Pasigraphy"

(1898), Schröder considered the algebra of relatives as a universal scientific language. At the same time, this pasigraphy was a foundational theory for prima facie every scientific domain. In this theory the basic or fundamental notions of 'pure mathematics' (logic, arithmetic and geometry) are introduced. From these notions further theories could be formulated. This is a different conception of universality, resembling previous ideas of Frege, concerning a consensual universality. The aim of this paper is to analyze Schröders conception of universality underlying his idea of a pasigraphy in its historical context, that is, as a chapter in the history of universal logic. Moreover, it will be discuss to what extent a general symbolic structure can be used as a basis for a universal language. It will be argued that the distinction between alternative senses of universality rests on alternative ideas of formal entities, whose ontological presuppositions should be elucidated. If a theory can be formal as far as it represents alternatively formal objects, on one side, and properties or structures, on the other side, then these two categories have to be clearly distinguished.

Completeness: actors, history, and meaning

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The completeness theorem is one of the best pointers of the balance, health and goodness of a formal system. A calculus without a semantic counterpart is a fruitless set of rules without a proper logical purpose. When and how is the necessity of a completeness proof born? When does it separate itself from a theorem concerning the decidability of satisfiability for a given logic? Russell and Whitehead were not in the position to properly distinguish and separate formal calculus and semantic issues, they do not believe that the semantical level constitute in itself a land for formal analysis. The early completeness demonstrations for Propositional Logics are intimately related to decidability and representation in terms of finite algebras. The original publications of Post, Stone, Quine, Tarski, and Gödel are relevant for this line. It was Gödel's idea not to presuppose decidability of the calculus and to search for completeness using a mechanism previously employed by Löwenheim and Skolem.

Henkin's completeness proof came two decades after, but it was soon adopted as "the method", since it was rather flexible and could be easily applied to other logical systems. In fact, Henkin himself created it for Type Theory, but shortly realized that it could also be used in first order logic. This universal method is at present our main research interest. Its genesis is what we are trying to clarify.

This historical overview help us to analyse the present equilibrium between models and calculi of a given logical system. Can we use as well Henkin's method in Universal Logic?

Completeness and interpolation with a standard abstract consequence relation

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Henkin's Completeness Proof deserves a place among those specific results lifted to broader logical contexts. Its advantages concerning flexibility and broad range of application have been widely appreciated. In the same vain, Robert Goldblatt has accomplished a sort of generalization of the method of Henkin's completeness proof, more recently. After having developed his nowadays-famous proofs, Henkin adapted his method to apply it to other targets, such as the Craig-Lyndon interpolation. Incidentally, potential generalizations of Henkin's proper extension of the Craig-Lyndon interpolation theorem using his method of constants deserve further consideration. We expect that such a generalization would shed light on the epistemological significance of conditions concerning completeness. This paper aims to stress the significance of a generalization of Henkin's method, similar to the generalization accomplished by Goldblatt some years ago, to understand certain facts relevant to seek for some logical universals.

Limitations of axiomatic dialetheic truth theory

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The best-known application of dialetheism is to semantic paradoxes such as the Liar. In particular, Graham Priest has advocated the adoption of an arithmetized axiomatic truth theory, which I will call 'PA*', in which contradictions arising from the Liar paradox can be accepted as theorems, thanks to the adoption of an underlying paraconsistent logic. It would be remarkable if the soundness of PA* could be proved from within PA*, but this has not been accomplished (and Hartry Field has shown that the obvious proof strategy cannot succeed). I argue that if such a proof were forthcoming, certain facts about the behavior of the provability predicate for PA* could not be established, whether in PA* or outside of it, on pain of trivializing the theory. I discuss drawbacks associated with this fact, while raising several other criticisms of the dialetheic strategy along the way. I conclude that it is not in virtue of its truth theory that dialetheism is to be considered an attractive position.

Adaptive Deontic Logics for Various Types of Normative Conflicts

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It is commonly known that Standard Deontic Logic leads to triviality when applied to normative conflicts. Over the past three decades, considerable attention has been paid to normative conflicts of the form "A is obliged and not-A is obliged". At this moment, a plethora of systems is available that can handle this particular form of deontic conflict. Other forms of normative conflicts have been largely (and in my opinion, unjustly) ignored.

The aim of this paper is threefold. First, I shall present a taxonomy of the different kinds of normative conflicts. Attention will be paid to the origin of the conflict as well as to the different kinds of "impossibility" that are involved. Next, I shall discuss different strategies to deal with the different kinds of normative conflicts and present general procedures to characterize the (monotonic) logics that are obtained by means of these strategies. Finally, I shall argue that these monotonic systems are too poor to deal with the different kinds of normative conflicts, but that the adaptive versions based on them lead to satisfactory results.

Tertium non datur in the Lambek-Grishin calculus

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The Lambek-Grishin calculus (LG) is a bilinear extension of the Syntactic Calculus obtained by complementing Lambek's multiplicative conjunction and the residual left and right implications with a multiplicative disjunction and residual left and right coimplications (difference operations). Interaction between these two families is governed by linear distributivity principles. These principles are structure-preserving: they respect the non-commutativity and non-associativity of the multiplicative operations–essential properties for a logic that wants to capture word order and phrase structure in natural language. In this talk, we add two pairs of negations to LG: a Galois connected pair, and by arrow reversal, a dual Galois connected pair. The composition of the Galois connected operations yields a closure operation (being expanding, monotone, and idempotent); from the composition of the dual Galois connected operations, one obtains an interior operation (contracting, monotone, idempotent). We generalize the distributivity principles relating the product and coproduct operations to the unary negations and show that (directional versions of) the law of the excluded middle are theorems in this setting. Because the distributivity principles have the status of irreversible rules, collapse into a one-negation system with an involutive negation and de Morgan interdefinability of the product and coproduct families is avoided. Instead, we find 'half' of the de Morgan laws. We show that LG extended with these negations has particularly simple analyses for a number of syntactic and semantic phenomena that are problematic for the original Lambek calculi.

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The Aristotelian p^q-semantics and their p^q-lattices

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"Universal logic" (UL) is said to deal essentially with the most abstract properties of "consequence". But if one acknowledges that logic is also concerned with "negation", things get more complex: since 2004 it has turned out that the pure formal study on negation opens to a whole new branch of mathematics, at the intersection of logic and geometry: "n-opposition theory" (NOT), the "geometry of oppositions" (negation being only a special kind of opposition). NOT teaches that the elementary oppositional structures (the logical "square", "hexagon", "cube",...) are just instances of "logical bi-simplexes of dim. m". And these, in turn, generalised by a game-theoretical "Aristotelian p^q-semantics" (generating "p^q-lattices"), are a particular case of "logical poly-simplexes of dim m". So, if NOT confirms the mathematical specificity of UL, its hyper-geometric nature changes UL's very architecture.

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Decision method for the paradoxes of material implication

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The aim of this study is to develop a method to select the occurrences of material implication in a formula of propositional logic which can be interpreted by the relation of consequence. It is well known that there are a lot of formulas of propositional logic, so that, despite their validity, they cannot be interpreted by correct arguments, because the material implication symbol cannot be interpreted by consequence (or deduction) relationship. These situations are known as paradoxes of material implication. On the other hand, there are formulas, as *modus ponens*, where the interpretation of material implication by consequence relation is possible without paradox. It results that the symbol of material implication can be interpreted by consequence relationship only conditioned. In this study, a test to distinguish between paradoxical and unparadoxical occurrences of material implication is presented.

Computer Aided Proof

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We examine the impact of computers on the notion of mathematical proof. Recent advances in the field of 'Interactive Proof Checking' with the associated development of powerful tools such as 'Proof Assistants' have given rise to an interesting consequence – viz. the practical feasibility of importing techniques developed in the computer science community and redeploying them to improve the main activity of the working mathematician, namely the process of proof development. At the core of such redeployed techniques lie the notions of formal systems, formal reasoning, and formal proofs. However the process of formalizing mathematics is a highly non-trivial task, and gives rise to a number of challenging and interesting issues which need to be addressed in order to make the discipline of computer assisted mathematics more prevalent in the future.

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Anti-realism, strict finitism and structural rules

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The conclusion of Michael Dummetts's manifestability argument is that intuitionistic logic satisfies the semantic requirements of antirealism. Some philosophers have argued that the traditional antirealist desideratum of decidability in principle is too weak. Semantic antirealism properly construed must be committed to effective decidability. As such, it either leads to strict finitism (Wright [1982] 1987),or to a much stronger kind of logical revisionism: substructural logics,and in particular linear logics, rather than intuitionistic logic,satisfy the semantic requirements of strict antirealism (Dubucs and Marion 2003). I shall develop two kinds of replies. The first kind of reply is that if we jettison the effectively vs.in principle distinction, as applied to manifestability-type arguments, we end up with an unsatisfactory explanation of how the meaning of statements covering the practically unsurveyable cases is fixed. I shall then look at two radical antirealist principles disqualifying structural rules: Token Preservation and Preservation of Local Feasibility. Against Bonnay and Cozic's criticisms (Bonnay and Cozic, forthcoming), I shall argue (i) that some conceptual support may be provided for Token Preservation, which doesn't rely on a causal misreading of the turnstile, and (ii) that the appeal to non feasible ways of doing feasible things is not a good way to argue for Preservation of Local Feasibility.

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The place of logic structure "question-answer" in dialogue theory of understanding

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Understanding question as a particular form of conclusion appeared in philosophy discussions of Antiquity. According to Socrates a dialogue is joint search for truth and is formed from questions (problems and contradictions) and answers (their solutions). Questioner and respondent in the process of dialogue exchanging thoughts express different views on the world. None of them is not true, but each of them can express his own understanding because the truth cannot be known by one person. In philosophy the hermeneutic method of understanding the sense of the text is the same as dialogue method where the structure "question-answer" is used. In his turn in informal logic D. Walton presenting a dialectical theory of explanation defined a successful explanation as a transfer of understanding from a respondent to a questioner in a dialogue. This theory combines two views on the nature of explanation: an explanation is seen as search for an answer to a question and is defined in terms of a concept called understanding. Such combination of different points of view is effectively for defining scientific explanations. Thus, logic of question is the main component in dialogue theory of understanding. Such coincidence of approaches to research of the procedure of understanding in philosophy and informal logic shows the importance of dialogue and its logic structure "question-answer" in developing of modern methodology of scientific cognition in whole.

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Truth and Falsehood Operators Logic from Point of View of Universal Logic

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This paper proposes the truth and falsehood operators logic. These operators are included in the language of this logic which allows the iteration of such operators. The truth and falsehood operators (T, F) are logically independent. It gives us an opportunity to consider the class of sentential logics in the language of this logic which have 2, 3, 4-valued interpretations. So this logic can be treated as a universal logic relatively to this class of sentential logics.

The truth and falsehood tetralemma [1]:

either (TS and ”FS), or (”TS and FS), or (TS and FS), or (”TS and ”FS).

Now let us consider language of logic with negation and implication. Truth and falsehood conditions for negation and implication are standard.

In spite of above, laws of classical logic are not valid. So we can assert axiomatic emptiness relative to languages of logics with negation and implication. Finally, bivalence principle is equivalent to T-biconditional.

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Preservationism and Truth

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This paper first introduces preservationism and gives it a precise formulation. Second, Field's discussion of the intuitive notion of validity is exposed. Field's analysis of validity will lead to a discussion of Field's own view on the matter of soundness and his thesis that validity should be considered a basic, intuitive notion, given that validity can't be captured, even extensionally, by necessary truth preservation. I will argue that Field's ideas can be made to fit with the preservationist program. The end of the paper ties the preservationist program to the theme of universal logic.

Some remarks about the status of second order logic

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Second order logic has been always considered problematic by modern logicians: so problematic that some of them refuse to call it logic at all. Lindström Theorem sets out a boundary between the "pure logicality" of first order logic and the "mathematicality" of second order logic: is the validity of completeness, compactness and Löwenheim-Skolem Theorem the only qualification to call a formal system "logic"? After all the lacking of expressive power of first order represented by the lacking of categoricity, could well be considered an important flaw too. Moreover, it could sound odd that, on the one hand we do not call second order a proper logic due to its being "uncontrollable", and on the other hand we state, as a corner stone of the "controllable" first order, the Löwenheim-Skolem Theorem, a theorem which states the incapability of a theory to "control" its models.

On decomposability in logical calculi

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In Computer Science, decomposition is a standard technique to reduce complexity of problems. In Logic, the notion of decomposition appears in numerous applications including the important field

of automated reasoning over theories. The main idea is to identify those fragments of a theory that are necessary and sufficient for testing a given property, thus reducing the search space and complexity of reasoning. There is a number of papers, in which modularization and decomposition methods for logical theories are considered.

In presence of interpolation, a natural approach is to consider signature partitions of theories.

Assume T is a theory, sig(T) is the signature of T, and Δ is a subset of sig(T). We call the theory T Δ-decomposable, if it is equivalent to a union of theories T_1 and T_2 such that the union of sig(T_1) and sig(T_2) equals sig(T) and the intersection of sig(T_1) and sig(T_2) is exactly Δ. In the case Δ is empty, we speak of a pure decomposition of T into signature-disjoint theories and if sig(T_1) or sig(T_2) equals Δ, we say that T is trivially Δ-decomposable. In fact, the components T_1 and T_2 induce a partition of the signature sig(T)\Δ which we call a signature Δ-decomposition of T. Note that T_1 or T_2 may happen to be non-trivially Δ-decomposable and thus, give a finer signature $Delta$-decomposition of T.

We consider the problem of deciding whether a given finite set of formulas is non-trivially Δ-decomposable for a given subsignature Δ. Which logical calculi allow for an algorithm to compute signature Δ-decompositions for an arbitrary given finite set of formulas T and a $\Delta \subset$ sig(T)? What properties should a calculus satisfy for every its set of formulas to have a unique finest signature Δ-decomposition for each subsignature Δ? In the talk, we give a partial answer to these questions on the example of a broad class of logical calculi. Unsurprisingly, the considered problems are closely related to important interpolation properties studied in logics: the Craig interpolation property and uniform interpolation property. The results covered in the talk are directly transferred to the classical, intuitionistic logic, and a wide range of modal logics.

Three adaptive logics of induction

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In this talk I want to present three basic adaptive logics of induction: LI, IL and G, defined within the standard format for adaptive logics from [2]. They provide three different qualitative accounts of how people might derive generalizations from a set of data. As expected, and as usual for most adaptive logics, they are nonmonotonic and have a dynamic proof theory.

The consequence sets of the three basic logics are closed under classical logic. For so-called "complete data", the basic logics all lead to the same consequence set. The difference lies in the conditional assumption each of them makes, which results in a different outlook of the proofs, and in significantly different consequences for some canonical "incomplete" premise sets. The logics can be ordered according to their strength (proofs have been provided). This leads to unexpected results when it comes to the need of instances or even positive instances to derive a generalization. It turns out that this extra condition actually strengthens the consequence relation, rather then weakening it.

Starting from some observations about G, I will show that yet another strengthening of this logic is both plausible and possible. I will rely on the Popperian idea that stronger generalizations should be privileged over less stronger ones, and apply this to G. This will result in a definition of SG, the strongest logic within the whole range of adaptive logics of induction studied so far.

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Disjunctive Syllogism, Lewis' modal logics and paraconsistency

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In logics where rules E$\wedge$ ($A \wedge B \vdash A, B$) and I$\wedge$ ($A, B \vdash A \wedge B$ hold, ECQ ("Ex contradictione quodlibet": $A, \neg A \vdash B$) is equivalently formulated in the form $A \wedge \neg A \vdash B$.

As E$\wedge$ and I$\wedge$ hold in all logics defined in this paper, by ECQ we shall refer to this second version of the rule.

As it is well-known (cf. [4]), a logic S is paraconsistent if ECQ is not a rule of S. Not less well-known is the fact that Lewis' logics are not paraconsistent. But this fact is, indeed, no accident. Actually, Lewis holds that B is deducible from $A \wedge \neg A$ (equivalently, $(A \wedge \neg A) \rightsquigarrow B$ — $\rightsquigarrow$ stands for strict implication), according to the argument in [3], p. 250, currently known as "Lewis' argument" (cf. [1] §16.1), which relies on the disjunctive syllogism (d.s) understood as a rule of inference.

The aim of this paper is not to discuss this challenged argument again (cf. e.g. [1] and [2]) but to define a series of paraconsistent logics included in Lewis' S4.

In all logics in this paper, d.s is understood as a rule of proof, not as a rule of inference. Therefore, ECQ does not hold in any of them. So, theories built upon these logics are not necessarily closed by d.s, and consequently, in case of inconsistency, triviality does not follow automatically. Nevertheless, consistent, prime theories are of course always closed by d.s.

All logics in this paper contain classical logic in the sense that all tautologies in $\wedge$, $\vee$ and $\neg$ are provable. Moreover, as d.s is a primitive rule in each one of them, Modus Ponens for $\supset$ is available. A Routley-Meyer type ternary relational semantics is provided for each one of these logics. Soundness and completeness theorems are proved.

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Deduction, induction, and abduction according to C.S.Peirce: necessity, probability, discovery

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According to C.S. Peirce, the essence of reasoning lies in the fact that where certain relations are observed to occur, certain others also obtain. So, there are three basic ways in which we can ascertain the logical connexion between premises and conclusions. Deduction is in first place the mode of necessary logical reasoning. It is the only mode of drawing necessary conclusions, for the truth of the premises grounds the truth of the conclusions. It can be described in its logical form as the well-known modus ponens or modus tollens. Induction is the type of reasoning that contrary to deduction doesn't draw necessary conclusions, for the truth of the premises doesn't necessarily warrant the truth of the conclusions, but only probably state it. Induction, therefore, doesn't allow for discovery, only for testing the conclusions we draw by deduction. Its specific feature is to allow us to see that certain characters belong to certain objects. Abduction is the type of reasoning that doesn't have any logical necessity and has the least probability of establishing a true relation between premises and conclusions, but is the only one with heuristic power. Its distinctive feature is that it tells us that certain objects might have certain characters. It can be described nonetheless as the logical fallacy of affirming the consequent, or *Post hoc ergo propter hoc*.

Knowing and applying rules: tableaux for mixed modal deontic-epistemic logics

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A game can be formalized in a multimodal logic with epistemic and deontic operators. Then, the game rules must be defined by means of associated epistemic and deontic actions, what means that we need a mixed modal deontic-epistemic logic to define a game as a set of rules in this way. Given a set of agents, models are constructed with two kinds of accessibility relations: equivalence relations for epistemic accessibility, and serial relations for deontic accessibility.

We will use labelled tableaux for dealing with this logic. Different labels are used according to the accessibility relation and the agent considered. Two kinds of rules are also used in this tableaux method that can be called common rules and inheritance rules. The former are used only once in the execution of the tableaux; the latter may be used so many times as we need and they guarantee that the accessibility relations have the desired properties. Indeed, we can deal with a S4 or a T system for epistemic operators just by changing the inheritance rules. Therefore, soundness is guaranteed by means of these inheritance rules. In such a system the operator's iteration can make the tableaux infinite. But this tableaux method solves the problem by allowing us to get finite models for interpreting mixed modal deontic-epistemic sentences that are satisfiable. To achieve this result we define accessibility relations for any deontic and epistemic interpretation of our modal operators and use this definition to develop construction rules for the tableaux with certain restrictions on world indices. This logic is very close to a dynamic interpretation of modal operators, since we can establish a relation between sentences that express strategic knowledge and some formulas of this logic.

Modal logic for qualitative dynamics

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Completeness in infinitary logic

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We investigate infinitary logics $\mathcal{L}_{\kappa,\lambda}$ and some more abstract logics, in view of the following general question: Given a theorem of the usual finitary logic $\mathcal{L}_{\omega,\omega}$, as Löwenheim–Skolem, Compactness, Completeness, Ultraproduct, Omitting Type, etc., for which κ, λ the logic $\mathcal{L}_{\kappa,\lambda}$ satisfies an appropriate analog of that theorem? Typically, the answer leads to large cardinals. A particular emphasis of the talk will be to our recent result on the Infinitary Completeness Theorem. We show that in fact Completeness is equivalent to Compactness: any consistent theory in $\mathcal{L}_{\kappa,\lambda}$ has a model if and only if κ is strongly compact, and any consistent theory in $\mathcal{L}_{\kappa,\lambda}$ using at most κ non-logical symbols has a model if and only if κ is weakly compact. Actually, this result is applicable to a wider class of infinitary logics, e.g., this holds for higher-order ones: any consistent theory in $\mathcal{L}_{\kappa,\lambda}^n$ has a model if and only if κ is extendible. We discuss also infinitary analogs of other logical theorems.

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Truth Tables as Metaphysics: sense, calculus and limits of expression in Wittgenstein's Tractatus

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The way of this research is to investigate how Wittgenstein's Tractatus can be read from its account of truth tables as a more adequate notational system for propositional expression, in comparison with the notation used in Principia Mathematica, for instance. In this way, I hold its truth table metaphysics as an specially illuminating key concept or reading strategy for understanding Tractatus. I defend that its truth table account is rich enough to allow a global reading of central tractarian theses about modeling paradigms in the nature of language.

Here follows an enumeration to these positive tractarian theses: 1) a strong principle of Frege or truthfunctional principle (every proposition is elementary or can be analysed in terms of elementary propositions); 2) a postulating of a complete propositional analysis; 3) sense propositional determinateness; 4) essential propositional bipolarity as a criterion for sense (a legitimate proposition should be able to be true and to be false); 5) a possible fully expression of reality. Besides, there are some specific theses about logic, namely: 6) logical propositions are tautologies; 7) they can be recognized by the symbol itself (where truth table could be a decision algorithm); 8) they are complex, i.e., a special articulation of elementary proposition; 9) logical propositions make manifest (zeigen) the inner structure of language; 10) logical operators do not denote anything in reality (its Grundgedanke!). This tentative approach shed light indirectly on the strong original relationship between tractarian theses and its specific account of logic. In this sense, it is easier to acknowledge that one of the major efforts made in Tractatus was the defense of an account of logic that could make logical propositions categorically different from scientific ones.

My point here is to show that when truth table is understood as an special notation, some theses in Tractatus can be elucidated both positively and negatively. In a positive way, as the topics above suggest. And in a negative way: revealing where and when these tractarian theses fail and bring down the whole tractarian project. We can read Tractatus from its truth table account emphasizing what Wittgenstein was trying to perform, i.e., to determinate an exhaustive horizon to propositional sense. This suggestion of reading can also exhibit tractarian failure, namely, the expressive incapacity of truthfunctional analysis deals with all empirical propositions, specially the ones which convey generality or hold any kind of gradation or series. In fact, Tractatus fails to deal satisfactorily with generalities (e.g. quantification in infinite domains), it lacks also subtleness in comparison to predicative negation (Tractatus holds negation as a propositional operator) and the exclusion by contrariety (e.g. ascription of color to visual points).

Tractatus fails where the truth table notation fails. It cannot deals with infinite generalities, predicative negation and exclusion by contrariety (only by contradiction). A fortiori, I defend that the failure ground to Tractatus is its strong compromise with the truth table as a more adequate notation to express propositions. Tractatus has bet too much on a technique which has a very short scope of expressivity because of its high level of abstraction in analysis. As a result, if we hold the truth table notation as a conceptual key to Tractatus we can fully understand its project and failure. Moreover, we can also explicit the relevant metaphysical compromises of a truth table technique in its own origin (greatly forgotten nowadays).

Pragmatic satisfaction and quasi-truth

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Newton da Costa and his collaborators (cf. [1]) have introduced the notion of quasi-truth by means of partial structures, where the relations within the structure are partial. Thus, the membership (or not) of a given tuple of the domain in such a relation is not always defined, and so any partial relation R is a triple of sets (P, M, D) where P is the set of tuples which effectively belong to R, M is the set of tuples which effectively do not belong to R, and D is the set of tuples whose membership to R is (still) undetermined. In this way, the "predicates as triples" approach provides a conceptual framework to analyse the use of (first-order) structures in science in contexts of informational incompleteness.

In this paper the notion of "predicates as triples" is extended recursively to any complex (i.e., non-atomic) formula of the first-order object language. Thus, the interpretation of any formula φ in a partial structure A inductively originates a triple, generalizing da Costa's approach to atomic formulas.

Moreover, this proposal generalizes the usual perspective of a given first-order formula φ (with at most n free variables) within a structure A seen as a relation R, which is defined inductively. From this, a new definition of quasi-truth via the notion of pragmatic satisfaction is obtained.

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Quantifier Elimination and Strong Isomorphism

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We generalize the classical theorem on quantifier elimination for infinitary languages, studying its consequences in respect to the notion of strong isomorphism of substructures of a given structure that is a model of our initial theory. Therefore, we obtain a kind of abstract version of Steinitz theorem of the classical field theory.

Some parametrization theorems for measurable sets with uncountable sections

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Every analytic set in the product of two Polish spaces with uncountable vertical sections contains a set in the product of Selivanowski sigma-algebra and Borel sigma-algebra with sections perfect. This generalizes similar result proved by Wesley (using forcing) and Cenzer-Mauldin. We use it to prove some parametrization theorems for measurable sets with uncountable sections.

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On institutions and proof events

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Proofs exist everywhere, not only in mathematics. *Proof events* are those social events generating proofs. They are actual experiences in the real world involving particular people [1]. Each proof can be studied, at least from two different viewpoints: 1) the formal or symbolic one and 2) the "proofs events" one. This distinction is applicable to all kinds of proofs regardless their mathematical nature. The formal approach focuses on sets of *rules of inference*, and claims that all proofs can be constructed as finite sequences of applications of such rules. A recent interesting development is the proof systems approach at the level of institutions, an abstract model theory with applications in logic and computer science [3].

Proof events have social nature and may include all kinds of expressions to convince the audience on the truth of a theorem. The standards of rigor, however, will depend on the audience of the proof event. A proof event minimally involves at least one person that has the relevant background and interest, and some mediating physical objects, such as spoken words, gestures, hand written formulae, 3D models, printed words, diagrams, or formulae [1]. As proofs we define the final outputs of proof events. We believe that a mathematical formulation of the concept of proof event

at the level of institutions will have expressive power. We claim that for *every proof event there is a corresponding formal representation of its output within an institution with proofs.* This can be summarily expressed with the motto: *formality of proofs comes a posteriori.*

We introduce an *institution of proof events.* Mediating physical objects are considered to be objects in the category of signatures. Σ-"*sentences*" are built from terms (proof processes), and terms are built using proof event operators and predicates. Such operations include a separate predicate *Proof*, which is used to describe the outputs of a proof event. The effects of proof processes are given using predicates such as *Initiates* and *Terminates* from the Event Calculus of [5]. Additionally, we suggest that a calculus of proof events should follow two extra criteria (a la David-son): *Two proof events are the same if and only if the have the same cause and the same effect (causal criterion).* Also, *two proof events are the same if and only if they occur at the same space and time and involve the same social group (socio-spatiotemporal criterion).* The first criterion corresponds to preordering of entailment systems [2]. As Σ-*models* we consider abstract proof systems at institutional level, and Σ-satisfaction is a relation between proof events and their "formal representation". To demonstrate the usefulness of the institution of proof events we provide some applications to trace based network proof systems [4].

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Combinatory Logics for Lambda Calculi with Patterns

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We introduce CLp, a combinatory logic system for a lambda-calculus with patterns (namely lambda-P), obtaining a consistent extension of classical combinatory logic (CL). Our goal is to find an appropriate bridge between the two formalisms, and take advantage of some of the positive aspects of each. As in classical CL, our system will avoid dealing with abstractions and bound variables, while allowing functions to impose restrictions over their arguments through pattern matching, in the same spirit as in lambda-P. We introduce back and forth translation rules which allow us to represent this pattern calculus within the language of combinators, simulating the abstraction mechanism and achieving combinatorial completeness. Since the full language becomes non-confluent - as does the unrestricted lambda-P - we provide a restriction to the set of patterns (based on the revised Rigid Pattern Condition of lambda-P) so that the whole system satisfies confluence and ensures the consistency of the underlying logical theory. We propose and study other variants of interest such as the introduction of both curried and first-order constructors for modeling data structures and the generalization of the matching mechanism. We also introduce a more general definition of pattern matching, characterizing a family of confluent variants. Finally, we propose two type systems and prove the fundamental properties.

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Adaptively Applying Modus Ponens in Conditional Logics of Normality

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Since the early eighties default reasoning, i.e., reasoning on the basis of what is normally or typically the case, has drawn much attention from philosophical logicians as well as scholars working in artificial intelligence. A promising logical representation has been given in form of conditional logics of normality (see e.g. [2], [3]). Compared to classical approaches such as default logic, circumscription, autoepistemic logic, these logics benefit from a natural, simple and unifying representation of default knowledge in form of conditionals A?B: "From A normally follows B". While these logics offer promising ways to reason about conditionals, what is missing is the ability to perform default inferencing itself. This talk presents a generic way to enhance given conditional logics of normality in order to allow for defeasible applications of modus ponens (MP) to conditionals. For that purpose an adaptive logic framework is introduced. Adaptive logics (see [1]) allow us to apply MP to a conditional A?B and a fact A on the condition that it is "safe" to do so, concerning the factual and conditional knowledge at hand. It is unsafe, for instance, if the factual information describes exceptional circumstances to a given rule. The two adaptive standard strategies are shown to correspond to different intuitions, a skeptical and a credulous one, that manifest themselves in the handling of so-called floating conclusions.

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Syntactic cut-elimination for modal fixed point logics

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Fixed point extensions of propositional modal logics occur naturally in many different contexts. In epistemic logic, for instance, common knowledge of a proposition is defined as the greatest fixed point of a certain positive operator. Also in temporal logics safety and liveness properties are formalized by least, respectively greatest, fixed points. Kozen's mu-calculus provides the general extension of modal logic with fixed points for arbitrary positive operators.

From a proof-theoretic perspective, syntactic cut-elimination is one of the major open problems for these logics. In this talk we will survey the problem of cut-elimination for modal fixed point logics. We will present a solution for the case of common knowledge which makes essential use of deep inference. Moreover, we discuss whether and how far this approach can be extended towards a cut-elimination procedure for the modal mu-calculus. We will show new results characterizing exactly the fragment of the mu-calculus for which the deep inference approach to cut-elimination works.

Some calculus for a logic of change

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We add to the language of sentential logic an operator C to be read as "It changes that ..." and present an axiom system in frame of classical logic to catch the intuitive meaning of the term "change". A typical axiom is e.g. ""CA implies Cnot-A", a basic rule is "From A you may infer not-CA" (theorems don't change). So this system is not regular. On the semantical level we introduce stages (in the development of some world, of some convictions of an agent or in some argumentation) on which a sentence may be true or false. In the book cited below we proved the completeness of this axiom system in respect to this semantics. It turns out that with the help of C an operator N can be defined whose intuitive meaning is "Next it will be that ..." and which behaves like A. Prior's F or $G.H$. von Wright's T in his "At next". The aim of our talk is to present this calculus, some metalogical results concerning it and to give some interpretations.

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Universal fragments of the logics of the strong contact and the strong connectedness

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Traditionally, the classical geometry and topology as an abstract kind of geometry are point-based in a sense that they take the notion of a point as one of the basic primitive notions. An alternative approach to the theories of the space goes back to Whitehead. It is based on the notion region as primitive and the binary predicate contact between regions. Usually the regions are regular closed sets in a given topological space (which form a Boolean algebra), the binary contact relation between regions is the relation 'non-empty intersection' and the unary relation connectedness is connectedness in topological sense. The binary relation strong contact between regions is defined as topological connectedness of the interior of the meet of non-empty subregions. The unary relation strong connectedness is defined as topological connectedness of the interior.

In the present talk axiomatizations of the universal fragment of the first-order theory of the Boolean algebra of regular closed subsets with the relations strong contact and strong connectedness for different classes of topological spaces are given. In particular we consider the so-called polytopes in Euclidean plane.

Pertinent Entailment

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We investigate infra-classical entailment relations that we call pertinent entailments. The notion of pertinence proposed here is induced by a binary accessibility relation on worlds establishing a link (representing some form of pertinence) between premiss and consequence. We show that this notion can be captured elegantly using a simple modal logic without nested modalities. One road to infra-classicality has been studied extensively, that of substructural logics, which weaken the generating engine of axioms and inference rules for producing entailment pairs (X,Y). Here we follow an alternative (not antagonistic) strategy: we first demand that X entails Y classically, and then, with supplementary information provided by an accessibility relation, more, trimming down the set of entailment pairs to infra-classicality. It turns out that our pertinent entailment relations restrict well-known 'paradoxes' avoided by relevance/relevant logic in an interesting way. We also show that they possess other non-classical properties, like paraconsistency. Moreover, we investigate how a notion of obligation can be captured elegantly and simply by our formalism. We also discuss the properties of these pertinent entailment relations with respect to inference rules traditionally considered in the literature.

From logical to metalogical pluralism

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The recent situation in logics features the stable proliferation of non-classical logical systems and this process, to all appearances, is irreversible by its nature. During more than two thousand years the scholars considered Aristotelian and Stoic logics solely; modern classical logic is a continuation of this tradition being different just by its means. The emerging of non-classical logics stroke seriously the logical investigations compelling to revaluate and cast doubt on many results which were took for granted so far. Though such a situation would be easily methodologically diagnosed as the well-known "monism vs. pluralism" dilemma but this by no means can help us to resolve a number of fundamental questions concerning the problem of plurality of non-classical logic responses on eternal disciplinary requests. This plurality makes important an issue of choice of the uniquely true among them while taking into account that there are no recipes and prescriptions for to-date. The strategy of overcoming this problem situation comes either to the quest for "paradise lost" (classical or other logic as the only recipe) or to the decisive acceptance of the point of view of the principal plurality of logical systems as the future prognostic perspective.

An interesting aspect of the confrontation considered is an issue of non-classical metalogics arising within the metalanguage formulation of logical consequence: from A follows B if and only if from A is true follows B is true. The second word "follows" points to the metalogical consequence and then a question arises: would this consequence be necessary the classical one? In fact, this condition is not necessary in many cases e.g. in relevant one but could we in this case reformulate the definition as "from A *relevantly* follows B if and only if from A is true *relevantly* follows B is true"? Here we have a relevant logic in metalevel. But then there arises a temptation to introduce formulations of such a kind: "from A *intuitionistically* follows B if and only if from A is true *intuitionistically* follows B is true", "from A *quantum logically* follows B if and only if from A is true *quantum logically* follows B is true" etc.

It worth noting that in a sense the choice of the respective metalogic renders the choice of non-classical universe: choosing either non-classical logic as metalogic we indirectly determine the result of deducibility of the formula by metalanguage reasons which are not directly relates to given logical system. The possible interconnection between metalogics would be evaluated from the perspective of the exploitation of the notion of global and local logics where "local logics" means a logical system formulated in object language whose conditions of deducibility are unambiguously defined in given metalogics i.e. in global logics.

If we will try to identify some formulation of type "from A in logic X_1 follows B if and only if from A is true in logic Y_1 follows B is true" and "from A in logic X_2 follows B if and only if from A is true in logic Y_2 follows B is true" then we will need a meta-metatheory for defining criteria of such an identification. The universal logic seems to be the good one for this aim since within it the cross mutual translatability of logical systems is considered. Then identification or connecting of formulations at hand would be carried out on the base, for example, the translation of logic (metalogic) Y_1 to the logic (metalogic) Y_2. The result will be as follows: "from A in logic X_1 follows B if and only if from A is true in logic Y_1 follows B is true" is true if and only if "from the translation of A in logic X_2 follows the translation of B if and only if from the translation of the condition of truth of A in logic Y_2 follows the condition of truth of B". If such an identification will succeed for a number of given translations then this should be considered as an argument in favour of local metalogical monism.

Piaget-like transformation in universal logic

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Piaget examined the behavior of classical propositional negation in the 50's. Examining the behavior of some negation induced transformations under composition, he found that they form a familiar 4-element group: the Klein group of symmetries. Piaget's analysis can be explained by the behavior of classical negation on a propositional letter: the integers modulo 2. This approach

also works for other kinds of logics: e.g., in the case of intuitionistic logic, one obtains a monoid of transformations.

We extend these ideas to the context of universal logic and other kinds of negation-like unary symbols. We consider two unary symbols and examine the effect of applying one externally and the other internally, up to equivalence. These transformations under composition form a Piaget-like monoid. The behavior of the underlying unary symbols under composition form (cyclic) monoids of transformations. These underlying monoids of transformations impose constraints on the corresponding Piaget-like monoid, which determine, to a large extent, its structure. Piaget-like monoids provide useful information on their logics. Much as the determinant provides some information about a matrix, the Piaget-like monoid gives a condensed view of the logic, which may be easier to handle. Distinct logics may present the same Piaget-like monoid, but logics with non-isomorphic Piaget-like monoids are non-isomorphic as well.

Piaget-like monoids are useful tools for analyzing and comparing logics.

Rich Set Theory using Adaptive Logic

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In this talk I present two solutions for the paradoxes of naive set theory. Naive set theory is based on the Abstraction axiom schema and the Extensionality axiom and leads to different paradoxes among which Curry's paradox, Russell's paradox and Cantor's paradox. These paradoxes and classical logic (CL) together trivialize naive set theory. The solutions presented here make use of so called adaptive logic (AL). This is a class of logics with a dynamic proof theory that elegantly formalize various kinds of complex defeasible reasoning forms. I discuss two classes of AL's as the underlying logic of a set theory based on the axioms of naive set theory. In this way, it is possible to block problematic (paradoxical) consequences of the axioms but allow for the unproblematic ones. The downside of going adaptive is a substantial increase in computational complexity. The philosophical purpose of this project is twofold. First, the process of searching for an appropriate AL solution might be seen as an intuitive explication for the defeasible reasoning process of overcoming the paradoxes of set theory. Secondly, the here presented adaptive set theories are interesting theories in their own respect. Most useful theorems in Zermelo Fraenkel set theory (ZF) are provable, but the adaptive theories are evidently non-trivial, in contrast to set theories like ZF. Moreover, the adaptive set theories start from no other than intuitive axioms on sets, unlike the rather counterintuitive set of axioms for ZF. One could argue that the adaptive set theories rely on weaker philosophical assumptions and thus make fewer metaphysical claims.

Nominalistic logic

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Nominalistic Logic (Jørgen Villadsen 2008) is a presentation of Intensional Type Theory (Paul Gilmore 2001) as a sequent calculus together with a succinct nominalization axiom (N) that permits names of predicates as individuals in certain cases. The logic has a flexible comprehension axiom, but no extensionality axiom and no infinity axiom, although axiom N is the key to the derivation of Peano's postulates for the natural numbers. We present a revised Nominalistic Logic with a new rule for application in the type theory such that each term has a unique type. We also add a choice axiom. The resulting logic provides a very concise foundation of mathematics.

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Completeness in infinitary modal logic

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We extend some results of Saveliev's "Completeness in Infinitary Logic" to infinitary modal logics. We show that the Canonical Model Theorem holds for all infinitary modal logics in the language $\mathcal{L}_{\kappa,1,\square}$ if and only if κ is a weakly compact cardinal. We show that some modal logics that are complete in $\mathcal{L}_{\omega,1,\square}$, e.g., $K, K4, S4$, remain complete in $\mathcal{L}_{\kappa,1,\square}$ for weakly compact κ. We discuss an analog of Salqvist's theorem. We discuss also Completeness of predicate modal logics in $\mathcal{L}_{\kappa,\lambda,\square}$. This is a joint work with Denis I. Saveliev.

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On consequence operations

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In the paper, the authors discuss two kinds of consequence operations characterized axiomatically. The first one are consequence operations of the type $Cn+$ in Tarski's sense that, in the intuitive sense, are infallible operations, always leading from accepted (true) sentences of a deductive system to accepted (true) sentences of the deductive system. The second kind are dual consequence operations of the type $Cn-$ that can be regarded as anti-infallible operations leading from non-accepted (rejected, false) sentences of a deductive system to non-accepted (rejected, false) sentences of the system (see Słupecki 1953, Wybraniec-Skardowska 2004). The operations of the types $Cn+$ and $Cn-$ can be usual finitistic consequence operations or the so-called unit consequence operations.

A deductive system can be characterized in two the following ways

by the triple: $(+,-)$ $\quad < S, Cn+, Cn- >$
or by the triple: $(-,+)$ $\quad < S, Cn-, Cn+ >$.

We compare axiom systems for operations of the types $Cn+$ and $Cn-$, give some methodological properties of deductive systems defined by means of these operations (e.g. consistency, completeness, decidability in Łukasiewicz's sense), as well as formulate different metatheorems concerning them.

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Acts of Withdrawing in Dynamic Logic of Propositional Commitments

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We usually withdraw our own assertions or concessions when our beliefs are revised. The effects of acts of withdrawing assertions or concessions, however, are significantly different from the effects of retractions studied in the AGM approach to belief revision; they do not satisfy AGM postulates for belief retractions. We develop a dynamic modal logic that can deal with effects of acts of withdrawing of assertions and concessions by extending a dynamic logic called DMPCL (Dynamified Multi-agent Propositional Commitment Logic), which is developed according to the same strategy that leads to the development of dynamic epistemic logics in the last two decades. Thus we first develop a propositional modal logic, MPCL, which deals with static structures of multi-agent propositional commitments, and then "dynamify" it by adding dynamic modalities that represent acts of asserting and conceding to MPCL. In the resulting logic DMPCL, acts of asserting and conceding are modeled as events that update propositional commitments borne by individual agents involved in a discourse. MPCL is axiomatized in a completely standard way, and DMPCL is axiomatized by adding a set of so-called "reduction axioms" to the proof system of MPCL. Acts of withdrawing assertions and withdrawing concessions can then be modeled as yet another kind of events that update agents' propositional commitments by adding dynamic modalities that represent acts of withdrawing assertions and withdrawing concessions to DMPCL. Unsurprisingly, the effects of acts of withdrawing are very difficult to capture, and the completeness problem for the extended logic, called DMPCL+, is still open. Yet the possibility of withdrawal seems to be a distinguishing characteristic common to a wide range of acts whose effects are conventional or institutional, and thus the logical dynamics of acts of withdrawing seems to be of great significance to the study of social interactions among rational agents.

The School

Tutorials

Consistency

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In classical logic the extended completeness theorem ($T \vDash \phi$ implies that $T \vdash \phi$ for any set of sentences T and any sentence ϕ) is frequently proved by the following two steps:

(CME) Every consistent set has a (classical) model; and

(RAA) If T cannot derive ϕ, then $T \cup \{\neg\phi\}$ is consistent (for any set T and any sentence ϕ).

Sometimes the former statement, as a major step of this approach, is called the extended completeness theorem (or the strong completeness theorem). This is not always a correct name of it because there are some non-classical logics satisfying CME, the classical model existence property (which means that consistency implies classical satisfiability).

In this course we investigate this meta-logical property CME. Since there are many different consistencies and in non-classical logics they are not always equivalent, we will study consistency first. (Note that with different consistencies the meaning of CME could be different.) Then we present ways to construct non-classical logics/proof systems (by selecting axioms or rules) which satisfy CME, and discuss how one can construct a weaker system still satisfying CME. These (propositional or predicate) logics include some (weak extensions of) paraconsistent logics, subintuitionistic logics, or substructural logics. Applications of CME include Glivenko-style theorem and pure implicational logic. Furthermore, we will also analyze the necessary-and-sufficient condition of CME (which is related to Left Resolution Gentzen system in [3]) and discuss other model existence property.

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Erotetic Logics

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The term *erotetic logic* is often understood as synonymous to *t*he logic of questions. There is no common agreement as to what erotetic logic should be. The most developed proposals will be overviewed and compared. Then a general setting, the Minimal Erotetic Semantics (MES), will

be presented; kinds of answers to questions, types of their presuppositions, basic relations between questions, and certain normative concepts pertaining to questions will be characterized in terms of MES. Next, conceptual foundations of Inferential Erotetic Logic (IEL) will be discussed. IEL focuses its attention on erotetic inferences, that is, roughly, inferences which have questions as 'conclusions'. Some of these inferences are intuitively valid; we will show how IEL explicates the relevant concept of validity.

We will also address some more technical issues. First, we will consider models of problem decomposition, offered by Hintikka's Interrogative Model of Inquiry and by IEL. Second, a certain proof method grounded in IEL, the Socratic proofs method, will be presented.

Finally, the idea of erotetic logic as a theory of internal question processing will be discussed.

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Fractals, Topologies and Logic

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This tutorial explores the connection between fractal geometry and topological intentional logic. The main cluster of results we present is a class proofs of completeness of the modal logic S4,and Intuitionist Propositional Logics with respect to the following fractals: Koch Snowflake/Curve, Sierpinski Carpet, Menger Cube, which all are well known classic fractals. The main corollary of the paper is a new proof of the completeness of afore mentioned logics for most significant n-dimensional Euclidean metric topological space R, R^2 and R^3. The latter results were originally obtained by Tarski and MacKinsey in [7], and much simplified and refined by Mints et al. and van Benthem et al. [examples, variations, and refinements can be found in [1], [2] and [4]]. Our new proof uses fractal techniques, that, as we will argue is the main contribution to the topological semantics for modal logic in recent years. The completeness for both Koch Curve and other fractals and R^n are best seen as examples of the power of the fractal techniques introduced here. Another somewhat original contribution is the use of the infinite binary tree with limits, Wilson tree. We prove that such a tree is complete for S4 and related logics in several extended languages. Such proof then facilitates topological transfer onto fractals and metric spaces. Although this tree has not to our knowledge been used previously in the modal logic community, and we have introduced it independently, it has since come to our attention that the tree has been used and named in category theory by Peter Freyd sometime in the late 1980s. [We follow his naming convention in calling the tree Wilson tree. Wilson tree is obtained by a kind of model saturation technique called 'sobering' from the more usual infinite binary branching tree. The name is then derived from the fact that Wilson was the founder of the 12 step recovery program.]

In this tutorial the following topics are considered.

We begin the tutorial by placing topological techniques and the techniques of fractal geometry in the pantheon of various spatial techniques in both universal approach to logic and in the full gamut of formal approaches to reasoning about space, space-time, and spatiotemporal dynamics. In the spirit of the school on universal logic, we will be emphasizing techniques that are portable and universally applicable over particular results and systems. We continue by introducing a class of self-similar fractals and discussing their usefulness in various model-construction techniques in topological modal logic. We look at a series of well-known trees, both finite and infinite discussing their fractal nature and demonstrating their connection with well known fractals, such as for instance, Koch Flake/Curve, Sierpinski Carpet, Menger Cube, and many others. We then explore various topological and logical properties of fractals. We will show that each of the three fractals mentioned, Koch, Sierpinski, and Menger, is in some sense universal for a class of model theoretic or topological objects. A distant topological relative of this result was first proved by Sierpinski in the early 1920s. We will explore logical relevance of this result. In showing the relevance we will use another three called Wilson Tree. This tree–like the infinite binary branching tree–ought to be well known, but it is not. We will explore the immense usefulness of the tree in reasoning about complete metric Euclidean spaces and speculate about the reasons why the tree is not more commonly known among modal (and intensional) logicians. As we mention in the introduction above, after we reinvented the tree, it turned out to be a rather standard object in Category Theory. We will explore some connections between results in category theory by P. Freyd and A. Scedrov and some well known results in topological semantics.

We will conclude the tutorial by looking at the class of intensional logics, (S4, S4u, S4 + time, Int. Logic, etc.) to which the fractal techniques introduced here apply. If we have some extra time, we will look at a recently introduced Probabilistic Modal Logics of Dana Scott.

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Geometry of Oppositions

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n-opposition theory (2004) generalises Aristotle's opposition theory (exemplified by the *logical square*) by introducing the notion of *logical bi-simplex of dimension n*.

This theory, relevant to both quantification theory and modal logic (both are tied to the logical square) shows that there exists a field, between logic and geometry, where logical-geometrical n-dimensional solids (highly symmetrical structures whose edges are implication arrows), instead of being limited to the square (the poorest and ugliest of them), develop into infinite growing orders according some relatively simple but generally unknown principles.

This field is related to modal logic, in so much such n-dimensional structures can be decorated (as can the square) with arbitrary modalities via some suited decorating techniques (as the *modal n(m)-graphs*, or the *setting-method*, for instance).

The theory's known applications, so far, concern mainly the study – from a new geometrical point of view – of the known modal systems (normal or non-normal, abstract or applied), but also the study of the non-logical formalisms inspired by the logical square in psychology (cognitive science and psychoanalysis), linguistics (semiotics and pragmatics), philosophy (analytical as continental) and others.

In this tutorial we will introduce to the theory of n-opposition, showing (rapidly) its historical roots (Aristotle, Vasil'ev, Sesmat, Blanché), then concentrating both on its classical (recently discovered) tenets and on its actual (open) research issues. At the end of the tutorial, according to the time left, we will discuss of some of this theory's possible applications.

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Graded Consequence

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Vagueness being an important area of investigation in many fields (e.g. logic, philosophy, computer science) nowadays, various logics have emerged during the past few decades to address the topic e.g., fuzzy logics, rough logics and the theory of graded consequence. But in most of the existing literature, be it of logic or philosophy or computer applications vagueness is thought to occur at the level of object-language only. The metalogical notions are usually considered to be crisp. There was a claim made by Pelta in [11] that many-valuedness at the metalevel had been first explicitly considered by Marraud in [10]. But this claim is incorrect. There is a series of papers viz. [2,3,4,5] dealing with the theory of gradation of metalinguistic concepts like 'consequence', 'consistency', 'tautologyhood', 'completeness' and the like. A Gentzen-type axiomatization for the consequence relation was introduced in [2] as follows:

A graded consequence relation $\vdash\!\sim$ is a fuzzy relation from $P(F)$ to F satisfying

i. if $\alpha \in X$ then $\mathrm{gr}(X \vdash\!\sim \alpha) = 1$ (overlap)

ii. if $X \subseteq Y$ then $\mathrm{gr}(X \vdash\!\sim \alpha) \leq \mathrm{gr}(Y \vdash\!\sim \alpha)$ (dilution)

iii. $\inf_{\beta \in Z} \mathrm{gr}(X \vdash\!\sim \beta) \wedge \mathrm{gr}(X \cup Z \vdash\!\sim \alpha) \leq \mathrm{gr}(X \vdash\!\sim \alpha)$ (cut)

where $P(F)$ is the power set of F, the set of well formed formulas, $X, Y \in P(F)$, $\alpha, \beta \in F$ and $\mathrm{gr}(X \vdash\!\sim \alpha)$ is an element of a Heyting algebra giving the 'grade' to which α is a consequence of the premise X.

The notion was further generalized in [5] by the use of the product operator '$*$' of a residuated lattice as the and-ing operator instead of the usual lattice meet ($\wedge$). In the same paper, the notion of graded consistency was introduced. The idea of partial consistency or consistency to a degree is frequent in everyday encounters. The axioms of graded (in)consistency are taken as:

i. if $X \subseteq Y$ then $\mathrm{INCONS}(X) \leq \mathrm{INCONS}(Y)$

ii. $\mathrm{INCONS}(X \cup Y) * \mathrm{INCONS}(X \cup \{\sim \beta\}) \leq \mathrm{INCONS}(X)$ for any $\beta \in Y$

iii. $k \leq \mathrm{INCONS}(\{\alpha, \sim \alpha\})$ for any wff α and some k, $0 < k$

where INCONS(X) is an element of the residuated lattice representing the 'degree' of inconsistency of the set X.

The interface between the notions viz. graded consequence and graded consistency has been investigated.

Although in Pavelka's seminal work [1] there are traces of these ideas, the main theme of his work being different, the above issues did not surface. Pavelka axiomatized consequence operator C in the Tarskian tradition in the following way:

   i. $X \subseteq C(X)$

   ii. $X \subseteq Y$ implies $C(X) \subseteq C(Y)$

   iii. $CC(X) = C(X)$

where X is a fuzzy subset of premises and $C(X)$ is the fuzzy subset of consequences of X.

The question, however, is

"What does the value $C(X)(\alpha)$ indicate? Is it the degree to which α follows from X? That is, is it a value of the process of derivation or is it a value that is assigned to α through the derivation process? C is after all a crisp function on the domain of fuzzy subsets."

The conceptual difference of the two makes significant difference in organizing a comprehensive theory of vagueness. Those who have followed and further developed the line of thought of Pavelka e.g. Hajek [9], Novak [6], Godo et al. [9] etc. have not taken up this issue either, except perhaps Gerla [7] who devoted a section on graded consequence in his book and Gottwald [12] who moved closer to this idea.

A comparison between Pavelka's approach and the approach of graded consequence and a scrutiny of the two reveal that

   1. these are not identical methods,

   2. there are confusions and arbitrariness in the usage of algebraic operators for evaluating values of meta-level expressions,

and more importantly,

   3 three levels are involved in the discourse of such logics, viz.

      a) object level, comprising of well formed formulae with terms interpreted generally but not necessarily, as vague notions;

      b) meta level I, required to talk about object level items, the predicates etc. of this level being in general vague as well;

      (a) meta level II, required to talk about and/or define the notions of meta level I.

These three levels are blurred in the discourses.

The present tutorial is an attempt to remove these confusions by

- clarifying the three levels,

- using appropriate symbolisms (viz. quotation mark ' ' for naming at level I and quotation mark ⌜ ⌝ for naming at level II) to distinguish the levels,

- identifying the predicates, functions, connectives and quantifiers required at each level,

- probing into the interrelation, both syntactic and semantic, between the levels.

The set of axioms for graded consequence under such symbolism takes the form

   i. $\mathrm{gr}\ulcorner{}'\alpha' \in {}'X'\urcorner \leq \mathrm{gr}\ulcorner{}'X'\mid\sim {}'\alpha'\urcorner$

   ii. $\mathrm{gr}\ulcorner{}'X' \subseteq {}'Y'\ \&\ {}'X'\mid\sim {}'\alpha'\urcorner \leq \mathrm{gr}\ulcorner{}'Y'\mid\sim {}'\alpha'\urcorner$

   iii. $\mathrm{gr}\ulcorner{}''v(v \in {}'Z' \to {}'X'\mid\sim v)\ \&\ {}'X \cup Z'\mid\sim {}'\alpha'\urcorner \leq \mathrm{gr}\ulcorner{}'X'\mid\sim {}'\alpha'\urcorner$

for all 'α', 'X', 'Y', 'Z'.

The expressions with the mark ' ' are terms of meta level I while expressions with the mark ⌜ ⌝ are names of wffs of level I in level II.

Axioms of graded inconsistency can also be presented similarly:

1. $\text{gr} \ulcorner \text{`}X\text{'} \subseteq \text{`}Y\text{'} \,\&\, \text{INCONS}(\text{`}X\text{'}) \urcorner \leq \text{gr} \ulcorner \text{INCONS}(\text{`}Y\text{'}) \urcorner$

2. $\text{gr} \ulcorner \text{INCONS}(\text{`}X \cup Y\text{'}) \,\&\, \text{INCONS}(\text{`}X \cup \{\sim \beta\}\text{'}) \urcorner \leq \text{gr} \ulcorner \text{INCONS}(\text{`}X\text{'}) \urcorner$ for any 'β' $\in$ 'Y'

3. there is some k, $0 < k$, s.t. $k \leq \text{gr} \ulcorner \text{INCONS}(\text{`}\{\alpha, \sim \alpha\}\text{'}) \urcorner$ for any 'α'.

Names of (truth) values such as ⌜k⌝ or ⌜0⌝ are also required at meta level II but dropped for simplicity in writing the above axioms. Similar is the case for the sign $\leq$. The complete list of symbols used at each level shall be presented in the full text of the paper.

Besides dispelling of above mentioned confusions, this study helps in the assessment of the two approaches viz. [1] and [2] with respect to the incorporation of degrees at the meta levels. At the metalevel II, all the notions are crisp. The study also suggests a general methodology for construction of logics with several tiers in which sentences of one tier 'speak' about objects of the next lower tier.

The tutorial is planned as follows :

Session 1 : Introducing the notion.

Session 2 : Comparison with Fuzzy Logics.

Session 3 : Illustrative examples with indications towards applications.

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How to Cut and Paste Logical Systems

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Combination of logics is still a fairly young subject. It arose from the study of some particular cases especially connected with modal logics, through the techniques of fusion, product and fibring. This was a first stage of the development of this new field of research, and afterwards the initial techniques (mainly fibring) were generalized to general logics. The techniques for combining logics help to the study of some fundamental phenomena of logic which are still not properly understood, connected to what a logic is and what are the relations between different formulations of a given logic. This tutorial, based on the book [2], is mainly devoted to the study of the so-called categorial fibring (or algebraic fibring), introduced in [1] and later on generalized to a wide class of logic systems such as modal (first-order) logics, higher-order logics and non-truth-functional logics, among others (see [2]).

The main topics to be analyzed herein are the following:
1. Fibring syntactically: The Hilbert calculi case
2. Fibring semantically: Interpretation systems and their fibring
3. Preservation results: Completeness and interpolation preservation by fibring
4. Heterogeneous fibring: Combining abstract proof systems
5. One step ahead: Fibring first-order (modal) logics
6. Still more generality: Fibring higher-order (modal) logics and non-truth-functional logics
7. The future: Graph-theoretic fibring

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Hybrid Logic

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These lectures introduce hybrid logic, a form of modal logic in which it is possible to name worlds (or times, or computational states, or situations, or nodes in parse trees, or people - indeed, whatever it is that the elements of Kripke Models are taken to represent).

The course has three major goals. The first is to convey, as clearly as possible, the ideas and intuitions that have guided the development of hybrid logic. The second is to introduce a concrete skill: tableau-based hybrid deduction. The third is to say a little about the history of the subject, and to link it to the philosophical work of Arthur Prior. No previous knowledge of hybrid logic is assumed.

The lecture outline is as follows:
Lecture 1: From modal logic to hybrid logic
Lecture 2: Hybrid deduction
Lecture 3: The Priorean perspective

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Ideospheres

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This tutorial will present formal structures which often structure the development of ideospheres in Human Sciences. An ideosphere (Barthes77) is initiated by a founding speech and helps establish correspondences to other such speeches and take commitment and refusal positions in a system that is unachieved. In pedopsychiatry, an ideosphere is focus on early interactions between the infant and his environment, and examine the processes of semiotization, as well as the use of representation abilities as a means to communicate (Golse 99,07) (Dor 02).

These structures and their organization within a general system can be formalized with category theory, as is done for instance when modelling computation systems and relating different models. We show under what conditions they correspond to a formalization within modal logic of the system in use; at this point we will make a comparison with what is done with categorial models which relate various logics. Finally we develop the concepts of autonomy and learning, and use them to illustrate the presented mathematical tools and methods; this will help model zig-zag processes between various formalizations.

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Instantiations

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Instantiation is an operation essential for performing logical reasoning, it is placed in the heart of its basis, when dealing with universal and existential quantifiers. In many logics it must appear explicitly in at least three of the four laws of introduction and elimination of quantifiers.

In spite of it, the study and presentation of this operation has been underestimated in most books on logic, usually only half of a page is dedicated to it. Beyond that, when it is presented, it is often wrongly confused with a particular case of another operation, named here replacement, which is used on formulation of some laws related to equivalence and equality. While replacement of a variable by a term considers all occurrences of variables which do not succeed a quantifier or a qualifier, instantiation consider only free occurrences of this variable; the former does not rename bound variables, while the latter, for a good working, needs renaming of bound variables in many of its forms.

By consequence of lack of carefulness in dealing with instantiation, when there is no renaming of bound variables, the presentation of the basic laws of introduction and elimination of quantifiers needs some patches, and these ones become more difficult reasoning perform from this point on.

In this tutorial all possible alternatives of instantiation are presented, and one of them, maybe the most promising one, is presented in detail, in order to make easier reasoning perform inside a logical system.

Institutions

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Institution theory is a major model theoretic trend of universal logic that formalizes within category

theory the intuitive notion of a logical system, including syntax, semantics, and the satisfaction relation between them. It arose within computing science, especially specification theory [1], as a response to the population explosion of logics there and where it has become the most important foundational theory. Later on institution theory has been successfully used in pure logic studies in the spirit of universal logic. This means the development of model and proof theory in the very abstract setting of arbitrary institutions, free of commitment to a particular logical system [2]. In this way we gain freedom to live without concrete models, sentences satisfaction, and so on, we gain another level of abstraction and generality and a deeper understanding of model theoretic phenomena not hindered by the largely irrelevant details of a particular logical system, but guided by structurally clean causality. The latter aspect is based upon the fact that concepts come naturally as presumed features that a "logic" might exhibit or not and are defined at the most appropriate level of abstraction; hypotheses are kept as general as possible and introduced on a by-need basis, and thus results and proofs are modular and easy to track down regardless of their depth. The continuous interplay between the specific and the general in institution theory brings a large array of new results for particular non-conventional, unifies several known results, produces new results in well-studied conventional areas, reveals previously unknown causality relations, and dismantles some which are usually assumed as natural. Access to highly non-trivial results is also considerably facilitated. The dynamic role played by institution theory within the wider universal logic project is illustrated by the fact that institution theory papers have come second and first, respectively, in the contests of the Montreux (2005) and Xi'an (2007) UNILOG, respectively.

In this tutorial we will start with a brief explanation of the historical and philosophical origins of institution theory, followed by a presentation of its basic mathematical concepts. We will also have a trip through the rather rich body of methods and results of the institution theoretic approach to logic and model theory. Although institution theory is primarily a model theoretic approach we will also discuss recent proof theoretic developments in the area. However our real emphasis will be not on the actual mathematical developments but on the non-substantialist way of thinking and the top-down methodologies promoted by institution theory, that contrast sharply the substantialist view and the bottom-up methodologies that pervade and underlie conventional logic, this being the most profound message of institution theory as a universal logic trend.

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Kripke's World

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Possible worlds models have been introduced by Saul Kripke in the early sixties. Basically they are graphs with labelled nodes and edges. Such models provide semantics for various modal logics such as temporal logics, logics of knowledge and belief, logics of programs, logics of action, logics of obligation, as well as for description logics. They also provide semantics for other nonclassical logics such as intuitionistic logics and conditional logics. Such logics have been studied intensively in philosophical and mathematical logic and in computer science, and have been applied in various domains such as theoretical computer science, artificial intelligence, and more recently as a basis of the semantic web. Semantic tableaux are the predominant reasoning tool for all these logics: given a formula (or a set of formulas) they allow to check whether it has a model or not. A tableau calculus uses a set of rules in order to build trees, and more generally graphs. If a tableau graph is contradiction free then it can be transformed into a model for the given formula.

The aim of the tutorial is to provide a step-by-step introduction to modal logics, both in terms of Kripke models and in terms of semantic tableaux. The different logics will be illustrated by means of examples. We will use the generic tableaux theorem prover Lotrec (http://www.irit.fr/Lotrec), which is a piece of software that allows to build models, check whether a given formula is true in a model, and check whether a given formula is valid in a given logic. Lotrec also allows to implement tableau systems for new logics by means of a simple interactive graph-based language accessible to users that are not computer scientists. The tutorial requires basic mathematical and

logic background (the basic definitions of graph theory, and the bases of classical propositional logic).

Contents:

1. graphs, Kripke frames and Kripke models; model checking;

2. the basic modal logic K and its tableaux; soundness, completeness and decidability;

3. description logics; the basic modal logics (KD, KT, S4, S5,...);

4. modal logics with transitive closure (PDL, logic of common knowledge).

Logics of Empirical Sciences

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In this tutorial, we present a general discussion on the relevance of the logical analysis of empirical theories, which is identified with the axiomatization of the relevant theories, giving special emphasis to physics. The concept of Suppes Predicate is introduced and some case studies are presented. The role of the 'background' set theory used for defining the predicate is discussed. In the last part, a particular topic involving quantum mechanics plus a metaphysics of non-individual entities is discussed.

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Logical Pluralism

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Up until the end of the 19th century, logic was typically seen as the art and science of correct reasoning, and in this sense it was not specific to any discipline or subject-matter (i.e. logic was seen as topic-neutral); moreover, even if there might have been competing systems, the general opinion was that there should be only one true logic. True enough, at different times the scope of logic went very much beyond correct reasoning in the sense of drawing inferences strictly speaking: in the Latin medieval period, for example, what we now call semantics and much of what we now call epistemology also belonged to the realm of logic. Still, even though there were different logical theories for different applications, when it came to reasoning, syllogistic inference remained the canon of correct reasoning for almost 2.500 years.

A bit over a century later, the status of logic as a discipline has changed dramatically: we now have different logics, specially designed for certain situations, topics or tasks – what one

could describe as a situation of 'logical plurality' – and the idea that there is only one correct way of reasoning and thus only one correct logic is no longer unanimously accepted - what is now referred to as the position of *logical pluralism*. In this context, a universal logic seems to be a welcome development: it should allow for the comparison between systems, and for the arbitration of the disputes between competing systems. However, this contemporary plethora gives rise to certain philosophical questions concerning logical pluralism and universality, in particular the kind of universality claimed by universal logic. The tutorial will look at logical pluralism against the background of the history of logic, and tie it to the recent debate started by Beall and Restall's book *Logical Pluralism* (2006, OUP). We will then look at a number of reactions to logical pluralism in the literature, and try to connect some of the questions to the project of Universal Logic.

Session (1) The Emergence of Logical Plurality and Logical Pluralism
Session (2) Contemporary Logical Pluralism
Session (3) Reactions to Logical Pluralism

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Logics of Plurality

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The correct logical analysis of plural terms such as the trees in the trees are similar or the trees are green is at the center of an important debate both in formal semantics and in philosophical logic. Two fundamentally distinct approaches can be distinguished, one on which the trees refers to a single collective entity, a plurality of trees, and one on which the trees refers plurally to the various individual trees. The first tradition is linked to the work of Link and related mereological approaches, the second to the work of Boolos and subsequent work in that tradition (Oliver, Yi, Rayo and others). This course will give an overview over the two kinds of approaches to the logical analysis of plural terms with its various developments and discusses the crucial linguistic empirical and conceptual motivations for the two kinds of approaches.

Session 1:
Reference to a plurality: The mereological approach
This session discusses the motivations and the development of the mereological approach such as that of Link and others. It presents a range of potential empirical and conceptual problems for that approach.
Session 2:
Plural Reference: The second-order approach
This session will discuss the seminal work of Boolos and subsequent developments such as the work of Oliver, Rayo, Yi. It focuses on the formal and conceptual aspects of that approach.
Session 3:
This session discusses potential extensions of the second approach, such as to to mass terms like courage, as in *courage is admirable*. It also discusses various ramifications of the plural reference approach and the challenges it faces from the point of view of natural language.

Natural Deductions

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The theory of natural deduction of pure intuitionistic logic for the $(\rightarrow, \wedge, A)$-fragment is very elegant. It includes a strong normalization theorem and a Church-Rosser property. This elegant treatment extends to pure second-order intuitionistic logic (Girard's polymorphic system F). The technical details are more complicated in this case because of the impredicativity of the second-order quantifier, but strong normalization (and Church-Rosser) still holds.

The other connectives (absurdity, disjunction, existential quantification), whose features are more typical of intuitionism, do not have such an elegant treatment. Girard sees their elimination rules as defective. In second-order logic, these connectives can be circumvented because they are definable in terms of the others. It is not widely known that these connectives can also be circumvented in the treatment of first-order logic if we embed it in a version of predicative second-order logic.

Our tutorial will explain these issues.

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Probabilistic Logic

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Logic is traditionally understood to be the study of *what follows from what* and probabilistic logic is no exception. But whereas the entailment problem typically asks whether a conclusion C is entailed by some premises $P_1, ..., P_n$, with respect to some specified entailment relation , written as $\vDash$

(1) $P_1, ..., P_n \vDash C$?

on our approach (Haenni et al. 2009) to probabilistic logic poses a slightly different question. In a probabilistic setting, attached to each premise P_i is a probability or set of probabilities, X_i. But it is rare to ask whether a set of probabilistic premises entails that a particular probability (set of probabilities) Y is assigned to C. Instead, the entailment problem in probabilistic logic typically concerns what probability (set of probabilities) are assigned to a particular conclusion, C, written as

(2) $P_1(X_1), ..., P_n(X_n) \vDash C$?

A surprisingly wide variety of probabilistic semantics can be plugged into (2), thereby providing different semantics for the entailment relation $\vDash$ and allowing those different systems to be studied as bona fide logics. In this respect we view (2) as the fundamental question of probabilistic logics and view it as the lynch pin to out proposal for unifying probabilistic logics. As for how to answer the fundamental question, we propose a unifying approximate proof procedure utilizing credal networks, which are probabilistic graphical models analogous to Bayesian networks but configured to handle sets of probability functions representing interval valued probabilities.,

This course introduces the progicnet framework for unifying probabilistic logic in three parts:

Day 1. An introduction to the fundamental question for probabilistic logic and an introduction to the most basic semantics, our generalization of the standard semantics of 'Bayesian probabilistic logic', as put forward in Ramsey (Ramsey 1926) and De Finetti (de Finetti 1937), and explicitly advocated by Howson, (Howson 2001, 2003), Morgan (Morgan 2000), and Halpern (Halpern 2003), to handle interval-valued probability assignments via sets of probabilities.

Day 2. An introduction to a semantics for handling relative frequency information, 'Evidential Probability' (EP) (Kyburg 1961, Kyburg and Teng 2001, Kyburg et al. 2007, Wheeler and Williamson 2009). EP is traditionally thought of as a logic of probability rather than a probabilistic logic (Levi 2007), and our framework helps to explain why this is so. Time permitting we will introduce some extensions to EP that utilize different features of the progicnet framework.

Day 3. An introduction to credal networks (Levi 1980, Cozman 2000) and approximate proof theory we develop with this machinery. Several open problems will be presented.

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Quantum Logic

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In this tutorial we start by reviewing the relevant concepts and results in linear algebra and operator theory. We also present Dirac notation and the postulates of quantum mechanics.

Then, we motivate he notion of orthocomplemented lattice and present the main results concerning Birkhoff and von Neumann quantum logics.

Moved by the emergence of new practical applications on quantum information and computation, we survey modern approaches to reason about quantum systems.

We focus on two approaches: exogenous quantum propositional logic (EQPL) by Mateus and Sernadas; and probabilistic modal logic (PML) by van der Meyden and Patra. We present several results concerning EQPL such as: complete Hilbert calculus; complexity of the model-checking and SAT problems; and how to deal with dynamic extensions. We illustrate both EQPL and PML by reasoning about quantum security protocols.

Finally, we briefly discuss how to extend a logic into a quantum logic.

Refutation

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Refutation systems are inference systems, just like traditional axiomatic systems, but they generate non-valid formulas rather than valid ones. They consist of refutation axioms (which are non-valid formulas) and refutation rules (which are rules preserving non-validity).

In this tutorial the following topics are considered.

- Examples of syntactic refutations in non-classical logics, and a general theory of refutation systems.

- Refutation systems and other standard methods (sequent systems, tableau procedures, model building constructions, the finite model property).

- Tools and techniques for proving syntactic completeness (characteristic formulas of finite algebras, normal forms and inductive completeness proofs).

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The World of Possible Logics

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Many logics are possible: from a logic where nothing is a consequence of nothing, to a logic where everything is a consequence of everything passing through a whole spectrum including classical logic, many-valued logic, turbo polar linear logic.

One can wonder if we are not then facing a wild jungle, from which some monsters like anti-classical logic, the complement of classical logic where a proposition is not a consequence of itself, have to be rejected.

In this tutorial we will discuss these questions. We will also present methodologies to construct logics and frameworks to compare them.

Many examples and concepts will be provided.

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Truth-Functionality

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In research areas so diverse as model theory, philosophy of language, formal and computational linguistics, algebraic logic, and the denotational semantics of programming languages, a common widely accepted meta-theoretical compositionality principle is to be found according to which the denotation of a complex expression is built up from the denotations of its parts. In proof theory and automated reasoning, also, two frequent ways in which such a structural connection is displayed between the whole and its parts, or between the premises and their conclusions, resides in the so-called subformula property and its connections to cut-elimination and interpolation, as well as in the notion of analyticity of proof formalisms and strategies. One of the most straightforward ways of realizing such principles, notions and properties is exactly through the intuitive and well-known notion of **truth-functionality**.

The first session of the tutorial will directly explore the characterization of truth-functionality from the viewpoint of Universal Logic. Abstract properties defining (single-conclusion) consequence relations that have adequate semantical counterparts in terms of truth-tables will be surveyed, and non-truth-functional LOGICS will be illustrated and carefully distinguished from logics that are still truth-functional but turn out to be circumstantially characterized by way of non-truth-functional SEMANTICS. Characterizability of logics by finite collections of truth-values and operators will also be touched upon.

The second session of the tutorial will be devoted to a reexamination of many-valued logics and fuzzy logics as inferential mechanisms based on truth-functionality. The so-called Suszko's Thesis, according to which a distinction can be made among 'algebraic' truth-values, on the one hand, and 'logical' values on the other, will be explained, and a constructive approach to the result, as applied to logics characterized by finite-valued truth-functional semantics will be exhibited. Applications to proof theory and rewrite systems, including generalizations of both the subformula property and the analyticity requirement, will next be illustrated.

The final session of the tutorial will recall some basic semantical results connected to functional completeness and pre-completeness, as well as to maximality of logics with a so-called standard truth-tabular semantics. Generalizations of truth-tabularity by the consideration of agents that behave truth-functionally in their own quarters but that appear not to do so when combined by way of a so-called *society semantics*, other *non-deterministic* versions of truth-tabularity, and also some controlled combinations of truth-tabular scenarios by way of the so-called *possible-translations semantics* will also be - Afficher le texte des messages précédents - illustrated, and shown to share some of the good computational behaviors of the standard approach to truth-functionality, in terms of the preservation or the enjoyment of important meta-theoretical properties such as decidability, compactness and modularity.

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Truth-Values

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The notion of a truth value has been introduced into logic and philosophy by Gottlob Frege in two seminal papers in 1891 and 1892. Truth values play a central role in logic and have been put to quite different uses. They have been characterized as:

- primitive abstract objects denoted by sentences in natural and formal languages,

- abstract entities hypostatized as the equivalence classes of sentences,

- what is aimed at in judgements,

- values indicating the degree of truth of sentences,

- entities that can be used to explain the vagueness of concepts,

- values that are preserved in valid inferences,

- idealizations of basic proof-theoretical properties,

- values that convey information concerning a given proposition.

Depending on their particular use, truth values have been treated as unanalyzed, as defined, as unstructured, or as structured entities.

In this tutorial, we will briefly comment on the history of the notion of a truth value and on the various uses of this notion in philosophy and logic. We will then focus on some selected topics including the famous slingshot argument and Suszko's Thesis. The slingshot argument may be

interpreted to call into question the view that sentences denote truth values, and Suszko's Thesis was meant to prove that there are but two logical values, Frege's the True and the False. Moreover, the tutorial will deal with generalized truth values as used in Belnap's useful four-valued logic (FDE) and extensions of FDE.

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