Handbook of the 5th World Congress and School on Universal Logic

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University of Istanbul
Turkey

Edited by
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Introduction

1 – Organizing and Scientific Committees

Organizing Committee

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- Gila Sher, University of California, San Diego, USA
- Vladimir Vasyukov, Academy of Sciences, Moscow, Russia
- Heinrich Wansing, Bochum University, Germany

2 – Aim of the event

This is the 5th World Congress and School on Universal Logic (UniLog 2015), gathering top researchers from all over the world, after

— 1st UniLog, Montreux, Switzerland, 2005;

— 2nd UniLog, Xi'an, China, 2007;

— 3rd UniLog, Lisbon, Portugal, 2010;

— 4th UniLog, Rio de Janeiro, Brazil, 2013.

All aspects of Universal Logic are under examination.
The UniLog 2015 is emphasizing an interdisciplinary perspective.

3 – What is Universal Logic?

In the same way that universal algebra is a general theory of algebraic structures, universal logic is a general theory of logical structures. During the 20th century, numerous logics have been created: intuitionistic logic, deontic logic, many-valued logic, relevant logic, linear logic, non monotonic logic, etc. Universal logic is not a new logic, it is a way of unifying this multiplicity of logics by developing general tools and concepts that can be applied to all logics.

One aim of universal logic is to determine the domain of validity of such and such metatheorem (e.g. the completeness theorem) and to give general formulations of metatheorems. This is very useful for applications and helps to make the distinction between what is really essential to a particular logic and what is not, and thus gives a better understanding of this particular logic. Universal logic can also be seen as a toolkit for producing a specific logic required for a given situation, e.g. a paraconsistent deontic temporal logic.

4 – Call for papers

Participants should submit full versions of their papers by email. People not taking part of the event are also welcome to submit a paper.

All participants are most welcome to send the full version of their talk for publication by September 30, 2015 to istanbul2015@uni-log.org.

All talks dealing with general aspects of logic are welcome, in particular those falling into the categories below. See also the workshops where you can submit your abstract if it is appropriate and the Contest. Participants of the School are also strongly encouraged to submit a contribution.

Tools and Techniques

- consequence operator
- diagrams
- multiple-conclusion logic
- labelled deductive systems
- Kripke structures
- logical matrices
- tableaux and trees
- universal algebra and categories
- abstract model theory
- combination of logics
- lambda calculus
- games

Scope of Validity
Domain of Applications of Fundamental Theorems

- completeness
- compactness
- cut-elimination
- deduction
- interpolation
- definability
- incompleteness
- decidability
- Lindenbaum lemma
- algebrization
- Dugundji's theorem

Study of Classes of Logics

- modal logics
- substructural logics
- linear logics
- relevant logics
- fuzzy logics
- non-monotonic logics
- paraconsistent logics
- intensional logics
- temporal logics
- many-valued logics
- high order logics
- free logics

Philosophy and History

- axioms and rules
- truth and fallacies
- identity
- lingua universalis vs. calculus ratiocinator
- pluralism
- origin of logic
- reasoning and computing
- discovery and creativity
- nature of metalogic
- deduction and induction
- definition
- paradoxes

Part II

5th World School on Universal Logic

5 – Aim of the school

For the 5th edition of this event there will be three kinds of tutorials: some dedicated to history of logic (Aristotle's logic, Jain Logic, Stoic Logic. Leibniz's logic, Kant's logic, Hegel's logic...), some dedicated to the relations/applications of logic to other fields (Logic and Music, Logic and Colour, Logic and Information, Logic and Fiction etc.), some dedicated to some important logical theorems (Gödel's Incompleteness theorem, Compactness theorem, Completeness theorem, Lindström's theorem, etc.)

ASL[1] student travel awards are available:
see http://www.aslonline.org/studenttravelawards.html.

For PhD students, postdoctoral students and young researchers interested in logic, artificial intelligence, mathematics, philosophy, linguistics and related fields, this will be a unique opportunity to get a solid background for their future researches.

The idea is to promote interaction between advanced students and researchers through the combination of the school and the congress (participants of the School are strongly encouraged to submit a paper for the Congress).

[1]Association for Symbolic Logic

6 – Tutorials

Opening Session: Why Study Logic?

This topic will be discussed by a variety of people in a round table animated by J.-Y. Beziau, UFRJ[1] & CNPq[2] (Brazil) / Visiting Scholar of UCSD[3] (USA), organizer of the School of Universal Logic since 2005, with the participation of

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Löwenheim-Skolem Theorem

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First Session: Downward Löwenheim-Skolem Theorem

— Prove the Downward LS for 1st order logic theorem in a countable language via Skolem functions.
— Talk about Mostowski collapse.
— Talk about Skolem paradox.
— Prove theorem for infinite languages (using Skolem functions).
— Introduce $L_{\kappa,\gamma}$.
— Show you need to take into account the size of the formula (i.e. even in countable language there is a sentence of $L_{\kappa,\omega}$ which only has sentences of size κ.
— Use a downward LS for set theory to prove downward LS for sentences of $L_{\infty,\omega}$.

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— Show that in $L_{\gamma^+,\gamma}$ there is a sentence with just equality which only has models of size $> \gamma$. Also show there is a sentence which only has models of size cofinality $> \gamma$ (if $\gamma > \omega$).
— Use downward LS for set theory to prove (a form of) downward LS theorem for $L_{\kappa,\gamma}$.
— Define about absolute logics.
— Give examples.
— Use the same techniques to prove a downward LS theorem for any absolute logic.

Second Session: Upward Löwenheim-Skolem Theorem

— Prove the upward LS theorem for 1st order logic using compactness.
— Define the Hanf number of an abstract logic.
— Show all (set sized) abstract logics have a Hanf number.
— Show the Hanf number of omitting types is $\beth_{\omega_1}$.
— Show the Hanf number of $L_{\omega_1,\omega} is \beth_{\omega_1}$.

Third Session: Downward Löwenheim-Skolem Theorem

First any spill over from the previous lecture.

— Reflection principle in L.
— Proof of GCH from the reflection principle.
— Observe that LS doesn't hold without choice.
— Show that it at least holds for Borel structures (i.e. if there is an infinite structure there is a Borel one on R)
— Chang conjecture.
— Connection to large cardinals.

Bibliography: Forthcoming.

Logic and Politics

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The tutorial includes a brief account of theories about the relations between logic and politics, then three lectures devoted to practical cases, illustrating the use of non-classical logics in political reasoning and public debate.

1. The relations between logic and politics: hypotheses and programs
2. Non-classical logics and politics:
   — Disagreements, gaps and gluts
   — Political pluralism and modal logics
   — Ideology as coherence without truth

Between politics and logics (like between politics and truth) there is traditional foreignness or even enmity. In 'realistic' perspective, political life is alleged to be refractory to logic and rationality, insofar as ruled by powers and interests. In normative political theory, logic is alleged to be damage more than advantage for public life, for instance because the compelling force of logical proofs may promote intolerant and context-insensitive attitudes, so it is hardly adaptable to the needs of political pluralism.

And yet, one may also see that there is close connection between logic, strictly intended as *formal theory of valid inference*, and politics, especially *democratic* politics. Democracy, in John S. Mill's famous definition, is "government by discussion", and human discussions are also if not primarily ruled, for good or ill, by the formal validity of arguments (I can hardly believe that p, if I believe that $p \to q$ and $\neg q$). So it is reasonable to admit that common citizens' and politicians' logical competence is one of the basic features of healthy associated life. But which sort of logical competence? And in which sense logic (as technique of formal validity) intersects politics in a significant way?

The adversaries of logic in political philosophy (see classically H. Arendt, J. Rawls, J. Habermas) generally have a fairly restricted conception of logic: they conceive it as study of classical forms as applied to mathematically-oriented ways of thinking. But this is not all what logic is and can be (like the "universal logic" enterprise is intended to show).

The importance of a certain kind of 'logic' for politics is admitted by some theorists of deliberative or direct democracy (see also the neo-Socratic approach to democracy proposed by M. Nussbaum and A. Sen). But usually, the 'logic' involved in these perspectives is not the formal theory of validity, but informal logic, or theory of argumentation, or critical thinking. All these techniques and disciplines can be useful, but only for a basic education of citizens in classical logic, and they do not seem to capture the real intersection of politics and logic, in the effective practice of political reasoning and arguing.

Some neo-structuralist thinkers (such as G. Deleuze or A. Badiou) have proposed a vision of political facts also including formal considerations, and have tried to apply some logical acquisitions (especially borrowed from structuralism) to a critical analysis of political facts. But these accounts usually have poor or null relation to the contemporary acquisitions of philosophical logic in the analytical tradition.

The tutorial is based on the idea that what is needed for politics (and political theory) is *philosophical logic*, in the current meaning of a series of logical inquiries concerning paradoxes, non-classical conceptions of truth and validity, and the connections between natural language and formal languages. As a matter of fact, it is not so difficult to see that political reasoning, especially in democratic perspective, is most often ruled (and should be ruled) by non-classical logics, and this can be seen in various ways, for instance:

— irreducible public conflicts usually involve under-determined or over- determined cases, so *paracomplete* and *paraconsistent* conceptions of truth may help in dealing with these sorts of conflicts;

— normative disagreements are based on conceptions of how the world is and could be, so logical awareness concerning *modality* — e.g. possible-worlds semantics — is highly helpful in understanding rival normative pictures of facts, saving pluralism while allowing truth-oriented confrontations;

— 'ideology' — in classical Marxian account — is a false system of beliefs that blocks any attempt at modifying reality to meet justice, hence *logical pluralism*, to say a ductile conception of validity (including classically deductive as well as Bayesian and relevant validity), provides a good antidote to ideological blindness.

The three lectures of the tutorial will deal with these three topics. Moving from the illustration of some particular cases, they will give attending people the preliminary elements for reflecting on how new acquisitions of philosophical logic may reverse the traditional judgement about the incompatibility, or enmity, or extraneousness, of logic and politics. Last but not least, they are also intended to suggest that the consideration of the real needs and occurrences of associated life can be heuristically useful for logical researches.

Bibliography (preliminary suggestions):

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Logic and Information

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The tutorial consists of three parts.

A first part covers the contrast between, on the one hand, the naturalness of characterisations of logical consequence in terms of information-containment, and, on the other hand, the perceived redundancy of informational notions in the formal characterisation of logical consequence. As part of this discussion, we review the seminal characterisation of semantic information due to Carnap & Bar-Hillel, Corcoran's information-theoretic characterisation, and informational interpretations of substructural logic.

A second part deals with the influence of the dynamic and interactive turn in logic on how we perceive the relation between logic and information. In particular we consider how a conception of information as a distributed commodity brings many informational or communicative actions within the scope of logic.

In a third part we identify common themes in the study of logic and information, explore new challenges for our thinking about logic and information that are inspired by the philosophy of information and the philosophy of computation.

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1. Patrick Allo, "Logical pluralism and semantic information", *Journal of Philosophical Logic*, vol. **36(6)**, 2007, pp. 659–694.

2. Patrick Allo and Edwin Mares, "Informational semantics as a third alternative?", *Erkenntnis*, vol. **77(2)**, 2012, pp. 167–185.

3. Rudolf Carnap and Yehoshua Bar-Hillel, "An Outline of a Theory of Semantic Information", *The Journal of Symbolic Logic*, vol. **19**, no. **3**, 1954.

4. John Corcoran, "Information-theoretic logic", in *Truth in Perspective*, edited by C. Martínez, U. Rivas and L. Villegas-Forero, Ashgate Publishing, Aldershot, England, 1998, pp. 113–135.

5. Luciano Floridi, *The Philosophy of Information*, Oxford University Press, 2011.

6. Edwin Mares, "Relevant logic and the theory of information", *Synthese*, vol. **109(3)**, 1997, pp. 345–360.

7. Edwin Mares, "The nature of information: A relevant approach", *Synthese*, vol. **175(1)**, 2010, pp. 111–132.

8. José M. Saguillo, "Methodological practice and complementary concepts of logical consequence: Tarski's model-theoretic consequence and Corcoran's information-theoretic consequence", *History and Philosophy of Logic*, vol. **30(1)**, 2009, pp. 21–48.

9. Johan van Benthem, *Logical Dynamics of Information and Interaction*, Cambridge University Press, 2011.

Linström's Theorem

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These lectures present Lindström's theorem, a seminal result in abstract model theory. Lindström's theorem characterizes first-order predicate logic a maximally expressive language that has the compactness property and the Löwenheim-Skolem property. Equivalently, the result shows that any attempt at increasing the expressive power of first-order logic in any way, for example by adding second-order quantifiers, cardinality quantifers, fixed point operators or infinitary connectives, must result in the loss of one or both of two classical theorems about first-order logic: the compactness theorem, or the Löwenheim-Skolem theorem.

This result has both conceptual and practical importance. Conceptually, Lindström's theorem tells us something about what makes first-order logic special, what is essential about first-order logic: the compactness result and the Löwenheim-Skolem theorem are considered two of the most fundamental results in first-order model theory. Lindström's theorem shows that they are, in a sense, *the* fundamental properties of first-order logic: given some conditions on what counts as a "logical system", these properties single out first-order logic uniquely.

On the practical side, there are good reasons to extend the expressive power of first-order logic: the inability of first-order logic to distinguish between infinite cardinalities leads to Skolem's paradox, i.e. although ZFC proves the existence of uncountable cardinals, its axioms are true (if consistent) in a countable universe. Also, the discovery of non-standard models arithmetic casts some doubt on whether first-order logic is suitable as a language for metamathematics. Lindström's theorem tells us something about our options here: it is not possible to remedy these issues without sacrificing either compactness or the Löwenheim-Skolem property.

The main purpose of the lectures is to present the proof of Lindström's theorem, without presupposing familiarity with anything beyond a basic course in first-order predicate logic. The basic concepts of abstract model theory will be introduced, and the model theoretic concepts required for the proof (in particular the technique of "back-and-forth" systems, or Ehrenfeucht-Fraïsse games) will be introduced. We will explain the topological point of view on Lindström's theorem and how it leads to variants for logics without classical negation. If time permits, some additional Lindström-style characterization theorems will be presented: a characterization of the infinitary logic $\mathcal{L}_{\infty\omega}$ due to Barwise, and two Lindström theorems for modal logic due to de Rijke and van Benthem.

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1. J. Barwise, "Axioms for abstract model theory", *Annals of Mathematical Logic*, vol. **7**, 1974, pp. 221–265.

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Logic and Language

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How is meaning possible? What indeed is meaning? Whether we speak Danish, Chinese, Maori, or English, we take for granted that we can express content and that others can grasp it. But what are the mechanisms that lie behind this? What is their structure? What must the world be like if language is to function as a way of communicating information? And — the key question for this course — what, if anything, does logic have to do with any of this?

This course attempts to provide some answers. We will examine how logical representations can be built up compositionally (that is, how word meanings can be plugged together Lego-style into logical representations for entire sentences) and how meanings make inference possible. The interplay of grammar, logic and ontology (in particular, the notion of event) will be emphasised. I will also discuss some basic ideas of what is called dynamic semantics.

Much of the course covers semantics, or literal meaning. But that is only a part — and perhaps not the most important part — of what meaning is. So I will also say something about pragmatics, or how meaning arises when we actually use language. And, needless to say, I will try to explain what logic has to do with pragmatic meaning.

The course is introductory. It is not intended for experts, but for novices, and I will not presuppose any prior knowledge of linguistics. But in order to give even a superficial overview of the topics just sketched, I will have to assume that attendees have at least a basic familiarity with propositional and first-order logic. That is, I will assume that I can write down basic expressions involving first-order quantifiers and the familiar sentential connectives and expect course participants to read and understand them.

I will give make more detailed reading suggestions during the course, and will be make a detailed set of slides available to all course participants. In the meantime, here are a four suggestions for background reading. You will get a lot more out of the course if you read this material in advance.

Preparatory Readings

1. "Discourse Representation Theory", by David Beaver and Bart Geurts, in *Stanford Encyclopedia of Philosophy*. I don't know a better introduction to Discourse Representation Theory (DRT) than this. DRT was the original, and is probably still the most best known, approach to dynamic semantics. Written by two leading semanticists/pragmatists, this article maps out a lot of territory of relevant to this course. Highly recommended.

2. *Meaning and Representation in Natural Language: A First Course in Computational Semantics*, by Patrick Blackburn and Johan Bos, CSLI Press, 2005. This is the most relevant book to browse before the course starts. In particular, if you are unsure of how much first-order logic you know, please check out Chapter 1. And definitely look at Chapter 2. This presents the lambda calculus, perhaps the basic technical tool used to define compositional semantics, and my first lecture will echo the presentation given here. One important remark. This book not only discusses the basic ideas of semantics, it also shows how to implement them (using the programming language Prolog). You do not — repeat not! — need to read the Prolog based material in this book. Indeed you do not — repeat not! — need any programming expertise whatsoever to follow this course.

3. "The Logical Form of Action Sentences", by Donald Davidson, in *The Logic of Decision and Action*, University of Pittsburgh, 1967. Widely available. This was not the first paper on events, but it was one of the most influential: all modern treatments of the topic take it as its starting point. A (slightly dated) classic. Not all of it is relevant to the course, but it has valuable lessons to teach about the role of ontology in inference. Give it a go.

4. "Logic and Conversation", by H.P. Grice, in *Syntax and semantics 3: Speech arts*, Cole et al., 1975, pp. 41–58. Another classic, and you can find it just about anywhere. It provides the background reading for our discussion of pragmatics. Essential.

Gödel's Incompleteness Theorems

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The goal of this tutorial is to introduce, review, and discuss selected concepts that play a key role in Gödel's two theorems on the incompleteness of consistent formal systems of arithmetic (= G1) and their inability to prove their own consistency (= G2). Each lecture will fall into two parts. A first shorter part will briefly review the traditional text book approach to Gödel's theorems and be accessible to everyone with a modest background in logic. As such, these parts double as either a self-contained introduction to or a refresher course in the incompleteness theorems and their proofs. A subsequent second part of each lecture will then introduce and review more recent and more advanced work pertinent to the lecture's topic (as indicated by its title). As such, the second part will require greater fluency in the language and the techniques of mathematical logic.

Lecture 1: Diagonalization, self-reference, and paradox. We start out with Gödel's proof for the fixed-point theorem and end with certain generalization in the framework of category theory.

Lecture 2: Models: weak and non-standard. We start out with some basic model-theoretic considerations of incompleteness and end with the role of cuts for proving G2.

Lecture 3: Provable closures and interpretability. We start out with the roles provable closure under modus ponens and provable $\Sigma 1$-completeness play for proving G2 and end with the question of whether G2 should better be framed in terms of interpretability of theories.

In the spirit of universal logic, some mention of non-classical alternatives will be made throughout the lectures.

Bibliography:

Background readings (free internet resources only):

1. Peter Smith, *Gödel without (too many) tears*.

2. Petr Hájek & Pavel Pudlák, *Metamathematics of First-Order Arithmetic*.

3. Per Lindström, *Aspects of Incompleteness*.

Tutorial readings (will be made available in June 2015):
`http://users.ipfw.edu/buldtb/goedel/index.html`.

Logic, quantum theory and natural language meaning (all in pictures)

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We introduce the idea of a process theory, as developed in the textbook [1]. The mathematical underpinning is entirely diagrammatic, in fact, it's category theory in disguise, although accessible pretty much to anyone who has a brain. Conceptually, it involves a logical stance that focusses on the interactions rather than on the description of the individual. It's successes so far are a high-level conceptual underpinning for quantum theory [2, 1], as well a framework to reason about meaning in natural language, solving the open problem on how to compute the meaning (not just true or false!) of a sentence given the meaning of its words [3, 2, 4].

This course has no prerequisites except maybe a little bit of linear algebra and an open mind, so in particular no background in quantum theory, nor category theory are required. A Long version is annually given at Oxford University as a first course both on diagrammatic reasoning and quantum computing.

After the course you will for example know what quantum teleportation, quantum non-locality, and quantum algorithms are about. And also how powerful diagrammatic reasoning is. You will also understand how meaning of words in natural language becomes meaning of sentences.

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2. B. Coecke, "The logic of quantum mechanics — Take II", 2012, arXiv:1204.3458.

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Dugundji's Theorem

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J. Dugundji's Theorem for Modal Logic is perhaps less popular than his famous result in Topology, but even so highly important. In 1940, Dugundji proves (see [6]) that no modal system between S1 and S5 can be characterized by a single finite logic matrix. Rarely cited in manuals of logic, it was Dugundji's result that boosted the development of new semantics for modal logic, such as R. Carnap's for S5 and S. Kripke's for S2, S3 and S4.

The method used to arrive at such negative result is not original. As pointed out by Dugundji himself, his result was inspired by a previous proof by K. Gödel [8] on a similar theorem concerning intuitionistic propositional logic IPC: Gödel proved that a wide family of logics encompassing IPC could not be characterized by finite logical matrices.

Starting with Gödel, passing through Dugundji's Theorem until arriving to paraconsistent logics, it will be shown that the results in [2, 3] support the thesis that, indeed, a comparatively small number of logic systems can be characterized by finite matrices (although there is an infinite number of many-valued logics). The first part of the present tutorial intends to trace a continuous line between the seven decades distancing these results.

Behind that line there exists, on the one hand, the attempt to generalize the semantics of finite matrices. By means of the notion of Nmatrices, it is possible to (non-deterministically) associate to each formula of a given language a set of possible truth-values, instead of a (deterministic) single truth-value. As proved in [1], Dugundji-like theorems even for Nmatrices persist for some systems of paraconsistent logics. In more precise terms: there is a vast family of paraconsistent systems such that no one of them can be semantically characterized by a single finite Nmatrix.

On the other hand, there have been attempts to generalize Dugundji's result in order to encompass the enormous amount of modal systems that have arisen since the publication of his theorem. Among them, it should be mentioned the results obtained by [9, 7, 5, 4].

The last part of this tutorial will be devoted to analyzing such generalizations of Dugundji's Theorem.

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Aristotle's Logic

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This tutorial on Aristotle's logic begins with a treatment of his demonstrative logic, the principal motivation for his interest in the field [4]. Demonstrative logic is the study of *demonstration* as opposed to *persuasion*. It presupposes the Socratic knowledge/opinion distinction between knowledge (beliefs that are known) and opinion (beliefs that are not known) [1].

Demonstrative logic is the focal subject of Aristotle's two-volume *Analytics*, as stated in its first sentence [8]. Many of Aristotle's examples are geometrical. Every demonstration produces (or confirms) *knowledge* of (the truth of) its *conclusion* for every person who comprehends the demonstration. Persuasion merely produces *opinion*. Aristotle presented a general *truth-and-consequence conception of demonstration* meant to apply to all demonstrations.

According to this conception, a demonstration is an extended argumentation [7] that begins with premises known to be *truths* and that involves a chain of reasoning showing by deductively evident steps that its conclusion is a *consequence* of its premises. In short, a demonstration is a *deduction* whose premises are known to be true. For Aristotle, starting with premises known to be true, the knower demonstrates a conclusion by *deducing* it from the premises. As Tarski emphasized, formal proof in the modern sense results from refinement and "formalization" of traditional Aristotelian demonstration.

Aristotle's general theory of demonstration required a prior general theory of deduction presented in *Prior Analytics* and embodied in his natural-deduction underlying logics. His general *immediate-deduction-chaining conception* of deduction was meant to apply to all deductions. According to this general conception, any deduction that is not immediately evident is an extended argumentation that involves a chaining of immediately evident steps that shows its final conclusion to follow logically from its premises. His theory of deduction recognizes both direct and indirect reasoning, an achievement not equaled before Jaśkowski's 1934 masterpiece [6].

To illustrate his *general* theory of deduction, Aristotle presented an ingeniously simple and mathematically precise *specialized* system traditionally known as the *categorical syllogistic*. With attention limited to propositions of the four so-called categorical forms, he painstakingly worked out exactly what those immediately evident deductive steps are and how they are chained. In his specialized theory, Aristotle explained how to deduce from *any* given categorical premise set, no matter how large, *any* categorical conclusion implied by the given set [10]. He did not extend this specialized treatment in general to cover non-categorical deductions, for example, those involving equations or proportionalities. Thus Aristotle set a program for future logicians, a program that continues to be pursued and that may never be completed.

We will also treat several metatheorems about his basic systems and about various extensions and subsystems. In particular, we show that one-one translation of Aristotle's syllogistic into a certain fragment of modern Hilbertian many-sorted symbolic logic yields a complete match in the following sense. In order for a conclusion to be a consequence of given premises according to Aristotle it is necessary and sufficient for the many-sorted translation of the conclusion to be a consequence of the many-sorted translation of the premises according to Hilbert [5].

If time permits we will review the high points of the vast literature [2] responding to the ground-breaking scholarship produced in the 1970s in Buffalo (NY) and Cambridge (UK) [3, 9].

Time will be set aside for student interaction. Students are encouraged to send questions in advance to the tutor. No prerequisites: knowledge of Greek and symbolic logic will not be needed.

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The Logic of Apuleius and Boethius

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In this course we are going to focus on ancient logic, in particular the contributions made by Apuleius (ca. 124–170 AD) and Boethius (480–524 AD) contributions to its development. As to Apuleius, we shall study the treatise named Peri Hermeneias, which has been attributed to him. As to Boethius, we discuss both treatises on categorical logic (in relation to his double commentary on Aristotle's *De Interpretatione* [5]) and his treatise on hypothetical logic. As it is likely that Apuleius and Boethius are depending on some earlier authors — some less known than them — we are also introducing those authors and their works (either they have survived or not) to recognize a history of this textual tradition. Apuleius and categorical logic: Here we discuss the Apuleian authenticity of the Peri Hermeneias treatise and its particular terminology. We define its main characteristics from a logical viewpoint, and discuss the similarity between this treatise and one written by Boethius (*De syllogismo categorico*). We resolve the issue of a possible textual dependence between Boethius and Apuleius [9], by pointing out a common exposition plan for categorical logic (proto-exposition of categorical logic).

Boethius and his categorical logic: Here we examine the logic of Boethius' exposition on categorical logic by integrating his twin treatises on categorical logic (*De syllogismo categorico* and *Introductio ad syllogismos categoricos*) to his twin commentaries on Aristotle's *De Interpretatione* [2]. We try to define Boethius' modus operandi and sources by discussing whether it is likely a textual guidance of Marius Victorinus, Alexander of Aphrodisias, Galen, Porphyry, Proclus and Ammonius Hermeias. We discuss Shiel's hypothesis [7] that Boethius is taking his comments and explanations from a Greek codex of Aristotle's Organon heavy annotated in the margins. We also state the differences between De syllogismo categorico and Introductio ad syllogismo categorico by adopting a critical view on Marenbon [4] and Thomsen Thornqvist [10, 11] that the latter treatise was a second review of the former one. We discuss the possibility of a categorical syllogistic with indefinite terms and whether Boethius could have had this material for intending to write a further review on categorical syllogistic.

Boethius and hypothetical logic: we study the main contents of *De hypotheticis syllogismis* [6] and whether they come from Stoic hypothetical logic. We defend the Peripatetic origin of Boethius hypothetical logic by defining its particular Peripatetic negation and semantic. We also discuss the unity of Aristotelian logic, once the hypothetical and categorical branches are accepted as originally Aristotelian, and we challenge Łukasiewicz's view that hypothetical logic is the logic of any other logical system [3], by pointing out that hypothetical and categorical syllogistic share a common set of axioms developed in [1].

Finally, we conclude by an overview of categorical and hypothetical Aristotelian logic by showing both its doctrinal and formal unity.

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Logic and Existence (Existence Predicates in Logic)

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The tutorial is divided into three sections:

In the first section, we first recapitulate the steps that led to an existential interpretation of the particular quantifier. Then, we critically evaluate some of Priest/Berto's arguments against such an interpretation. Finally, we also engage some of their philosophical arguments in favor of the introduction of an existence predicate.

In the second session of the tutorial we present some standard examples of logics with existence predicates, viz. the three major families of free logics â€“negative, neuter and positive. We also study the dialogical approach, that provides a very general framework to reformulate those logics and according to which their existence predicates are entirely dispensable and gives a new perspective on the issue of ontological commitment.

In the third session we present another family of examples of logics, which come principally from mathematics and specifically from category theory, in which existence predicates are seemingly needed or, at least, provide the most natural and convenient treatment of certain issues. We extend the dialogical approach of the second session to cover those logics.

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Logic and Nonsense

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"It is the basic idea of [Bertrand Russell's] theory that the division of linguistic expressions into true and false is not sufficient, that a third category must be introduced which includes meaningless expressions. It seems to me that this is one of the deepest and soundest discoveries of modern logic." (Hans Reichenbach, Bertrand Russell's Logic)

Overview: During the early 20th century, the problem of nonsense arose in a number of contexts. For example, Bertrand Russell solved the paradoxical sentences of naive class theory by dismissing them as *meaningless*, while positivists like Carl Hempel made similar assertions concerning any statements not empirically verifiable. Granted the existence of grammatical-yet-meaningless sentences, the question how a theory of deduction should approach such sentences becomes important. It is arguable that the classical Frege-Russell account of logic is ill-equipped to deal with such sentences. For example, there is a case to be made that such sentences are neither true nor false; should such sentences be grammatical while not possessing a truth value, then the semantics of classical logic cannot account for such sentences.

A number of responses to apparently nonsensical sentences within the bounds of classical logic have been proposed. Some — like Rudolph Carnap — have diagnosed all such sentences as artifacts of an "incorrectly constructed language", suggesting that a *category mistake* such as "Socrates is an even number" would ever enter into a "logically perfect language". Others — such as Willard Van Orman Quine — suggested that while such sentences are grammatical, classical logic is equipped to handle them by either treating them as "don't cares" by assigning them truth values arbitrarily, or by uniformly assigning such sentences the value of falsehood. Each camp asserts that no revision of logic is necessitated, on the one hand by diagnosing nonsensical sentences as *ill-formed* and on the other by diagnosing these statements as irrelevant.

But just as many have argued that the existence of nonsensical sentences indeed demands a revision of the classical principles of inference, suggesting that a logic of nonsense provides a natural solution to many of the problems of early 20th century analytic philosophy. It is against this backdrop that logicians such as Dmitri Bochvar and Sören Halldén developed their programs of nonsense logics. Such programs aim to clear up the relationship between deduction and nonsense by examining how logic must be adapted in order to provide a theory of inference that acknowledges nonsensical sentences. Once the need for such a project is conceded, a new host of philosophical questions arise with respect to how to implement such a program: How should an operator “x is nonsense” behave? What should the appropriate generalization of semantic consequence? Is the negation of a nonsensical statement itself nonsensical? It is questions such as these that distinguish the major systems of nonsense logic, such as Bochvar’s Σ and Halldén’s C.

The aim of this tutorial is first to acquaint attendees with the primary philosophical problems giving rise to problems of logic and nonsense and the debates concerning their resolution. Then, we will introduce the most well-known logics of nonsense at the propositional level while examining the philosophical positions that motivate their definitions. Finally, we will review functional and first-order extensions of such systems, placing an emphasis on the relationship between Russell’s theory of types and the theories of predication posited by the proponents of nonsense logics.

Philosophical Topics Concerning Nonsense: We will discuss a number of the philosophical concerns that underlie the worry about the relationship between logic and nonsense. Emphasis will be placed on Russell’s comments on meaninglessness in the *Principia Mathematica* and on the empiricist theories of meaning forwarded by the logical positivists, as well as their respective resolutions to the apparent problem of grammatical-yet-nonsensical sentences. We will then proceed to examine the question of whether a revision of classical, bivalent logic is necessitated in light of such concerns.

Propositional Nonsense Logics: Assuming that a revisions of classical logic is needed in light of nonsensical sentences, we will first examine the philosophical matters that arise when formalizing logics of nonsense at the propositional level. We will discuss the philosophical differences between distinct schools of nonsense logic with respect to challenges such as how to define semantic consequence in this context and how to generalize the propositional connectives of the *Principia Mathematica*. One the basis of these distinctions, we will first discuss the primary propositional logics of nonsense, i.e., Bochvar’s logic Σ and Halldén’s C, drawing on their commentators. Finally, we will consider some of the alternative accounts of logics of nonsense offered by Bochvar and Halldén’s successors.

First-Order Nonsense Logics: Many of the philosophical considerations with respect to nonsense lead naturally to questions concerning predicates, e.g. the notion of a category mistake is precisely the suggestion that some properties do not meaningfully apply to objects of certain types. In this session, we will review some of the accounts of predication and quantification formalized first order logics of nonsense. We will discuss Russell’s theory of types to provide some of the formal apparatus, before discussing the particular theses of Halldén, Goddard and Routley on the matter of predicates and properties in logics of nonsense.

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Hegel's Logic

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The tutorial is an introduction to Hegel's logic from the point of view of the history and philosophy of logic.

The first part is about the meaning of Hegelian dialectic within the history of logic, and focuses on Hegel's own reflections on the relation between his dialectical logic to Aristotle's, Leibniz's and Kant's logic.

The second is devoted to the role of Hegel's logic within the philosophy of logic, considering Hegel's view on some basic concepts such as "logic", "truth" and "validity".

The third focuses on the strictly formal import of Hegel's logic, briefly presenting the main attempts at formalizing Hegel's dialectic, and examining the behaviour of dialectical negation, conjunction and contradiction from a paraconsistent point of view.

First Session: Hegel within the History of Logic.

Second Session: Hegel Within the Philosophy of Logic.

Third Session: Formalizing Hegel.

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Lindenbaum Maximalization Theorem

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Lindenbaum’s Theorem plays a small, but essential role within the canonical proof of the Completeness Theorem in logic: Lindenbaum’s Theorem provides maximal consistent extensions of consistent theories and allows, due to the maximality of those extensions, the construction of models.

Usually, Lindenbaum’s Theorem is proved by an application of a set-theoretical theorem, as the Axiom of Choice or Zorn’s Lemma. Thereby, Lindenbaum’s Theorem is a non-constructive method for obtaining special logically relevant objects, similarly to theorems of other disciplines of mathematics such as, for example, Hahn-Banach Theorem in functional analysis, Krull’s Theorem in ring theory et cetera.

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– First Session –

In the first session, we follow Tarski's abstract approach to logic and introduce *deductive systems* and *consequence operations*. In this context, we are able to present the original version of Lindenbaum's Theorem (provable in set theory without the Axiom of Choice).

Due to the abstract approach, we find a great number of versions of this theorem: within logic we find versions of Lindenbaum's Theorem of different logical strength related to specific kinds of logic (as, for example, propositional logic, classical logic or intuitionistic logic). But we also can identify versions of Lindenbaum's Theorem with respect to non-standard interpretation of deductive systems.

In this tutorial we are especially interested in some special versions of Lindenbaum's Theorem, formulated in the abstract context of (slightly generalised) deductive systems, all equivalent to the Axiom of Choice. By these equivalences we have an indirect prove (via the axiom of choice) that the special versions are all equivalent among each other.

– Second Session –

In the second session, we investigate a problem raised by Miller. He demands to re-prove the equivalence of the special versions of Lindenbaum's Theorem, but allows only proofs without an application of the Axiom of Choice.

We discuss the standard set-theoretical approach of Miller and Gazzari to this mathematical problem and provide some partial solutions based on special constructions of deductive systems. From a mathematical point of view, those partial solutions are unproblematic.

But from a more philosophical perspective, we observe a problem: having only an informal description of our restrictions on proofs it is not clear, whether those solutions satisfy our own demand for avoiding the Axiom of Choice or not. There are crucial philosophical questions, which need some (formal) clarification in order to get an adequate criterion for our problem. In particular:

- What does it mean to avoid an axiom in a proof?
- What does it mean to avoid a detour in a proof?
- Which formulae qualify to be a version of Lindenbaum's Theorem?

We sketch our ideas towards a formal criterion for our demands under the light of the philosophical problems and discuss the relationship of this criterion to pure, direct or simple proofs.

– Third Session –

In the third session, we discuss a proof-theoretical approach to find a proof of the equivalence of the special versions of Lindenbaum's Theorem such that the Axiom of Choice is not applied.

We investigate composed proofs of the desired equivalence, which first prove a variant of the Axiom of Choice out of a special version of Lindenbaum's Theorem before inferring another version of Lindenbaum's Theorem. Clearly, such proofs do not satisfy our demands.

But analysing the normalisation of proofs, we will argue that the normal form of the composed proof has changed relevant properties. Our claim is that this resulting proof satisfies any adequate formalisation of our intuitive demands with respect to the application of the Axiom of Choice.

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Jain Logic

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The field of investigation traditionally referred to as 'Indian logic' lies at the junction between theories of knowledge, theories of argumentation and theories of meaning. It consists of the study of persuasive reasoning as a reliable source of knowledge.

In classical India, logicians from different schools, especially Buddhist, Naiyāyika (of Hindu obedience) and Jain logicians, built a common area of discussions within which an agreement on philosophical issues could be achieved. The 11th century CE can be seen as the final stage of a period rich in such debates, period in which a pan-Indian inter-doctrinal consensus on what counts as a canonical presentation of a satisfactory justification was achieved.

But whereas Buddhist and Naiyāyika's conceptions are well known, Jain theses have a marginal position in this general picture of classical Indian philosophies. However, Jainism is an ancient religion and a broad philosophical system whose specificity mainly pertains both to the way it takes into account the context of an assertion and to its interest for more formal issues, especially the structure of an assertion and its impact on truth values. This double specialty makes it worth studying from a contemporary perspective:

First, it is worth confronting Jain classical philosophy, in which the concrete argumentative situation serves as a basis for the semantic notions at stake, with the actual developments in the field of the semantic-pragmatic interface. More precisely, Jain philosophers give sets of rules of application of words (theory of angles of analysis, *nikṣepavāda*) and of sentences (theory of viewpoints, *nayavāda*) in given argumentative contexts. Moreover, they give sets of maxims that one has to take into consideration when the disputants in a debate cannot agree on ontology. And finally, their theory of modes of predication (*saptabhaṅgī*) can be seen as the exhaustive list of types of end-states a process of ideal deliberation might have.

Second, it is worth confronting the Jain conception of logic in terms of interaction between agents with the actual discussions concerning the links between logic and proof in science, because Jain logic can be seen as an example of an original treatment of these links, since Jain philosophers focus on the procedural aspect of inference. In this dynamic, logic is conceived in terms of interaction between agents and is more adapted to empirical situations.

Session 1: Introduction to Jain theory of inference.

Session 2: Introduction to the theory of viewpoints, a set of guidelines for construing an adequate representation of the meaning of a uttered sentence that is especially concerned with the way to deal with existential presuppositions.

Session 3: Case-study of a debate between Jain and Hindu logicians, and comparison with a similar one between Buddhist and Hindu logicians.

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An Introduction to Stoic Logic

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The view expressed in the nineteenth century about Stoic logic, namely that it either copies Aristotle's syllogistic or develops it in a vacuous and pedantic way, was accepted unanimously both in the histories of ancient philosophy and in the works focusing on the history of logic. From the early decades of the twentieth century on, however, given the important advances in the field of formal logic, it has finally become obvious that the Stoic logical system differed essentially from the Aristotelian and should be studied on its own merits. Indeed, the first reactions to the negative appraisal of Stoic logic came mainly from logicians, who were interested in the development of ancient logic and noticed the similarities between Stoic and propositional logic. The articles and books on the Stoics' contribution to logic published since then, have managed to reconstruct in detail the Stoic logical calculus and to show its significance in the history of ancient logic.

These tutorials are an introduction to Stoic logic. My aim is to present the context from which Stoic logic emerged, to give as clearly as possible its basic features, and to assess its importance by comparing it to the Aristotelian syllogistic. No knowledge of ancient Greek is assumed.

- Session 1: The background of Stoic logic.
- Session 2: The Stoic logical system.
- Session 3: Stoic vs. Aristotelian syllogistic.

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Cut-Elimination's Theorem

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Cut-elimination Theorem (CET) is one of the most important results of modern proof theory established in the framework of Sequent Calculi (SC). Gerhard Gentzen introduced SC for Intuitionistic and Classical Logic in his groundbreaking paper from 1934 devoted to Natural Deduction (ND). Although SC was treated originally as a technical tool for investigations on the properties of ND-proofs soon it became of interest in its own, for researchers in proof theory. Since then many variants of SC were devised suitable for dealing with various non-classical logics and formal theories. Also a lot of generalized versions of SC were provided like Display Calculi, Hypersequent Calculi, Many-sided Sequent Calculi. SC was also influential in the field of investigation on automated proof search leading to invention of early forms of Tableau Calculi.

The rule of Cut is in a sense essential for any SC; Gentzen needed it for showing equivalence of SC with Axiomatic Calculi, namely for simulation of applications of Modus Ponens. This is not all - depending on the interpretation of sequents, Cut rule may be seen as expressing transitivity of consequence relation induced by SC, or as encoding the process of using lemmata in proof construction, or even as expressing the principle of bivalence. On the other hand it is welcome not to have it as a primitive rule of SC. Gentzen was well aware of the importance of CET and used it for showing the existence of normal ND-proofs, consistency and decidability of Propositional Intuitionistic and Classical Logic. The list of important consequences of CET may be enlarged with many technical (e.g. automated deduction, interpolation) and philosophical (e.g. analytic proofs, proof-theoretical semantics).

Despite the variety of applications of CET, the methods of proving it are also of interest. The original Gentzen's proof is brilliant yet quite complicated. Since then a lot of other proof methods for CET were proposed. One can divide them roughly into indirect (semantic) and direct (constructive) proofs. The former show how to obtain cut-free proofs just from the beginning, whereas the latter are based on syntactical transformations of proofs either of local (e.g. Dragalin, Schutte) or of global character (e.g. Curry, Buss). Some of the methods were suitably abstracted and generalized in order to apply them in the framework of nonstandard variants of SC, and for many non-classical logics.

The tutorial is rather self-contained but familiarity with standard results of mathematical and philosophical logic may help. It is divided into three parts:

1. We briefly present a history of SC, the main variants of this kind of deduction system, and relations of SC to other systems. We explain the importance of CET and state a number of preliminary results concerning properties of rules of SC.

2. We focus on the methods of proving CET and discuss the differences as well as the scope of applications of them.

3. We take a look at some generalizations of SC and related forms of CET.

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Husserl's Conception of Logic

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Although the logico-mathematical background of phenomenology belongs nowadays to common knowledge about history of early analytic philosophy, the scope of Husserl's solutions to epistemic and epistemological questions raised by the use of symbols in formal systems remains largely to be exploited. At this point, his concept of pure logic developed at the turn of the 20th century is of the greatest significance. Its structuring will be the subject of the tutorial.

The structuring of pure logic is conceived by Husserl as to handle the requirement of an epistemic and epistemological justification of the formal logic that grounds (via axiomatization) the contents of knowledge (objective, ideal, symbolically given) of mathematics. In the Prolegomena (1900), it is stratified on three levels: on one hand, the inferior level of categorical morphologies, on the other hand, the superior level of formal logic subdivided into the theoretical level of objectively valid theories (grounded in the categories and laws of categorical connections of the inferior level) and the meta-theoretical level of ultimate logico-categorical generality (conceived as the science of all theories of the theoretical level); and those three levels are then split transversally into two planes, the one of syntax (or 'apophantics'), the other of semantics (or 'ontology'). The purpose of this tutorial is to provide a formal systematic reconstruction of that structuring of pure logic by putting it into the perspective of the semiotics of the Logical Investigations (1901) — combining thereby Husserl's philosophical work on mathematical logic with the phenomenological theme of intentionality.

After a historical introduction contextualizing the evolution of Husserl's philosophico-mathematical investigations, the tutorial will run on three sessions:

S1. **The Level of Formal Logic.** Systems of axioms and formal manifolds; The problem of imaginary in mathematics and the question of the expansion of axioms systems; The notions of 'definiteness' and 'completeness'.

S2. **The Level of Categorical Morphologies.** The pure morphology of significations and the theory of parts and wholes; Morphologies as theoretical systems; An epistemic-epistemological foundation of formal logic.

S3. **The Structuring of Pure Logic.** Stratification and components of pure logic; The internal linkages of pure logic; From a semiotic point of view.

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Boole's Logic

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George Boole undertook the algebrization of a modified syllogistic logic, extended to conditionals and probabilities. Boolean operators are known worldwide today because of the search engine pathways they make possible electronically. The electronic circuits by which search engine software transact the complex traffic of electronic signals across a circuit switching board are also generally described as Boolean. Boole's logic is worth exploring both as a chapter in the history of nineteenth century logic, in the movement from Aristotelian term logic to a more flexible universal algebraic logic that was to find full fruition only later in the century and beginning of the twentieth century in Gottlob Frege's *Begriffsschrift* and *Grundgesetze der Arithmetik*. Boole and Frege nevertheless have rather different visions both of logic as algebra, which is to say of how in general terms logic should be algebrized, and, more importantly, of the base logic to be cast as an algebraic formalism. Roughly, Boole takes a modified extended and amplified Aristotelian syllogistic term logic as given, and, himself a highly accomplished adept of mathematical algebra, makes use of all the substitution and simplification devices, often unspoken in his own complex derivations, in order to drive inferences of an algebraic logic of categoricals. Boole reinterprets syllogistic+plus in algebraic terms and adapts the tools of arithmetical algebra and trigonometry in his 1847 book, *The Mathematical Analysis of Logic*, and later in his much expanded 1854 treatise, *An Investigation of the Laws of Thought on which are founded the mathematical theories of logic and probabilities*, also known simply as the *Laws of Thought*.

The purpose of this tutorial is to (1) introduce and explain Boole's basic concepts and his model for the reconstruction of syllogistic as an algebra rather than logic of terms with selections from and commentary on Boole's two main books of interest to logicians; (2) compare and contrast Boole's logic with the more familiar functional calculus or predicate-quantificational logic as developed by Frege, by offering a close reading of Frege's unpublished *Nachlaß* essays, translated as, 'Boole's Logical Calculus and the Concept-Script' (written c. 1880–1881) and a revised version of the essay 'Boole's Logical Formula-Language and My Concept-Script' (1882); (3) consider Boole's philosophical interest and importance for contemporary logic, and in particular for such topics in philosophy of logic as the psychologism that seems to be implied by Boole's reference to logic as laws of thought.

Tutorial Sessions:

1. Principles and Mechanics of Boole's Algebraic Syllogistic Logic
2. Frege's Comparison of Boole's Logic with *Begriffsschrift* (1881–1882)
3. Logical and Philosophical Investigations of Boolean Algebras

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Logic and Colour

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Part 1

In the course of history, there have been many attempts to capture patterns of perceptual colour opposition in diagrammatic representations. In the first lecture, we shall trace some of the history of these attempts and argue that the postulated patterns of opposition between cardinal colours — represented by such lexical items as red, green, blue, yellow, black, white, magenta and cyan, though dated in several respects, are in their basics sursprisingly similar to the relational pattern that has been proposed between the logical operators that define predicate logic, represented by the lexical items all/every, some/any, no. We shall see that some of the historical debates in both domains were actually variants the same discussion about two different realms of the natural language lexicon.

Part 2

In this lecture we turn to the formal evidence showing that Wittgenstein's intuition about a logic of colour relations is to be taken near-literally. We will show with a Smessaert-type bitstring algebra that definitions for logical operators are transferable to basic colour categories and describe relations such as those between complementary colours, etc. in formal detail. We will go into linguistic data, where the pattern imposes a distinction between natural and non-natural lexicalization (such as *nand and *nall in the lexis of logic; cyan and magenta in the field of colour terms).

Part 3

In the third part, we will argue in favour of an internalist view on colour and on what are called linguistic functional categories. It will be shown that the pattern established is extendable to other functional domains such as morphological tense distinctions in English, person and number, as well as to several non-functional categories [11].

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Kant's Logic

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The tutorial will give an introduction into the main features of Kant's formal and transcendental logic, their modern formalization, and their impact on the development of logic.

Logic had been, according to Kant, on a "sure path of a science" since antiquity, but had not reached its pure scientific (systematic) form, being often rhapsodical, and non-systematically mixed with other kinds of knowledge. Kant's program is to give logic a systematic form, founded on the first principles of our knowledge, and that in two steps: (1) reduction of basic logic to formal logic by means of a functional account of logical forms; (2) establishing of a logic of knowledge ("transcendental logic") on the basis of formal logic and analytic of space and time, with independent verification of logical forms and concepts on a fixed (empirically defined) model.

According to Kant's functional account, logical forms should be conceived in the sense of "bringing different representations under a common one" by means of abstract acts (operations) of our faculty of understanding (formal apperception, consciousness). For example, it will be shown how for Kant categorical, hypothetical and disjunctive judgments (as well as their modal counterparts: problematic, assertoric and apodictic judgments) gradually strengthen the conditions of bringing our representations under the formal objective unity of apperception, and in this way gradually implement logical laws. The foundational approach to Kant's logic as based on the theory of formal unity of apperception was developed in a seminal work in [18].

By means of modern formalization, it will be shown that Kant's formal logic has features of paraconsistent and paracomplete logic (see [11], together with an axiomatization of the propositional part in [17]). A formalization of Kant's logical forms and formal unity of apperception with the use of geometric logic and geometric implication is elaborated in [6].

Kant's transcendental logic is a sort of philosophical logic to which, according to Kant's view, formal ontology should be reduced. It will be shown that Kant's transcendental logic includes elements of model theory and type theory, by means of which he solves, for instance, cosmological antinomies. In this part, we will build especially on the ideas in [20].

Session 1. Kant's formal logic. Functional account of logical forms. Concept, judgment, inference. Relation of judgment (condition of objectivity — assertion): subject — predicate, antecedent — consequent, whole — members. Laws of non-contradiction, sufficient reason, and excluded middle. Modalities. Foundations of logic: formal unity of apperception (analytic, synthetic).

Session 2. Formalization of Kant's predicate logic by means of modal logic and generalized quantifiers (formal system and semantics). Paraconsistency and paracompleteness in Kant's logic.

Session 3. Transcendental logic. A priori — a posteriori, analytic — synthetic. Categories and transcendental ideas, transcendental and empirical reality, antinomies. Formal system and empirical model, type-theoretical distinctions in Kant's transcendental logic. Influences on the posterior history of logic (e.g., Frege, Hilbert, Brouwer, Gödel).

The tutorial is self-contained, not presupposing anything beyond the elementary knowledge of classical and modal logic.

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Leibniz's logic

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The logic of G. W. Leibniz (1646–1716) is usually considered as a pivot between traditional syllogistic and modern algebra of sets. Many historiographers believe that although Leibniz intended "to produce a calculus wider than traditional logic [...] he never succeeded in producing a calculus which covered even the whole theory of the syllogism" ([1], p. 337). As a matter of fact, however, Leibniz not only discovered a fully axiomatized algebra of concepts (provably equivalent to Boolean algebra of sets), but he also anticipated important principles of contemporary systems of set-theory, quantifier logic, and modal propositional calculi.

Description of the contents of the tutorial:

This tutorial aims at reconstructing the following main components of Leibniz's logic:
1) The algebra of concepts, L1, which can be viewed as the "intensional" counterpart of the ordinary ("extensional") algebra of sets;
2) The extension of L1 by means of "indefinite concepts" which function as (second order) quantifiers;
3) A genius mapping of L1 into an algebra of propositions which gives rise to a calculus of strict implication;
4) The syntax and semantics of alethic and deontic modal logic.

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Logic and Grammar Contesting the Semantics-Pragmatics Divide

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Abstract

Grice's theory of conversational implicature offers an influential way to account for the interpretation of utterances in terms of logical meanings, on the assumption that speakers' purposes shape the understanding of utterances in conversation. This course provides an overview and critical discussion of Grice's theory and of subsequent developments in philosophy, linguistics, psychology and artificial intelligence. While acknowledging the central place for collaboration in language use, we find that further linguistic rules and interpretive processes are often implicated in recognizing alleged implicatures. These factors necessitate important changes to Grice's views. By highlighting researchers' opposing takes on key issues — the interpretive constraints imposed by linguistic knowledge, the nature of pragmatic inference, the diversity of imaginative perspective taking, and the psychological mechanisms of social understandingâ€"we hope to provide students with a road map to develop their own views about the subject matter.

Motivation

The category of CONVERSATIONAL IMPLICATURES, introduced by Grice in his 1969 William James Lectures at Harvard, is a fundamental conceptual tool for getting clear on the relationship between logic and language. It has helped to cement the modern perspective that ordinary language, warts and all, is rule-governed and amenable to analysis with formal tools, and has sparked a diverse and exciting range of follow-up research across cognitive science. It can be justifiably viewed as a breakthrough in linguistics and philosophy. Grice's ideas have been profoundly influential — by now, they have inspired a wide variety of modified and extended accounts. These frameworks turn out to conceptualize pragmatic principles and pragmatic reasoning in diverse ways, and as a result they make strikingly different claims both about what, in fact, pragmatic reasoning contributes to interpretation, and about how it does it.

In the first two meetings of the course, we survey the accounts of CIs developed by Grice himself and those explored in subsequent research in philosophy, linguistics, psychology and artificial intelligence. We shall reach the conclusion that, while Grice's theory of CIs provides the foundation for most of modern pragmatics, the place and status of the semantics-pragmatic divide remains deeply contested. The goal of this part of the course is for students to appreciate the challenge of getting clear on the relationship between semantics and pragmatics. There are difficult empirical questions to answer about the nature and scope of the rules of language. But it's not enough to distinguish the interpretive effects of different kinds of knowledge and reasoning in language use. The interpretation of these empirical results depends on such philosophical considerations as the nature of content and representation, the relationship of meaning and agency, and the bases for human interaction and collaboration.

The agenda for the rest of course is then to bring empirical characterizations of utterance interpretation into closer contact with the philosophical issues and arguments that inform how natural language meaning should be conceptualized and formalized. In particular, on days three and four of the course, we survey what's known about the status of various interpretive effects without prejudice to the overarching theoretical and philosophical questions, and develop a broader and better-informed perspective.

Day three gives an overview of the rules of language that potentially affect the status of interpretive effects as implicatures. Some rules, governing DISCOURSE COHERENCE, seem to explicitly specify the possible actions that particular utterances can be used for. Other rules, governing PRESUPPOSITION, seem to explicitly specify the possible contexts in which particular utterances can be used. Still more rules, governing INFORMATION STRUCTURE, seem to explicitly specify what relationships particular utterances can bear to salient alternatives that have been or might be uttered in the ongoing conversation. Ultimately, conventional meaning seems so eclectic and variable that we need an explicit methodological justification for how researchers have been able to characterize these interpretive effects as linguistic.

Day four, meanwhile, explores speakers' diverse and particular ways of engaging with imagery, through interpretive effects such as metaphor and irony. For example, we will argue, particularly following [2, 1], that metaphorical interpretation involves a distinctive process of PERSPECTIVE TAKING. Metaphor invites us to organize our thinking about something through an analogical correspondence with something it is not. Any explanation of the import of metaphorical utterances will need to appeal to this distinctive perspective-taking operation. General pragmatic principles will not explain metaphor on their own.

Day five distills the consequences of these considerations for semantics and pragmatics. Semantics, on our view, can be taken to include all the linguistic information — truth conditional or otherwise — that speakers use to recover the content contributed to conversation through utterances.

Pragmatics, meanwhile, is best characterized as a process of disambiguation: the identification of the linguistic structure that the speaker had in mind and the associated rules that are taken to govern its content. Neither semantics nor pragmatics exhausts interpretation, which also requires interlocutors to approach the content speakers present through appropriate practices of imaginative engagement. And even this broad sense of interpretation does not exhaust understanding. Even after we interpret an utterance we may still reason further in an attempt to better understand the speaker. Thus, in place of Grice's uniform pragmatics and its associated notion of conversational implicature, we have a much more nuanced taxonomy.

We don't expect our perspective to be definitive. However, by exposing students to this taxonomy, we expect to enable students — whether they agree with us or not — to pursue more robust research into interpretation, and to engage more productively with interdisciplinary audiences in presenting their ideas.

Our course draws closely on my and Matthew Stone's recent book *Imagination and Convention: Distinguishing Grammar and Inference in Language*, to be published by Oxford University Press in Fall 2014. The course emphasizes key views that students should be familiar with, but, with the book as a resource, students will easily be able to build on what we present, and relate it to current debates on questions such as the grammatical status of scalar implicatures, the role of meaning in metaphorical interpretation, the role of speaker intentions in disambiguation, and the limits of logic in capturing any of these interpretive effects.

Outline

Day 1

The landscape of pragmatic inference. Overview of the course. [4, 13, 14]
Pragmatic inference: linguistic and psychological approaches. [5, 7, 10, 12]

Day 2

The interpretive effects of linguistic rules. Reconciling pragmatic arguments with empirical accounts of discourse structure, presupposition, anaphora and information structure. [16, 6, 15]

Day 3

Varieties of interpretive inference. Reconciling pragmatic arguments with empirical accounts of metaphor and irony. [1, 2]

Theorizing semantics and pragmatics. Communicative intentions, the conversational record, context dependence and the semantics-pragmatics divide. [3, 9, 8, 11]

Expected Level

The course will present its arguments from scratch, so no prior experience is expected. We imagine the course will be most attractive to MS students and early PhD students, but of course we'd welcome more advanced students as well.

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Theorem of Completeness

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1 The Completeness Theorem

To reconcile the syntactic and semantic presentations of consequence is at the core of any logic (be it pure or applied, classical or non classical). This aim can also be seen as the goal to compare (and relate), side by side, the expressive capacity of a formal language, with the computational power of a particular presentation (calculus).

Together, the soundness and completeness theorems, establish the equivalence between the syntactic (associated to a given calculi) and semantic (associated to a given class of models) notions of consequence, for a given language. Intuitively, the semantic notion of 'truth' helps us select the set of all the sentences of a given language that are always true in all structures or models — this set of formulas are usually called 'logically valid', **VAL**. On the other hand, we can also describe a set of formulas using a purely syntactic definition in the form of a deductive calculus. Such calculus would define when a formula 'logically follows' from others. In particular, the set of formulas which 'logically follows' from the empty set is call the set of logical theorems, **THEO**.

Are these two sets the same?

That is the exact question addressed by the soundness and completeness theorems.

2 Relevance of the Completeness Theorem

Theoretical Relevance We can say that we don't know a logic till we haven't identified its set of valid formulas. Intuitively, we can say that the logicality of a given formal language resides in the set **VAL** of valid sentences. Each model $\mathcal{A}$ for a given signature select from the set of all sentences those which are true under this particular interpretation. This set of formulas is usually called the theory of the structure, $\mathrm{Th}(\mathcal{A})$, and it characterizes the structure $\mathcal{A}$. But all such theories share a common nucleus which is the set **VAL**.

Does this set characterize something?

The answer is yes, **VAL** characterizes the logic in question itself. It represents what the logic 'has to say' about any arbitrary structure. If we are able to 'generate' this set easily, we would have finally capture the essence of a logic, its *perfume*.

Practical Relevance As we just discussed, the semantic notion of truth is at the core of a given logic. But because of its generality, it is very difficult to manipulate. For example, the semantic notion of consequence perfectly defines when a formula φ follows from a given set of formulas Γ (it is 'enough' to verify that φ is true in all models where Γ is true); but it does not provide for an 'algorithm' that helps us verify this relation. This is when the set **THEO** of theorems, and the notion of completeness, come to our help. In particular, we can establish a chain of inference from the premises in Γ to the conclusion φ. Actually, this operational definition of consequence seems even more adequate and closer to the intuitive notion of inference, given that it reflects the discursive character of the process.

3 The Completeness of the First-order Calculus

Completeness theorem is proved in its strong sense, $\Gamma \vDash \varphi$ implies $\Gamma \vdash \varphi$, for any Γ, φ such that $\Gamma \cup \{\varphi\} \subseteq \mathsf{Sent}(L)$. One prove completeness and its corollaries following the path:

$$\left.\begin{array}{l}\text{Lindenbaum's lemma}\\ +\\ \text{Henkin's lemma}\end{array}\right\} \to \begin{array}{l}\text{Henkin's theorem}\\ \downarrow\\ \text{Strong completeness}\\ \downarrow\\ \text{Weak completeness}\end{array} \quad \begin{array}{l}\nearrow\ \text{Compactness}\\ \searrow\ \text{Löwenheim-Skolem}\end{array}$$

These theorems are understood as follows:

- *Lindenbaum lemma:* If $\Gamma \subseteq \mathsf{Sent}(L)$ is consistent, there exists Γ^* such that $\Gamma \subseteq \Gamma^* \subseteq \mathsf{Sent}(L^*)$, Γ^* is maximally consistent and contains witnesses.
- *Henkin's lemma:* If Γ^* is a maximally consistent set of sentences and contains witnesses, then Γ^* has a countable model.
- *Henkin's theorem:* If $\Gamma \subseteq \mathsf{Sent}(L)$ is consistent, then Γ has a model whose domain is countable.
- *Strong completeness:* If $\Gamma \vDash \varphi$ then $\Gamma \vdash \varphi$, for any $\Gamma \cup \{\varphi\} \subseteq \mathsf{Sent}(L)$.
- *Weak completeness:* If $\vDash \varphi$ then $\vdash \varphi$, for any $\varphi \in \mathsf{Sent}(L)$.
- *Compactness theorem:* Γ has a model iff every finite subset of it has a model, for any $\Gamma \subseteq \mathsf{Sent}(L)$.

- *Löwenheim-Skolem:* If Γ has a model, then it has a countable model, for any $\Gamma \subseteq \mathsf{Sent}(L)$.

4 Completeness Notions

In logical writings we find the term 'completeness' applied either to theories, to deductive calculuses, or just to logics. In all three cases we wish to express some kind of *sufficiency* of the rules (or *completeness* of the final product) but, as we shall see, these concepts differ in some aspects having to do with the resources and methods needed to establish them.

In [14] we focus on the evolution of the notion of completeness in contemporary logic. We discuss the differences between the notions of completeness of a theory, the completeness of a calculus, and the completeness of a logic in the light of Gödel's and Tarski's crucial contributions. As far as first-order logic is concerned, *our thesis is that the contemporary understanding of completeness of a calculus was born as a generalization of the concept of completeness of a theory.*

There are three main lines that I want to consider in the tutorial:

1. *Origin:* When and how is the necessity of a completeness proof born? When does it separate itself from a theorem concerning the decidability of satisfiability for a given logic? For example, the first completeness proofs for Propositional Logics are intimately related to decidability and representation in terms of finite algebras. The original publications of Post, Stone, Quine, Tarski, and Gödel are relevant for this line.

2. It seems natural to think that Henkin's completeness theorem for first-order logic was proved before the completeness for type theory. Surprisingly, in his 1996 paper he stated that he obtained the proof of completeness of first-order logic by readapting the argument found for the theory of types, not the other way around.

3. *Evolution of Henkin Completeness Proof:* The original proof of completeness for classical logic resulted extremely versatile, and its fundamental idea of using witnesses during the model construction can be used for many other logics. In [1] we use a similar idea. It is specially interesting to establish the relation between Henkin proof and the use of rigid designators in hybrid logics [2].

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Logic and Music — The Logic of Chords and Harmony

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The tutorial provides a detailed introduction into a very new approach to a formal theory of music: the logic of chords and their internal harmony. The logic of chords is closely related to mathematical theories of music but by far not identical with them. The constitutive decisions creating our logic of chords are (a) fixing the logical space by specifying a scale, (b) indicating chords as our well-structured basic elements and (c) introducing n-ary chord operators. The participants are invited to formulate and discuss advanced ideas to develop a calculus of chords.

Chords possess a complex interval-related inner structure. *Internal* harmony is the totality of formal (context-free) relations between chords which is given without fixing any point on the scale. It can be shown that tonality needs such a fixed point.

(a) Our *logical space* is given by the *scale of integers*. Each integer will be interpreted as a (different) *simple tone*. Tone intervals are ordered pairs of tones. Each interval has a characteristic positive *length* l with $l > 0$.

(b) A key feature of the logic of chords is that this formal theory is not an atomic one. The basic elements are chords consisting of at least three tones, two basic intervals and one reference interval. A *basic interval* is the relation between directly adjacent tones. The *reference interval* is the relation between the highest and the deepest tone of any chord. A chord is a *molecular* expression characterized not only by its tones but mainly by its *matrix of interval lengths*. Each chord can be uniquely identified solely by its *inner* structure. A class of (partially or totally tone-different) chords — e.g., the class of 3-tone-major-chords in root position — can be identified simply by knowing its characteristic matrix of interval lengths. *Internal harmony* is nothing else than the relation between two or more chords based solely on the inner structure of the chords. In this sense “chord” as well as “harmony” are formal concepts. Euphony is not necessary. E.g., we have of course chords and harmony in twelve-tone music (dodecaphony) and free jazz.

(c) An n-ary *chord operator* takes an n-tuple of chords as input and yields a chord as its output. *Unary operators* are *negations* (complete inversions of basic interval lengths relative to tone-related or interval-related fixed points), other *interval permutation operators*, *barré operators* (outputs with isomorphic matrices) and *inversion operators*. If it comes to more complex harmonic constructions like sequences consisting of tonic, subdominant and dominant we need at least *binary operators* to create them (cadence operators).

Session 1
The inner logical form of chords

We start with the introduction of our symbolism to describe tones, intervals, interval lengths and interval classes. We define several relations between intervals. The most important one is the relation of *directly connected* intervals. Chords are multi-dimensional sequences of directly connected intervals. *The general form of chords* will be explained with emphasis on the inner complexity of the pattern of intervals. We differentiate between *basic* and *intermediate intervals* (in chords with 4 and more tones) as well as *the reference interval*. With respect to characteristic matrices of interval lengths it is possible to characterize *classes of chords* solely with respect to their inner structure. No further context is needed. Chord classes with sufficient complex matrices of interval lengths contain submatrices which characterize other chord classes. Such sub-matrices can be connected or disconnected. Finally we discuss the interesting cases of *interval length perfect* (e.g., all-interval tetrachord) and *interval length disjoint chords*.

Session 2
Unary chord operators

We start with unary chord operators like *negations of chords*. One type of chord negation is characterized by the complete inversion of the order of all basic intervals within a fixed reference interval. Another type is the complete inversion of this order with a fixed middle tone (if the chord contains an odd number of tones) or a fixed middle basic interval (if the chord contains an even number of tones). We show that these negations are analogies of the (partial) negation in the logic of first degree entailments. We will sketch a proof that either of these negations of an arbitrary major chord yield a corresponding minor chord and vice versa. Using both negations alternately we get the major and minor chords with fixed matrices of all basic tonalities. If time permits we discuss other unary chord operators like barré operators and inversion operators.

Session 3
Internal harmony, tonality, binary chord operators and family resemblance

The simplest form of *internal harmony* is the (context-free) relation between two chords with respect to their inner formal structure alone. The application of a unary operator creates necessarily an internal harmony between its input and its output. Creating internal harmony depends on the logical behavior of the chord operator as well as the inner structure of the argument(s). It is an inspiring research question for the logic of music which aspects of tonality can be characterized as internal harmony. A known candidate is the asymmetric relation "is the *parallel minor* of" (but not *tonic parallel*). A novelly defined concept is the symmetric relation "is the X-*dominant* of" with X is empty ("dominant") or replaced by "sub" ("subdominant"). To determine a chord which is located between two other chords we need *binary chord operators*. But n-ary operators are insufficient to determine *tonic chords* as internally harmonious. To get this we have to extend our approach by a fixed point on the scale. The range of both kinds of theory is still an open question. But there is also a *creative* aspect here: *composing* new music. One possibility is the creation of family-like sequences of chords. Adapting Wittgenstein we can say that "the strength of the thread [harmony in a sequence of chords] does not reside in the fact that some one fibre [interval length] runs through its whole length [whole sequence], but in the overlapping of many fibres [crisscrossing of interval lengths]." (*Philosophical Investigations*, vol. **67**). An audio example will be given and formally explained.

Primary text for the tutorial

Well enough in advance of our tutorial an extended handout will be hyperlinked here!

Secondary sources

- `http://en.wikipedia.org/wiki`
  - All-interval tetrachord, Chord (music);
  - Chromatic scale;
  - Diatonic function, Harmony;
  - Hexachord, Interval (music);
  - Inversion (music);
  - Neapolitan chord, Octave;
  - Riemannian theory, Set theory (music);
  - Tonality, Trichord, Tristan chord.
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Lewis Carroll's Symbolic Logic

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Several expressions have been coined (and many are still in use) to name the new logic that was developed from the mid-nineteenth-century onwards in the footsteps of logicians such as George Boole and Gottlob Frege. It has been known as mathematical logic, algebraic logic, algorithmic logic, logistic, new logic, modern logic, symbolic logic, etc. The latter was chosen in the 1930s by the founders of the Association and Journal of Symbolic Logic, which consequently contributed to its survival and circulation. Though earlier occurrences do exist, this expression seems to have been popularised by John Venn in the second volume of his trilogy: Symbolic Logic (1881, second edition in 1894). This expression had the advantage of indicating clearly what was seen as the most perceptible feature of this new logic: its thorough use of symbols. From this perspective, the very idea of symbolic logic does not assume any a priori relation to mathematics. All that is required for a logic to be recognised as symbolic, is a broad use of symbols to represent logical operations.

The aim of this tutorial is to discuss this symbolization process through the singular case of Lewis Carroll's logic. What makes the author of the Alice tales so special is that he believed in the utility of symbolic logic in daily life, and as such refused to simplify his logical system for the sake of convenience, as most his colleagues did and overtly admitted. He regarded his logic treatise as a "work for God", that would help to reason in everyday circumstances. Consequently, he designed his logical theory in such a way as to agree both with the "accepted facts of Logic" and the "actual facts of life". This principle differentiates his logic from most of the symbolic systems of the time. The tutorial will run on three parts, of about one hour each:

I. Logical Symbolism

This part will be devoted to giving an overview of how symbolic logic was developed and what logical notations, diagrams included, were used. The idea is to see how the evolution of those symbolisms led slowly to the standard notation (if any) we use today, notably after Peirce, Peano and Russell. The point is to highlight the difficulties raised by the introduction of symbolism in logic and to identify the criteria that determined the choice, the design and the neglect of specific logic notations. This is an essential point to understand what symbolic logicians, Carroll included, were doing at the time.

II. Carroll's Logical Theory

This part will be devoted to the exposition of Carroll's logical theory, mostly as it is exposed in his main work: Symbolic Logic (4th edition, 1897). We will examine his typology of propositions and his logic notation. Then we will pay particular attention to some specific features, notably the existential import of propositions and the theory of non-existent objects and classes. We will see that Carroll explored some unusual paths that made him solve some uneasy problems that faced his colleagues, but that also prevented him from making significant advances due to the complexity of the logical notation and system he got.

III. Inferring

In this last part, we will discuss the raison d'être of Carroll's symbolic logic: the problem of elimination. That is how to find the conclusion that is to be drawn from a set of premises, regarding the relation between given terms, by eliminating the "middle terms". It is for the purpose of solving this central problem that mid-nineteenth century logicians invented symbolic, diagrammatic, and sometimes mechanical, devices. We will expose some of Carroll's methods for handling such "logic sequences", as he called them. Finally, we will briefly discuss two papers on hypotheticals that Carroll published in the journal Mind: "A logical paradox" (1894) and "What the Tortoise said to Achilles" (1895). These papers have been widely discussed by nineteenth and twentieth century logicians (Peirce, Russell, Ryle, Prior, Quine, etc.). The first paper is often mentioned as a good illustration of the paradoxes of material implication while the second gave rise to what is known as the paradox of inference.

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Logic and Category — Or Planar Heyting Algebras for Children

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One way to explain intuitionistic logic to a non-logician is this. The usual truth-values are just 0 and 1, and we will change that by decreeing that the new truth-values will be certain diagrams with several 0s and 1s. We choose a subset D of $\mathbb{N}^2$, for example $\therefore$, and we say that a *modal truth-value on D* is a way of assigning 0s and 1s to the points of D. A modal truth value is *unstable* when it has a 1 immediately above a 0, for example $\begin{smallmatrix} & 1 & \\ 1 & & 0 \end{smallmatrix}$ is unstable, and an *intuitionistic truth-value on D* is a stable modal truth-value on D. Now that we have defined our (intuitionistic) truth-values we explain to our non-logician friend how to interpret $\top, \bot, \land, \lor, \to$ on them, and we show that if $P = \begin{smallmatrix} & 0 & \\ 1 & & 1 \end{smallmatrix}$, then $P \neq \neg\neg P = \begin{smallmatrix} & 1 & \\ 1 & & 1 \end{smallmatrix}$, and some other classical theorems also do not hold. We then explain some logical axioms and rules that do hold in this system, define intuitionistic propositional logic from them, show how this particular case based on D generalizes, present the standard terminology, and so on.

When we do this we are using several tricks — finding an insightful particular case, doing things in the particular and in the general cases in parallel using diagrams with the same shapes, lifting proofs from the particular case to the general one, and this didactic method can be defined precisely. In the terminology of [Ochs2013] this logic on subsets of $\mathbb{N}^2$ and DAGs on them (ZSets and ZDAGs) is an *archetypal model* for intuitionistic propositional logic ("IPL"). If we abbreviate "explaining and formalizing (something) via an archetypal model" as "(that something) for children", then the contents of the tutorial become easy to state.

Heyting Algebras for children. When a ZDAG D doesn't have three independent points, then the open sets of $(D, O(D))$ are in bijection with the *points* of another ZDAG, D'. This D' is a Heyting Algebra, and our way of interpreting Intuitionistic Predicate Logic on open sets of D translates into a way of interpreting IPL on the points of D'. The operation $D \mapsto D'$ gives us lots of examples of *planar* Heyting Algebras — but not all.

Take any ZDAG D whose points all have the same parity. There is a simple, visual criterion that can tell us very quickly whether D is a HA or not. The ZHAs are the Ds that obey this criterion and the parity condition; an arbitrary ZDAG D is a HA iff it is isomorphic to a ZHA, and this is also easy to check. This gives us all planar Heyting Algebras.

There is a system of coordinates that we can put on a ZHA — the (l, r) coordinates — that make $\top, \bot, \land, \lor, \to$ trivial to calculate. We will present a computer library that does all these calculations, and that can produce ascii and LaTeX diagrams for both ZDAGs and functions on them.

Heyting algebra modalities for children. A *modality* is an operation $*$ on the points of a HA obeying $P \leq P^* = P^{**}$ and $(P \land Q)^* = P^* \land Q^*$. The operations $B_\bot(P) = \neg\neg P$, $B_Q(P) \mapsto ((P \to Q) \to Q)$, $J_Q(P) = Q \lor P$, $J^Q(P) = Q \to P$, are modalities, and our computer library can show visually how they behave on the points of a ZHA and how they can be composed in several ways (as in [FourmanScott79], p. 331) to obtain new modalities.

The usual way of presenting HA modalities in the literature is by showing first some basic consequences of the axioms, then how modalities interact with $\land, \lor, \to$, then theorems about how the algebra of modalities behave; I have always found this approach quite opaque. By using ZHAs we can explain these theorems and exhibit countermodels for all non-theorems visually — and it turns out that modalities on a ZHA D correspond to ways of cutting D into equivalence classes using diagonal lines. This visual way of thinking *complements* the usual formal way... but how, exactly? *Can we make that precise?*

Categories for children. For our purposes, the archetypal category is **Set**, and in most examples we can use only finite subsets of $\mathbb{N}$ as its objects in diagrams. This lets us introduce quickly two flavors of typed λ-calculus, the distinction between *structure* and *properties*, a trick to focus only on structure and leave the "properties" part for a second moment, and a way to regard having a terminal, binary products, and exponentials — the cartesian-closed structure — as extra structure on **Set**. A *CCC* is a category with that extra structure, and by regarding **Set** as the archetypal case we get a way to interpret the simply-typed λ-calculus formally in any CCC.

It turns out that ZDAGs are categories, and ZHAs are CCCs, both archetypal in slightly weaker ways than Set. By interpreting λ-calculus in ZHAs and making a series of changes in the notation we get the categorical interpretation of intuitionistic predicate calculus, plus Natural Deduction, and Curry-Howard.

Toposes for children. Let D be a ZDAG; for example, $D = \therefore$. The category of functors $\mathbf{Set}^D$ is a topos — a *ZTopos*, and its objects are $\therefore$-shaped diagrams that are easy to draw explicitly. CCCs are categories with extra structure that lets us interpret simply-typed λ-calculus on them; toposes are CCCs with extra structure, that lets us interpret *Intuitionistic Set Theory* (IST) on them. By regarding both **Set** and our '$\mathbf{Set}^D$'s as archetypal toposes we can start topos theory from the "internal language", i.e., from the way of interpreting all operations of IST in terms of basic categorical operations; our approach lets us begin by examples that show how each operation of IST ought to behave, then guessing a formalization, than proving that it works and thus toposes are models for IST, then proving other facts about toposes that are less logical and more algebraic in character.

Sheaves for children. Each modality on a Heyting Algebra D' induces a notion of "sheafness" on a ZTopos, plus a quotient from it into a "smaller" topos, which has an adjoint that is a functor from the "smaller" topos back into the "bigger" one; the "sheaves" are the objects of $\mathbf{Set}D'$ that are in the image of that adjoint functor.

Using ZToposes as our archetypal toposes we can understand how all these entities and definitions behave by generalizing a few examples where the diagrams are not too big. One nice example — of the *logical* definition of sheaf — shows how the notion of sheafness induced by $\mathrm{B}_\perp$ booleanizes the logic of a topos; another example, motivated by the *topological* definition of sheaf, shows how sheafification and étalification are adjoint functors, using an order topology.

The possibilities for exposing technicalities using archetypal cases are endless, but we will dedicate the best part of our energy in this tutorial not to them, but to something more general and more useful: how to use archetypal cases to make the literature more accessible, and to create bridges between different notations.

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The Compactness Theorem

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Let us say that a logic has the *compactness property* if a set of sentences of the logic has a model whenever every finite subset of the set has a model. For present purposes, **the Compactness Theorem** is that first-order logic has the compactness property. This theorem is fundamental to model theory. However, Hodges's comprehensive twelve-chapter 1993 volume *Model Theory* finds no need to state and prove the theorem until Chapter 6. It is worthwhile to think about what needs Compactness and what does not.

One consequence of the Compactness Theorem is that a set of sentences with arbitrarily large finite models must have an infinite model. A more purely mathematical consequence is the **Prime Ideal Theorem**: *every nontrivial commutative ring has a prime ideal.* One can prove this by noting first that every *maximal* ideal is prime. Moreover, every *countable* ring has a maximal ideal; for we can obtain a generating set of such an ideal by considering the elements of the ring one by one. In particular then, every finitely generated subring of a given ring has a maximal ideal, because every finitely generated ring is countable. By the Compactness Theorem then, the original ring must have an ideal that is at least prime, although it might not be maximal. The point here is that primeness is a "local" property, while maximality is not.

It is usually understood that every nontrivial commutative ring has, not just a prime ideal, but a maximal ideal. To make it easy to prove such results, Zorn stated in 1935 the result now known by his name. However, Zorn's Lemma relies on the Axiom of Choice. The Compactness Theorem is strictly weaker than this, with respect to ZF (Zermelo-Fraenkel set theory without Choice). For, Compactness is also a *consequence* of the Prime Ideal Theorem, even the Boolean Prime Ideal Theorem; and *this* is strictly weaker than the Axiom of Choice (as shown by Halpern and Lévy in 1971).

The Compactness Theorem for *countable* sets of sentences needs nothing beyond ZF. Skolem showed this implicitly in 1922 when he established the paradox that Zermelo's axioms for set theory must have a countable model, if they have a model at all. In 1930, Gödel proved countable Compactness explicitly, though not by that name. Mal'tsev stated the full Compactness Theorem as the General Local Theorem in 1941, having proved it implicitly in 1936; he used it to prove algebraic results in the way we proved the Prime Ideal Theorem above.

In his 1950 address to the International Congress of Mathematicians, Tarski gave the Compactness Theorem its current name and noted its topological meaning. But this meaning is not generally well expressed in today's textbooks of model theory.

The class of structures having a given signature can be given a topology, although the closed "sets" in this topology are not sets, but proper classes (except for the empty set): they are the classes of models of sets of sentences. The space of all structures has a Kolmogorov (T_0) quotient that is a set: it is the space of complete theories of structures. If one replaces sentences with their logical equivalence classes, then the set of sentences becomes a Boolean algebra, called a Lindenbaum algebra; and the complete theories of structures become ultrafilters of the Lindenbaum algebra. By means of the Boolean Prime Ideal Theorem, the Stone space consisting of all ultrafilters of the Lindenbaum algebra is easily shown to be compact. Or one could look instead at the spectrum, consisting of the prime ideals of the corresponding Boolean ring; the spectrum of every ring is compact. The Compactness Theorem says more: every ultrafilter of the Lindenbaum algebra is derived from the complete theory of a structure.

The compactness theorem for *propositional logic* can be seen as a version of the theorem that the product of two-element discrete spaces (or indeed any compact Hausdorff spaces) is compact. The Compactness Theorem for first-order logic does not follow so readily, though it can be seen to result from a kind of reduction of first-order logic to propositional logic. Then Lindström's Theorem is roughly that there is no such reduction for certain more expressive logics — but see that tutorial for more. Sometimes the Compactness Theorem is derived from the Completeness Theorem: see *that* tutorial for more. Meanwhile, the present tutorial is intended to fill out the foregoing sketch of the Compactness Theorem as such.

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Logic and the Theory of Relativity

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There are several reasons why both special and general relativity theories are interesting from the point of view of logic. For example, both theories contain many surprising predictions and interesting concepts (such as the Twin Paradox, curved space and time, etc.). Also general relativity predicts self-referential situations (possibility of time travel) in spacetimes as Gödel’s rotating universe.

These predictions, concepts and models clearly deserve some deep logical investigation and understanding. That is why our research school lead by Hajnal Andréka and István Németi aims to develop a logic based foundation for relativity theories.

Among other we aim to axiomatize relativity theories using simple, comprehensible and transparent basic assumptions (axioms); and to prove all the surprising predictions (theorems) of relativity theories using a minimal number of convincing axioms. However, we are not aiming to have one axiom system, but we are building a whole net-like hierarchy of axiom systems and logical connections between them. And we not only axiomatize relativity theories, but also analyze their logical and conceptual structures.

Some of the questions we study to investigate relativity theories are:

- What is believed and why?
- Which axioms are responsible for certain predictions?

- Can we change the axioms and at what price?
- What happens if we discard some axioms?

A novelty in our approach is that we try to keep the transition from special relativity to general relativity logically transparent and illuminating. We are going to "derive" the axioms of general relativity from those of special relativity in two natural steps. First we extend special relativity of inertial observers to a theory of accelerated observers. Then we eliminate the difference between inertial and non inertial observers in the level of axioms.

Among others, logical analysis makes relativity theory modular: we can replace some axioms with other ones, and our logical machinery ensures that we can continue working in the modified theory. This modularity might come handy, e.g., when we want to extend general relativity and quantum theory to a unified theory of quantum gravity.

- Session 1: Axiomatic theory of special relativity
- Session 2: Axiomatic theory of accelerated observers
- Session 3: Axiomatic theory of general relativity

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Logic and Fiction

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We will consider the impact of three ontologically parsimonious principles or assumptions on the logic of fiction and on the efforts of others to produce an intellectually satisfying natural language semantics for fictional discourse.

Parmenides' Law: There is nothing whatever that doesn't exist.

Kripke's Law: No referring expression refers unless there is something to which it refers.

The Fiction Law: There is no object that any object of fiction is. The objects of fiction don't exist.

Our particular purpose will be to determine whether a plausible semantics of fiction is possible under these tight constraints.

Three options will be considered:

- A double-aspect semantics
- An inferentialist semantics
- A no-ambiguity semantics

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Here some notes for attendees to read the notes as either background or at least concurrent material.

Part III

5th World Congress on Universal Logic

7 – Talks of Invited Keynote Speakers

A Dual Representation Theory of Quantum Systems and its Ontological Consequences

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In the framework of formal philosophy, we present the formal ontology of the natural realism (NR) as a formalization of the ontology of the quantum field theory (QFT) in fundamental physics, as far as QFT is based on a dual (algebra/coalgebra) representation theory of quantum systems. Indeed, the QFT — differently from the quantum mechanics (QM), and the so-called "standard interpretation" of QFT based on QM — is fundamentally a thermal field theory. It offers, therefore, one only theoretical framework to fundamental physics, from the microscopic relativistic realm of quantum physics (Standard Model), to the macroscopic realm of the condensed matter physics, the physics of biological and neural systems included. Because of the Third Principle of Thermodynamics, indeed, no quantum system can be considered as energetically closed, because it is always "open" to the irreducible fluctuations of the underlying quantum vacuum (QV). In this way, the theoretical framework of QFT is based on the "doubling of the degrees of freedom" (DDF) between the system and its thermal bath, represented through the "duality", in the category theory (CT) sense, between a q-deformed Hopf algebra (representing the system) and its opposite q-deformed Hopf coalgebra (its thermal bath), where q is a thermal parameter (the cosmological evolution parameter), and the functor projecting each structure onto its dual opposite (the mapping "signature") is the functor G of the Bogoliubov thermal transform. Therefore, the notion of duality between Universal Algebra (UA) — Universal Coalgebra (UC), independently developed in theoretical computer science (TCS), has an immediate relevance for quantum logic and ontology. In TCS, indeed, the semantics of the different steps of a given program, as far as satisfiable in a sequence of physical states of a computing system (interpreted as a labeled "state transition system" (STS)), can be formalized as the duality between an initial (sub-)algebra and its final (sub-)coalgebra. This depends on the definition of the UC structures on Aczel's non-wellfounded sets, allowing the set self-inclusion, and therefore the existence of unbounded sequences of set inclusions with no-total ordering among sets. The immediate relevance of UC for quantum logic and ontology is related with the possibility of formally defining in the

[1] Keynote speaker of the workshop "Representation and Reality: Humans, Animals and Machines" (page 152).

UC framework, the non-extensional notions of "bisimilarity" (homomorphism up to isomorphism) and hence of "observational equivalence" that are dual to the extensional notion of "equivalence by congruence" in UA. Consequently, the powerful method of proof by "coinduction" is allowed in UC, as dual to the proof by "induction" in UA. From the formal ontology standpoint, these notions explain why it is possible to give in UC a formal proof of completeness of Kripke's relational semantics in ML, extended to the possibility of infinite inclusions among Kripke models, so to justify the TCS dictum that "modal logics are coalgebraic". The ML of NR is therefore the **KD45** system, defined on Aczel non-wellfounded sets, allowing a non-transitivity of the inclusion relations, and hence their "branching", by using the Euclidean relation on them (the ML axiom **5**). In such a way, we can formally justify, in the quantified ML semantics of NR, the use of evaluations based on bounded morphisms (bisimilarities) among Kripke models, the logical completeness of their unbounded sequences, and the consequent theory of stratified rigidity. These notions constitute the core of the NR ontology, as far as formalizing the causal inclusions of natural kinds (genus/species branching) of ever more complex physical systems, as characteristic of the QFT evolutionary cosmology, based on the DDF between q-deformed Hopf algebras/coalgebras. More generally, the NR ontology can formalize, in terms of the CT dualities algebras/coalgebras, with "the inversion of arrows and of compositions" characterizing them, the duality between the logical implication (direct implication, in its modal version: "it is impossible that the premise is true and the consequence is false"), and the ontic implication (converse implication: "it is impossible that the effect exists, without its cause exist"), originally suggested by Aquinas in XIII cent. for justifying the ontological bi-conditional ($\leftrightarrows$), as distinguished from the logical bi-conditional ($\leftrightarrow$).

There is no logical negation: Confessions of a former logical exhaustivist

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I used to think that the paradoxes teach us that there are gluts, much as the pioneer Asenjo and later Asenjo-Tamburino — and later, and clearer, and much better known Routley/Sylvan, Mortensen, Priest — maintained. The argument appealed to the exhaustive behavior of logical negation: by logic's lights, at least one of A and its logical negation $\neg A$ is true. Throw in typical paradoxes of circularity, together with foundational principles governing the target notions, and the gluts seem to roll out. Of course, just as Asenjo and later glut theorists noted, true glutty theories are

[1]Keynote speaker of the session "Philosophy" (page 335).

absurd if logical negation is also exclusive, that is, if by logic's lights every sentence A together with its logical negation (logically) entails every sentence (including the scary ones, the absurd ones, and so on). (This makes the negation operator exclusive in the sense of excluding the possibility of closed theories that are negation-inconsistent but short of containing all sentences, where negation-inconsistency is defined as containing a negation using the given negation operator together with the given negatum.)

But why not the dual response, as is more standard (e.g., the Strong Kleene type approaches, including recently Kripke, Horsten, Field, and others)? Why not instead think that the familiar paradoxes tell against exhaustion (but not against exclusion)? For a long time, I've had no satisfactory answer to this question, but nonetheless kept clinging to my 'logical exhaustivist' ways (according to which logical negation is exhaustive).

I still have no satisfactory answer. But my lack of answer has finally pushed me to go straight: namely, reject both exhaustion and exclusion. If you can justify giving up one or the other feature (exhaustive, exclusive) but can't — in a honest, satisfying fashion — justify giving up one particular side over the other (say, exhaustion over exclusion, or vice versa), then give up both (provided the initial, antecedent justification remains intact). And so I now do, openly and publicly: I reject that logical negation is either exclusive or exhaustive (by logic's lights). That's my confession.

The question is: where does this leave us? My talk gives some answers to this question by answering — or gesturing at answers to — the following questions. (My answers/gestures are in fact snippets from a bigger project.)

Q1. Are there arguments for going weaker than the standard K3 or LP subclassical logics — say, FDE-ish?

A1. Yes. One argument stems from aesthetic considerations of logic (qua topic-neutral universal closure relation for our theories): each of K3 and LP give a lopsided picture of logic, whereas something like FDE doesn't. Another argument (related to the first): each of the K3 and LP approaches to paradox have dual virtues and dual problems, and so neither (lopsided-logic) framework has clear benefits over the other; and so a more balanced FDE picture is motivated. (And the FDE-ish picture seems to be able to accommodate all of the virtues of the lopsided pictures without gaining any vices.)

Q2. What's left of logical negation?

A2. Not much. Indeed, in an important sense, there just is no interesting negation connective that logic characterizes on its own. (E.g., there is no connective that, according to logic, has stand-alone behavior characterized by, say, the classical sequent rules for negation.) My current view on logic is roughly this: That whatever arguments we had for thinking that logic is subclassical remain (and arguments in this vicinity are tricky indeed); but giving up the exclusion/exhaustive view of negation puts logic closer to Anderson-Belnap FDE than to any of its stronger extensions (e.g., K3 or LP extensions). (Terminology: saying that logic is subclassical is to say, roughly, that if an argument is logically valid, then it's valid according to classical logic.)

Q3. But how then do we explain the apparent ubiquity of classical negation?

A3. If it looks like classical negation, it isn't *logical* negation; it is instead some operator whose apparent classical-negation behavior is delivered by some more-than-logical entailment relation. (Logic, as per A2, won't give such behavior. So, the behavior comes about not from the base, ubiquitous logical entailment operator — the base, universal closure operator on theories — but rather via some stronger entailment relation serving as a closure relation for theories in which the given classical-negation-looking operator appears. (See my work on 'shrieking theories'. In effect, the apparent classical-negation behavior is behavior imposed by theory-specify entailment or closure relations; it isn't imposed by logic itself.))

Q4. If logic doesn't impose exhaustive behavior on all of our theories (e.g., excluded middle isn't valid according to logic), we lose all motivation for gluts. No?

A4. No. One argument, which I in fact find plausible, is from naturalness. I will briefly discuss this in the talk. Such an argument will not be as prima facie powerful as invoking logic; but that is the price of having a weak logic — namely, that the existence gluts, 'gaps', etc is a matter on which logic remains nicely neutral.

Q5. If logic is so utterly weak, what possible connection could it have to reasoning – to our rational acceptance and/or rejection behavior?

A5. The answer is that it has its traditional connection: it rules out various patterns of 'change in view' or 'acceptance/rejection patterns', but doesn't force one to accept much (if any) at all. (In this respect, lessons from Gilbert Harman were largely right.)

Q6. If logic provides so very little by way of constraining our theories (neither exclusivist or exhaustivist constraints), then how do we come up with constraints on our true theories?

A6. The answer is the one our parents taught us: rational life is hard; it's as if being on a raft. (See Neurath, Quine, and many others.)

From syllogisms to syllogistic consequences: a turning point in the history of logic

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The aim of this paper is to weight to theoretical and historical drawbacks of the inclusion of syllogistic within a general theory of consequences especially as observed in the English logical tradition that culminated in North Italy in the 14th and the 15th centuries, while full commentaries on the Posterior Analytics kept on being continuously produced in the same time, whether in scholastic or humanistic circles. The result is not so much the eclipse of the Prior Analytics, not commented upon any longer during the 15th century in Italy, and reduced to the sole assertoric and modal syllogistics, but the very idea of considering syllogisms as consequences, be they formally valid. This means that important features syllogisms have not as inferences but as arguments (which they are) are left apart, and that the two Analytics are cut apart; many arguments that connect the two Analytics as a two-steps theory of deduction and proof are left out of sight, such as the analytical logic of proof and discovery.

Paraconsistent probability theory: betting rationally under contradiction

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I intend to show how probability measures can be neatly introduced in connection to certain (paraconsistent) Logics of Formal Inconsistency (LFIs), and how such measures can be viewed as degrees of rational belief under contradiction. I argue that the distribution of degrees of belief under the pressure of contradictions by an ideally rational agent can be supported by paraconsistent probability axioms, and that this philosophical stance leads to a new, simpler and yet useful, paraconsistent theory of probability.

Among other features, probabilistic reasoning under contradictions can be naturally extended to appropriate notions of conditional probability and updating, via a version of Bayes' Theorem for conditionalization. It will be shown how those paraconsistent probabilities can be identified, as much as in the classical case, with expectations of

[1]Keynote speaker of the workshop "Medieval Logic" (page 191).
[2]Keynote speaker of the session "Paraconsistency" (page 269).

truth values of sentences, and how they can provide more relaxed, but still pragmatically meaningful, constraints on rational belief. It will be also shown that the dissimilarity between the notions of contradiction and inconsistency, one of the pillars of LFIs, plays a central role in this proposed extension of the notion of probability and poses interesting problems related to paraconsistent versions of Dutch Book Arguments.

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Hypothetical Syllogism in Avicenna

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Even though one of the controversial topics in Arabic logic is about Avicenna’s works and from 10th to 14th century it is seen an Aristotelian effect on the writings of Arabic logicians, they are not wholly a rethinking or commenting of Aristotle. Hypothetical Syllogism, in this respect, comes to light and shows its importance beneath the studies of Avicenna. Although there may seem the first related issues in Aristotle’s *Prior Analytics* with the statement of “syllogism from hypothesis”, the more fundamental and systematic studies are seen within the era between Peripatetic school and Stoics. As a medieval Arabic philosopher, Avicenna, deals with the hypothetical syllogism in his *The Book of Healing* (as-Sifa) and makes several striking claims on conditional propositions and syllogisms. He develops a non-Aristotelian tradition into his treatment of syllogistic, especially on categorical and conditional. The quantified hypothetical propositions, analysis of inferences of categorical and hypothetical syllogism and his understanding of wholly hypothetical conjunctive syllogistic with quantified condition can be shown as the main titles of this issue. Decidedly, though his ideas are inherited from Ancient Greek and particularly Alfarabi in some respect, my aim at this point is (1) to show the key points of this heritage from Aristotle to Alfarabi and (2) to state the originality of Avicenna within the concept of hypothetical syllogism.

[1] Keynote speaker of the session “History” (page 362).

The theory of topos-theoretic ‘bridges’, five years later

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The methodology of topos-theoretic ‘bridges’ was introduced in [1]. This technique allows to effectively use Grothendieck toposes as unifying spaces for transferring notions, properties and results across different mathematical theories having an equivalent or a strictly related semantic content. Throughout the past five years this theory has generated many applications in different mathematical areas, such as model theory, proof theory, algebra, topology, functional analysis and algebraic geometry. We shall review the basics of the theory and make a survey of the most significant applications obtained so far (such as [2, 3, 4]).

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Is the Church-Turing Thesis the new Pythagoreanism?

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The Church-Turing Thesis (CTT) is a philosophical claim that the class of ‘effectively’ computable functions is precisely the class of functions computable by a Turing machine. Since this hypothesis has not been violated for almost 80 years now, it motivates me to think about a suitable ontological and epistemological philosophy of mathematics compatible with CTT.

[1]Keynote speaker of the session “Algebra and Category” (page 369).
[2]Keynote speaker of the session “Computation” (page 346).

In a broad sense, Pythagoreanism says that everything in the universe can be expressed with the natural numbers and their ratios. Therefore, Pythagoreanism gives a complete description of the universe and the objects in it via natural numbers. Similarly, CTT says that the complete description of our intuitive computation is provided by Turing machines. Neither Pythagoreanism nor CTT allows further expansion towards inexpressibility and the possibility of the existence of objects, in their domain, beyond their expressive barrier. One may say that both Pythagoreanism and CTT are *complete in expressibility*. Respectively, one regarding the complete expressibility of the universe itself via natural numbers, the other regarding the complete expressibility of the intuitive notion of computation via Turing machines. The relationship between these two will be based on this type of completeness. Hence, one may say that CTT holds a *new-Pythagoreanistic* position, in the new era of formalization and computability, in a sense that computational knowledge is acquired by formulations which can be explained by mechanical meanings.

I shall investigate a possible ontological philosophy of mathematics taking CTT as a primary assumption. The main focus will be on a theme called *idealistic abstractionism*. Idealism in this context, as a consequence of the formalistic nature of CTT, is based on the rejection of the existence of a pre-determined consistent Platonic universe of mathematical objects. Abstractionism, as an Aristotelian view, provides a framework for the ontological status of mathematical objects regarding where they are originated from, assuming CTT. The investigated philosophy can be seen to be Pythagoreanistic due to the relationship between Pythagoreanism and CTT based on the expressive-completeness of both views.

Identity Statements, Doxastic Co-Indexation and Frege's Puzzle

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[1]Keynote speaker of the workshop "Frege's Puzzle" (page 237).

Representation and Reality: Humans, Animals and Machines

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Symposium Representation and Reality: Humans, Animals and Machines at UNILOG 2015 can be seen as the continuation of the previous two symposia organized by Gordana Dodig-Crnkovic and Raffaela Giovagnoli: "Computing Nature", on the AISB/IACAP World Congress 2012, that resulted with the book "Computing Nature" published by Springer in 2013, and "Representation: Humans, Animals and Machines" at the AISB50 Convention at Goldsmith 2014, that will result in a new book published by Springer in 2015 under the title "Representation and Reality: Humans, Animals and Machines".

Contributors are Terrence Deacon, Jan van Leeuwen and Jiří Wiedermann, Clare Horsman, Susan Stepney, Viv Kendon, David Zarebsky, Dean Petters, John Hummel, Martin Jüttner, Ellie Wakui, Jules Davidoff, Gianfranco Basti, Gordana Dodig-Crnkovic, Marcin Miłkowski, Hector Zenil, Marcin J. Schroeder, Raffaela Giovagnoli, Rajakishore Nath, Philip Larrey, Matej Hoffmann, Vincent Müller, Ferdinando Cavaliere, Yasemin J. Erden, Roberta Lanfredini, Tom Froese, Jean-Yves Beziau, Angela Ales Bello, Jesus Ezquerro, Mauricio Iza, Henri Prade and Gilles Richard.

They address diverse facets of the relationships between representation and reality in humans, animals and machines such as: life versus engineering; knowledge, representation and the dynamics of computation; the role of representation in signal processing in biological systems; the realism of human and machine cognitive ontologies; the visual representations used for object recognition in middle childhood and adulthood; the quantum field theory (QFT) dual paradigm in fundamental physics and the semantic information content and measure in cognitive sciences; reality construction in cognitive agent through info-computation; modeling empty representations: the case of computational models of hallucination; cognition, information & subjective computation; information integration; the social dimension of human representation and its relevance for the web; mind & machine; machine super-human intelligence; trade-offs in exploiting body morphology for control: from simple bodies and model-based control to complex ones with model-free distributed control schemes; a "distinctive" logic for ontologies and semantic search engines; models, maps and metaphors and why the brain is not a computer; matter, representation and motion in the phenomenology of the mind; enactive criticisms of info-computationalism; rationality and representation; on the difference among animals, humans and machines; cognitive processing; a general representation setting for capturing homogeneity and heterogeneity.

In my keynote address I will give a short account of different contributions to the

[1]Keynote speaker of the workshop of the same title of this abstract (page 152).

book and their relationship to the topic of the symposium and to each other and focus on my view of the process of reality construction in cognitive agent through info-computation within the framework of info-computational constructivism. Cognition in this framework is capacity of every living organism. Even a single cell while alive constantly cognizes, that is registers inputs from the world and its own body, ensures its own continuous existence through metabolism and food hunting while avoiding dangers that could cause its disintegration or damage.

Unicellular organisms such as bacteria communicate and build swarms or films with far more advanced capabilities formed through social cognition. In general groups of smaller systems cluster into bigger ones and this layered organization provides information processing benefits. Brains in animals also consist of many cells mutually communicating. Interesting and unexpected is the fact that single neuron is a relatively simple information processor, while the whole brain possess much more advanced cognizing capacities.

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The ontology of logical form: formal ontology vs. formal deontology

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The intuition of formality is a methodological principle traditionally used to demarcate the boundaries of logic. A variety of interpretations of this intuition is possible. As J.-Y. Beziau put it, "form is like a multi-headed dragon: cut one head, three more heads grow" (Beziau 2008, 20). The main aim of this paper is to show the advantages

[1]Keynote speaker of the workshop "The Idea of Logic: Historical Perspectives" (page 113).

of shifting focus from substantial towards dynamic model of formality, i.e. from formal ontology (the domain of higher order formal objects, e.g. hypostases of structurally invariant properties of models) to formal deontology (the domain of rules-governed and goals-directed activity). Substantial model goes back to the Aristotelian form versus matter dichotomy, though, as J. MacFarlane pointed out, "the father of both formal logic and hylomorphism was not the father of logical hylomorphism" (MacFarlane 2000, 255). Substantial hylomorphism considers logic as a theory of formal relations which takes their general properties and turns them into general laws of reasoning. I am going to systematize the variety of substantial hylomorphism according to different types of formal relations, e.g. transcendental relations (scholasticism); psychological relations (E. Husserl); ideal relations (A. Meinong); relations of ideas in themselves (B. Bolzano); metalogical relations (N. Vasiliev) and logical relations (A. Tarski). For example, Tarski explained the concept of logical notions as exactly those which are invariant under arbitrary permutations of the underlying domain of individuals. He proposed the following general philosophical interpretation of his invariance criterion, "our logic is logic of cardinality". Because of the overgeneration of the criterion (Tarski's criterion assimilates logic to set theory; see McGee 1996, Feferman 2010) and its undergeneration with respect to modal logics (MacFarlane, 2000, Dutilh Novaes 2014) permutation invariance cannot be considered as necessary and sufficient criterion of logicality. According to C. Dutilh Novaes, "[p]ermutation invariance is above all an adequate formal explanans for the notions of quantity and quantification" (Dutilh Novaes 2014, 86). In this paper I argue that Tarski's thesis of 'our logic' as 'logic of cardinality' is not correct even for the theory of polyadic quantification. Distinguishing relations of equal power polyadic quantifiers take into account not only the cardinality of classes of numerically identical individual, but also the structures or types of the ordering of the universe. Furthermore, the model-theoretical approach based on the relation of language to model structures is incapable of recording the dynamic of such ordering. The treatment of the types of isomorphism as formal objects does not involve the application of formality characteristics to the activity as a result of which formal objects arise. Switching attention to this activity means in turn a transition from substantive to dynamic model of formality. The dynamic model of formality goes back to the scholastic conception of logic as a formal art. The dynamic formality characterizes a special way to following the rule. Thus, its various modifications may be classified into two clusters according to J. Rawls and J. Searle's dichotomy of constitutive and regulative rules. The explication of the constitutive formality in Wittgenstein's project of philosophical grammar will be sketched. I am going to compare substantial (model-theoretical) and dynamic (game-theoretical) approaches to the exegesis of Wittgenstein's thesis that colors possess logical structures, focusing on his 'puzzle proposition' that "there can be a bluish green but not a reddish green" (Wittgenstein 1977). What is gained, then, is a new game-theoretical framework for the logic of 'forbidden' (e.g., reddish green and bluish yellow) colors.
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Cut-free proofs for more and more logics

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Perhaps universally, if a formal logic has a proof theory, it will have an axiomatic one. But axiomatic systems are difficult to use, do not yield important metatheoretic results like interpolation, and are generally useless for proof automation. Ever since they were introduced in the 1930's, *cut free* proof systems have been the preferred alternative to axiomatic systems. They provide the insights and formal consequences that axiomatics does not. Generally speaking, cut free systems come in two forms, forward reasoning (sequent calculi) and backward reasoning (tableau systems). These are dual to each other and can be seen as notational variants.

Unfortunately, most logics do not (or are not known to) possess cut free proof systems. I want to tell a story about my own work on this, stretching over more years than I prefer to remember. I am hardly the only person who has contributed to this, but it is impossible in a short time to cover the whole history. (I must mention the work of Jan von Plato and Sara Negri here, though I will not discuss it in my talk.)

[1] Keynote speaker of the session "Universal" (page 239).

Confining the discussion to a more-or-less personal history provides a coherence that may help those unfamiliar with the subject, or so it is hoped.

Early on there were tableau and sequent calculus systems for a small number of classically based modal logics, say half a dozen or so. Formal machinery was expanded using *prefixes* for tableaus (1970's) and *nesting* for sequent systems (2000's). This encompassed around two dozen familiar modal logics. Quite recently this machinery has been further enhanced using *set prefixes* for tableaus and *indexed nesting* for sequents (current work). We now are able to handle infinitely many modal logics. The work extends naturally to intuitionistic based modal logics as well, but I will not cover this.

We probably will never know which formal logics have cut free proof systems. Indeed, it is not even a well-formulated question, since one can add machinery (as above) in unexpected ways. One recognizes a cut free system when one has it, but there is no good abstract definition that I know of. We progress, but cannot know when the work is done.

In search for a conceptual logic of information

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Philosophy is often concerned with structural ways of analysing systems and their dynamics. Before the advent of modern mathematical logic, such structural studies could simply be considered as part of logic. Today, this would be very misleading. But so would be the use of labels such as "informal logic" and "philosophical logic", already appropriated by other branches of philosophy. So, for lack of a better expression, I shall refer to them as "conceptual logics". Modernity has been dominated by two main conceptual logics, Kant's and Hegel's. Kant's transcendental logic concerns the study of the conditions of possibility of a system under investigation. It is therefore consistent with causal and genetic forms of reasoning, with the identification of necessary and sufficient conditions, with past-oriented analyses of what must have been the case for something else to be the case, and with the natural sciences. Hegel's dialectical logic concerns the study of dynamic equilibria. It is therefore more easily associated with polarised and procedural ways of reasoning, with the identification of contrasts and their resolutions, with present-oriented analyses of processes and mutual interactions, and with the social sciences. The two conceptual logics can be seen at work in philosophers such as Marx (Hegel) and Husserl (Kant). They are not incompatible and can easily be found interacting, e.g. in Foucault. Both investigate systems as something given, whether in the natural universe or in human history. And both move from the system to the model understood as a description of it, so to speak, looking for generalities and patterns. Neither is a conceptual logic of construction, which moves from

[1] Keynote speaker of the workshop "Logic and Information" (page 219).

the model, now understood as a prescription (blueprint), to its implementation as a realised system. Such a gap in our philosophical reasoning is pressing today because of the rising importance of computer science as a poietic science, which does not just describe its objects, but actually builds them, and empowers other sciences to build their own, through simulations, algorithms, and big-data-based research. In my presentation I shall address such a gap. I shall explore the possibility that the conceptual logic of construction that we need may be the conceptual logic of information, and that this may be consistent with design-oriented forms of reasoning, with the identification of requirements and constraints, with future-oriented analyses of what could work, and with the engineering sciences. Such a poietic logic would concern the study of design projects.

First-Order Logic and First-Order Functions

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In the first part of my talk I will expound basic ideas and goals of the theory of first-order functions. First-order functions were introduced by me as the proper first-order correlate of truth-functions. I think the study of those functions has shed new light on first-order logic and on its relation with propositional logic. Secondly, I would like to show how the introduction of first-order functions gives us new directions of inquiry in first-order logic.

Is there a Logical Reasoning Module in the Brain?

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Logically, deductive reasoning is a closed system and thus a good candidate for a cognitive module. However, neuropsychological research into the neural basis of reasoning has failed to identify a coherent module specifically activated during logical reasoning. Rather, the data point to a fractionated system that is dynamically configured in response to certain task and environmental cues. We have explored four lines of demarcation (Goel, 2007): (a) systems for processing familiar and unfamiliar content; (b) conflict detection/resolution systems; (c) systems for dealing with determinate and indeterminate inferences; and (d) systems for dealing with emotionally laden content.

[1] Keynote speaker of the session "Tools and Results" (page 309).
[2] Keynote speaker of the session "Cognition" (page 252).

Furthermore, meta-analysis studies indicate that different logical forms (e.g. categorical syllogisms, conditionals, and transitive inferences) also recruit different neural systems. I will review this evidence and discuss the implications for the psychology of logic.

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Truth-functional alternative to epistemic logic (and its application to Fitch's paradox)

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It is common to formalize the expressions of the form "agent a knows x" by the use of an epistemic operator $K_a x$. Hintikka (1962) provides a non-functional semantic interpretation of this operator in terms of possible worlds semantics. His interpretation is intuitively clear when the formalization of the fact of knowing something is represented as syntactic operator K. My aim here is to introduce an epistemic system, in which the K-operator does not appear, but the fact of knowing or not knowing some truths (or the falsity of some statement) can be defined truth-functionally. In order to obtain this system, we propose a four-valued logic, that we call the logic of a rational agent. The valuations in this logic are intuitively understood as follows: "true and known to be true" ($T1$),"true and unknown to be true" ($T0$),"false and known to be false" ($F1$) and "false and unknown to be false" ($F0$). Thus, the fact of knowing something is formalized at the level of valuations, without the use of K-operator. On the base of this semantics, a sound and complete system with two distinct truth-functional negations (an "ontological" and an "epistemic" one) is provided. These negations allow us to express the statements about knowing or not knowing something by an agent at the syntactic level. Moreover, such a system may be applied to the analysis of Fitch's paradox: if we accept the thesis that all truths are knowable, then all truths are already known. In particular, we show that the paradox is not derivable in terms of the logic of a rational agent.

[1]Keynote speaker of the session "Paradox" (page 300).

Discourse and logical form

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Traditionally, pronouns are treated as ambiguous between bound and demonstrative uses. Bound uses are non-referential and function as bound variables; and demonstrative uses are referential, and pick out objects as determined by their linguistic meaning and surrounding non-linguistic cues — e.g., an accompanying demonstration or an appropriate and adequately transparent speaker's intention. In this paper, we challenge tradition and argue that both demonstrative and bound pronouns are dependent on, and co-vary with, antecedent expressions. Moreover, the semantic value of a pronoun is never determined, even partly, by extra-linguistic cues;, it is fixed, invariably and unambiguously, by features of its context of use governed entirely by linguistic rules. We exploit the mechanisms of Centering and Coherence theories to develop a precise and general meta-semantics for pronouns, according to which the semantic value of a pronoun is determined by what is at the center of attention in a coherent discourse. Since the notions of attention and coherence are, we argue, governed by linguistic rules, we can give a uniform analysis of pronoun resolution that covers bound, demonstrative, and even discourse bound ("E-type") readings. Just as the semantic value of the first-person pronoun 'I' is conventionally set by a particular feature of its context of useâ€"namely, the speaker — so too, we argue, the semantic value, e.g., of 'he' is conventionally set by a particular feature of its context of use. In this paper, we elucidate what this feature is and how it works.

Leon Henkin on Completeness

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This research has been possible thanks to the research project sustained by Ministerio de Ciencia e Innovación of Spain with reference FFI2013-47126-P.

The Completeness of Formal Systems is the title of the thesis that Henkin presented at Princeton in 1947, and his director was Alonzo Church. His renowned results on completeness for both *type theory* and *first order logic* are part of his thesis. It is interesting to note that he obtained the proof of completeness of first order logic readapting the argument found for the theory of types.

[1]Keynote speaker of the session "Language" (page 294).
[2]Keynote speaker of the session "Completeness" (page 356).

In 1963 Henkin published a completeness proof for propositional type theory, *A Theory of Propositional Types,* where he devised yet another method not directly based on his completeness proof for the whole theory of types.

It is surprising that the first-order proof of completeness that Henkin explained in class was not his own but was developed by using Herbrand's theorem and the completeness of propositional logic.

"Since we use the completeness of sentential logic in our proof, we effectively reduce the completeness problem for first order logic to that of sentential logic."

We conclude this paper by pointing two of the many influences of his completeness proofs, one is the completeness of *basic hybrid type theory* and the other is in correspondence theory, as developed in *Extensions of First-order Logic*.

In the book **The Life and Work of Leon Henkin**, recently published, there is a complete chapter devoted to this issue, *Henkin on Completeness*.

1 The completeness of FOL in Henkin's course

The story behind this is that of María Manzano, who during the academic year of 1977-1978 attended his class of *metamathematics* for doctorate students at Berkeley. Before each class Henkin would give us a text of some 4-5 pages that summarized what was to be addressed in the class. The texts were printed in purple ink, done with the old multicopiers that we called *"Vietnamese copiers"* and that were so often used to (illegally) print pamphlets in our past revolutionary days in Spain against Franco regime.

It is surprising that the first-order completeness proof that Henkin explained in class was not his own but was developed by using Herbrand's theorem and the completeness of propositional logic. In what follows I will summarize the proof, but trying to maintain close to the spirits of Henkin's purple notes.

Theorem 1 (Herbrand's Theorem)**.** For each first-order sentence A there exist an (infinite) set of sentences of propositional logic Ψ such that: $\vdash A$ in FOL iff there is some $H \in \Psi$ such that $\vdash H$ in LP ($\vdash_{PL}$ means that we just use sentential axioms and detachment).

The above result can be regarded as a special case of the following

Theorem 2. Let L be a first order language: We can extend L to L' by adjoining a set $\mathcal{C}$ of individual constants, and we can effectively give a set Δ of sentences of L' with the following property: For any set of sentences $\Gamma \cup \{A\} \subseteq Sent(L)$,

$$\Gamma \vdash A \text{ iff } \Gamma \cup \Delta \vdash_{PL} A.$$

Proof. In the first place, we build a set Δ, where

$$\Delta = \Delta_1 \cup \Delta_2 \cup \Delta_3.$$

Δ_1 consists of the sentences $\exists x_i B \to B(c_{i,B})$ (for each $\exists x_i B \in Sent(L')$). Δ_2 consists of various formal axioms for quantifiers (from first order logic), and Δ_3 consists of various formal axioms for the equality symbol (if there is one in the language L, otherwise is $\varnothing$).

In the spirit of Herbrand's theorem, an effective method of transforming any given derivation of A from $\Gamma \cup \Delta$ in PL into a formal derivation of A from Γ in FOL was given, which solves half of the theorem.

$$\Gamma \cup \Delta \vdash_{PL} A \text{ implies } \Gamma \vdash A.$$

As for the other half,

$$\Gamma \vdash A \text{ implies } \Gamma \cup \Delta \vdash_{PL} A.$$

Suppose now that we do not have $\Gamma \cup \Delta \nvdash_{LP} A$. Then if we use completeness of propositional logic, $\Gamma \cup \Delta \not\models_{PL} A$ and we conclude that there is an assignment g for atoms of L'_0 that extends to an interpretation $\mathfrak{I}$ such that $\mathfrak{I}(A) = F$, but $\mathfrak{I}(\Gamma \cup \Delta) = T$.

In order to prove the theorem, from this interpretation $\mathfrak{I}$ we obtain a first order structure $\mathcal{A}$ such that $\mathcal{A} \models \Gamma$, but $\mathcal{A} \not\models A$, and so $\Gamma \not\models A$.

By soundness of first order logic, $\Gamma \nvdash A$. □

Predicate Logic — Reduction to Sentential Logic: Using the previous theorem we effectively reduce the completeness problem for first order logic to that of sentential logic. To this effect the following proposition was proved.

Proposition 3. Theorem 2 and completeness of PL implies completeness of FOL.

Note that a proof of the kind described above, provides a completeness proof for first order logic. For the theorem shows

$$\Gamma \nvdash A \text{ implies } \Gamma \cup \Delta \nvdash_{PL} A.$$

On the other hand, using the structure $\mathcal{A}$ we show that

$$\Gamma \cup \Delta \nvdash_{PL} A \text{ implies } \Gamma \not\models A.$$

Therefore, $\Gamma \models A$ implies $\Gamma \vdash A$, which is completeness for first order logic.

Another completeness proof he also developed in class was his result based on Craig's interpolation theorem [7].

2 His renowned proof of the completeness

The theorem of completeness establishes the correspondence between deductive calculus and semantics. Gödel had solved it positively for first-order logic and negatively for any logical system able to contain arithmetic. The lambda calculus for the theory of types [2], with the usual semantics over a standard hierarchy of types, was able to express arithmetic and hence could only be incomplete. Henkin showed that if the formulae were interpreted in a less rigid way, accepting other hierarchies of types that did not necessarily have to contain all the functions but at least the definable ones, it is easily seen that all consequences of a set of hypotheses are provable in the calculus. The valid formulae with this new semantics, called general semantics, are reduced to coincide with those generated by the rules of calculus.

As is well known, Henkin's completeness theorem rests on the proof that every consistent set of formulae has a model. Surprisingly, the model uses the expressions themselves as objects; in particular their elements are equivalence classes of expressions, the equivalence relationship being that of formal derivability of equality.

Hierarchy of types The types are structured in a hierarchy that has the following as basic types:

- $\mathcal{D}_1$ is a non-empty set; that of individuals of the hierarchy.
- $\mathcal{D}_0$ is the domain of truth values (since we are in binary logic, these values are reduced to T and F).
- The other domains are constructed from the basic types as follows: if $\mathcal{D}_\alpha$ and $\mathcal{D}_\beta$ have already been constructed, we define $\mathcal{D}_{(\alpha\beta)}$ as the domain formed by all the functions from $\mathcal{D}_\beta$ to $\mathcal{D}_\alpha$.

To talk about this hierarchy, a formal language is introduced.

The hierarchy described above is standard, and the completeness result depended on accepting other hierarchies of types that did not necessarily have to contain all the functions but at least the definable ones. In particular, his main theorem reads:

Theorem 4. If Λ is any consistent set of cffs (sentences), there is a general model (in which each domain $\mathcal{D}_\alpha$ of $\mathcal{M}$ is denumerable), with respect to which Λ is satisfiable.

To prove this theorem the set Λ is extended to a maximal consistent set which serves both as an oracle and as building bricks for the model. Specifically, to identify objects named by using M^α and N^α he made use of a criterion based on the calculus, in particular the fact that $\vdash M^\alpha = N^\alpha$.

How does Henkin construct type hierarchies? On page 86 of *Completeness in the Theory of Types* he says this:

> We now define by induction on a a frame of domains $\{D_a\}$ and simultaneously a one-to-one mapping Φ of equivalence classes onto the domains D_a such that $\Phi([\alpha_a])$ is in D_a.

Surprisingly, he obtained the proof of completeness of first-order logic later, readapting the argument found for the theory of types. Another interesting aspect that Henkin himself pointed out is the non-constructive nature of the proof, despite coming from a tradition as tightly bound to proofs with a constructive nature as those developed by Church.

In 1963 Henkin published the paper *An Extension of the Craig-Lyndon Interpolation Theorem*, where we can find a different proof of completeness for first order logic. Craig had shown the following theorem:

Theorem 5. If A and C are any formulas of predicate logic such that $A \vdash C$, then there is a formula B such that (i) $A \vdash B$ and $B \vdash C$, and (ii) each predicate symbol occurring in B occurs both in A and in C.

Henkin recalls that due to the fact that the relations $\vdash$ and $\models$ coincide in extension (by the strong completeness theorem), the above theorem is also valid if we replace the syntactic notion of derivability by the semantical notion of consequence. However, his idea was to obtain completeness from a slightly modified version of Craig's theorem:

> Notice, however, that if we alter Craig's theorem by replacing the symbol "$\vdash$" with "$\models$" in the hypothesis, but leaving "$\vdash$" unchanged in condition (i) of the conclusion, then the resulting proposition yields the completeness theorem as an immediate corollary.

The main theorem to be proved is:

Theorem 6. Let Γ and Δ any sets of nnf's (negation normal formula) such that $\Gamma \models \Delta$. There is a nnf B such that (i) $\Gamma \vdash B$ and $B \vdash \Delta$, and (ii) any predicate symbol with a positive or negative occurrence in B has an occurrence of the same sign in some formula of Γ and in some formula of Δ.

The strong completeness theorem is implied by the previous one.

The proof of the theorem is done by contraposition and to arrive to the conclusion that $\Gamma \not\models \Delta$ Henkin inductively builds two sets of sentences and define a model based on them using the technique he himself developed in his classical completeness proof [4].

3 Two results based on Henkin's ideas

Let us highlight how Henkin's *general models* are related to the *theory of representation*, or in other words: the *correspondence theory* and *non-standard models*. A more detailed examination of this can be consulted in the article by Manzano entitled *"Divergencia y rivalidad entre Lógicas"* [9] or in her book *Extensions of First Order Logic* [8]. Currently, the proliferation of logics used in Philosophy, Informatics, Linguistics and Mathematics make it crucial to achieve an operative reduction for all of them. We attribute most of the ideas handled in the reduction to many-sorted logic

[8] to two articles by Henkin: *"Completeness in the theory of types"* from 1950, and the one from 1953, *"Banishing the rule of substitution for functional variables"*. Nevertheless, with all the foregoing we do not wish to deceive possible readers. In the article from 1950, there are no translations of formulae, and the language and many-sorted calculus do not even appear explicitly. Regarding higher-order logic, as far as is known many-sorted calculus appears for the first time in the 1953 article. In it, Henkin proposes the axiom of comprehension as an alternative to the substitution rule used in the calculuses previously proposed for higher-order logic. If the axiom of comprehension is removed from this calculus, one obtains the MSL calculus. There is also another idea — this time from the 1953 article — that is also interesting and is as follows: If we weaken the axiom of comprehension (for example, we restrict it to first-order formulae or to translations of dynamic or modal formulae or to any other recursive set), we obtain calculuses in between MSL and SOL. And it is easy to find their corresponding semantics. Naturally, the class of structures corresponding to them will be situated in between $\mathcal{F}$ and $\mathcal{GS}$. The new logic, let us call it XL, will also be complete. The reason is because this class of models is axiomatizable.

In [1] a Basic Hybrid Type Theory is introduced. The goal of this paper is to investigate whether basic hybridization also leads to simple Henkin-style completeness proofs in the setting of (classical) higher-order modal logic (that is, modal logics built over Church's simple theory of types [2], and as we shall show, the answer is "yes". The crucial idea is to use $@_i$ as a rigidifier for arbitrary types. We shall interpret $@_i\alpha_a$, where α_a is an expression of any type a, to be an expression of type a that rigidly returns the value that α_a receives at the i-world. As we show, this enables us to construct a description of the required model inside a single MCS and hence to prove (generalized) completeness for higher-order hybrid logic.

We now come back to Henkin's crucial idea for taming higher-order logic. The standard semantics (ignore for the moment the modal and hybrid components) is the usual semantics for higher-order logic and it is logically intractable: if we define validity as truth in all standard structures, we have a complex (indeed, provably unaxiomatizable) notion of validity. His notion of general interpretations simultaneously lowers the logical complexity of validity (as there are more general structures than standard ones, it is, so to speak, easier for a formula to be falsified, and indeed, higher-order validity becomes recursively enumerable) and makes clear just why those plausible looking axiomatizations were so plausible: they are complete with respect to Henkin's general semantics.

Our completeness theorem is essentially an adaptation of Henkin's hierarchy construction, using the rigidity and truth equivalence classes introduced at the end of the previous section.

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Consequiland: on logics with many dimensions

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Universal Logic aims at studying features shared by all sorts of *logical structures*. But *what are & what should be* logical structures? This talk will promote a promenade beyond the study of structures in which a set of sentences is endowed with some fixed notion of consequence and have a peek at a number of alternatives that allow for a single given logical system to combine several *modes of reasoning* defined over the same set of sentences, incorporating variegated dimensions in which one might sensibly ask *what-follows-from-what*. Among the alternatives that I shall survey, one consists in having a logic associated to many notions of consequence sharing their perspectives on validity and unsatisfiability, and hierarchically organized into an appropriate lattice-like structure; another one portrays a logic as associated to several notions of consequence, each of which aiming to represent the preservation of some convenient notion of logical value; yet another path is to abandon truth-values as primitive entities and consider a single richer notion of consequence that embodies at once the several logical dimensions that we are interested upon, each one recoverable at any given time simply by taking the appropriate viewpoint on the whole. Of course, all the alternatives have their qualities and their shortcomings. The latter alternative, that I call *B-consequence*, will receive the biggest amount of attention, for reasons that will be explained in the course of the talk.

Connexive Logic based on an Incompatibility Operator

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A new dyadic propositional operator "$\downarrow$" is introduced, where "$A \downarrow B$" is read "A is incompatible with B". The connexive implication operator "$\rightarrow$" is defined as follows:

$$A \rightarrow B =_{\mathrm{df.}} A \downarrow {\sim} B,$$

and a semantic tableau formulation of connexive logic based on "$\downarrow$" and "$\sim$" is constructed.

[1] Keynote speaker of the workshop "Non-Classical Abstract Logics" (page 166).
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Invitation to Non-Classical Mathematics

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We take as starting point a theorem from Archimedes on the area of a circle. We prove this in a setting where some inconsistency is permissible, using paraconsistent reasoning. The new proof emphasizes that the famous method of exhaustion gives approximations of areas closer than any *consistent* quantity. This is equivalent to the classical theorem in a classical context, but not in a paraconsistent context, where it is possible that there are inconsistent infinitesimals. That the core of Archimedes' idea still works in a weaker logic is evidence that the integral calculus, analysis, and mathematics more generally are still practicable even in the event of inconsistency. The role of equality is central, and leads to investigation of the idea of distinguishability, rather than identity, as a primitive notion.

Next, we begin to explore a natural approach to mathematics, akin to (but simultaneously dual to) Brouwer's, where the logic arises out of the mathematics, rather than the mathematics out of the logic. The approach shows how a number of key concepts and results from classical analysis can be broken down and reconsidered when more care is taken with contradiction. Contrary to Hardy, proof by contradiction may turn out *not* be a far finer gambit than any chess gambit after all—in fact, it may be characterized as the crudest of all gambits. Direct proof plays a much larger role, and mathematics may be more characterized as a dynamic, organic enterprise rather than a sterile, clinical science where theorems are immutable.

On the Way to Modern Logic — the Case of Polish Logic

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Polish logicians played a significant role in the development of logic in the twentieth century — Warsaw School of Logic with its leaders Jan Łukaszewicz, Stanisław Leśniewski, Alfred Tarski and many others are known to any historian of logic. They developed logic in its modern form, in the form of formal, mathematical logic as an autonomous discipline having its own subject and methods. Nevertheless interest for logic existed in Poland much earlier though it has been understood and developed in a

[1]Keynote speaker of the workshop "20 Years of Inconsistent Mathematics" (page 234).
[2]Keynote speaker of the workshop "The Idea of Logic: Historical Perspectives" (page 113).

traditional way. In this paper we shall present two scholars: Henryk Struve (1814–1912) and Władysław Biegański (1857–1917) who can be considered as representatives of the traditional pre-mathematical approach to logic standing on the threshold of the new paradigm.

Henryk Struve was a philosopher living at the turn of the 19th and the 20th centuries. He is regarded as one of the most important figures of Polish logic in the 19th – yet he has been forgotten. According to him the object of logic was principles and rules of thinking. Logic concerns objective reality; nonetheless, it does not concern it directly – the mediator between logic and the world is the thought. Struve did not value the role and significance of symbolic and mathematised formal logic but he stressed psychological questions.

Władysław Biegański was by profession a general practitioner, held a medical doctorate and had his own medical practice. However, his true passion was logic. He was rather a philosophical logician in the standpoint formulated by Łukasiewicz. According to him the laws of logic concern the relationships of mental phenomena because of its aim, which is true cognition. Logic is the art of argumentation, it has a normative character and is an applied science. He stressed also the autonomy of logic and psychology.

A Model of Dialectic

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In this talk I give a formal model of dialectical progression, as found in Hegel and Marx. The model is outlined in the first half of the paper, and deploys the tools of a formal paraconsistent logic. In the second half, I discuss a number of examples of dialectical progressions to be found in Hegel and Marx, showing how they fit the model.

[1]Keynote speaker of the workshop "Philosophy of non-classical logics: Towards problems of paraconsistency and paracompleteness" (page 123).

Creativity and Visualisations in Mathematics

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This paper aims to show that creative reasoning with visualisations can facilitate development of a new approach and new mathematical concepts.

The case study for supporting this claim is taken from geometric group theory. This began as an application of some geometric ideas in combinatorial group theory, but a novel geometric perspective developed in the 1980s was so fruitful in results about groups that geometric group theory acquired the status of a branch of mathematical research in its own right. The case study demonstrates that representing groups as Cayley graphs, and then representing the latter as metric spaces, facilitated studying groups by geometric methods and led to the discovery of a number of geometric properties of groups. As a result, many combinatorial problems were solved through the application of geometry. On top of that, new interesting concepts expressing the geometric properties of groups were developed.

Groups are not completely alien to geometry. They were first studied as symmetries of geometric objects. Then in 1870s Klein's Erlangen Programme aimed to provide a unifying framework for various geometries applying the notion of a transformation group, viewing each geometry as a space of points with a group of transformations acting on it. That was a case of applying algebraic methods, specifically group theory, to obtain results in geometry. In the case of geometric group theory, the direction is reversed – algebraic groups are now considered as geometric objects as such, namely metric spaces, and studied by geometric methods to obtain new results about groups.

Groups, even equipped with a simple metric, have relatively un-interesting structure, and it is hardly possible to grasp their geometric properties. However, when finitely generated groups are represented by their Cayley graphs, the visualisations of graphs allow for a creative move. When the Cayley graphs are "observed from distance", so that only their large-scale structure is visible and the detailed structure is effaced, the similarity between them and familiar metric spaces becomes evident. For example, for the group $\langle \mathbb{Z}, + \rangle$, the integers look like a straight line and the graphs of free groups remind one of hyperbolic spaces. All the necessary mathematics still ought to be provided, but this scaling experiment unfolds the whole idea with its fascinating opportunities.

Though it sounds puzzling, many known groups turn out to be hyperbolic, or negatively curved, in a precisely defined generalization of this familiar geometric property. Hyperbolicity is traditionally defined through the sum of the inner angles of a triangle from the given metric space. For example, in the Euclidean metric space, it amounts

[1] Keynote speaker of the workshop "Computational Creativity, Concept Invention and General Intelligence" (C3GI) (page 163).

to π. Nevertheless when finitely generated groups are represented by their Cayley graphs, one can define triangles on the Cayley graphs, i.e. triangles with no angles! In fact, these triangles can be identified as hyperbolic in virtue of a non-angle-related property. Hyperbolic "angle-less" triangles defined on graphs are no doubt far from our habitual image of a triangle, and the use of an angle-independent definition of hyperbolicity is rare. This innovative approach involves an innovative use of visualisations: graphs have to be seen for a moment not anymore as combinatorial but as geometric objects. For example, edges are seen as lines and triplets of connected vertices can be seen as triangles. All the necessary mathematics still ought to be provided, but these new practices of using visualisation of a graph in a geometric way helped to unfold a whole area of research. This paper gives a step-by-step epistemic analysis of how such creative use of visualisations can be performed.

One's Modus Ponens: Classical Logic and Semantics for Modality

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Recently, several authors have independently touted counterexamples to some of the most entrenched classical rules of inference; viz., *Modus Ponens* (MP) and *Modus Tollens* (MT) (e.g. [3, 2, 5]). Here's one from [5]:

> Take an urn with a 100 marbles. 10 of them are big and blue, 30 big and red, 50 small and blue, and 10 are small and red. One marble is randomly selected and hidden (you do not know which). Given this setup, (1) and (2) are licensed, but, surely, (3) does not follow:
>
> 1. If the marble is big then it is likely red.
> 2. The marble is not likely red.
> 3. So, the marble is not big.

The growing consensus among semanticists is that this failure of MT (and other classical rules of inference) reflects a tension between the semantics for modal vocabulary and classical logic, and that the lesson is that we need a revision of the standard semantics for modals, that would invalidate these patterns. This is the course taken by some relativists (Kolodny and MacFarlane 2010), some expressivists [6, 5, 4], and even some (dynamic) contextualists [1]. I will argue, contra these theorists, that the real lesson of the apparent counterexamples is not the one the critics have drawn, but rather one they have missed: namely, that (and how) a discourse context impacts the interpretation of modal language. I propose a theory of context change that explains the appearance of counterexamples, while (provably) preserving classically valid patterns of inference.

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Argumentation Semantics for Adaptive Logics

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In this talk we will associate adaptive logics with argumentation frameworks in the tradition of Dung [3].

The standard format for adaptive logics [2, 7] offers a generic framework for defeasible reasoning (e.g., inconsistency handling, inductive generalizations, abduction, normative reasoning, etc.). Its dynamic proof theory extends a monotonic, reflexive and transitive core logic (**L**) with a set of retractable inferences which are associated with defeasible assumptions. More specifically, these assumptions are sets of formulas Δ of a predefined 'abnormal' form that are assumed to be false in the given inference. When an assumption turns out to be dubious in view of a premise set Γ, e.g. when some $A \in \Delta$ is derived from Γ in **L** as part of a minimal disjunction of abnormalities, the inference associated with it gets retracted. Various adaptive strategies offer mechanisms for this retraction of inferences, some following a more cautions rationale then others.

Examples are inconsistency-adaptive logics based on the paraconsistent core logic **CLuN** where abnormalities are contradictions $A \wedge \neg A$. One retractable inference rule is disjunctive syllogism (DS): $\neg A, A \vee B$ implies B on the assumption that $A \wedge \neg A$ is false. Take the premise set $\Gamma = \{\neg p, p, \neg r, p \vee s, r \vee q\}$. While applying DS to $\neg r$ and $r \vee q$, assuming that $r \wedge \neg r$ is false, is non-retractable, applying DS to $\neg p$ and $p \vee s$ will be retracted since $p \wedge \neg p$ is derivable.

[1] Keynote speaker of the session "Argumentation" (page 283).

Argumentation frameworks are one of the central paradigms in A.I. for the modelling of defeasible reasoning. Arguments are arranged in directed graphs $\langle \mathcal{A}, \rightarrow \rangle$ in which $\mathcal{A}$ is a set of arguments and $\rightarrow \subseteq \mathcal{A} \times \mathcal{A}$ represents argumentative attacks (e.g. Pollock's rebuttals and undercuts). Given such a graph, argumentation semantics specify criteria for selecting sets of arguments that represent stances of rational discussants. While this often has been studied on an abstract level where the concrete logical structure of arguments is left unspecified, in recent years we have seen a renewed interest in frameworks with structured arguments (see e.g., [1, 4, 5, 6]).

In this talk we associate a premise set Γ, a core logic $\mathbf{L}$, and a set of abnormalities Ω with an argumentation framework $AF^{\Omega}_{\mathbf{L}}(\Gamma) = \langle \mathcal{A}, \rightarrow \rangle$. More precisely, we define $\mathcal{A} = \{\langle A, \Delta \rangle : A \vee \bigvee \Delta \in Cn_{\mathbf{L}}(\Gamma), \Delta \subseteq \Omega\}$ and $\langle A, \Delta \rangle \rightarrow \langle B, \Theta \rangle$ iff $A \in \Theta$. We call A the conclusion of an argument $\langle A, \Delta \rangle$ whereas Δ is the assumption under which it was derived. Various adaptive strategies will be associated with different argumentation semantics. Continuing the example given above, e.g.,

$$\langle \neg p, \varnothing \rangle, \langle p, \varnothing \rangle, \langle p \wedge \neg p, \varnothing \rangle, \langle s, \{p \wedge \neg p\} \rangle \in AF^{\Omega}_{\mathbf{CLuN}}(\Gamma),$$

where Ω is the set of contradictions) and $\langle p \wedge \neg p, \varnothing \rangle \rightarrow \langle s, \{p \wedge \neg p\} \rangle$.

Accordingly, $\langle s, \{p \wedge \neg p\} \rangle$ is not selected since it cannot be defended from this attack. We will also show how each stage of an adaptive proof from Γ can be associated with sub-graphs of $AF^{\Omega}_{\mathbf{L}}(\Gamma)$.

This research complements [8] where it was proceeded vice versa: abstract argumentation was represented by means of adaptive logics.

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Ontology of Programs

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Computer science appears to be a strange blend of mathematics and technology; one in which abstract objects and concepts are gradually massaged into concrete devices that perform physical computations.

On the face of it programs are formal objects that have a syntactic structure and semantic content. The latter is most often expressed in terms of their impact upon abstract machines. Moreover, the activity of programming involves mathematical reasoning about this impact. This cannot be avoided. However informal or implicit this reasoning might be, at some level, programs do not come into existence without it. Moreover, high level programs are constructed not in terms of their impact upon a low level physical machine but in terms of their impact upon a high level virtual or abstract one. Contemporary computer science is dependent upon the level of abstraction that these machines provide. This activity is formal or even mathematical in nature. Indeed, if matters stopped here one might suggest that computer science is little more than mathematics.

However, the central activity of computer science involves the specification, design and construction of programs and software. While this is partially a mathematical endeavour it is also a technological one. Although part of this process involves abstract notions, the other part is engineering design rather than mathematical creation. Certainly, many of the methodologies involved in software construction belong to engineering. More to the point, the final objective of computer science involves the construction of physical devices that induce computations on actual machines. What might start out as abstract notions end up as physical devices. In this sense, computer science blends and merges abstract notions with concrete ones. In a different form, this observation is widely acknowledged.

Much of the philosophical literature has it that programs have both a symbolic representation and a physical manifestation. The earliest example of this is the following (from [4]):

[1]Keynote speaker of the workshop "Philosophy of Computer Science" (page 200).

It is important to remember that computer programs can be understood on the physical level as well as the symbolic level. The programming of early digital computers was commonly done by plugging in wires and throwing switches. Some analogue computers are still programmed in this way. The resulting programs are clearly as physical and as much a part of the computer system as any other part. Today digital machines usually store a program internally to speed up the execution of the program. A program in such a form is certainly physical and part of the computer system.

A later expression of much the same observation, but put it in terms of text and machine, is given by [1]:

Software seems to be at once both textual and machine-like. After all, when one looks at a printout of a program one sees a lot of statements written in a formal language. But when one holds the same program on a floppy disk in one's hand, one feels the weight of a piece of a machine.

The following are of more recent origin, and explicitly asserts the duality thesis (from [4,1]):

Many philosophers and computer scientists share the intuition that software has a dual nature.

According to [3]:

It appears that software is both an algorithm, a set of instructions, and a concrete object or a physical causal process.

From [2]:

Software is a rather unique entity. On the one hand it can be considered a mathematical object — its component parts and operations of construction are rigorously defined, and the output result of a piece of software can be predicted precisely, at least in principle. On the other hand, it is also an empirical object — a piece of software executing on a machine is a physical object that can, as most of us have experienced on many occasions, produce unexpected and unforeseen behavior.

On the face of it programs have both symbolic and physical manifestations; somehow they have both. But this raises a question about how one thing can exhibit such different guises. This ontological dilemma expresses [1] as follows:

While computer scientists often enthusiastically embrace this duality, a metaphysician will view it as a puzzle to be explained. How can something, namely a computer program, be at once concrete and abstract?

To address this question we introduce a central concept from the philosophy of technology that will unite these two aspects of programs and wrap them up in a single concept. This will provide us with a way of conceptualizing the ontology of programs; a way that embraces both their mathematical and technological natures.

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Constructing Universal Logic of Development? Hundred Years after James Mark Baldwin

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Simple but basic ideas take long time to reach human sciences. The focus on issues of embryology (Hans Driesch), philosophy (Henri Bergson) and development (James Mark Baldwin) in the 1890s led to the efforts to formalize a logic of development ("genetic logic") by James Mark Baldwin in the years 1906–1915. Hundred years have passed and Baldwin's efforts have not been revitalized or developed further. I will outline basic premises of Baldwin's efforts and show how these could be reconstructed to create a universal logic of unique transformations (ULUT).

Seduced and Abandoned in the Chinese Room

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Toward a Logic for Realistic Reasoning in Humans and Computers

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Logic should reestablish its focus on valid reasoning in realistic situations. Since classical logic only covers valid reasoning in a highly idealized situation, there is a demand for new logics that are based on more realistic assumptions, while still keeping the general, formal, and normative nature of logic. A constructive example is NARS, which is a system that explores the possibility of building a logic as the "law of thought", both in humans and in machines.

NARS is based on the theory that "intelligence" means "adaptation with insufficient knowledge and resources", which requires the system to depend on finite computational power, to work in real time, to open to unanticipated tasks, and to learn from its experience. Working in such an environment, the validity of inference in NARS is justified by adaptivity, and the system uses an experience-grounded semantics. The formal language and inference rules are formalized in the framework of term logic, and can uniformly handle multiple types of uncertainty (randomness, fuzziness, ignorance, inconsistency, etc.), as well as multiple types of inference (deduction, induction, abduction, analogy, revision, etc.).

NARS has been mostly implemented as an open-source project. Though the system is still under testing and tuning, it already shows many novel properties, and the results address many topics in the study of logic, artificial intelligence, and cognitive science.

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8 – Workshops

The Idea of Logic: Historical Perspectives

This workshop is organized by

Juliette Lemaire
Centre Léon Robin, Paris Sorbonne University, France

Amirouche Moktefi
Tallinn University of Technology, Estonia

Throughout most of the history of Western philosophy, there has been a closely related (sub-)discipline called 'logic'. However, the common name should not conceal the marked differences among what counted as logic at different times. In other words, despite the stable name, logic as a discipline is not characterized by a stable scope throughout its history. True enough, the historical influence of Aristotelian logic over the centuries is something of a common denominator, but even within the Aristotelian tradition there is significant variability. Furthermore, as is well known, in the 19th century logic as a discipline underwent a radical modification, with the birth of mathematical logic. The current situation is of logic having strong connections with multiple disciplines — philosophy, mathematics, computer science, linguistics — which again illustrates its multifaceted nature.

The changing scope of logic through its history also has important philosophical implications: is there such a thing as the essence of logic, permeating all these different developments? Or is the unity of logic as a discipline an illusion? What can the study of the changing scope of logic through its history tell us about the nature of logic as such? What do the different languages used for logical inquiry — regimented natural languages, diagrams, logical formalisms — mean for the practices and results obtained?

The invited keynote speakers is this workshop are Elena Dragalina-Chernaya (page 88) and Roman Murawski (page 102).

Call for papers

This special UNILOG session will focus on both the diversity and the unity of logic through time. Topics may include:

- Historical analyses on what specific logicians or logic traditions considered to be the nature and scope of logic.
- Historical analyses illustrating differences in scope and techniques with respect to the current conception of logic, but also suggesting points of contact and commonalities between these past traditions and current developments (possibly by means of formalizations).

- Historical and philosophical discussions on the place of logic among the sciences and its applications/relations with other disciplines, now and then.
- Discussions of the logical monism vs. logical pluralism issue in view of the historical diversity/unity of logic over time.
- General philosophical reflections on what (if anything) the diversity of scope and practice in the history of logic can tell us about the nature of logic and the role of universal logic as such.

Abstracts for this workshop should be sent to moktefi@unistra.fr.

Logical and Non-Logical Lexicons: Was Tarski is right that there are no objective grounds to draw a sharp boundary between them?

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Inductive logic may be the closest cousin to the high and mighty bastion of logic, viz. deductive logic, and yet even then it has almost become a philosophical lore that, unlike induction, deduction is fairly unproblematic and the two modes of argumentation are in certain significant ways dissimilar. However, appearances aside, there may be more similarities than dissimilarities between them, and deduction may be as much open to the same type of problem(s) as induction is purported to face. Indeed, Rudolf Carnap once remarked that the epistemological situation in inductive logic is not worse than that in deductive logic, but indeed quite analogous to it. The purpose of this paper is, therefore, to pursue parallelisms between induction and deduction in terms of the issue of justification against the wider context of Alfred Tarski's scepticism about the bifurcation of lexicons into logical and non-logical. David Hume famously argued that induction faces a crisis of rationality as it cannot be justified either deductively or inductively. Apparently, a direct analogue could be constructed for deduction where it will be shown that deduction suffers from the same problem of justification: that is, it cannot be justified either inductively or deductively. The discussion then focuses on the limitations and shortcomings of claims made by, for example, Michael Dummett and Dag Prawitz that, unlike induction, at least some fragments of deductive logic can be justified. However, an attempt will be made to show that the parallelism could still be extended to an additional and more interesting analogy between deduction and induction in terms of Nelson Goodman's 'the new riddle of induction', i.e., the problem of "grue" predicate. It will be contended that likewise a "new riddle" could be formulated for deduction in terms of Arthur Prior's "tonk" connective. Yet, it may be objected, especially in the case of the latter parallelism, that there is a sharp disparity between deduction and induction, and the similarity is only skin-deep. For, again unlike induction, in deduction there are, for example, proof-theoretic principles such as Emil

Post's "conservative extension", or some model-theoretic variants of it, that proscribe the introduction of such rogue logical connectives. Indeed, it may be claimed that such constraints not only prevent the occurrence of tonk-type connectives, but could also be deployed to demarcate deductive logics from inductive ones. The remainder of the paper will, therefore, be devoted in a Carnapian spirit to an examination of this twofold claim, and in particular the status of those constraints themselves. Generalising the discussion, it may be argued that, after all, Tarski may have been right to contend that there are no objective grounds to permit one to draw a sharp boundary between logical and non-logical lexicons.

Logic as Semeiotic: Peirce's Philosophy of Logic

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In his later years, Peirce devoted much energy to the project of a book on logic, whose intended title was "Logic, considered as Semeiotic". That the science of logic is better considered as semeiotic is indeed one of the most fundamental tenets of Peirce's mature philosophy of logic. But what is the primary motivation for considering logic as semeiotic and what advantages did Peirce see in doing so? This question has largely remained unsettled in Peirce scholarship. If logic is to be considered as semeiotic, this can only mean that its objects and their functioning are to be described in purely semeiotical terms. But did Peirce succeed in providing such a description? This paper deals with the semeiotical functioning of the classical triad of logic: terms, propositions, and arguments. In particular, the paper focuses on the highly articulated semiotic structure of arguments.

According to Peirce, a term is a sign whose object and interpretant are implicit, for a term is a sign of qualitative possibility only. A proposition is a sign whose object is explicit but whose interpretant is implicit, for a proposition is a sign of actual existence and of nothing more. An argument is a sign whose object and interpretant are explicit, for an argument is a sign which is also represented as being a sign. This of course requires a division of interpretants, for "the sign not only determines the interpretant to represent [...] the object, but also determines the interpretant to represent the sign" (Peirce to Welby, 1906). Peirce thus distinguishes the immediate interpretant, which is a further sign of the object created or expressed by the sign, from the representative interpretant, which is the representation of the manner in which the sign represents its object.

The argument, which is for Peirce the perfect sign, is a body of premises that represents a conclusion. The conclusion is the immediate interpretant. This is, at bottom, the Stoic conception of sign or sēmeion ("a pre-antecedent statement in a sound conditional, revelatory of the consequent", Pyrrh. Hyp. II, 104). But reasoning for Peirce cannot consist in simply deriving a proposition (the conclusion) from other propositions (the premises); rather, it involves the approval of the rationality of the derivation, or the judgment that the reasoning is valid because it is an instance of a class of valid arguments. In semeiotical terms: an argument is a sign that separately represents its immediate interpretant and whose representative interpretant represents it as representing its immediate interpretant according to a valid logical principle.

A large part of Peirce's later (1903–1914) discussions and divisions of signs and interpretants had as ultimate objective the description of the semiotic functioning of the principal logical signs (the classical triad). More than a general theory that would cover the entire domain of signs, Peirce's semeiotic is better understood as his peculiar way of presenting his philosophy of logic.

Logic and its place in philosophy. T.H. Green and the idealistic view

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Speaking of 19th-century works on logic, R. Adamson once wrote in the /em Encyclopedia Britannica that "in tone, in method, in aim, in fundamental principles, in extent of field, they diverge so widely as to appear, not so many expositions of the same science, but so many different sciences". That makes 19th-century logic a difficult field for historians of philosophy, since they have to deal with a subject whose nature itself is highly controversial, and which tends to include issues that nowadays would not be regarded as belonging to logic. The case of idealistic logics is particularly interesting, since they differ both from a formal conception of logic (ultimately stemming from Aristotle) and from what came to be called a "psychologistic" approach (famously exemplified by J.S. Mill). Whithin the British brand of 19th-century idealism, the case of T.H. Green is most representative, in that his idealistic views are in a way more orthodox (i.e., closer both to the German antecedents and to the "centre of gravity" of the whole movement of British idealism) than those of such later authors such as F.H. Bradley or J. McTaggart. Furthermore, in his thoughtful attempt at acclimatizing German philosophical culture to his own country, Green paid special attention to assess the new views against the background of the British tradition in philosophy: in his Lectures on Logic (1874-75) the idealistic conception of logic is put forth by means of a sustained criticism of different strands of British logic, mainly formal logicians such as W. Hamilton and H.L. Mansel on the one hand, and J.S. Mill on the other.

One could surmise that Green wanted to place himself midway between opposite errors. A widespread historiographic scheme, inspired by M. Dummett, according to which one of the main features of post-Fregean analytic philosophy is its rejection of psychologism, seems to corroborate such a surmise: British idealists reacted against psychologism, but their reaction was not radical enough, and fell short of devising a fully formal conception of logic (that was also G.E. Moore's criticism of Bradley's philosophy). But such an interpretation would be simplistic and ultimately incorrect. In fact, according to Green: (i) formal logicians and Mill share a common error; (ii) formal logicians are more wrong than Mill. Concerning (ii), in his criticism of formal logicians, Green did not recoil from using some of Mill's arguments against the Scholastic tradition in logic, especially with regard to classification and definition. The major part of the debate revolved around the question of the "sterility" of logic. That was course an old charge made against logic by Renaissance and early modern thinkers, and it was very common within the British empiricist tradition. So common indeed, that even a staunch defender of the formal conception of logic in 19th-century Britain such as R. Whately felt himself obliged to make in some way more acceptable the apparent sterility of logic deriving from its formal character. In this sense it is clear that Mill could become an ally in the struggle against formal logicians. Yet Green thought (in accordance with the general bent of Hegelian idealism) that Mill and the empiricist tradition could be accused of the same mistake â€" though in a somewhat attenuated form â€" of the formal logicians. The main root of the common error lies in the central role attributed to abstraction as the fundamental operation of thought. The criticism of abstraction (which is to be replaced by the opposite process of "accretion") thus becomes the crucial move in Green's effort to establish a correct understanding of logic.

Probably the clearest account of such criticism can be found in Green's *Essay on Aristotle* (1866), where the modern debates on these issues are transposed to ancient times, and where Aristotle seems to play a role similar to that of Mill: Aristotle clearly understood the barrenness of Plato's "abstract" ideas, but he did not succeed in freeing himself completely from the shackles of the old way of thinking (and, what is worse, his most precious insights got lost in the later Scholastic tradition).

According to Green's positive views (similar to Hegel's), there is no place for logic as distinct from metaphysics, since logic really concerns the way in which knowledge is constituted by consciousness, by way of accretion. Such a process of constitution, as performed by the so-called "eternal consciousness", is identical with the constitution of reality itself. Perhaps a relative independence of logic could be envisioned with reference to the thought processes of individual minds, which are just "vehicles" of the eternal consciousness. However, the critical character of Green's writings on this subject does not leave enough room for the development of these suggestions.

More room for a relatively independent logic can be found in Bradley's philosophy. That is in part due to accidental circumstances (Bradley devoted a whole book *The Principles of Logic*, to a systematic, and not merely critical, exposition of logical doctrines), but probably also to philosophical reasons. First, Bradley elaborated a theory of the degrees of truth (which is only implicit in Green's thought), and within such a framework the relative independence of logic with respect to metaphysics can be easily accommodated. Second, but that is just a suggestion to be further developed, Bradley's opposition between immediate experience and thought (which is wholly alien to Green and to more orthodox forms of idealism) might have opened a somewhat more comfortable space for logic.

Metaphysics as "Natural Logic" in Hegel

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In my talk I focus on what Hegel explicitly says on the concept of logic and its relation to metaphysics in some passages of the *Science of Logic*, the *Encyclopaedia Logic* and the *Lectures on Logic and Metaphysics*. These texts show that Hegel aimed at both tracing metaphysics back to its logical roots and logic back to its metaphysical roots, thus developing the idea of an interplay between the two disciplines. In particular, I focus on the Hegelian idea that both logic and metaphysics have a "natural" level (or phase), distinguished from the theoretical one. Natural logic is the logic (the form) of natural reasoning; natural metaphysics is natural thought about reality and being. The two are strictly connected, as natural logic is based on metaphysical notions, such as unity, identity, causal connection. Conversely, natural metaphysics is based on the natural use of language and thought, as the human practice of reasoning needs the notions of unity, identity, etc. This dialectical connection remains for Hegel also at the theoretical level, when both metaphysics and logic become theories and disciplines, so that, for Hegel, there is no logic without metaphysics, and vice versa. My claim is that this theory is one of the main reasons of Hegel's importance for contemporary debates in philosophy of logic and metaphysics.

The Notion of Logical Form and its Application in Boole and Jevons

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Keywords: *History of Logic, Notion of Form, Algebraic Logic, Reasoning and Computing, Logic Machines, George Boole, John Venn, William Stanley Jevons.*

Today's notion of formal logic is coined in the 19th Century. This concerns the form of logic: its shape and ways of representation. But it also relates to the conception of logical form: its interpretation and the scope of applications. Thus, a new notion of the formal originates from seminal changes concerning both the form of logic and the 'logic' of 'form'.

In the 1880s, John Venn distinguishes three views of the logical form of propositions shaped 'A is B.' The first is characterized by a predicative interpretation. To say that A is B means to specify a subject A by predication of an attribute B. The second point of view is to conceive of propositions as expressing relations between classes. Given two classes A and B, either A is contained in B or B is contained in A, or A and B overlap, or they are coextensive. A negative proposition indicates that they are disjoint. The third approach departs from splitting A into compartments one of which does involve B while the second does not. Similar case-by-case analyses pertain to each term of logical problems of higher complexity.

Around the mid-century, George Boole launches an algebraic model of logic. Its distinctive shape is determined by the use of functions and variables. Its methods derive from calculating techniques based on explicit transformation rules, most importantly from the idea of 'expansion' of a function. A function $f(x)$ is developed into an expression of the form $ax+b(1-x)$. To determine a and b, the function is factored in $x = 1$ and $x = 0$, which translates into equating the variable to the totality of elements in a given domain, and to the absence of any of them alternately. The expansion will then be of the form $f(1)x + f(0)(1 - x)$; given functions of more than one variable, their expansions become more complex. As one variable may always be expressed in terms of the others, the resulting system allows for solutions with respect to each of them. Venn's principle of case-by-case compartmental analyses is mirrored in the differentiation of each constituent according to the function's values for 0 and 1.

Some ten years later, William Stanley Jevons proposes an understanding of logical method which is grounded in a classification of the differentiated cases. For instance, a consideration of four terms A, B, C and D yields their 16 possible combinations: $ABCD$, $ABCd$, $ABcD$, $ABcd$, $AbCD$, $AbCd$, $AbcD$, $Abcd$, $aBCD$, $aBCd$, $aBcD$, $aBcd$, $abCD$, $abCd$, $abcD$, $abcd$ (where lower case italics represent negatives). Now as a premise, introduce, e.g., the implication 'From A follows B'. All combinations not containing A are temporarily sorted out as irrelevant. The remainder is split up according as they do or do not contain both A and B. The latter contradict the given premise, while the former comply with it. The joint set of the former and the temporarily excluded combinations gives the number of cases which prove consistent with the premise. They may then be subject to subsequent selections according to further premises such as 'From C follows D'. This technique of crossing out empty compartments from the list of all possible junctions of terms Jevons calls a method of 'indirect deduction'.

The proposed contribution to the Universal Logic Conference aims at showing that both Boole's and Jevons's approaches are based on the methodical principle of compartmental analyses derived from Venn's third understanding of logical form. However, they differ in their possibilities of symbolical generalization and to the respective kind of algorithms. While Boole's ideas give direction to design and application of logical calculi, Jevons's proposal may be seen in relation to diagrammatic and tabular methods. Nowadays, Boole's name is remembered in the history of computing and computer science. However, Jevons's method allowed for implementation in a mechanical logic machine even in his own lifetime.

Logic as Physics. On Logic and the Aristotelian Concept of Historia, according to the Paduan Philosopher Jacobus Zabarella

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My paper will focus on the Paduan philosopher Jacobus Zabarella (1533-1589); probably the most prominent and influential Neoaristotelian in the field of logic in the sixteenth century. In his logical writings Zabarella developed the Aristotelian distinction between factual knowledge and causal knowledge. The first kind of knowledge goes back to the senses, cognitio sensilis, and was, perhaps surprisingly, called historia. The other one goes back to ratio and was called scientia in theoretical philosophy. This Aristotelian concept of historia, distinct, as it is, from the chronological concept of history, was rapidly disseminated in the republic of letters, but it is almost totally neglected by modern research. None the less, it has striking consequences for our understanding of the structure of early modern Aristotelian logic and of the academic disciplines.

The distinction, mentioned above, is, possibly, most clearly displayed in the theoretical discipline of physics or natural philosophy, of which methodology became a somewhat imperialistic paradigm for other disciplines. In physics, natural history provided the discipline with empirical data, observations, or historiae. By means of induction, data were transformed into general and "true" statements, in order to be used in natural philosophy proper as premises in deductions or syllogisms. Logic, applied on a discipline, was, according to the Paduan philosopher, transformed to very discipline itself. When the logical methods were applied to physics, logic was transformed into physics. Logic became physics. In such a case, logic was called applied logic, i.e. logica applicata or logica utens, in contrast the logic as a discipline, logica docens.

In the Aristotelian epistemological paradigm, historiae formed the empirical foundation of any discipline, which had to do with the sensual part of reality. On that foundation, the philosophical part of the same discipline was built. Consequently, when logic was applied to physics, logic as physics was dependent upon natural history, and both natural history and natural philosophy constituted vital parts in forming the integrated discipline of natural philosophy or physics.

In my lecture, I will delineate the Aristotelian concept of historia according to Zabarella and explain how histories, i.e. factual knowledge, related to logic and other disciplines on which logic was applied.

Frege and Aristotle about the logical foundation of mathematical functions, or Aristotle translated by Frege

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The aim of this presentation is to understand the relation between Aristotle's thought and Frege's foundation of logic. Particularly, it is focused here the enormous debt that Frege's notions of concept (Begriff) and "to fall under concept" (**fällt unter Begriff**) has to Aristotle's foundation of logic.

Of course, Frege takes the concept of concept from German tradition, from Kant's tradition and from Hegel's philosophy, in both these thinkers, this word appears abundantly. Specially in the last, the word "Begriff" acquired a more intensional sense. This word frequently translates the universal and the second substance(Aristotle's concepts that came to us by *Categories* and *De Interpretatione*). In Frege's formula this word appears in relation to another "object" (Gegenstand).

"Ein Gegenstand fällt unter einen Begriff". By this formula, Frege, as Aristotle in his own way, makes a dialectical approach toward extensional (Logik des Umfangs) and intensional logic(Logik des Inhalts) and he can surpass this dicotomy, that he pointed in his essay "**Ausführungen uber Sinn und Bedeutung**".

Supposing the concept of house, we can geometrically find infinite "solutions"(objects) for this **Begriff**.

It is to remark that Frege sees the absence of maths in Aristotle's theory **Logik in Mathematik** [1]), but his foundation of mathematical function is not other thing than saving the Aristotle's substantial proposition:

"Ein Gegenstand fällt unter einen Begriff"

is a very good translation for the aristotelical formula:

"first substance is second substance".

"Socrates (Plato and a large number of persons) is man [$F(m)$]":
"$F(m) = x$", where "x" is one man.

Congratulations, Herr Frege, you very well translated Aristotle!

[1] "Wenn man nun die Logik zur Philosophie rechned, so ergibt sich hieraus das Bestehen einer besonders engen Verbindungzwischen Mathematik und Philosophie, was duch die Geschichte der Wissenschaften bestätigt wird. (Plato, Descartes, Leibnitz, Newton, Kant)." Maybe we discovered this "Verbindung" (connection) only lately concerning Aristotle! Or, at least, after Frege's inventions. Of course, concerning others aspects, Łukasiewicz has already pointed the presence of mathematicism in Aristotle's logic.

Logic, Judgement and Inference. What Frege Should Have Said about Illogical Thought

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Frege held the view that logic is a normative science, because he saw logic as being concerned with setting forth norms for thinking — judging and inferring — correctly, and not with merely describing how thinking takes place. Not abiding by logical norms will thus make one think wrongly, or incorrectly. This view forms the core motivation for Frege's polemic against psychologistic conceptions of logic, widespread during his lifetime.

I argue that this normative view of logic motivates Frege's response to the 'logical aliens' scenario, as discussed in the introduction to the Grundgesetze. Logical aliens are beings who reject (basic) laws of logic, and therefore manifest illogical thought. Given his normative understanding of logical laws, Frege seems compelled to say that logical aliens do think — judge and infer — albeit wrongly, or incorrectly. As he puts it, our acceptance of logic "hinders us not at all in supposing beings who do reject it; where it hinders us is in supposing that these beings are right in so doing, it hinders us in having doubts whether we or they are right".

This is what Frege actually said about the possibility of illogical thought. Some scholars have thought that Frege's diagnosis amounts to an understatement, however; someone who rejects basic logical laws — one could argue — does not simply think incorrectly, but fails to think altogether. In the remainder of this paper I explore and expand two lines of thought — originally developed by James Conant and Joan Weiner — according to which, even from a Fregean understanding of the nature of logic, the very idea of illogical thought should be rejected as not constituting a genuine possibility. These thus amount to two different accounts of what Frege should have said about illogical thought.

According to the former account Frege should reject the possibility of (deeply) illogical thought, in so far as he believes logic to be an 'arbiter' that establishes the possibility of agreement and disagreement. But if the framework of logic is let go, then illogical thought could not be recognised as contradicting ours, and could not even be recognised as thought or reasoning at all. According to the latter account, Frege saw an internal relation between judgement and inference; in particular, he was committed to the view that the content of a judgement is not independent of the (correct) inferences in which the judgement in question can figure, and so any judgement already presupposes some acceptance of logical laws, which are laws of correct inference. Rejecting logical laws thus undermines the very possibility of judgement. While the former line of thought is unsuccessful, the latter is successful, and constitutes an important Fregean correction to Frege's own response to the possibility of illogical thought.

Husserl's Idea of Pure Logic: Constructive or Axiomatic?

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In this presentation, I aim to show that despite of some textual evidences from Husserl's work, his main approach is in contrast with the axiomatic method of Frege-Hilbert-Russell; rather, it is, in its basic philosophical grounds, in accordance with the overall attitude of Peirce and of Brouwer, regarding their actual practice in mathematics and logic. That Husserl does not support algebraic view toward logic and Peirce does, and that Brouwer does not give primacy to logic over mathematics and Husserl, in some sense, does, I hope to show, are only secondary and rather terminological issues.

According to the conception of logic that transcendental phenomenology endorses we may speak of the *interplay* between axiomatic method and constructive method. This is based upon the interplay between intuition and construction, more precisely, between *intuitive* intention and *signitive* intention while the latter serves in *categorial synthesis*. I would argue for the claim that not only phenomenology admits constructive approach toward logic, but also it provides a philosophical ground in order to develop constructive logic; yet the emphasis that Husserl puts on axiomatization should be interpreted in its own peculiar way.

Philosophy of non-classical logics: Towards problems of paraconsistency and paracompleteness

This workshop is organized by

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Ingolf Max
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There is an ongoing philosophical and logical debate about motivations in accepting or rejecting the principle (law) of (non-)contradiction and the principle (law) of excluded middle. A logic rejecting the principle of non-contradiction is called paraconsistent and a logic rejecting the principle of excluded middle is called paracomplete. If both principles are dual of each other we have some reason to reject both principles and get paranormal systems. But what does it really mean to reject a classical principle (law)?

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And what are the philosophical consequences for this refusal? In which sense would it still be possible to defend nowadays that there is just one true logic, if we have such a great diversity of logics?

Among the famous logical systems which are paraconsistent but not paracomplete are, for instance, the da Costa systems. Intuitionist logics are paracomplete but not paraconsistent. And a lot of systems of relevant logic are paraconsistent as well as paracomplete. To evaluate these systems' philosophical relevance, we have to inter alia examine the logical form of their atomic formulas, the logical behavior of their negation, conjunction and disjunction as well as the properties of logical consequence relations. From a philosophical point of view it is very important to understand which elements are responsible for such deviations from classical logic. E.g., do we have only local reasons? In the case of Jaśkowski's version of paraconsistent logic we have to change the conjunction. In the da Costa systems mainly negation is under attack. Or do we have global reasons like in systems of first degree entailments? (Belnap, Dunn, Priest). What is the position of paracomplete, intuitionist approaches (Brouwer, Heyting and their followers)?

This workshop shall represent a privileged platform to evaluate proposals for a more integrated and general approach to philosophical motivations and consequences in the emergence of non-classical logics.

The invited keynote speaker of this workshop is Graham Priest (page 103).

Call for papers

Topics may include:

- logical monism & logical pluralism
- philosophical motivations for creating non-classical logics (dialethism, anti-realism, relevantism, etc.)
- local vs. global and formal vs. application-oriented reasons for paraconsistency and/or paracompleteness
- non-explosiveness of logical consequence
- trivialization strategies and classical logic
- philosophy of contradiction and inconsistency (Hegel, Wittgenstein, Meinong, Heraclitus, Indian Philosophy, etc.)
- philosophy of constructivism (Poincaré, Brouwer, Heyting, Kolmogorov, Wittgenstein, Lorenzen, Dummett, Prawitz, etc.)
- philosophical relations between paraconsistency and paracompleteness

Abstracts (500 words maximum) should be sent via e-mail before November 15th 2014 to istanbul2015philosophyncl@gmail.com.

Notification of acceptance: December 1st 2014.

Hard and soft logical information

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The problem of accounting for acceptable uses of classically valid but paraconsistently invalid arguments is a recurrent theme in the history of paraconsistent logics. In particular, the invalidity of the disjunctive syllogism (DS) and modus ponens (MP) in, for instance, the logic of paradox **LP**, has attracted much attention.

In a number of recent publications, Jc Beall has explicitly defended the rejection of these inference-forms, and has suggested that their acceptable uses cannot be warranted on purely logical grounds [1, 2]. Some uses of DS and MP can lead us from truth to falsehood in the presence of contradictions, and are therefore not generally or infallibly applicable [3].

Not much can be objected to this view: if one accepts **LP**, then MP and DS can only be conditionally reintroduced by either

1. opting for Beall's multiple-conclusion presentation of **LP** ($\mathbf{LP}^+$) which only gives us

$$\begin{aligned} A, A \supset B &\vdash_{\mathbf{LP}^+} B, A \wedge \neg A, \text{ and} \\ \neg A, A \vee B &\vdash_{\mathbf{LP}^+} B, A \wedge \neg A \end{aligned}$$

2. by treating MP and DS as default rules.

The latter strategy was initiated by inconsistency adaptive logics [4, 5], and implemented for the logic **LP** under the name *Minimally inconsistent LP* or **MiLP** [6].

The gap between these two options is not as wide as it may seem: The restricted versions of MP and DS that are valid in **LP** are the motor of the default classicality of **MiLP**. The only difference is that the restricted version only give us *logical options* (Beall speaks of 'strict choice validities'), whereas default classicality presupposes a preference among these options (unless shown otherwise, we must assume that contradictions are false).

A cursory look at the debate between Beall and Priest [3, 7] may suggest that not much can be added to their disagreement. However, if we focus on the contrast between the *mere choices* of $\mathbf{LP}^+$ and the *ordering* of these choices in **MiLP**, we can tap into the formal and conceptual resources of modal epistemic and doxastic logic to provide a deeper analysis [8]. We can thus develop the following analogy:

$\mathbf{LP}^+$ is motivated by the view that logical consequence is a strict conditional modality, and is therefore knowledge-like. Using a slightly more general terminology: all logical information is *hard information*.

MiLP is motivated by the acceptance of forms of logical consequence that are variable conditional modalities, and are therefore belief-like. With the same more general information: some logical information is *soft information*.

This presentation still gives the upper-hand to Beall's stance (shouldn't logical consequences be necessary consequences?), but only barely so. The upshot of this talk is to motivate the views that (i) the soft information that underlies the functioning of **MiLP** can be seen as a global as well as formal property of a logical space, and is therefore more logical than we may initially expect, and that (ii) adding a preference among logical options can be seen as a legitimate and perhaps even desirable step in a process of logical revision.

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Logical pluralism from the perspective of logical expressivism

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I will attempt to tackle the question whether there can be more "right" logics by first considering what the criteria for being a logic might be. The relatively easy way to logical pluralism is to assert the plurality of purposes a logic might serve to fulfil and derive the plurality of legitimate logics from it. It is, of course, only good for a given logic if it can do more jobs at a time, yet it will not be very surprising if we would actually need different logics for different tasks. This thesis can be easily trivialized, when we formulate the purposes of various logics in ways favorable to particular systems. Thus intuitionistic logic clearly serves to study inference in constructivist mathematics, while the classical logic in the non-constructivist and so on.

I wish to concentrate, though, on one particular thing some philosophers think a logic should deliver, namely to make explicit the implicit rules of inference that we apply in our discourse. According to this Brandomian view, logic plays first of all an expressive role. The key notion of an implicit rule of inference is far from clear, though. My main question is whether such an approach is compatible with logical pluralism and what the sources and reasons for logical pluralism might in this particular case be.

On the one hand, it is clear that the implicit rules should be here before logic, which comes to express them. They are autonomous. But does the process of making them explicit involve creativity? To some degree it is simply given what is supposed to be expressed but perhaps more ways of expression might be possible due to different possible interpretations of these rules. Can it be therefore said that a given implicit rule can be interpreted to "state" various different things? If so, does such a plurality of interpretation enforce logical pluralism? I am inclined that say that it does not. First of all, the implicit rules are normative statuses of actions rather than normative statements, so the talk of their interpretation does not make much sense. Furthermore, it is not clear what the criteria of equivalence of formulations would be, whether two interpretations could really differ while being about the same implicit rule. But this issue has to be examined more closely.

Whatever the answer to the question about the possible plurality of interpretations of implicit rules might be there still remains a possibility of logical pluralism because there might be more ways how to explicate the same rules. If logic is a tool for explication, then perhaps it can come in more shapes which are just as satisfactory for the relevant purposes. How much can such logics differ, if at all? I will examine the proof-theoretic demarcations of the domain of logic from this expressivist perspective and show how they help to clarify this problem.

Finally the focus should shift rather from the rules of inference to the very activity of making them explicit. Debates between adherents of different logics can be seen as debates about the nature of this expressive rationality and the tools, such as the conditional or negation, which we use to perform the expressive task.

A piece of logical handicraft illustrating a philosophical position

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I shall defend a position with respect to three topics from the Call for Papers: (i) logical monism and logical pluralism, (ii) philosophical motivations for creating non-classical logics, and (iii) local vs. global and formal vs. application-oriented reasons for paraconsistency and/or paracompleteness.

Rather than extending the cluster of available theoretical arguments, I shall illustrate my position by discussing a specific problem solving process, namely, the elaboration of an adaptive Fregean set theory, or rather of a set of such theories. Those theories should enable one to remain faithful to the full richness of Frege's underlying ideas while avoiding triviality. The process leading to the theories should be helpful to understand the transition from Frege's trivial theory to apparently non-trivial set theories.

In preparation, I shall refer to published results by Peter Verdée and offer some reasons for the quest for different theories. Next, I shall show that the desired theories cannot be obtained by applying a general and a priori method, but requires a content-guided procedure in the sense of Dudley Shapere. The specific problem solving process will enable me to tell a concrete and detailed story, which has an interest of its own. It is my hope that, first and foremost, the story will clarify my philosophical position on those three topics, in particular on logical pluralism. Once that is done, the strength of the position should be apparent.

A Paraconsistent Defense of Logical Pluralism and Relativism

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From the emergence of paraconsistent logics, there were several criticisms claiming that they should not be called logics. In the same way it was argued that paraconsistent negation was not a "real" negation and hence when paraconsistent logicians talk about contradiction, they are not talking about logical contradiction but something else.

These criticisms lead to several abstracts discussion regarding the nature of negation, consequence relation and logic itself. But most of this debate neglects what we think is essential to comprehend this situation, namely: the acknowledgment of a logical pluralism and relativism. Hence, the aim of this communication is discuss how paraconsistent logics lead not only to logical pluralism as some claim [3, 4] but also to logical relativism. It should be mention that we'll be dealing not with a single form of pluralism or relativism, but with a (possibly overlapping) family of both.

In addition to that, we'll discuss how to extend this logical relativism to a mathematical relativism. Bell [2] argues that in Category Theory - by rejecting in a sense the set-theoretic discourse, in which every concept is reduced to an absolute idea of set-mathematical concepts no longer have absolute meaning, but become relative to local frameworks.

From that perspective we developed a Paraconsistent Functor that enables us to finding, for a given logic, its paraconsistent counterpart. This functor allows us to shift from one local framework to another, establishing a connection between distinct structures, providing us with a conceptual framework for possible comparisons among these structures.

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The Use of Definitions and their Logical Representation in Paradox Derivation

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We will start by examining the role and usage of definitions in the derivation of paradoxes, both set-theoretic and semantic. Then, we examine the various ways that such definitions have been logically represented in practice.

There are three features that are special about these definitions in these contexts. The first is the use of self-reference between the definiens and the definiendum, the second is the generality of context in the definiendum, and the third is the underdetermination and overdetermination that occurs as a result of these definitions. We examine the impact of these three features on the logical representation of definitions and how this representation then leads to a uniform paradox solution.

The first has an effect on the representation of definition, as the usual differentiation between definiens and definiendum is broken down. A new symbol is usually introduced by definition, being defined as some more complex expression, and every instance of this expression is then replaceable by the new symbol. In this usual case, the new symbol is equated with its definiendum as a shorthand form, and one is still operating within the range of symbols under consideration. A circularity here would introduce an infinite regress of expressions, created out of just one expression, such as occurs in the Liar Paradox, where L is defined as 'L is false'. So, one needs to go beyond the level of expressions to deal with definitions such as this self-referential one. Indeed, one needs to go to the next step of dealing with meanings rather than just expressions, so that the definitions are meaning equivalences rather than expression equivalences.

The second requires a logic representing meaning equivalence to be included in the definiendum, because of its generality. Meaning equivalence is naturally broken up into a conjunction of two meaning containments, one in each direction. The logic MC of meaning containment has been developed by Brady in Universal Logic, CSLI, 2006, but subsequently tweaked by dropping the distribution axiom in favour of its rule form. As such, the logic MC of meaning containment is well enough conceptualized to be able to work out how to apply the logic. This is better than a logic which is technically determined but without a clear concept.

The third feature of a self-referring definition is the prospect of underdetermination and overdetermination, due to the lack of specificity or overspecificity produced by the self-reference. Underdetermination is ubiquitous in logical reasoning, whilst overdetermination requires a resolution of a conceptual clash. We identify the conceptual clashes that occur in various current accounts that leave the contradictions in place and we will indicate how these clashes can be resolved to finally produce a uniform paradox solution. That this solution is possible is essentially due to the fact that the logic does not contain the Law of Excluded Middle or any forms of Contraction. Thus, this paper serves to explain the philosophy behind the simple consistency results in Universal Logic and other sources.

An epistemic approach to paraconsistency: dealing with evidence and truth

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The aim of this paper is to offer an alternative way to paraconsistency, besides metaphysical neutrality and dialetheism. If we assume that there are no 'real contradictions', hence, no pair of contradictory true propositions, and yet a pair of propositions A and $\neg A$ is accepted in some context of reasoning, it is on us to say what it means, if it is not the case that they are both true.

The basic idea is that contradictions occurring in several contexts of reasoning have epistemic character in the sense that their occurrence is related to thought and reason alone. Although the topic of epistemic contradictions is not new, and its origins can be traced back at least to Kant, the issue has not been properly developed yet, neither in its philosophical motivations, nor in terms of a paraconsistent non-dialetheist formal system designed to represent contexts of reasoning in which contradictions have a strictly epistemic character.

Our supposition is that logic is not restricted to the idea of truth preservation. Classical logic is a very good account of strict truth preservation — perhaps the best possible account — but sometimes truth is not only what is at stake. The guiding intuition here defended is that the acceptance of a pair of contradictory propositions A and $\neg A$ does not commit us to their truth. Rather, we understand it as some kind of conflicting information about A, namely, that there is 'conflicting evidence' about A. Evidence for A is understood in broad terms as reasons for believing that A is true. Evidence is clearly a notion weaker than truth, in the sense that if one knows that a proposition A is true, one has evidence that A is true, but not vice-versa.

This paper introduces a natural deduction system designed to express preservation of evidence rather than preservation of truth. The system is paraconsistent and paracomplete, since neither explosion nor excluded middle hold, although double negation equivalence holds. The inference rules for $\vee$, $\wedge$ and $\rightarrow$ are obtained in two steps. First, we ask about the sufficient conditions for having evidence that a given proposition is true. Then, we ask what would be sufficient conditions for having evidence that a given proposition is false. Each step produce rules whose conclusions are disjunctions, conjunctions, conditionals and negations of these formulas. Once we have the introduction rules, the elimination rules are obtained, as suggested by Gentzen, as 'consequences' of the introduction rules.

Although the system so obtained is able to express the notion of preservation of evidence, and not preservation of truth, by applying the resources of the logics of formal inconsistency, classical logic is recovered within the domain of propositions whose truth value has already been conclusively established. Once classical logic is recovered, the system turns out to be able to give an account of preservation of truth.

On the exclusivity of logical negation

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Let *glut theories* be theories postulating the existence of gluts on the face of semantical, and perhaps soritical, paradoxes. A standard criticism to glut theorist — but here I limit ourselves to Priest's dialetheism — concerns the difficulty of expressing the notion of *exclusivity* — exclusive truth or exclusive false — dialetheically. *Only true*, or *just true* is usually understood as "true and not false", where the mentioned *not* in "true and not false" should be *exclusive*, a *resource* which is not available for a dialetheist. We say that the *not* is *exclusive* if, in virtue of its very meaning, for any proposition A, the truth of both A and $\neg A$ is excluded in virtue of the very meaning of negation. The exclusive negation is called by Priest *Boolean negation*.

However, Priest contends that, like dialetheic negation, even classical logical negation is *non exclusive* (see 1, cap. 4): the very meaning of *classical logical negation* would fail, according to him, to guarantee that two propositions A, $\neg A$ are *incompatible*, i.e. that a contradiction like $(A \wedge \neg A)$ cannot be true. The reason would be, in a nutshell, that what classical logic "can say about a contradiction is expressed by the rule of *ex falso quodlibet* (EFQ): from a contradiction like $(A \wedge \neg A)$ everything follows. And classical logic *fails to exclude* that everything is true, i.e. *triviality*.

With Priest's words: "A dialetheist [glut theorist] can express the claim that something, α, is not true in those very words, $\neg \mathsf{T}(\alpha)$. What she cannot do is ensure that the words she utters behave consistently: even if $\neg \mathsf{T}(\alpha)$ holds, $\alpha \wedge \neg \mathsf{T}(\alpha)$ may yet hold. But in fact, a classical logician [or any explosive logic theorist] can do no better. He can endorse $\neg \mathsf{T}(\alpha)$, but this does not prevent his endorsing α as well... . [C]lassical logic, as such, is no guard against this... . [A]ll the classical logician can do by way of saying something to indicate that α is not to be accepted is to assert something that will collapse things into triviality if he does accept α. But the dialetheist can do this too. She can assert $\alpha \rightarrow \perp$" (1, p. 291).

Here $\rightarrow$ is Priest's *entailment connective* in the tradition of relevance logic, and $\perp$ some *explosive sentence*, i.e. a sentence that implies all sentences.

In this paper I discuss Priest's claim that even the classical negation cannot guarantee consistency. I will argue that, though there is a precise sense according to which the claim is acceptable even by a classical logician, nevertheless the exclusivity of negation is essential to classical logic and its semantics. Furthermore, I will argue that even a dialetheist cannot avoid some use of exclusive negation and this fact is in contrast with Priest thesis that the meaning of logical constants *is to be the same* both in the object language and in the meta-language.

I conclude that:

- A classical logician can develop any classical theory adopting in the object language a non-exclusive negation.
- The use of the exclusive negation in the metalanguage is essential for a classicist, as well as for a dialetheist: Priest himself needs the exclusive negation for developing his dialetheic semantics.

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The Paradox of Singularity. Contradiction and Individuals between Aristotle and Hegel

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The main thesis of this paper is that the problem of contradiction is tightly related, from an ontological point of view, to the question of the status of individual being. In order to discuss that I will take into account two thinkers, Aristotle and Hegel, who are traditionally pointed out as the main exponents of two rival positions: on one side the thought of non-contradiction, on the other the idea of contradiction as regula veri; on one side analytical, on the other dialectical logic.

First of all, I'll try to prove that it's questionable that such an incompatibility can be found between the standpoints of these two philosophers: this not just because, as more than once stated during the last century, it's not so evident that Hegel really wanted to negate the principium firmissimum stated by Aristotle, but also — and mostly — because Aristotle himself recognises, as the origin of philosophy, the very same factor pointed out by Hegel: the aporetic nature of experience and the appearance of contradiction. Reading thoroughly the IV book of Metaphysics, in fact, it's possible to underline a deep agreement between the aristotelian standpoint, that easily acknowledges the evidence of an apparently contradictory world of becoming, and the hegelian one, which makes of it the dialectical engine of individual and collective human experience. It's also hard to negate the further agreement of the two thinkers about the sake of philosophy: it has its origin in the appearing of contradiction and aims to overcome it. But if Aristotle and Hegel agree on these topics, and about the general essence of philosophy itself, they are indeed opposed with respect to the solution given in order to satisfy the need for consistency: on one side we find the attempt to "save the phenomena", on the other the statement that only the Whole is true. In Aristotle's case the result is an ontology of independent, identical individual being, which should guarantee the meaningfulness of language against the apparent paradoxicality of experience. In Hegel's case, on the contrary, the finite is inconsistent but is not true being: the only true being is the Whole, whose nature is to be free from contradiction. In both cases the study of contradiction leads to an ontology of individual being: I'll argue that it is only here that Aristotle and Hegel's philosophies are really opposed to each other.

But the aristotelian attempt to save individuals from contradiction is doomed to failure. In the second part of my paper I will discuss the main aristotelian arguments defending the consistency of individuals, in order to show their insufficiency. I'll so defend the hegelian standpoint, understood as the statement of the consistency of the Whole against the inconsistency of abstract finitude. The result is the acnkowledgement of the paradoxical nature of individuality, and of the incompatibility between this latter — understood as in Aristotle and in the aristotelian tradition until today — and consistency of being.

Denial Won't Get You Anywhere

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Glut theorists think that there are sentences that are both true and false, among them the liar sentence that says of itself that it's false. They have certain expressive advantages over competing non-classical approaches to the liar paradox. However, glut theorists face an expressibility problem of their own. Consider a version of the strengthened liar sentence that says of itself that it's not just true. Since the glut theorists' distinction between just true, just false, an both true and false sentences is pairwise exclusive and jointly exhaustive, this strengthened liar produces triviality if the glut theorists had a 'just true' predicate in their language. Nonetheless, glut theorists might want to be able to express of a sentence that it's just true. This is the so-called "just true" problem: Glut theorists are unable to fully say what they think regarding sentences that are just true. Surely, even glut theorists don't think that every sentence is both true and false and that, for example, many sentences not involving the notion of truth, are either just true or just false. However, glut theorists have trouble saying of a just true sentence that it's just true. They can't simply say that the sentence is true. For that, according to their own lights, doesn't rule out that it's also false, as in the case of the liar.

One response to the "just true" problem that's popular among glut theorists (see [8, 11, 3, 4, 6, 9, 10]) is the one that involves the speech act of denial (as well as its mental cognate, rejection). Denial is unlike assertion of a negation for it is governed by the rule that one ought to deny something only if it is just false (whereas one ought to assert a negation only if its negatum is either just false or both true and false). Denial allows glut theorists to express that a sentence is just true by denying its negation. However, the distinction between denial and assertion of the negation just made relies on the notion of just false, which in turn relies on the notion of just true.

In this paper, I argue that from within the glut theorists' own logic, the notion of denial presupposes a solution to the "just true" problem and thus can't be invoked in a solution to that problem. I also show that, due to contingent facts about our assertive practices, there isn't an analogous problem for paracomplete logicians. The result is that it's very easy for champions of paracomplete logics such as K3 to regain classical reasoning in select domains, whereas champions of paraconsistent logics such as LP a much harder time doing so. The paper ends with a discussion of proposed solutions to the "just true" problem based on the notions of shrieking (see [8, 5]) and Gricean implicatures (see [2], pg. 512, and [1], pg. 168) and finds these proposals wanting as well.

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The explication of paraconsistency, dialetheism and paracompleteness in classical logic syntactically extended by functorial variables

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The ongoing debate on paraconsistency, dialetheism and paracompleteness seems to be based on two fundamental assumptions: (1) The *logical* explication of *paraconsistency* and/or *paracompleteness* forces us to use or develop concrete *non-classical* calculi. (2) There is a *direct* connection between being a dialetheist and accepting/using paraconsistent logics: A dialetheist has to be a paraconsistentist (but no vice versa). In this talk it will be shown that both assumptions are not correct.

To reject (1) we develop a *syntactically extended* version of classical propositional logic. We introduce four restricted unary functorial variables which have exactly two — not necessarily different — classical functors as their substitutable values. These functorial variables can be interpreted as 4 different types of negations. This relatively minor extension yields a remarkable enhancement of our *expressive power* which allows to explicate a whole bunch of concepts precisely. But nevertheless it is still a *classical* logic because the concepts "being a theorem" and "being a tautology" of our enriched structures remain explicitly definable using only the classical ones.

To reject (2) we show that from our point of view there is *no direct comparability* of *paraconsistency* with *dialetheism* if we differentiate clearly between the formal concept "paraconsistency" which can be explicated *formally* without considering some sort of *application*. The concept "true contradiction" tries to combine the expressions "true" and "contradiction" within one and the same dimension but they belong not to the same one. We can keep our classical understanding of *contradiction* ("0" in all assignments) in the first dimension. "True" in "true contradiction" is *not* represented by "1" or any special truth-value like "both". It indicates that we try to understand contradiction not logically but in the context of some application which gives rise to a second dimension where we can have the technical value "1". A "1" in the second dimension should not be read "true" but it can be interpreted, e.g., as "applicable", "usable" etc. *Gluts* does not indicate a value but the interaction of dimensions with respect to possible applications.

To make this criticisms clear a syntactically extended version of classical propositional logic will be sketched. The aim is not to vote for this relatively weak system as an exceptionally nice logic. The aim is to show that a lot a relevant concepts and their interrelations can be explicated within one and the same object language. We can use this system philosophically to reject several misapprehensions.

The introduction of functorial variables (variable functors, in German: Funktorenvariablen) dates from S. Leśniewski (1929). Another famous Polish logician, J. Łukasiewicz (1951) connected this new syntactic tool with special substitution rules. By restricting the values of such variable functors he himself, C.A. Meredith and A.N. Prior (1956, 1963, 1965) and W. Stelzner (1977, 1979) created new areas of *application*.

Let F^1 be a symbol within the object language of the new syntactic type *unary functorial variable*. The substitutable values of F^1 are the unary classical functors ϕ^1_1, ϕ^1_2, ϕ^1_3 and ϕ^1_4 characterized by the following value-tables:

p	$\phi^1_1 p$	$\phi^1_2 p$	$\phi^1_3 p$ $(\neg)$	$\phi^1_4 p$
1	1	1	0	0
1	1	0	1	0

If there are several occurrences of F^1 in a formula X we have to substitute the same classical functor for each occurrence of F^1 (abbreviated by $[F^1/\phi^1_i]$). Now we are able to express extensionality within the object language: as *theorem* $\vdash_4 (p \equiv q) \supset (F^1 p \equiv F^1 q)$ or as *tautology* $\models_4 (p \equiv q) \supset (F^1 p \equiv F^1 q)$. "$\vdash_4$" ("$\models_4$") indicates that the four substitution instances without functorial variables are classical theorems (tautologies).

If we restrict each functorial variable to exactly two substitutable values ϕ^1_i and ϕ^1_j in that order it has the form $F^1_{i,j}$. Łukasiewics showed that, e.g., $F^1_{4,2}$ is a good candidate for explicating *necessity* (without Gödel-rule) and $F^1_{1,2}$ for *possibility*. Each formula X containing n occurrences of functorial variables of the forms $F^1_{i_1,j_1}, \ldots, F^1_{i_n,j_n}$ represents exactly two classical formulas A_1 and A_2: A_1 is created by the simultaneous substitution of the form $[F^1_{i_1,j_1}/\phi^1_{i_1}, \ldots, F^1_{i_n,j_n}/\phi^1_{i_n}]$ and A_2 by $[F^1_{i_1,j_1}/\phi^1_{j_1}, \ldots, F^1_{i_n,j_n}/\phi^1_{j_n}]$.

We interpret each functorial variable of the form $F^1_{3,j}$ as *negation*. The i-dimension (first dimension) gives it the classical characterization as "negation". We can freely choose the j-dimension with $1 \leq j \leq 4$. Therefore, we get four functorial variables as *negations* characterized by the second dimensions in the following way:

— $F^1_{3,1}$: *tautological* negation
— $F^1_{3,2}$: *assertoric* negation
— $F^1_{3,3}$: *negative* negation (given the full logic it behaves like the functor ϕ^1_3 $(\neg)$)
— $F^1_{3,4}$: *contradictory* negation.

It will be shown how *tautological negation* can be used for representing paraconsistency but not paracompleteness, *contradictory negation* for representing paracompleteness but not paraconsistency and *assertoric negation* for representing both.

Finally, we discuss *dialetheism* with respect to the constructions $p \wedge F^1_{3,1} p$ and $p \wedge F^1_{3,2} p$ and the fact that $\models_2 (p \wedge F^1_{3,1} p) \equiv F^1_{4,2} p$ as well as $\models_2 (p \wedge F^1_{3,2} p) \equiv F^1_{4,2} p$ — each representing on both sides of "$\equiv$" a Łukasiewicz-type of *necessity* and not a "true contradiction".

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Hegel: contradiction as a property of language. A Hegelian way towards paraconsistency

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We know it at least since Plato's Sophist: speaking, as identifying the different, has organically something to do with contradiction. We will sustain that the Hegelian Wesenslogik offers an explicit theorisation of these topics and that these texts significantly meet contemporary problems in paraconsistent logics.

Let's try to determinate the linguistic stakes of Hegel's Logic in what concerns identity, difference and contradiction. If the logic of essence appears to be the always presupposed "truth" of the logic of being, it's not only in the sense of an external metalanguage. On the opposite, we see in the logic of Grund that Hegel is totally aware of the possibility of a regressio ad infinitum from languages-objects to metalanguages, anticipating, could we say, some Gödelian problems. To avoid this "vortex", Hegel will rather accept a kind of reflexivity, major logical theme by him. For Hegel, every linguistic act, whose basic form is $A = A$, is an interlacing (in the Platonician sense of Symploké) of identity and difference. Identity's function appears to presuppose difference and difference's function identity as a positive element of itself (like a name, to identify a thing, to put it as something [als Bestimmtsein], has to be different from this thing, to negate it, while remaining a constituting element of this determined thing). We could also summarize the Hegelian development in this way: Identity = identity and difference; difference = difference and identity; so, difference = identity, which is precisely a new category: contradiction. We see that the Hegelian contradiction is not simply trivial, as it always involves in itself two levels of language. But these two ranks are not in a fixed hierarchy of meta-language and language-object, their roles reverse, which is precisely the — contradictory — process of reflexion.

If we compare the Hegelian trial to more recent logical problems, we can discover that it matches the category of paraconsistency. In order to avoid paradoxes, Russell will forbid this kind of reflexivity, will forbid that a category (a totality) possesses a part defined in terms of this category, a part that presupposes this category. Tarski will also refuse the reflexivity of "natural languages". But after works such as the one of Jaśkowski we can imagine a formal logic dealing with these problems. A basic formalisation of the previous dialectics could be the following:

1. $\sim(A \wedge \sim A)$;
2. $A = (A \wedge \sim A)$;
3. $\sim A = (\sim A \wedge A)$;
4. so $A \wedge \sim A$;
5. $(A \wedge \sim A) = B$;
6. $\sim(B \wedge \sim B)$.

And so on. We see the properly paradoxical and "reflexive" character of this logic. We will develop the questions about this logical system in three complementary directions:

— What do we have to accept concerning the different levels of language in order not to make this logic trivial?
— What are the formal implications and the range of such a logic?
— Can we say that we managed to formalize the dialectics?

The Non-classical Side of Classical Logic

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The aim of this paper is to make more widely known an old result that shows that there are non-classical phenomena, indeed paraconsistent phenomena, just below the surface of conventional classical logic, and to investigate how far the result can be generalized to non-classical logics.

The result alluded to is a slight variation (and, in an uncensorious sense, rectification) of Tarski's observation that 'the formal relation of the calculus of systems to the ordinary calculus of classes is exactly the same as the relation of Heyting's sentential calculus to the ordinary [classical] sentential calculus' (1935–1936, § 2, p. 352). By the calculus of deductive systems is here meant the calculus of axiomatizable and unaxiomatizable deductive theories, which is described by Tarski as representing 'a very essential extension of the sentential algorithm' (*ibidem*, p. 350).

A classical *deductive theory* is a set of sentences closed under a finitary consequence operation $\mathfrak{Cn}$ that contains at least the whole of classical sentential logic. Tarski defines the *logical product* and *logical sum* of two theories $\mathbf{X}$ and $\mathbf{Z}$ as $\mathbf{X} \cap \mathbf{Z}$ and $\mathfrak{Cn}(\mathbf{X} \cup \mathbf{Z})$ respectively. Evidently the product of two theories, so defined, is in general weaker than either, and the sum is stronger. A smoother integration of sentences and theories is obtained by interchanging these definitions, that is, by defining $\mathbf{X} \vee \mathbf{Z}$ as $\mathbf{X} \cap \mathbf{Z}$ and $\mathbf{X} \wedge \mathbf{Z}$ as $\mathfrak{Cn}(\mathbf{X} \cup \mathbf{Z})$, so that $\mathbf{X} \wedge \mathbf{Z} \vdash \{\mathbf{X}, \mathbf{Z}\} \vdash \mathbf{X} \vee \mathbf{Z}$, as in the sentential case. Arbitrary conjunctions and disjunctions of theories can be defined similarly. The negation of a theory $\mathbf{Y}$ can then be defined as $\bigwedge\{\mathbf{Z} \mid \vdash \mathbf{Y} \vee \mathbf{Z}\}$, which is the dual of one form of the definition that Tarski gives for negation. Whereas what Tarski defines is the *pseudocomplement* $\bar{\mathbf{Y}}$ of $\mathbf{Y}$, which satisfies the law of non-contradiction but in general may violate the law of excluded middle, what is defined here is the *authocomplement* $\mathbf{Y}'$, which satisfies the law of excluded middle but violates the law of non-contradiction whenever the theory $\mathbf{Y}$ is not finitely axiomatizable. Indeed, if $\boldsymbol{\Omega}$ is a maximal theory that is not axiomatizable, then $\boldsymbol{\Omega} \wedge \boldsymbol{\Omega}' = \boldsymbol{\Omega}$, which implies that the law of explosion fails. In short, the logic of classical deductive theories is paraconsistent.

This result calls stridently into question the view promulgated by Quine (1970), p. 81, that the advocate of paraconsistent logic hardly 'knows what he is talking about', since the notation '~', ceases to be recognizable as a notation for negation when 'some conjunctions in the form "*p* . ~*p*" [are regarded] as true', and 'such sentences [are not regarded] as implying all others. ... [The paraconsistent logician] only changes the subject'. For the calculus of deductive systems is no more than a natural generalization of sentential calculus. Looked at in reverse, indeed, sentential calculus is just the special case of the calculus of deductive systems in which authocomplementation becomes classical. In no way does it involve any change of subject matter.

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First steps towards non-classical logic of informal provability

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Mathematicians prove theorems. They don't do that in any particular axiomatic system. Rather, they reason in a semi-formal setting, providing what we'll call *informal proofs*. There are quite a few reasons not to reduce informal provability to *formal provability* within some appropriate axiomatic theory [6, 5].

The main worry about identifying informal provability with formal provability starts with the following observation. We have a strong intuition that whatever is informally provable is true. Thus, we are committed to all instances of the so-called *reflection schema* $P(\ulcorner\phi\urcorner)\phi$ (where $\ulcorner\phi\urcorner$ is the number coding formula ϕ and P is the informal provability predicate).

Yet, not all such instances for *formal* provability (in standard Peano Arithmetic, henceforth PA) are provable in PA. Even worse, a sufficiently strong arithmetical theory T resulting from adding to PA (or any sufficiently strong arithmetic) all instances of the reflection schema for provability in T will be inconsistent (assuming derivability conditions for provability in T are provable in T). Thus, something else has to be done.

The main idea behind most of the current approaches [7, 2, 3] is to extend the language with a new informal provability predicate or operator, and include all instances of the reflection schema for it. Contradiction is avoided at the price of dropping one of the derivability conditions. Thus, various options regarding trade-offs between various principles which all seem convincing are studied.

In order to overcome some of the resulting difficulties and arbitrariness we investigate the strategy which changes the underlying logic and treats informal provability as a partial notion, just like Kripke's theory of truth [4] treats truth as a partial notion (one that clearly applies to some sentences, clearly doesn't apply to some other sentences, but is undecided about the remaining ones). The intuition is that at a given stage, certain claims are clearly informally provable, some are clearly informally disprovable, whereas the status of the remaining ones is undecided.

In Kripke-style truth theories strong Kleene three-valued logic is usually used — which seems adequate for interpreting truth as a partial notion. Yet, we will argue that no well-known three-valued logic can do a similar job for informal provability. The main reason is that the value of a complex formula in those logics is always a function of the values of its components. This fails to capture the fact that, for instance, some informally provable disjunctions of mathematical claims have informally provable disjuncts, while some other don't.

We develop a non-functional many-valued logic which avoids this problem and captures our intuitions about informal provability. The logic is inspired by paraconsistent logic CLuN (see e.g. [1]), in whose standard semantics the value of a negation is not determined by the value of its argument. We describe the semantics of our logic and some of its properties. We argue that it does a much better job when it comes to reasoning with informal provability predicate in formalized theories built over arithmetic.

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Philosophical elucidation of implication

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Implication is semantically explained, either from a classical or from an intuitionistic perspective, as a connective which expresses the fact that from a proof of the antecedent a proof of the consequent can be obtained. But, we will argue, this is only a necessary condition for being an implication, it is not sufficient. At least, the explanation is not completely clear before explaining how proofs are conceived.

What would be a falsitymaker for the principle of non-contradiction?

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The basic idea of truth as correspondence is that the truth of a proposition must be grounded in reality. The so-called *truthmaker scheme*, p is true if and only if there is an s such that s makes p true, intends to capture the fundamental tenet of truth as correspondence. Dually, we may say that if a proposition p is false, there is something that makes it false, i.e., whatever is false, there must be a falsitymaker in virtue of which it is false. Falsity, as well as truth, must be grounded in reality. Whether or not the principle of non-contradiction is false — and only false, not simultaneously true and false — is obviously a central question for paraconsistent logicians, especially dialetheists. But what would be a falsitymaker for the principle of non-contradiction?

In *Metaphysics* Aristotle presents three versions of the *principle of non-contradiction* (from now on, *PNC*), that may be rephrased as follows:

(1) A property cannot at the same time belong and not belong to the same object.
(2) Two beliefs which correspond to two contradictory propositions cannot obtain in the same consciousness.
(3) Two contradictory propositions cannot be true at the same time.

These versions are talking about (1) objects and their properties, (2) beliefs, and (3) propositions. We call them, respectively, ontological, epistemological and linguistic. The epistemological version, as it stands, is plainly false, since it is a fact that in various circumstances people have contradictory beliefs. The ontological version corresponds to the theorem-scheme of first order logic $\forall x\, \neg(Px \wedge \neg Px)$, and is based on a categorization of reality in terms of objects and properties that has been central in philosophy and is present in logic since its beginnings. The linguistic formulation, although talking about language, also has an ontological vein because of the link between reality and the notion of truth. Furthermore, if we accept that every proposition 'says something about something', there is no important difference between the ontological and the linguistic versions of *PNC*. So, it is reasonable to consider that these two versions collapse.

Thus, in order to show a falsitymaker for *PNC* one needs an object a and a property P such that a has and does not have P. It is very unlikely that such a contradictory object is to be found in mathematics. With respect to the empirical sciences, there is an extensive literature about the occurrence of contradictions in empirical theories. Up to the present day there is no clear indication that these contradictions are grounded in the nature of reality — or, borrowing a Kantian terminology, that they belong to the *things-in-themselves* and not only to *phenomena*.

However, there still remains to be analyzed two alleged counterexamples to *PNC*: Russell's set and the Liar sentence. We argue here that none of them provide a legitimate falsitymaker for the *PNC*.

Philosophical elucidation of implication

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Implication is semantically explained, either from a classical or from an intuitionistic perspective, as a connective which expresses the fact that from a proof of the antecedent a proof of the consequent can be obtained. But, we will argue, this is only a necessary condition for being an implication, it is not sufficient. At least, the explanation is not completely clear before explaining how proofs are conceived.

Catuskoti: Paracomplete, Paraconsistent, Both, or None?

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We deal here with the logical import of the Buddhist Logic of *Catuskoti* (tetralemma), before arguing for its dual relationship with the Jain Logic of *Saptabhangi* (seven-fold theory of predication). Let us symbolize these by **LC** and **LS**, respectively. Should **LC** be taken as a paracomplete, or a paraconsistent logic? Can it be both, otherwise? Why not none, even leading to an increasingly set of paradoxical combinations between these four options?

Recall that, for any sentences p,q, an arbitrary logic **L** is said to be paraconsistent if the following Principle of Explosion fails:

$$p, {\sim}p \models_{\mathbf{L}} q$$

and **L** is said to be paracomplete if the following Principle of Implosion fails:

$$p \models_{\mathbf{L}} q, {\sim}q$$

The talk consists of three steps.

Firstly, we display the "logic" behind **LC** and **LS**, arguing that these are more like single sets of sentences than logical systems in the Tarskian sense of a truth-preserving theory.

Secondly, we display a number of strategies proposed in the literature to make sense of **LC** and **LS**, including Priest's five-valued system for the *Catuskoti*.

Thirdly, we question the logical nature of **LC** and **LS** in their usual form of combined literals $p/{\sim}p$ with paradoxical valuations.

Our constructive answer is semantic and relies upon a number of logical concepts including logical form, sentences, logical constants (negation, especially), model, truth-values, and the core relation of consequence. The latter plays a central role in the talk, since the traditional question whether the initial Buddhist proposition p is true, false, both, or none is replaced by a more general question about whether the Buddhist system is consistent, inconsistent, both, or none.

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On the Justification of Logical Principles

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How to justify logical principles? Some would rather put this question aside and take logical principles — for example, the principle of bivalence (a well-formed assertion is either true or false independently of us being in a position to actually decide which) or non-contradiction — on a par with the axioms of any formal mathematical discipline (e.g. formal geometry), which in themselves do not have any intrinsic meaning or truth, acquiring both only upon interpretation. We would then be free to chose the principles by which a logic is to abide, whose scope of validity would be circumscribed by the interpretations that validate these principles. From this perspective, logical principles are not justified, but freely chosen.

But this is not the traditional sense of logic. For Frege, logic has to do with truth to the same extent zoology has to do with animals; for Husserl, logic is the a priori theory of science, the science of science. Traditionally, logic is identified with reason and its most basic notion is that of truth. Logic, together with intuition, a direct grasp of truth in the same sense that perception is a direct grasp of objects (which does not rule out cases of misperception), are the only forms of rational justification available. In mathematics, both types have a place, intuition for the basic truths, the axioms, and logical reasoning, mixed or not with intuition, for the rest. From this perspective logical principles can and must be justified. Obviously, we cannot justify logical principles logically, since logic rests on its principles, but we can look for a justification either in intuition (by clarifying intuitively the concept of truth) or by unveiling yet more basic facts concerning truth on which logical principles rest. I take these approaches as essentially the same.

Concrete logic, if I am permitted the term, is to formal logic as concrete geometry is to formal geometry. Whereas the latter deals with an abstract notion of space, concrete geometry has to do with perceptual space. Analogously, concrete logic is the theory of the concept of truth; its principles spell out our apprehension of this concept to the same extent that concrete geometry spells out our notion of perceptual space.

There are further analogies between logic and geometry. Although geometry was born out of our interest on perceptual space, the space of *mathematical* geometry is not space as directly perceived; geometrical space is an idealization of perceptual space. Analogously, there is a more fundamental concept of truth and an idealized one, based on it. On the most basic level, truth is an *experience*, that of the adequation between what is said and what is directly experienced (*adequatio rei ad intellectum* as an *experience*); on a more abstract, theoretical level, it is an *idealization* of such an experience (*adequatio rei ad intellectum* as an *ideal*). There is then the logic of the (immediate) experience of truth (to the same extent there is a geometry of our immediate perception of space, which may or may not take a mathematical form) and the logic of an idealized conception of truth (the counterpart of the mathematical theory of idealized perceptual space). Both have a role in science.

The concept of truth involves, on the one hand, assertions and, on the other, that to which assertions refer, either experience, in the case of the logic of experience or, in the case of abstract logic, that which underlies experience, which I call the "world" or "reality", in the most general sense of these terms, which encompasses, in particular, mathematical realms and worlds of fantasy. Experience is the subject privileged access to reality but it cannot give it the whole of reality; the realm of (actualizable) experience is by its very nature constantly open to new experiences. "Reality" is the *ideal* realm, considered as a being-in-itself, of all *possible* contents of experience, in a sense of possibility that goes beyond mere actualizability. An (actualized) experience opens a window to the real, and no chunk of reality is *in principle* incapable of being given in direct experience.

Although looking a lot like a metaphysical principle, the last sentence above is instead a presupposition, but one of a very peculiar nature; not something that can be put to test and be found to be untenable, not a *hypothesis*, *but a true constitutive aspect of reality as intentionally posited*. Reality, as a *scientific notion*, is an intentional construct, a background against which experiences acquire a sense, an ideal pole to which experiences converge, that which science strives to represent in a way that better makes sense of experience. The logic of assertions as referring to facts of reality (not necessarily actually experienceable facts), that is, abstract logic, the logic of the real, depends, then, on the sense with which reality is intentionally posited.

I'll point out the most relevant aspects of reality as an intentional object. 1) Reality is an ontologically complete being-in-itself, i.e. no possible situation remains indefinite as to its factuality (on the side of the subject this translates into the presupposition that the subject can ideally — maybe not actually — verify any assertion that it can meaningfully assert); 2) reality is an ontologically stable domain, i.e. the same fact, with the same objects in the same relations, can be the content of different experiences, with different intentional senses and modes (intuitively given or only symbolically represented, as a content of perception or one of memory, for instance).

Now we have reached the crux of the matter, *logical principles are justified by these presuppositions*. The principles of *bivalence* and *identity*, in particular, rest on, respectively, presuppositions (1) and (2) above (which implies that the logic of reality, as we characterized this notion here, is the so-called *classical* logic).

Husserl's *Formal and Transcendental Logic* makes this point very clearly (§92b):

> [...] logic, by its relation to a real world, presupposes not only a real world being-in-itself but also the possibility, existing "in itself", of acquiring cognition of a world as genuine knowledge, genuine science, either empirically or a priori [...] all of that is claimed as an Apriori.

There are presuppositions concerning experiences too. Although (1) is not valid for the realm of (actual or actualizable) experiences, since this does not constitute an ontologically complete realm of being (there are experiences in principle *possible* that are not in principle *actualizable* — keep in mind that logic, any logic, as an a priori science, can consider only matters of principle, not fact; supposing, of course, that one does not believe, with Brouwer, that logic is a posteriori), (2) arguably still stands. But there is a third presupposition worth mentioning: (3) experience is consistent, i.e. the subject cannot experience simultaneously both a *fact* and *that* this fact is not experienceable; this is inscribed in our very *conception* of experience. Since experiences necessarily involve a subject considered abstractly and ideally, idealizing presuppositions concerning the subject will necessarily be reflected on the principles of the logic of experience; (3) is one of these presuppositions, on it rests the *principle of non-contradiction* (which implies that a logic of experience, as we understand this notion, cannot do without non-contradiction).

In conclusion, these considerations purport to show that the justification of logical principles is a task for a transcendental sort of inquiry into intentional constitutive experiences, and that formal logic demands *philosophical* completion in a sort of transcendental logic that digs below experience and reality into the intentional constitution of the conceptions of experience and reality.

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Paraconsistency and External Justification

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Graham Priest often argues for the non-classical (in particular, paraconsistent) nature of logical consequence by showing a discrepancy between classical principles and how we rationally reason. This methodology for justifying logical principles is often considered to be flawed. In this paper, I will argue that, for Priest's argument to succeed, the logical principles governing consequence relations must not only be internally coherent (and thereby non-trivial) but also externally justified. In other words, the justification of paraconsistent logic involves establishing non-triviality as well as the adequacy of someone reasoning paraconsistently in relation to the external circumstance that prompted that reasoning. I shall defend Priest's methodology by articulating and defending the role of external justification of logical principles for justifying paraconsistent logic. I shall thereby make a new case for the paraconsistent nature of logical consequence. Moreover, given that logical principles are not thought to be responsive to the external world, the argument advanced in the paper challenges not only classical logic but also the formal conception of logic presupposed by most contemporary logicians.

Abstract Duality and Co-Constructive Logic

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This paper investigates an abstract duality existing between paracomplete and paraconsistent logics, with the suggestion that these can be understood as co-constructive logics of proofs and refutations. In the current literature, dual-logics (including self-dual logics) are typically held together by some form of coherence principle, where, roughly, if a formula is a theorem (or provable) in one system then the dual sentence will be a counter-theorem (or refutable) in the dual system. We formalize this by means of a Galois connection between dual logics, and show that coherence holds for: classical logic; Greg Restall's [2] inferentialist approach to logic by means of assertion and denial; general logics of proofs and refutations (e.g. [3, 4, 5]); Dummett's [1] consideration of a dual falsificationism logic to verificationism; Urbas' [6] analysis of dual-intuitionistic logic. In so doing, it is also simple to see why the coherence principle renders such systems fairly uninteresting (for example, bi-intuitionistic logic, which combines intuitionistic and dual-intuitionistic logic contains theorems which are constructively unacceptable). By syntactically separating dual calculi for intuitionistic and co-intuitionistic logic, we then investigate structures where the coherence principle does not hold unrestrictedly, and which generate non-trivial inferentialist semantics. Philosophically, we understand the relation between dual calculi in terms of a dialogue between "prover" and "refuter", allowing for both potential and conclusive proofs (refutations).

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Hegel and The Idea of Negative Self-Relatedness

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Hegel's adoption of the medieval principle *omnis determinatio est negatio* which he attributes to Spinoza is widely acknowledged as one of his most important philosophical commitments and an essential aspect of his thought. Recently, Brandom has argued that the concept of determinate negation which Hegel developed based on this principle is Hegel's fundamental conceptual tool ([1], p. 180) and that the relations of determinate negation articulate the basic structure of Hegel's metaphysics ([1], p. 49). According to this, determinateness of a property requires exclusion of the incompatible properties and this allows definition of incompatibility and consequence relations. The proposition or property p entails q just in case everything incompatible with q is incompatible with p. For instance, having the property square entails having the property polygonal as everything materially incompatible with square is incompatible with polygonal ([1], p. 49). Brandom made a convincing case that this particular aspect of Hegel's concept of determinate negation can be domesticated into the context of contemporary philosophy and receive new life.

On the other hand, another important dimension of Hegel's further development of the same concept which is no less important for his speculative thought is often missed. This is the negative self-relatedness of every finite determination. In Hegel's treatment of determinate being in Science of Logic, we learn that something is determinate being by excluding its other from itself. Hegel claims that this exclusion of the other from itself is at the same time is its inclusion of its own limit. According to this, limit of something does not fall outside it but belongs to its very own determination. Without its limit, it would not be a determinate being but indeterminate being. But limit is nothing but the non-being of something. Consequently, something includes its own non-being within itself and is determinate and therefore something only by virtue of this. Finitude and therefore being determinate is not being limited in general but inclusion of its own limit or its own non-being within itself:

The something, posited with its immanent limit as the **contradiction of itself** by virtue of which it is directed and driven out and beyond itself, is the *finite* ([2], p. 101).

This inclusion of its own non-being, within itself is the negative self-relatedness of everything finite and is an essential aspect of being determinate:

Finite things *are*, but in their reference to themselves they refer to themselves *negatively* — in this very self-reference they propel themselves beyond themselves, beyond their being ([2], p. 101).

In my view, negative self-relatedness of every finite determination is the core idea defining the dialectical moment in Hegel's thought. The objective of this paper is to present this revolutionary concept and evaluate its prospects for being domesticated to contemporary philosophy.

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Against the World

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In previous works (e.g. [2]), I've developed a naive theory of truth (i.e. a theory that validates the intersubstitutability between φ and '‘φ' is true'). The theory validates the law of excluded middle (LEM) and the law of non-contradiction (LNC), and solves the semantic paradoxes (e.g. the Liar paradox) by restricting instead the structural property of contraction; more in detail, the base sentential logic of the theory is the multiplicative fragment of sentential affine logic. Thus, the principle of distributivity of conjunction over disjunction (D) fails in the theory—in fact, even the weaker principle of modularity (M) fails. In other previous works (e.g. [1]), I've developed a naive theory of vagueness (i.e. a theory that validates the intersubstitutability between, say, 'n is small' and '$n+1$ is small'). Again, the theory validates LEM and LNC, and solves the paradoxes of vagueness (e.g. the Sorites paradox) by restricting instead the structural property of transitivity; more in detail, the logic of the theory is defined over a class of non-modular lattices. Thus, again, D fails in the theory — in fact, again, M also fails. The failures of D and M in both theories is certainly surprising: after all, while many logical moves have been tried out to solve the semantic paradoxes and the paradoxes of vagueness, to the best of my knowledge no antecedent solution to either the semantic paradoxes or the paradoxes of vagueness has ever envisaged failures of D and M. And that does not seem a mere historical accident: neither kind of paradox seems to involve those principles in the first place. How could it be that, in this respect, familiar liars and heaps deviate from the laws of classical logic in an even deeper way than the exoteric objects of quantum mechanics? It might then seem that the solutions I've proposed feature unnecessarily weak logics. I'll argue that these appearances are deceiving: if the broad naive approaches I've proposed are on anything like the right track — in particular, if they are correct in upholding LEM and LNC — D and M just cannot be had. I'll then draw out a philosophically significant consequence of the failure of D and M in the solutions I've proposed: on both solutions, there is no true statement that, for every φ, either entails φ or entails $\neg\varphi$; in other words, there is no complete way things are. Since the world is supposed to be just that — the complete way things are — it thus turns out that, on both solutions, there is no world.

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Representation and Reality: Humans, Animals and Machines

This workshop is organized by

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Our workshop could be considered as the continuation of a part of the symposium "Computing Nature", organized by Gordana Dodig-Crnkovic and Raffaela Giovagnoli, in the AISB/IACAP World Congress 2012 and "Representation: Humans, Animals and Machines" in the AISB50 Convention at Goldsmith 2015. We would like to offer a further occasion to discuss the problem of "representation" in humans, other animals and machines. It is closely related to the question what capacities can be plausibly computed and what are the most promising approaches that try to solve the problem.

The invited keynote speakers of this workshop are Gordana Dodig-Crnkovic (page 87) and Gianfranco Basti (page 79).

Call for papers

The following interesting topics related to the problem of representation are welcome:

- The point of view of connectionism and dynamical systems (Scheutz, Clark, Juarrero, Kaneko and Tsuda, O'Brien, Horgan, Trenholme) namely the different proposals about the possibility to rule out representation.
- A plausible strategy to analyze the problem of representation from a philosophical perspective implies the comparison between human and machine capacities and skills. Searle presented an interesting theory of representation based on the mind's capacities to represent objects and to the linguistic capacities to extend the representation to social entities. For machine representation current results in AI and cognitive robotics are of interest.

- Evolutionary aspects of the development of increasingly complex capacities in (embodied, embedded) living organisms to process information in the interaction with the environment and as a consequence develop new morphological structures — morphogenesis, meta-morphogenesis.

Abstracts should be sent to raffa.giovagnoli@tiscali.it.

The possibility of superminds. A response to Bringsjord's argument on infinitary logic, by Florent Franchette

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The term *supermind* refers to human brains that exceed Turing machines limits, and was vastly discussed in the literature — e.g., in [7, 8, 4, 3, 10, 9]. [2], in particular, provided an argument in favour of the possibility of superminds based on infinitary logic. According to the argument, the infinitary nature of mathematical expertise is a proof of the impossibility for human expertise to be formalised by a formal system, i.e., to be computed by a Turing machine (TM). **My talk, however, is meant to provide a response to Bringsjord's argument on infinitary logic, and thus to minimize the possibility of superminds.**

To show that human expertise cannot be computed by TMs, Bringsjord focuses on a specific infinitary system called $\mathcal{L}_{w_1w}$. Such a system is based on the first-order logic and is able to express some mathematical concepts – e.g. the concepts of finite interpretation or ordinary arithmetic – that cannot be expressed in the first-order logic system $\mathcal{L}_I$ ([6]). According to Bringsjord, since human expertise can deal with $\mathcal{L}_{w_1w}$, which is a system capable of expressing some concepts that cannot be expressed in first-order logic — and so by TMs —, human expertise exceeds Turing machines limits.

A couple of objections was raised against Bringsjord's argument though. Among these is the following objection which was discussed by Bringsjord himself:

> Here you switch from describing infinite sentences to somehow using them [...] Surely it is quite possible that a human expert mathematician uses some finite mental representations to reason *about* $\mathcal{L}_{w_1w}$. Not even you, Bringsjord, can be reasoning *with* $\mathcal{L}_{w_1w}$ ([2], pp. 22–23).

Although Bringsjord provided a reply to the objection, its entire force remains. In my opinion, the distinction between to reason *about* and to reason *with* is crucial for deciding whether superminds are possible. **I argue, specifically, that human minds cannot reason *with* $\mathcal{L}_{w_1w}$, even though they may reason *about* $\mathcal{L}_{w_1w}$.** To this end, I propose the following definitions:

Definition 1. To reason *about* and *with* a deductive system.
— To reason *about* a deductive system means inferring theorems of the system by using verifiable and non-verifiable proofs.
— To reason *with* a deductive system means inferring theorems of the system by using verifiable proofs only.

Definition 2. (Verifiable proofs and rules).
— A proof is verifiable if it only contains verifiable rules.
— A rule is verifiable if a human computer is able to verify the correctness of every elementary step of the rule.

Example 1. (Verifiable and non-verifiable rules).
— An example of a verifiable rule can be found in Peano arithmetic: if n is a natural number then its successor is also a natural number. Formally, $N(n) \Longrightarrow N(s(n))$ where N is the property of being a natural number and $\Longrightarrow$ the logical inference. The correctness of every elementary step of the rule can be verified by a human computer because the successor function $s(n)$ is a primitive recursive function, i.e., there is an effective procedure computing $s(n)$ for all n.
— An example of a non-verifiable rule is the ω-rule of the ω-logic. The ω-logic is a deductive system which both contains all axioms and rules of the first-order logic and the ω-rule: for every formula $P(x)$ where x is a free variable, $P(0), P(1), P(2), \ldots \Longrightarrow \forall x\ (N(x) \rightarrow P(x))$. The ω-rule is not verifiable because the correctness of the infinite number of elementary steps included in the rule cannot be verified by a human computer.

It is now possible from these definitions to specify my response to Bringsjord's argument: human minds can reason *about* $\mathcal{L}_{w_1 w}$, for they are able to infer theorems such as Scott's Isomorphism theorem by using non-verifiable rules such as the ones allowing to construct infinite conjunctions ([5]). However, human minds cannot reason *with* $\mathcal{L}_{w_1 w}$, for they are not able to infer all known theorems of $\mathcal{L}_{w_1 w}$ from verifiable rules only. The system requires, in particular, the use of infinite long proofs that are not verifiable ([1]).

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The Relevance of Language for the Problem of Representation

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The present contribution deals with the relationship between representation and language that becomes more relevant if we do not intend the process of forming internal representations of reality but rather the representative function of language.

According to Searle, a belief is a "representation" (not in the sense of having an "idea") that has a propositional content and a psychological mode: the propositional content or intentional content determines a set of conditions of satisfaction under certain aspects and the psychological mode determines the direction of fit the propositional content [9].

Thoughts can be true or false but sentences do not express them "randomly". Sentences express thoughts as related to contexts of use in which they acquire their truth-value i.e. they are true or false. For instance the sentence "That is a funny play" can be true or false depending on the context of use. We can grasp thoughts but Frege does not present an analysis of the "grasping" because he thinks that this implies a psychological order of explanation [3]. Searle rather gives an account of the grasping through his brilliant account of the functioning of background based on intentionality. We can therefore show the complementarity between the description of the functioning of the cognitive grasping of the content of beliefs and the "normative" objective content that represent the ground of shared beliefs.

There is a different interpretation of the Fregean semantics, which is bound to the concepts use in ordinary language along the line of Davidson, Dummett and Sellars [1, 2]. This theoretical option cannot be discussed in the ambit of cognitive sciences (in particular cognitive psychology, developmental psychology, animal psychology and artificial intelligence).

We'll show the human representation of knowledge through the use of language by following a peculiar logical process. It entails three steps:

1. the differentiation between "labeling" and "describing";
2. the separation between "force" and "content";
3. the formation of complex predicates.

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Supervaluation Semantics for Modeling Spatial Representations in Humans and Robots

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Spatial knowledge plays an essential role in human reasoning, permeating tasks such as locating objects in the world (including oneself), reasoning about everyday actions and describing perceptual information. In this work, we propose to investigate the possibilities for both a unifying theory for spatial expressions representations in humans and a working system with which to equip an intelligent tool (such as a robot) with the ability to understand and use natural language spatial relations to refer to objects in the environment. Ideally, these relations are to be formalized as to be the closest in meaning to the human usage of spatial expressions in natural language. Although there has been a long tradition in cognitive psychology on human understanding of such expressions [2], attempts in representing them in artificial languages rarely takes natural languages semantics into account.

Using spatial expressions in their absolute form (e.g. Cartesian coordinates or cardinal points) is not common in spoken language, specially when it comes to instructions for locating objects in space. Natural speech most often mentions interactions between objects: "The book is on the table"; "The spotlight above the sofa", etc. Such descriptions imply some cognitive aspects of the relationships between located objects and reference objects. Other expressions, although propositionally analogous to the same state of affairs, would sound strange and are therefore rarely verbalized: "? The table is under the book"; "? The sofa below the spotlight". Furthermore, simple notions of distance between objects are also meaningless without a context [2].

Finally, another feature to consider is that of the spatial axes biases in human representation of space [5]. In short, humans tend to favor the lower portion of vertical space (as pointing towards the feet) and, horizontally, the front. In this work we propose a novel formalism for space representation that takes into account this asymmetry on the human perception. This formalism is built upon current methods for Qualitative Spatial Reasoning [1; 6] and takes supervaluation semantics [4] as the base logical framework to represent the distinct preferences taken by the human perceptual system as standpoints within a consistent theory. Supervaluation semantics is commonly used to model vagueness in a language as a set of distinct precise versions of it. Each of these versions is called a *precisification* of the language. In this work, we provide a new definition to precisification that applies to the distinct ways a particular predicate can be applied on the same situation, but with distinct *angle* (or standpoint) of it. For instance, the term "below" can have distinct precisifications given the particular arrangement of objects, observers and observers' poses involved when applying it.

The resulting formalism will be applied on a robot capable of interacting verbally with a human operator. Usability tests shall be fully conducted to evaluate the quality of the formalism on the human-robot interaction [3].

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The quantum strategy of completeness

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The thesis is: The Gödel incompleteness can be modeled on the alleged incompleteness of quantum mechanics. Then the proved completeness of quantum mechanics can be reversely interpreted as a strategy of completeness as to the foundation of mathematics.

That argument supposes that the Gödel incompleteness originates from the deficiency of the mathematical structure, on which it is grounded. Furthermore, one can point out that generalized structure, on which completeness is provable and thus it can serve as a reliable fundament of mathematics.

Set theory and arithmetic were what was put as the base of mathematics. However, it is a random historical fact appealing to intuition or to intellectual authorities such as Cantor, Frege, Russell, Hilbert, "Nicolas Bourbaki", etc. rather than to a mathematical proof. Even more, the so-called Gödel incompleteness theorems demonstrated that set theory and arithmetic are irrelevant as the ground of mathematics rather than no relevant branch of mathematics allowing of self-grounding though the orthodox view.

One can utilize an analogy to the so-called fundamental theorem of algebra: It needs a more general structure than the real numbers, within which it can be proved. Analogically, the self-foundation of mathematics needs some more general structure than the positive integers in order to be provable.

The key for a relevant structure is Einstein's failure to show that quantum mechanics is incomplete. The incompleteness of set theory and arithmetic and the alleged incompleteness of quantum mechanics can be linked. The close friendship of the Princeton refugees Gödel and Einstein might address that fact. However, Gödel came to Princeton in 1940 much after the beginning of Einstein's attempts to reveal that and how quantum mechanics was incomplete. In particular, the famous triple article of Einstein, Podolsky, and Rosen "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" pointed out as a kind of theoretical forecast as to the phenomena of entanglement and thus of quantum information was published in 1935. So, there should exist a common mathematical structure underlying both "incompletenesses" and in turn interpretable as each of them.

The mathematical formalism of quantum mechanics is based on the complex Hilbert space featuring by a few important properties relevant to that structure apt to underlie mathematics:

1. It is a generalization of positive integers: Thus it involves infinity.
2. It is both discrete and continuous (even smooth): Thus it can unify arithmetic and geometry.
3. It is invariant to the axiom of choice: Thus it can unify as the externality and internality of an infinite set as the probabilistic and deterministic consideration of the modeled reality as well as even model and reality in general.

The target of the presentation is:

I. Those three properties of the complex Hilbert space to be demonstrated.

II. A simple mathematical structure underlying both the Gödel incompleteness and the alleged incompleteness of quantum mechanics to be described explicitly.

III. The undecidable statements according to the Gödel incompleteness theorems to be demonstrated as decidable in that generalized structure of Hilbert space.

IV. The so-called Gödel first incompleteness theorem to be interpreted as allowing of the self-foundation of mathematics.

Representation and Reality by Language

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Reality as if is doubled in relation to language: The one counterpart of reality is within the language as the representation of the other counterpart of reality being outside the language and existing by itself. Both representation and metaphor are called to support the correspondence between the two twins as an "image and simile".

The mechanism of that correspondence and its formal conditions are investigated by the following construction: Language is reduced to an infinite countable set (A) of its units of meaning, either words or propositions, or whatever others. It includes all possible meanings, which can be ever expressed in the language rather than the existing till now, which would always a finite set.

The external twin of reality is introduced by another set (B) such that its intersection with the above set of language to be empty. The union of them ($C = A \cup B$) exists always so that a one-to-one mapping ($f\colon C \longleftrightarrow A$) should exist under the condition of the axiom of choice. The mapping (f) produces an image ($B(f)$) of the latter set (B) within the former set (A). That image ($B(f)$) serves as the other twin of reality to model the reality within the language as the exact representation of the reality out of language (modeled as the set B). In the model, the necessity and sufficient condition of that representation between reality both within and out of the language is just the axiom of choice: If the axiom of choice does not hold, the relation between the sets $B(f)$ and B cannot be defined rigorously as an exact representation but rather as some simile and the vehicle between the two twins can be only metaphor.

Furthermore the metaphor can be anyway defined to a set of one-to-one representations of the only similar external twin into a set of internal "twins", each of which is a different interpretation of the external "twin" so that a different metaphor is generated in each case. The representation seems to be vague, defocused, after which the image is bifurcate and necessary described by some metaphors within the language.

Consequently reality is in an indefinite, bifurcate position to language according to the choice formalized in the axiom of choice. If that choice is granted, the language generates an exact image of reality in itself; if not, only some simile can exist expressible within it only by metaphors.

If the axiom of choice does not hold, language and reality converge, e.g. as 'ontology': Ontology utilizing metaphors can describe being as an inseparable unity of language and reality within language abandoning representations and the conception of truth as the adequacy of language to reality. Furthermore, those metaphors should coincide with reality (and with physical reality in particular) in virtue of the ontological viewpoint.

Furthermore, language can be formally defined by representation after the latter is in turn defined as a one-to-one mapping between two infinite sets, one of which is defined as reality and the other, as its image. Language is namely the natural interpretation of that image.

The advantage of that approach is to link the representation of the human being supplied by language to the representation by a machine (e.g. a computer), which should be formally modeled to be constructed. Another point of interest is the following: That mathematics, which is underlain by the mapping between sets, can be related to language by link of representation.

Internal inconsistencies: how can an information system ontology be both realistic and common-sense friendly?

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The robustness of some high-level *cognitive categories* and their subsequent representational constraints across various domains is one of the striking facts about human cognition. In this way, no matter the scale of the domain from chemistry to astrophysics human common sense remains ontologically committed to the existence of time-proof entities (individuals) which may 1) possess some common essence, 2) be affected by processes some of which depend causally on others, etc. In other words, these intuitive high-level representational constraints constitute the human *Cognitive Ontology* (CO).

Ontologies, similarly understood as conceptual structures, became also relevant in the context of *Artificial Intelligence* and *Knowledge Engineering* Information System Ontologies (ISO). In such a context, the ability to fit our CO is a secondary goal, for the main one consists in developing an efficient artificial agent able to deal with complex situations. However, interestingly, some upper level ISO are claimed to be consistent with our human common sense representations, either explicitly as the *Descriptive Ontology for Linguistic and Cognitive Engineering* (Gangemi et al. 2002) or implicitly as the *Basic Formal Ontology* [7, 6]. This specific feature is usually not perceived as a treat for the realistic significance of these ISOs. As it is the case in some formal ontological investigations, the ontology is believed to grasp genuine ontological differences and, by chance, being consistent with our CO (see [2]).

Supporting together a realist ontological claim and consistency with common sense is usually a difficult, nay an impossible, task. The two main reasons for this are a) the partiality of the middle-sized objects of a typical human environment, thus the fact that, as an example, some intuitive ontological categories might not be relevant in the sub-atomic domain and its quantum phenomena, b) the internal inconsistencies of CO, i.e. the existence of conflicting formats of mental representation in human understanding [5, 4]. We will address these two issues and evaluate in what extent their current answers the *Theory of Granular Partitions* [1] and the *Realist Perspectivalism* [8] fulfill the requirement of a classical realist position.

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4th International Workshop on Computational Creativity, Concept Invention and General Intelligence (C3GI)

The invited keynote speakers of this workshop are Tony Veale (page 110) and Irina Starikova (page 104).

For every kind of workshop-related question, please mail to c3gi@cogsci.uos.de.

The Workshop's Mission Statement: Why and what for?

Over the last years, an old AI dream has seen its renaissance: "Thinking machines". Having been almost completely abandoned for decades, more and more researchers have recognized the necessity — and feasibility — of returning to the original goal of creating systems with human-like intelligence. Increasingly, there is a call for confronting the more difficult issues of human-level intelligence, addressing the artificial (re)creation of high-level cognitive capacities. Within the range of these capacities, due to their elusive and nonetheless indispensable nature, creativity in all its facets (e.g. in engineering, science, mathematics, business processes), concept invention, concept formation, creative problem solving, the production of art, and the like are assigned a special status.

Researchers in several communities are trying to understand the basic principles underlying these special abilities, working on computational models of their functioning, and also their utilization in different contexts and applications (e.g. applications of computational creativity frameworks with respect to mathematical invention and inventions in engineering, to the creation of poems, drawings, and music, to product design and development, to architecture etc.). In particular, a variety of different methodologies are used in such contexts ranging from logic-based frameworks to probabilistic and neuro-inspired approaches. Although the different approaches to questions concerning aspects of computational creativity, concept invention, and artificial general intelligence do share significant overlap in underlying ideas, the cooperation between the respective communities is still in an early stage, and can greatly profit from interaction and discussion between people from the respective fields, forming trans- and interdisciplinary alliances in research and application.

The workshop shall offer a platform for scientists and professional users within relevant areas, on the one hand presenting actual and ongoing work in research, on the other hand also offering a chance for obtaining feedback and input from applications and use-case studies. The format of the workshop will leave ample space for interaction and discussion, complementing talks highlighting the key points of the accepted paper submissions with dedicated discussion phases and special contributed "flash talks" by renowned people in the field. Furthermore, this workshop explicitly encourages controversial position papers about open problems, ongoing discussions, and projections to the future of computational creativity.

A Path is a Path is a Path

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Image schemas are recognised as a fundamental ingredient in human cognition, learning, abstraction, and creative thought. They have been studied extensively in areas such as cognitive linguistics. However, the very notion of an image schemas is still ill-defined, with varying terminology and definitions throughout the literature.

For the purpose of formalising image schemas in order to exploit their role in computational creative systems, we here study the viability of an idea to formalise image schemas as graphs of interlinked theories. We discuss in particular a selection of image schemas related to the notion of 'path' and show how they can be mapped to a formalised family of micro theories reflecting the different aspects of path following.

Analogical Inference and Transfer for Creativity

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Analogies are often considered a mechanism that allows to explain phenomena of creativity. An analogical mapping prepares the transfer of concepts and ideas between different domains. The imported knowledge can provide new insights into the target, leading to a reconceptualization of a domain or showing solutions to a problem.

While the process of mapping between domains is in the focus of many works on analogy, the actual transfer receives much less attention. We argue, that this lack of interest is inappropriate as transfer is not a straight forward continuation of the mapping process but a complex phenomenon with close connections to the field of conceptual change. We further claim that there are different ways in which analogical transfer can contribute to the introduction of new ideas into a domain and suggest a classification of transfer types based on a formal analogy model.

"Seeing as" and Re-Representation: Their Relation to Insight, Creative Problem-Solving and Types of Creativity

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Re-representation and restructuring are processes relevant to creativity research. These are related to "seeing as" defined as the ability to represent features as different meaningful objects, and select and group objects as relevant structures for the problem at hand.

Creative problem-solving, insight and the three types of creativity proposed by Boden are explored from the perspective of these terms. A set of essential questions to be answered by the cognitive systems discipline from the perspective of re-representational ability is put forward. Some of the implications of enabling re-representation and evaluating systems based on re-representation are then explored.

The Input, Coherence, Generativity (ICG) Factors: Towards a Model of Cognitive Informativity Measures for Productive Cognitive Systems

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Classical thinking on information and informativity considers the informee as a perfect information receiver. However, when studying productive natural and artificial cognitive systems, cognitively based models of informativity need to be formulated.

Three factors relevant to cognitive informativity measures are proposed: Input, Coherence and Generativity (ICG). These factors take into account the type of Input which can be stored, the Coherence of the system after acquiring the information, and the Generativity of the system after the new information was integrated.

Mathematical Style as Expression of the Art of Proving

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Mathematicians talk of 'proofs' as if they were real things. However, the only things that can actually happen in the real world are proof-events, or provings, which are actual experiences, each occurring at a particular place and time, and involving particular people. Proof-events are social events that generate proofs presented in different styles.

Styles describe specific mathematical practices and characterize different cultures or schools that may differ in their views of rigor. A style can be personal for a mathematician, or for the school he belongs or for a whole tradition; it may be also mimicry of the style of a renowned authority. In general, it is considered as principal indicator of the art of proving.

In our view, style can be defined as a meta-code that determines the individual mode of integration (selection, combination, blending) of concepts into a narrative structure (proof). Thus, style depends on the chosen mode of signification (semiosis), the selected code and the underlying semiotic space (algebraic, geometric, probabilistic, λ-calculus, etc.). Styles perform certain functions that concern not only the elegance of exposition of a proof (the way of writing a proof), but might also facilitate or obstruct communication and understanding of a mathematical proof, depending on the metaphors used in the narrative (semiotic) space as well as the communicational functions of the codes and metaphors chosen.

In this paper, we attempt to analyze the communicative functions of mathematical proving styles by appealing to Roman Jakobsons communication model. This model was initially conceived for describing the communicative functions of language. However, it can be modified and specified for use in any medium of communication, in particular in the medium of mathematical proving, computer-generated proving, Web-based proving, etc. In the framework of this model, aesthetic pleasure gained from mathematical proving can be associated with the poetic function. It is an ideal for a (pure) mathematician (prover) to find an elegant proof and cause the reader aesthetic pleasure from his proving activity and its "stylized" outcome.

Non-Classical Abstract Logics

This workshop is organized by

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According to the "classical" view, a logic is a theory of consequence $\langle L, Cn \rangle$, where Cn is taken to be either a relation between sets of formulas in a language L or an operator on formulas. By "classical" here, we do not mean the view of logic which is defined by a semantic model with only two truth-values (truth and falsity). Rather, we mean the broader sense in which the foundations of a logic have to do with a basic relation of truth-preservation. This workshop will question this view, and investigate alternatives.

There are a number of suggestions available in the literature regarding what shape such a non-classical "abstract logic" might take. For example, from the point of view of semantics, Shramko and Wansing suggest a generalized theory of truth-values and entailment relations which do not solely preserve truth. From the point of view of dialogue, reasoning and inferentialism, a number of authors (e.g. Dutilh-Novaes; Restall; Ripley) both investigate the foundations of logical deduction and question the centrality of truth-preservation in the construction of logical systems.

We think that there are three predominant attitudes that can be identified in the investigation of the abstract properties of logical systems. The first contends that every logical system has ultimately to do with different ways to preserve truth from premises to conclusion. The second questions the centrality of truth and makes room for more relations of consequence between premises and conclusion. Finally, the third is even more radical by questioning the very relation of consequence. Should the latter always be seen as a cornerstone in any abstract study of logic? In this vein, falsification, and relations of rejection, have been studied on a equal footing with truth-preservation (e.g. Slupecki, Skura). Such investigations may be understood as initiating a broader view of logical relations that could lead to a more comprehensive reflection on the discipline. In this regard, consequence, rejection, inference, or even mere difference (in the context of a wider reading of the logical concept of opposition) may be considered to be equally basic notions to investigate the foundations of logic.

The invited keynote speaker of this workshop is João Marcos (page 101).

Call for papers

Any contribution to the renewal of abstract logic (both in a "classical" or "non-classical" trend) will be welcome in this workshop, utilising various working methods (algebraic semantics, proof theory, sequent calculus, dialogues), and aiming at a unifying abstract theory of logic of the form $\langle L, Cn \rangle$. This includes questions concerning:

- the notion of consequence and its various facets;
- the notion of rejection, as both a dual of consequence or a more complex (and independent) relation "non"-classical abstract relations, like rejection or difference;
- logic and language-games;
- investigations regarding consequence and implication, opposition and duality, and pure negation.

Abstracts should be sent via email to schang.fabien@voila.fr.

Towards a pragmatic logic for denial

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In a classical theory of denial to deny A is equivalent to asserting $\neg A$:

Classical denial. A is correctly denied iff $\neg A$ is correctly asserted.

Glut theorists reject the right-to-left direction of the *Classical denial*: asserting $\neg A$ must not commit one to denying A, i.e. denial must not be reducible to the assertion of $\neg A$. In particular, the paraconsistent denial of A is *stronger* than the assertion of $\neg A$. Unlike paraconsistent negation, which allows for overlap between truth and falsity, denial is assumed to be *exclusive*: assertion and denial are mutually incompatible speech acts (For a general background on denial in non-classical theories, see Ripley [2].)

Our paper starts from this basic idea i.e. that assertion and denial are mutually incompatible speech acts and from the fact that we have a logic for assertion as a speech act, i.e. the *logic for pragmatics* (LP), a logic proposed by Dalla Pozza and Garola in (1). Question: Is it possibile to extend LP to include also the speech act of denial? Aim of the paper is to defend a negative answer to the question. Doing so, we also skectch some positive requirements for a *logic of denial*.

In a nutshell, the basic idea of LP is to follow Frege's idea of distinguishing propositions from judgments: A proposition is either true or false, while a judgment, expressed through the speech act of an assertion, is either justified (J) or unjustified (U). A justified assertion is defined in terms of the existence of a proof that the asserted content is true. Although the concept of *proof* is meant to be *intuitive* and *unspecified*, it must always be understood as *correct*: a proof is a proof of the truth. Elementary sentences of LP are built up using only the sign of pragmatic mood of assertion, $\vdash$. So, for example, if α_1 and α_2 are propositional formulas, then $\vdash \alpha_1$ and $\vdash \alpha_2$ are elementary assertions, while $\vdash \alpha_1 \cap \vdash \alpha_2$ or $\vdash \alpha_1 \cup \vdash \alpha_2$, $\sim\vdash \alpha_1$ are complex assertions of LP.

Let us suppose that it is possible to extend LP to the speech act of denial; one could think that the easiest way for doing so is to say that:

(i) $v(\dashv A) = \mathsf{J}$ *iff* $v(\vdash \neg A) = \mathsf{J}$,

where '$\dashv$' is a symbol for *denial*. The informal meaning of (i) is that it is justified to deny A if and only if it is justified to assert $\neg A$. *Prima face*, (i) is simply the translation, in an extension of LP, of our before mentioned basic idea: assertion and denial are mutually incompatible speech acts. But, in LP, $\vdash \neg A$ entails $\sim\vdash A$, where the last formula means that there is a proof that A has not been proved. If so, the direction from left to right of (i) does not work: it does not seem to be necessary to have a proof that A is not proven to deny A. A may be denied without a so strong reason. Moreover, observe that (i) should be taken distinct from (ii):

(ii) $v(\dashv A) = \mathsf{J}$ *iff* $v(\vdash A) = \mathsf{U}$,

where the informal meaning of (ii) is that it is justified to deny A if and only if there is not a proof for the truth of A. But (ii) does not work too. It is indeed too weak: if there is no conclusive proof for A why should I have to deny it? We formulate lots of scientific hypotheses without a conclusive proof, and we accept them for some other/different reasons. If (i) and (ii) are respectively too strong and too weak, what can we say on assertion and denial as speech acts modeled on LP?

In the paper we consider a proposal based on the dual notions of *what is rational to accept* and *what is rational to reject*. Briefly put: It seems adequate to argue that if $v(\vdash A) = \mathsf{J}$ then it is rational to accept A and, on the same line, if $v(\dashv A) = \mathsf{J}$ it is rational to reject A. Moreover, if $v(\vdash A) = \mathsf{J}$ then it is rational to reject $\neg A$. We analyse this different characterization of denial and LP. The new proposal seems to capture a basic requirement for the extension of LP: if to assert a certain proposition you need a proof of it in LP, to reject it you need something like a 'disproof' of the same proposition. Such extension, its *pro and cons* will be analysed in the paper.

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Anti-intuitionism as a logic of refutation

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There has been a relatively small but recurring interest in the paraconsistent logic that is dual to intuitionistic logic (and called anti-intuitionistic-, dual-intuitionistic-, Brouwer- or even co-intuitionistic logic; we will use the first appellation). Yet, no approach has established itself as a standard one. Some view anti-intuitionism as a logic suited for refutation (see [7], [4] or [8]) but, to my knowledge, a formulation of it as a pure refutation system (in the sense of, say, [9]) is still lacking. Others have studied it as a logic dual to intuitionism and have tried to define it by various algebraic, syntactic, Gentzen-like or category-theoretic methods, but without considering its potential as a refutation system.

In our work, we try to take the idea of a refutation system seriously and offer a unified account of anti-intuitionism. We describe it as a refutation system in three ways: (1) as a Hilbert-like formal system with counter-axioms and a "reverse" *modus ponens*,(2) as a sequent calculus where a sequent actually represents the process of refuting the succedent on the assumption that the antecedent is refuted and (3) as a logic with a Kripke-style semantics based on the value False.

From a category-theoretic point of view, the resulting logic then appears as a natural particular case of the internal logic of a complement topos (see [6]), thus elucidating the link between the logical, algebraic and category-theoretic aspects of anti-intuitionism. As a consequence, a higher-order anti-intuitionistic set theory becomes readily available and anti-intuitionism reveals itself as a *constructive* logic of refutation. Another consequence of our approach is that the status of the logical operator dual to implication (and called pseudo-difference) becomes clear in a refutative framework.

Finally, anti-intuitionism can be seen as a concrete, "experimental" proof that a genuine logic based on falsity and refutation is possible and can be potentially as useful and philosophically interesting as intuitionism proved to be.

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Dualizing q-consequence operations

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q-consequence relations were proposed in [3] by Grzegorz Malinowski, in the context of a discussion raised by the Polish logician Roman Suszko. According to Malinowski, q-consequence relations are based on Łukasiewicz's notion of rejection and try to reflect the process of reasoning behind scientific investigation. In contrast to the canonical notion of entailment, namely the preservation of designated values from the premises to the conclusion, a q-consequence is valid iff there is no interpretation that assigns a non-rejected value to the premises but a non-designated value to the conclusion. This is due to a tripartition of the set of truth-values into designated, rejected and neither designated nor rejected values. Such tripartition defines what Malinowski called a q-matrix. Later on, Frankowski proposed in [2], based on Malinowski's notion of a q-matrix, another non-canonical notion of entailment, which he called p-entailment (plausible entailment). A p-consequence is valid iff there is no interpretation that assigns a designated value to the premises but a rejected value to the conclusion.

In [1], based on a bi-dimensional approach to entailment, the authors proposed a general procedure to explore the duality between noncanonical notions of consequence relations. However, the duality between rules and operators from the point of view of q-logics and p-logics are not explored. By following a general requirement for duality proposed in [4, 5], we will explore the duality of operators and its consequences within p- and q-logics. Furthermore, we will investigate in what sense p-logics can be recognized as the dual of q-logics.

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Bayesian Networks on Transition Systems

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Let $TS = (S, E, T, I)$ be a transition system, where S is the non-empty set of states, E is the set of actions, $T \subseteq S \times E \times S$ is the transition relation, $I \subseteq S$ is the set of initial states. Let us extend the transition system TS to the following information system: $\mathcal{S} = (S, At, \{V_a : a \in At\}, \{I_a : a \in At\})$, where S is a finite nonempty set of objects called universe associated with the set of all states of TS, At is a finite nonempty set of attributes which express properties of $s \in S$ (such as color, intensity, chemical formula of attractants, etc.), V_a is a nonempty set of values $v \in V_a$ for $a \in At$ (such as type of color, type of intensity, type of chemical formula, etc.), $I_a : S \to V_a$ is an information function that maps an object in S to a value of $v \in V_a$ for an attribute $a \in At$ (e.g. this attractant $s \in S$ is blue). Now, let us build a standard logical language $\mathcal{L}_S$ closed over Boolean compositions of atomic formulas (a, v). The meaning $||\Phi||_S$ of formulas $\Phi \in \mathcal{L}_S$ is defined by induction:

$$||(a, v)||_S = \{s \in S : I_a(s) = v\}, a \in At, v \in V_a;$$

$$||\Phi \vee \Psi||_S = ||\Phi||_S \bigcup ||\Psi||_S;$$

$$||\Phi \wedge \Psi||_S = ||\Phi||_S \bigcap ||\Psi||_S;$$

$$||\neg\Phi||_S = S - ||\Phi||_S.$$

In the language $\mathcal{L}_S$, we can define decision rules in S as follows. Assume, each formula $\Phi \in \mathcal{L}_S$ is considered a node of the directed, acyclic graph. Then a decision rule in S is a graph $\Phi \to \Psi$, where Φ is a parent and Ψ is a child, that is interpreted as an appropriate conditional probability:

$$\pi_S(\Psi|\Phi) = p_S(\|\Psi\|_S \mid \|\Phi\|_S) = \frac{card(\|\Psi\|_S \cap \|\Phi\|_S)}{card(\|\Phi\|_S)},$$

$$\text{where} \|\Phi\|_S \neq \varnothing.$$

In this way, the direct cause $\{\Phi \to \Psi\}$ in decision is expressed by $\pi_S(\Psi|\Phi)$, the indirect cause $\{\Phi \to \Psi, \Psi \to \Theta\}$ by $\pi_S(\Psi|\Phi) \cdot \pi_S(\Theta|\Psi)$, the common cause $\{\Phi \to \Psi, \Theta \to \Psi\}$ by $\pi_S(\Psi|\Phi, \Theta)$, the common effect $\{\Phi \to \Psi, \Phi \to \Theta\}$ by $\pi_S(\Psi|\Phi) \cdot \pi_S(\Theta|\Phi)$.

For each formula $\Phi \in \mathcal{L}_S$ with k atomic parents, we have 2^k rows for the combinations of parent values $v \in V_a$. Each row gives a number $p \in [0,1]$ if Φ is true, and it gives a number $1 - p$ if Φ is false. If each formula has no more than k parents, the complete network requires $O(n \cdot 2^k)$ numbers.

So, we can construct Bayesian networks in $\mathcal{L}_S$ by using the following Bayes' formula:

$$\pi_S(\Psi|\Phi) = \frac{\pi_S(\Psi) \cdot \pi_S(\Phi|\Psi)}{\pi_S(\Phi|\Psi) \cdot \pi_S(\Psi) + \pi_S(\Phi|\neg\Psi) \cdot \pi_S(\neg\Psi)},$$

where $\pi_S(\Psi|\Phi)$ is the *a posteriori* probability of Ψ given Φ, $\pi_S(\Psi)$ is the *a priori* probability of Ψ, and $\pi_S(\Phi|\Psi)$ is the likelihood of Φ with respect to Ψ. Hence, the Bayes' formula allows us to infer the *a posteriori* probability $\pi_S(\Psi|\Phi)$ from the *a priori* probability $\pi_S(\Psi)$ through the likelihood $\pi_S(\Phi|\Psi)$.

Uniqueness without reflexivity or transitivity

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The goal of this paper is to explore the idea of a set of rules *uniquely* pinning down the meaning of a piece of vocabulary. This is an idea that might be of interest for anyone, but it looms large in at least two different domains.

First, uniqueness is of concern to *inferentialist* theories of meaning, where rules are taken to be meaning-giving. It is natural to think, and many have thought, that in order to be meaning-giving, a set of rules must succeed in giving a *unique* meaning.

Second, uniqueness is of concern to anyone interested in *combining* logics. It is well-known, for example, that intuitionist negation and classical negation cannot live happily together in the same language, at least in many familiar settings. This is because the rules that specify intuitionist negation succeed in characterizing it *uniquely*. But these rules also apply to classical negation. As a result, the two negations collapse into each other; there is a merely syntactic distinction between them.

A common criterion given for uniqueness of connectives is the following. A set of rules characterizes an $n$ary connective $\star$ uniquely iff: supposing that the rules apply to $\star$ and supposing that they also apply to $\star'$, we can show $\star(A_1, \ldots, A_n) \dashv\vdash \star'(A_1, \ldots, A_n)$ for any formulas $A_1, \ldots, A_n$. Call this the *reflexivity* criterion.

The reflexivity criterion works well in many tame settings. But it depends crucially (as a criterion for uniqueness) on both *reflexivity* and *transitivity*. Regarding reflexivity: if $\star$ sentences do not entail *themselves*, then entailing their $\star'$ relatives is not a *sign* of uniqueness; it is a guarantee that uniqueness *fails*! Regarding transitivity: knowing that $\star(A_1, \ldots, A_n) \vdash \star'(A_1, \ldots, A_n)$ only allows us to substitute $\star$ for $\star'$ (or vice versa) in the presence of *transitivity*. But surely if the rules for $\star$ succeed in uniquely pinning down a meaning, we ought to be able to substitute one thing obeying those rules for another freely; after all, they ought to mean the same.

So the reflexivity criterion cannot work as a criterion for uniqueness in settings where either reflexivity or transitivity of consequence is not assumed. In this talk, I will press the above argument in more detail, and explore other options for a uniqueness criterion, options that *can* continue to work in the absence of reflexivity and transitivity.

Lindström Theorem for First-Order Modal Logic

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Lindström style theorems are used to show the maximal expressive power of logics in terms of model theory. In 1969, Per Lindström proved that any abstract logic extending first-order logic with the compactness and the Löwenheim-Skolem property is not more expressive than first-order logic.

The first Lindström theorem for propositional modal logic was proved by de Rijke [1]. He showed that any abstract logic extending modal logic with finite depth property or equivalently by preservation under ω-ultraproducts is equivalent to modal logic. In [5] van Benthem improved de Rijke result and showed that modal logic is the strongest logic satisfying compactness and relativization property and is invariant under bisimulation. Otto and Piro [3] proved the Lindström theorem for modal logic with global modality and guarded fragment of first-order logic by using compactness, corresponding bisimulation invariance and Tarski union property.

In this talk we will first review the notion of abstract logic for first-order normal modal logic and show that we can improve three methods of various versions of Lindström theorem in [4] to have following results:

Theorem. Any abstract logic L containing first-order normal modal logic is equivalent to it if and only if

- it is invariant under bisimulation and preserved under ultraproducts over ω,
- it has the Löwenheim-Skolem, compactness and the Tarski union property and is invariant under bisimulation,
- it has the Löwenheim-Skolem and the compactness property and is invariant under bisimulation.

We are looking for methods to let us have Lindström theorem for first-order classical modal logic (neighbourhood semantics), especially for monotonic case. It should be noted that Venema and Kurz [2] used the coalgebraic method for proving the Lindström theorem that can be used for propositional neighbourhood modal logic.

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Emergent Computational Logics

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Various general logics have been formalised in the history of philosophy and mathematics, like propositional, predicate, higher-order or fuzzy logics. Artificial intelligence should posses the capability for generating formal logics automatically from any given data set. Such data could for instance include corpora of information collected in interactions with the environment or any other approach for extracting some logic from data or statistics. An emergent logic should represent the logic of a particular environment, from which it was generated. As the environment widens towards the global environment, emergent logics should give up the flexibilities they have retained, in favor to general properties and ultimately converge to universal logics. Any work related to automated generation of some logic or the automated verification of such logic is welcome. Application areas for such techniques are hybrid systems of emergent and symbolic systems. Related research is typically found in the literature on computational intelligence and symbolic artificial intelligence that focuses on logic.

The invited keynote speaker of this workshop is Pei Wang (page 111).

Call for papers

Any work related to automated generation of some logic or the automated verification of such logic is welcome. Application areas for such techniques are hybrid systems of emergent and symbolic systems. Related research is typically found in the literature on computational intelligence and symbolic artificial intelligence that focuses on logic. Topics of interest include (but are not limited to) the following:

- logic discovery
- logics generated by computational intelligence
- logics with uncertainties
- logics extracted from probabilistic and/or possibilistic systems
- logics of emergent systems
- logics of behaviours
- logics of cognitive systems
- logics generated by neural systems
- logics emerging from evolutionary computation
- mining logics
- subjective logics
- machine learning and logic generation
- statistical/probabilistic ontologies
- logics of abduction
- logics of biological systems
- learning logics of dynamic systems

Submissions of extended abstracts should be sent by May 1st 2015 to borakumova@iyte.edu.tr.

Algorithmically Verifiable Quantum Functions *vis à vis* Algorithmically Computable Classical Functions: A suggested mathematical perspective for the *EPR* argument

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We suggest that the paradoxical element which surfaced as a result of the *EPR* argument (due to the perceived conflict implied by Bell's inequality between the, seemingly essential, non-locality required by current interpretations of Quantum Mechanics, and the essential locality required by current interpretations of Classical Mechanics) may reflect merely lack of recognition of classically definable mathematical expressions that could represent—as deterministic—the unpredictable characteristics of quantum behaviour. The anomaly may dissolve if a physicist could cogently argue that:

(i) All properties of physical reality are deterministic, but not necessarily mathematically pre-determined—in the sense that any physical property could have one, and only one, value at any time $\mathrm{t}(n)$, where the value is completely determined by some natural law which need not, however, be representable by algorithmically computable expressions (and therefore be mathematically predictable).

(ii) There are elements of such a physical reality whose properties at any time $t(n)$ are determined completely in terms of their putative properties at some earlier time $t(0)$. Such properties are predictable mathematically since they are representable by algorithmically computable functions. The values of any two such functions with respect to their variables are, by definition, independent of each other and must, therefore, obey Bell's inequality. The Laws of Classical Mechanics describe the nature and behaviour of such physical reality only.

(iii) There could be elements of such a physical reality whose properties at any time t(n) cannot be theoretically determined completely from their putative properties at some earlier time $t(0)$.

Such properties are unpredictable mathematically since they are only representable mathematically by algorithmically verifiable, but not algorithmically computable, functions. The values of any two such functions with respect to their variables may, by definition, be dependent on each other and need not, therefore, obey Bell's inequality. The Laws of Quantum Mechanics describe the nature and behaviour of such physical reality. In this paper we formally define such functions, and suggest how they could provide an alternative perspective from which to view philosophical issues underlying some current concepts of quantum phenomena such as indeterminacy, fundamental dimensionless constants, conjugate properties, uncertainty, entanglement, *EPR* paradox, Bell's inequalities, and Schrödinger's cat paradox.

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Fuzzy-Syllogistic Reasoning with Ontologies

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The Fuzzy Syllogistic System (FSS) is our attempt to develop an application for automated reasoning. As a core of the reasoning mechanism we used deductive scheme known as categorical syllogism. During the FSS — a project implementation — a mathematical model of whole syllogistic system was developed and it was designed the algorithm for calculating any syllogistic properties like truth or falsity for a particular syllogism. After structural analysis of syllogisms, 25 valid syllogistic structures out of 256 possible combinations were found. Since the vast majority of possible forms of syllogism are invalid, we had to introduce the term of fuzzy-syllogistic reasoning (FSR) to generalize reasoning scheme and extend the possible number of valid syllogistic structures. According to the structure of syllogism we applied the fuzzyfication in two ways by using fuzzy quantifiers and defining fuzzy sets. After implementation of fuzzyfication, the FSS became more flexible in terms of quantity of possible quantifiers, which has been increased, and numbers of valid syllogistic forms. The developed system can be applied for different domains as a major component of reasoning mechanism.

The main aim of our current project is to implement the syllogistic reasoning under ontology to generate new ontology which consist of not only conceptions and relations between them but the results of performed reasoning. It can be considered from the viewpoint of internal structure of intelligent agent as a kind of emergent learning process, so it can potentially be used as part of learning component for symbolic and emergent cognitive architectures.

Since our goal is to analyse relationships in given ontology, on the other hand, creating of a new ontology is time and resource consuming process, we do not deal with creating of source ontology directly by ourself and assume that input for FSR is an existing ontology. Among the various possible ways to construct ontologies for given domain, the most widely used approaches related to generation of an ontology from text-based sources. There are several open-source tools for ontology generation from text corpora are available for research purposes, such as Text2Onto, WebKB or DLLearner. We found that the most convenient for our purposes is the use of Text2Onto, because it gives us opportunity to generate ontology in automatic mode and the quality of of the generated ontology satisfies our requirements.

To generate the source ontology it is necessary to prepare text corpora for the given domain. In case of use of Text2Onto, the text corpora may be a set of plain text documents, html pages and other unstructured or semistructured text sources. The integration of this tool with the web search engine seems to be an optimal solution for collecting and preparing text corpora for given domain. Further, as a result of successive steps to input data, such as extraction of concepts and properties, we obtain source ontology for a particular domain, defined by input text corpora. Since as we have a ready source ontology, we can build a graph of dependencies which reflects the relationship between concepts in original ontology in a form suitable for applying the FSR. Having the graph on dependencies as input for FSS, it is possible to perform reasoning for each triple of conceptions and match each triple with the set of appropriate syllogistic forms. In some cases, there is ability to remove a conception from the triple, which represent the middle term in categorical syllogism and actually not included in conclusion, so this allows to reduce the complexity of ontology and achieve a higher level of abstraction by removing details.

As was mentioned above, in FSS we operate with 256 possible inference schemes that allow a some degree of fuzzyfication, so we can consider our system as approach for approximate reasoning. Generally speaking, inference in approximate reasoning related to computation with fuzzy sets that represents the meaning of a particular set associated with fuzzy quantifier. In the current system the fuzzy approach is implemented in two levels: fuzzy quantifiers and fuzzy sets.

Regarding the categorical syllogism we can apply fuzzy â€“ quantification in very strict manner. The are 4 possible crisp quantifiers for categorical syllogisms: All, Not All, Some and Not Some. In certain situations, taking some assumptions, we can replace one quantifier by another. More specifically, we can introduce 2 quantifiers as

Almost All and Almost None. Obviously, quantifier Almost All can be considered as the special case of quantifier Some, in the same way, quantifier Almost None is the special case of quantifier Not Some. It is possible to introduce some thresholds for ratio of number of elements satisfying given conditions, to cardinality of whole set, for example, for quantifier Almost All it should be very close to 1 and for quantifier Almost None it should be close to 0. Taking this into account we can replace quantifier All by Some and None by Not Some respectively according to the cardinality of given set. Our result shows that in some cases we may obtain significant increase in number of valid syllogistic forms. This approach can be considered as pseudoâ€“fuzzy, because we use fuzzyfication implicitly actually without introducing new quantifiers, we just try to improve number of valid syllogistic forms based on cardinality input sets.

From the point of view of using fuzzy sets we modified the existing quantifiers. Due the fact that this approach can be applied only in terms of exclusive logic, we combined quantifiers Some and Not Some into general quantifier *Some and expanded possible values of this quantifier to Most, Many, Half, Few, Several. Our next challenge is finding a method for calculating the ratio of validity for fuzzy quantifiers. The problem can be solved by defining of membership functions for all fuzzy quantifiers, as in pure fuzzy solutions.

Despite the fact that we use deductive schemes as a main form of logical inferencing in proposed solution, it seems that it can be used for applying reasoning in opposite direction as implicit form of abduction. Actually, after performing FSR for each triple of conceptions we have obtained a kind of mapping to predefined syllogistic forms. Considering the fact that structure of each syllogistic form is strictly fixed, based on the conclusion we can predict what the premises are. In general, for weaker fuzzy quantifiers we can find out the conditions necessary for them to became more stronger. If given, a syllogistic structure contains fuzzy quantifiers in premises, for example several, we can determine the conditions for the replacement of current quantifier on Few or Half.

The designed system have some limitations caused by used logics. Currently we can use only two premises to infer conclusion, so we have to decompose input data, such as ontologies, on triple sets. Removing of this restriction will allow the system to became a universal mechanism for modelling of decision making.

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Can Machines Learn Logics?

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To consider the question *"Can machines learn logics?"*, suppose the following problem. There is an agent $\mathcal{A}$ and a machine $\mathcal{M}$. The agent $\mathcal{A}$, which could be a human or a computer, is capable of deductive reasoning: it has a set $\mathcal{L}$ of axioms and inference rules in classical logic. Given a (finite) set S of formulas as an input, the agent $\mathcal{A}$ produces a (finite) set of formulas T such that $T \subset Th(S)$ where $Th(S)$ is the set of logical consequences of S. On the other hand, the machine $\mathcal{M}$ has no axiomatic system for deduction, while it is equipped with a machine learning algorithm $\mathcal{C}$. Given input-output pairs $(S_1, T_1), \ldots, (S_i, T_i), \ldots$ (where $T_i \subset Th(S_i)$) of $\mathcal{A}$ as an input to $\mathcal{M}$, the problem is whether one can develop an algorithm $\mathcal{C}$ which successfully produces an axiomatic system $\mathcal{K}$ for deduction. An algorithm $\mathcal{C}$ is *sound* with respect to $\mathcal{L}$ if it produces an axiomatic system $\mathcal{K}$ such that $\mathcal{K} \subseteq \mathcal{L}$. An algorithm $\mathcal{C}$ is *complete* wrt $\mathcal{L}$ if it produces an axiomatic system $\mathcal{K}$ such that $\mathcal{L} \subseteq \mathcal{K}$. Designing a sound and complete algorithm $\mathcal{C}$ is called a problem of *learning logics*. In this framework, an agent $\mathcal{A}$ plays the role of a teacher who provides training examples representing premises along with entailed consequences. The output $\mathcal{K}$ is refined by incrementally providing examples. We consider a deduction system $\mathcal{L}$ while it could be a system of arbitrary logic, e.g. nonmonotonic logic, modal logic, fuzzy logic, as far as it has a formal system of inference. Alternatively, we can consider a framework in which a teacher agent $\mathcal{A}$ is absent. In this case, given input-output pairs (S_i, T_i) as data, the problem is whether a machine $\mathcal{M}$ can find an unknown logic (or axiomatic system) that produces a consequence T_i from a premise S_i.

The abstract framework provided in this study has challenging issues of AI including the questions:

1. *Can we develop a sound and complete algorithm $\mathcal{C}$ for learning a classical or non-classical logic $\mathcal{L}$?*

2. *Is there any difference between learning axioms and learning inference rules?*

3. *Does a machine $\mathcal{M}$ discover a new axiomatic system $\mathcal{K}$ such that $\mathcal{K} \vdash F$ iff $\mathcal{L} \vdash F$ for any formula F?*

The first question concerns the possibility of designing machine learning algorithms that can learn existing logics from given formulas. The second question concerns differences between learning Gentzen-style logics and Hilbert-style logics. The third question is more ambitious: it asks the possibility of AI's discovering new logics that are unknown to human mathematicians.

In this study, we provide simple case studies concerning the first question. To this end, we represent a formal system $\mathcal{L}$ using *metalogic programming* which allows object-level and meta-level representation to be amalgamated (Bowen and Kowalski, 1983). We also argue the possibility of learning non-deductive inference such as abduction (Peirce, 1932) or conversational implicature (Grice, 1975). An extended version of this study will be published in (Sakama and Inoue, 2015).

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n-Valued Refined Neutrosophic Logic and Its Applications to Physics

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In this paper we present a short history of logics: from particular cases of 2-symbol or numerical valued logic to the general case of n-symbol or numerical valued logic. We show generalizations of 2-valued Boolean logic to fuzzy logic, also from the Kleene's and Lukasiewicz's 3-symbol valued logics or Belnap's 4-symbol valued logic to the most general n-symbol or numerical valued refined neutrosophic logic. Two classes of neutrosophic norm (n-norm) and neutrosophic conorm (n-conorm) are defined. Examples of applications of neutrosophic logic to physics are listed in the last section. Similar generalizations can be done for n-Valued Refined Neutrosophic Set, and respectively n-Valued Refined Neutrosophic Probability.

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Utopian Thinking and Logic-s

This workshop is organized by

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Following the symposium organized in July 26 at Andros Island, Greece, by Stefaneas (National Technical University of Athens) and Magioglou (EPoPs/FMSH, Paris), on "Logic and Utopia", this workshop focuses on the **Plural logics of Utopian thinking**. Utopian thinking will addressed from an *interdisciplinary* perspective.

Utopias as projects of future societies have historically been present since ancient Greece, with the example of Plato's and Aristotle's Polities, but the term is created by Thomas Moore in England. Desire, perfectionism and (im)possibility to become reality have been some of its aspects in the past, as well as a form of criticism of the present situation. In this way, they are impregnated by culture, socially and historically constructed and open to revision. Utopias are also present in a less elaborate way, in the projects and social representations of political and social actors, part of social and political identities.

How does utopian thinking use "logic"?

The concept of logic will be is addressed in its plurality: not only (a) "logos", philosophical reasoning and (b) language, but also (c) as "social logic" of people who are not "experts", knowledgeable or powerful on a particular field, but instead, "ordinary citizens". The workshop focuses particularly on the perspective of the social and political actors in different cultural contexts, in other words, the utopian logic-s of lay or everyday thinking.

The plural aspect of logic, is present in the notion of utopia, and particularly the political utopias as prospective of better worlds. The common good, the notion of the Polis and "Demos", will be discussed and confronted to studies that show concrete initiatives taken by local actors, in the direction of change. What distinguishes utopias from dystopias could be our representation of the common good, in other words, the objectives chosen.

Democracy, as well as economy, are examples of utopias related to the rationality of the modernity. They are imbricated in the contemporary societies to traditions (still active or reinvented) and religion. They allow new combinations, hybrids of meaning that multiply in the context of "global politics", as producers of utopias, moral geographies, and imaginary identities that we are going to question.

Organizing Dimensions of the debate:

— Utopia in Imaginaries, Social representations and everyday/lay thinking
— The logic-s of power in everyday and political discourse
— Culture and subjectivities in utopias
— Utopian Temporalities
— Social Logics and forms of reasoning

The invited keynote speaker of this workshop is Jaan Valsiner (page 110).

Call for papers

Abstracts should be sent via email before January 31th 2015 to thalia.magioglou@msh-paris.fr.

Contemporary imaginaries of women elected in positions of power in the temporality of globalization

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The presentation, based on field work and a tradition of political anthropology (Abélès), will focus on the link between a neo-liberal form of utopia and the social and political "logic" of female elected leaders. These leaders are addressed as "ordinary citizens", who arrive at concrete initiatives and innovative ways of action inspired by "performative imaginaries". These collective imaginaries are embedded in the debate of "sensible" topics such as cultural forms of resistance to different types of institutional violence, as well as new forms of governance, citizenship and social representations of egalitarian norms. The continuity between discourses, myths of social identity construction related to a common origin will be questioned, as well as a form of pessimism related to a "global menace" which is projected toward future societies.

The time-space of the "halka", or narrative circle in Marrakech: utopia or heterotopia?

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This paper questions the plurality of the public space at the place Djema El Fna of Marrakech, through the concept of halka as time-space of a utopia. The halka presents a variability of geometries which follow the representations and forms of a public spectacle. Halka, or circle of the spectators is taking different meanings: spatial, symbolic and political. The Halka involves two or three narrators and dancers who

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engage in a form of dialogue with each-other and with the public, in order to encourage its participation in the story. Finally the public offers a form of retribution to the artists. These theatrical and chorographical or acrobatic representations use the Halka as a topos of an ordinary utopia, and will also be considered as possible illustrations of a "heterotopia", or alternatives social logics.

Different materials, such as photos and collective interviews will be used to illustrate this reflection on the relationship between the body of the artists and the public space, with its different public languages, as vectors of "social logics".

The Oppositional Geometry of Badiou's Political Revolutions

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The philosopher Alain Badiou is known for being a brilliant representative of radical leftist utopia, joining high-level mathematics and logic (see [2]) to Lacanian, Heideggerian and Marxist philosophical concepts (see [3, 4]). He denounces Western democracy (as structurally committed to belligerent capitalism) and tries to lay the bases of a future general revolution. Despite polemics and the accusation of being politically irresponsible, Badiou's excellent reputation and aura as a contemporary philosopher comes from the fact that he built an impressive theoretical model of "exceptional collective agency", called "theory of Truth procedures" (or γ-scheme), which is based on mathematical concepts and concerns four fields of exceptional action: love, politics, art and science. In several of his recent texts about politics (notably in [5], p. 31, and [6], p. 70), Badiou says warningly that in current society there are four "segments" of people, or "forces" (i.e. social groups) to be taken particularly into account: the students, the young people of the suburbs (banlieues), the common workers and the illegal workers (clandestine immigrates). Badiou adds: (1) the State makes it, by all means, that these four groups remain, two by two, deeply unrelated; (2) for the State knows that the day any two of these four groups enter into deep contact (i.e. communicate and collaborate), the State as such will run the risk of collapsing under a serious and structured revolution. What Badiou describes here is a situation of "opposition", where a starting oppositional structure for some reason changes, with dramatic political consequences. Since a century, oppositions, which were previously handled formally through the "logical square" (which articulates contradiction, contrariety, subcontrariety and subalternation), are reduced by logicians and analytical philosophers to the concept of "logical negation" (i.e. contradiction, forgetting contrariety), therefore relying on a discussion about the principles of non-contradiction and of excluded-middle.

In this paper we want to offer a formal model of Badiou's strange but appealing idea through a new young branch of mathematics: "oppositional geometry". This is an approach to opposition alternative to the logical ones, which gains considerable conceptual power by generalising successfully the otherwise mysterious notions of logical square and hexagon (cf. [1, 7, 9, 11, 12]). Oppositional geometry is based on the notion of "oppositional bi-simplex of dimension m" ("simplexes" being the geometrical counterpart of numbers) and originated in the framework of universal logic from a discussion of the roots of paraconsistency (see [8]). The idea here is to read Badiou's intriguing remark in terms of the key notions of oppositional geometry: the disunion of the four political forces will be read as a blue tetrahedron of 4-contrariety, whereas the active communication (and solidarity) of the same four forces will be read as a green tetrahedron of subcontrariety. The general problem then becomes: how to think, inside oppositional geometry, that a blue simplex of contrariety *becomes* a green simplex of subcontrariety? This is both interesting and problematic: interesting, because the idea of a (graded) "metamorphosis" of opposition is new and commits to a new chapter of the theory, "oppositional dynamics"; problematic, because currently it is not yet clear how to build mathematically such a transformation. In this paper we propose to explore this issue by relying on the notions of "oppositional *poly*-simplex of dimension m" (see [7]) and of "*hybrid* oppositional structure" (see [10]). If successful, our enquiry will offer to Badiou a formal model of political revolution which could be called an "Empedoclean lattice of oppositional metamorphosis".

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Utopian thinking in the case of the Greek Youth. The Use of linguistic connectors for two different "logics"

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Utopian thinking is conceptualized in this paper as a form of ***lay thinking*** which aligns an objective of a "good" or "***better world***", to a positive social identity of a group/person, and a form of action, drawing from a the societal and cultural political psychology perspective (`http://epops.hypotheses.org`), which is inspired by the social representations theory and the advances of cultural psychology (Moscovici; Jodelet; Valsiner). Democracy, Economy and Religion are approached as "hegemonic social representations" a notion first used by Moscovici, which has been further questioned by Magioglou and Obadia in our EHESS seminar in 2014. We have drawn not only on the work of Moscovici, but have also been in dialogue with the work of Gramsci, Castoriadis and Abélès especially when it comes to the importance of these notions for the opposing dynamics of the global-political. The content of these notions can be very elusive, and still, culturally and historically constructed. They are shared by the members of a group, as a social representation, are objectified in institutions, rituals, objects, but still, their vagueness allows the creation of opposing groups and social identities. Their construction could obey different styles and "***social logics***".

The presentation will focus on results from a longitudinal study conducted with non-directive interviews, on the social representation of democracy in Greece. A discourse analysis based on the use of linguistic connectors will be presented, following the work of

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Ducrot and Carel in pragmatics. This analysis has revealed two "***ways of thinking***", or ***social logics***, used to construct the meaning of democracy. The first is characterized by the use of oppositions such as "***but***", and ***negations***, the second, is characterized by the use of *connectors of reformulation* such as, "***in other words***". These social ***logics*** are associated to "utopic" representations of democracy as a form of common good, which oppose each other, but also, more "dominant" and ideological representations and logics of the global-political (Abélès).

Medieval Logic

This workshop is organized by

Rodrigio Guerizoli
Federal University of Rio de Janeiro, Brazil

As scholars are becoming more and more aware, one of logic's most fruitful periods filed the five centuries between, roughly speaking, 1100 and 1600. This medieval tradition on logic exhibits an extraordinary richness extending its reach from creative reinterpretations of the syllogism, researches on the proprieties of the terms, logical consequence, inference, quantification, formalization, paradoxes, fallacies, to treatments of the relation between logic and natural languages. Since a couple of decades the material medieval logicians produced are being object of critical editions, on the basis of which new researches are on their way. Has little chance of losing who bet that there are quite a number of interesting logical reasonings waiting for us to be discussed in those texts.

This UNILOG'2015 workshop will focus on the various and diversified contributions logical questions received in the medieval period.

The invited keynote speaker of this workshop is Julie Brumberg-Chaumont (page 83).

Call for papers

We invite scholars to submit abstracts of papers they would like to present (30-minutes including discussion) along the following themes:

- Consequences
- Deduction and Induction
- Definition
- Epistemic Logic
- Fallacies
- Formalization
- History of Medieval Logic
- Inference
- Logic and Dialectic

- Logic and Grammar
- Logic and Natural Language
- Modal Logic
- Obligations
- Paradoxes
- Praedicabilia
- Proprietates terminorum
- Quantification
- Semantic puzzles
- Sophismata literature
- Square of oppositions
- Syllogisms
- Syncategoremata
- Temporal logic
- The scope of logic
- Truth and Logic

Abstracts should be sent via e-mail before November 15th 2014 to rguerizoli@ufrj.br, along with a brief biographical paragraph that includes your institutional affiliation.

Notification of acceptance: December 1st 2015.

Conceivability and possibility in Abelard's theory of modality

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Many contemporary philosophers interested in modality face the question whether or not our ability to conceive, think or imagine is to be taken as a reliable guide for distinguishing what is possible from what is not. The origin of the idea that conceivability could entail — or provide evidence for — possibility is usually backtracked to modern philosophers, as in the case of Hume or Descartes (see [2] and [4]). Yet, conceivability-based accounts of possibility are to be already found in some medieval logicians (see [1] on this), and I think that a similar idea can be found, in an early stage of development, in Abelard's theory of modality.

In Aristotelian modal theory, a property, in order to belong *necessarily* to a subject, had to be either an essential property or a *proprium* of the subject. Late ancient philosophers, from Porphyry to Boethius, considered these accidental but necessary predications of a subject [such as "being able to laugh" for a man] as being, although inseparable *in actu*, still *separable in the mind, through reason*, from their subject (see [3] on this). They admitted, then, the conceivability of a man unable to laugh, i.e. of things which were considered impossible in Aristotelian modal theory. Martin stated in [3] that, for these authors, this account of conceivability (understood as *conceptual* *separability*) did not entail possibility in any sense. However, I want to show that Abelard admitted a connection between conceivability and possibility, and agreed that there is a sense of possibility in which it is *possible* that men are not able to laugh, the justification of which being our ability to conceive or imagine it so. This investigation of Abelard's analysis of *propria* will lead us to a have a better understanding of the multiplicity of interpretations he gave to the modality of possibility and, in general, of his modal semantic.

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The Medieval Octagons: Analogies and Differences

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The Medieval Octagons of opposition and equivalence for sentences with quantified predicates, quantified modal sentences and sentences in oblique case present a strong analogy respect to its logical form. In this paper I present the Medieval Octagons using a special notation designed by Walter Redmond. This notation allows us to identify common features present in the three octagons. The basic pattern, which is in common to the three octagons, is the quantification of predicates. This is the Octagon, where "h" stands for "*human*" and "a" for "*animal*". Brackets and parentheses are particular and universal quantifiers respectively. "/" stands for internal negation.

$$\begin{array}{cc} (h)(a) & (h)/(a) \\ (h)[a] & (h)/[a] \\ [h](a) & [h]/(a) \\ [h][a] & [h]/[a] \end{array}$$

The octagon in the genitive case presents this form:

$$\begin{array}{ll} \langle\mathfrak{R}(h)(d)\rangle[r] & \langle\mathfrak{R}(h)/(d)\rangle[r] \\ \langle\mathfrak{R}(h)[d]\rangle[r] & \langle\mathfrak{R}(h)/[d]\rangle[r] \\ \langle\mathfrak{R}[h](d)\rangle[r] & \langle\mathfrak{R}[h]/(d)\rangle[r] \\ \langle\mathfrak{R}[h][d]\rangle[r] & \langle\mathfrak{R}[h]/[d]\rangle[r] \end{array}$$

where "h" stands for "*human*", "d" for "*donkey*", "r" for the predicate "*to run*" and "$\mathfrak{R}$" stands for the special relation of the genitive case, "$\langle$" and "$\rangle$" encapsulate the expression inside in order to be taken as a unit. $\langle\mathfrak{R}(h)(d)\rangle$ means "*of every human every donkey*", it constitutes the subject of the sentence being "*run*" or "*is running*" the predicate. The common pattern is shown by brackets and parentheses. But I will also show the differences among the Medieval Octagons.

Logical Consequence in Avicenna's Theory

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How does Avicenna view and define the consequence relation in his system? To answer this question, I will consider especially his hypothetical logic. Unlike al-Fārābī, who presents some hypothetical arguments comparable to the Stoics' ones, Avicenna presents a whole theory about the hypothetical syllogism, which almost duplicates the categorical syllogistic when the conditional propositions are considered and contains mixed premises, when the disjunctive propositions are introduced.

However, his analysis of the implication considers several meanings, which are more or less strong depending on the kind of link between the antecedent and the consequent. He thus holds a weak meaning called "'ittifāq" (translated by "chance connection") which looks more like a conjunction and a strong meaning called "luzūm", which involves a strong and causal relation between the antecedent and the consequent. In addition, he also introduces universal and particular quantifications in his hypothetical logic by using the words "kullamā" (whenever) and "qad yakūn" (maybe). When the word "kullamā" is used as in "whenever the sun rises, it is daytime" there is either a perpetual or a necessary connection between the antecedent and the consequent. While the word "qad yakūn" does not express such a strong relation between the two elements of the conditional.

So it seems natural to say that the luzūm corresponds to the universal quantified conditional proposition expressed by "$(\forall s)(Ps \rightarrow Qs)$" or "$(\forall t)(Pt \rightarrow Qt)$" if one quantifies on times as in N. Rescher's interpretation). But is the luzūm what Avicenna means by the consequence relation? Despite its strength and the fact that Avicenna calls this kind of implication the "real implication", it does not seem to be the consequence relation in his theory. At best, the universal quantification is a kind of "material" consequence relation (in the medieval sense), because of the necessary link between the antecedent and the consequent, which makes it truth preserving. But the real consequence relation is formal and is illustrated by the hypothetical syllogism itself. For Avicenna deduces conclusions only by means of the hypothetical syllogisms which contain two premises each, as in categorical syllogistic. These hypothetical syllogisms may contain both conditional and disjunctive propositions and may be viewed as relevant implications, because of the presence of common variables in the antecedent and the consequent. However, their validity is not always clear, as we will show in this contribution. This is due to the lack of formalization and the intuitive character of the theory.

Non normal modal logics in Thomas Aquinas

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Aquinas's theory of future contingents have been largely studied by historians of philosophy, but little attention has been devoted to the underlying logic of this doctrine. According to Aquinas, God knows all future contingents. It is necessary that, if God knows x, then x is the case. From this Aquinas concludes that future contingents are truly contingent de re, even though the whole of the proposition 'if God knows x, then x is the case' is necessary. There is, however, a further difficulty, because Aquinas maintains that whatever God knows, God knows it necessarily. By virtue of the K axiom — $\Box(A \to B) \to (\Box A \to \Box B)$ — it follows that if God knows x, x is necessarily the case (because Aquinas's theology supposes the theorem 'if God knows x, then necessarily: God knows x'). Aquinas's claim that future events are truly contingent may be defended only by rejecting the K axioms. The claim of this paper is that many semantic observations scattered in Aquinas's works support the idea that if we were to give a possible worlds semantics for Aquinas's modal logic, we should have a semantics that included impossible worlds too. Evidence for this is offered by two texts:

A) While discussing Aristotle's proof for the validity of the principle of non-contradiction (PNC), Aquinas notes that the rejection of the PNC may not be thought, but can be expressed with words. This 'expression' of the rejection of the PNC is an (impossible) world in which $\Box$PNC is false. (Aquinas's remarks on the impossibility of thinking something like that may be regarded as a psychological remark on how our mind is structured; the remark does not rule out 'impossible worlds').

B) Aquinas talks about 'imaginabilia' as distinct from the possible ways in which God might have created the world. Among the thing we can think of, there is the change of the past — an event which is incompatible with the PNC.

These texts suggest that Aquinas was open to subscribing to non-normal modal logics.

Marsilius of Inghen's Consequentiae

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My talk will focus on Marsilius of Inghen's Consequentiae; I will proceed with an overview of the (still critically unedited) text, then with an analysis of some relevant aspects of the theory, and collocating Marilius' theory of consequence within the contemporary discussions. I will introduce my provisional edition of the text and then examine some interesting features of Marsilius' theory of consequences and its relations to the contemporary discussions on the subject. I will focus in particular on Marsilius' definition of consequentia bona, and his accounts of formal and material consequences. I will try to place Marsilius' doctrine within the still little known framework of the XIV century theories of consequences, in particular but (not only) in correlation to the "continental" ones (Buridan, PsṠcotus, Albert of Saxony). In doing so, on the one hand, I will try to identify Marsilius' probable sources and the positions that have some influence on him, therefore clarifying some aspects of Marsilius' theory. On the other hand, a more detailed, grounded, and systematic analysis of Marsilius' Consequentiae might be a precious contribution to give a more precise and detailed picture of the articulation of the complex XIV century debates around consequences.

Where Medieval Logicians Feared to Tread. Syllogismus falsigraphus according to Medieval Latin Sources

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Falsigraphies are a very peculiar kind of fallacious reasonings. For one thing, they are demonstrations of sorts, that is, they are set out, as Aristotle repeatedly says in his Topics and Sophistical Refutations, in accordance with given science principles (e.g., geometry or medicine); for another, it takes an expert to expose them (e.g., a geometer or a physician). Despite the fact that, both in their capacity as Aristotelian commentators and as upholder of scientific rigor and consistency, Medieval Logicians have shown a keen interest in what they called falsigraphic syllogisms, these have not received all the attention they deserve. Relying on a variety of edited and unedited sources, we would like to show that the way Medieval authors handled falsigraphies tells us much about how sophisticated their views were on what counts as sound argument and why.

In order to do so, we will provide first a classification of falsigraphies as they occur both in commentaries on Aristotle (namely his Posterior analytics, Topics and Sophistical Refutations) and in logical treatises (summulae and tractatus). We will explore then their most distinctive features as well as their relationship with other types of syllogisms, particularly the critical or inquisitive ones (which they sometimes replace in medieval taxonomies).

Because of its interest and ubiquity, special attention will be granted to the problem of the circles quadrature, whose different methods have been discussed in great detail almost every time (Ancient and) Medieval Logicians have been tackling the issue of scientific paralogisms.

John Buridan on the Structure of Definitions: Quaestiones Topicorum I.6–8

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The paper I would like to present is part of a bigger project, recently started, whose aim is to reconstruct and to analyze the medieval reception of the Aristotelian theory of definition in commentaries on the 'Topics' from the 13th and 14th Centuries. In my talk I would like to focus on the commentary written by John Buridan (ca. 1300-1361) in the first half of the 14th Century. This commentary has the form of a set of questions on Aristotle's text, and in my paper I shall concentrate in three questions formulated on the basis of the teaching of Aristotle in the first book of the 'Topics'. Those questions build a unit, I argue, whose task is to provide reliable answers to three issues concerning the structure of definitions, namely: a) the problem whether we can express what is it do be a definition, b) the precise explanation of what definitions are, and c) the justification of the status of definitions as predicates. I hope that the analysis of those questions may provide an interesting introduction to the atmosphere of discussions on the notion of definitions carried out into the tradition of medieval commentaries on the 'Topics'.

Thomas Manlevelt: Ockham and beyond

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Not much is known of the fourteenth-century logician Thomas Manlevelt, but his work is remarkable enough. His fame rests chiefly on his *parva logicalia*, comprising *De suppositionibus*, *De confusionibus* and *De consequentiis*. Widely popular in the fourteen hundreds, they were in use as textbooks and commented upon at universities all over the continent.

In this paper I will rather concentrate on his *Questiones libri Porphirii*, an extensive commentary on Porphyry's *Isagoge*. It is edited in full, with introduction and indices, by the presenter of this paper, as volume 113 in the series *Studien und Texte zur Geistesgeschichte des Mittelalters* (STGM), Brill, 2014.

Following in the footsteps of William of Ockham, Manlevelt stresses the individual nature of all things existing in the outside world. In this paper, I will illustrate how he radically challenges our conceptual framework. He applies Ockham's razor in an unscrupulous manner to do away with all entities not deemed necessary for preservation. In the end, Manlevelt even maintains that substance does not exist.

With Manlevelt, early Ockhamism is being pushed to its extremes.

In fact, in his commentary on the *Isagoge*,Thomas Manlevelt applies the tactics of extending Ockhamist tenures and insights to any logical, and if need be metaphysical or theological subject matter. We are confronted with a radical variety of nominalism, outdoing Ockham in a number of ways. The individualizing tendency is stretched to its limits on the subject's as well as on the object's side, in an untiring effort to work out the primacy of the individual over the universal in any kind of detail. Manlevelt not only stresses the capacity of each individual instance (or 'token') of a term to stand for individual things in the outside world, he also stresses the token character of each instance of rational activity in itself. As each instance of a term — be it a genus, a species, or any of the remaining five universals — is an accident of the individual human mind doing the thinking, our author's 'singularising' of the domain of the universals is coupled with an 'accidentalising' of this same domain. The link between terms and reality may thus look disturbingly thin, as the linking takes place on an accidental level only.

The Origin of the Distribution Doctrine

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In the light of an analytic historical approach, I will try to prove Buridan's vital contribution to the doctrine of distribution which is mostly forgotten or rarely underlined. Buridan's version of the doctrine will be presented on the basis of his two strict criteria used in dividing the common personal supposition. Consequently, It will be described that modern authors' version of the distribution doctrine can be understood as a consistent simplification of Buridan's version. We will also explain how different views and presumptions about the constitutional types of the distribution doctrine cause to form different historical approaches to the history and origin of the doctrine. Finally, Peter Geach's and Terence Parsons' views about the doctrine will be compared with the provided conceptual-historical view in this paper. The former will be rejected and the latter will be accepted solely as a story of the origin of 'distributed term'.

Paradoxes of Signification

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In [1], Ian Rumfitt has drawn our attention to a couple of paradoxes of signification, claiming that although Thomas Bradwardine's "multiple-meanings" account of truth and signification can solve the first of them, it cannot solve the second. Bradwardine's solution of the paradoxes of truth in his *Insolubilia* [2] appears to turn on a distinction between the principal and the consequential signification of an utterance. Whereas, for example, Socrates' utterance of 'Socrates says something false' (and nothing else) principally signifies that what Socrates said is false, it only consequentially signifies that what Socrates said is true, as a result of Bradwardine's claim that an utterance signifies everything that follows from it. Thus Socrates' utterance is self-contradictory and so simply false and not true.

Once this distinction between principal and consequential signification is admitted, however, the second of Rumfitt's paradoxes bites and seems to leave Bradwardine with no response. Both paradoxes were discussed extensively in the fourteenth century in the decades after Bradwardine's treatise was written, by Roger Swyneshed, William Heytesbury, Robert Fland and Ralph Strode.

It is shown that the distinction between the principal and the consequential signification is made not by Bradwardine but by his opponents, and is not required for Bradwardine's solution to work. In fact, it dissolves on further examination, and the problematic paradox with it.

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Philosophy of Computer Science

This workshop is organized by

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The Philosophy of Computer Science is concerned with epistemological, methodological, and ontological analyses of methods and techniques of inquiry involved in computer science as a discipline. The philosophy of computer science should be carefully distinguished from traditional philosophy of computing, dealing with ontological and epistemological issues arisen with the discovery of computable functions. The philosophy of computer science focuses on methods, developed in computer science and software engineering, in the design, specification, programming, verification, implementation, and testing of computational physical machines. As such, the philosophy of computer science is not to be conceived as a branch of philosophy of mathematics and philosophy of logics but rather as an independent, separate, discipline sharing interests with the philosophy of mathematics, the philosophy of empirical sciences, and the philosophy of technology.

Contemporary philosophy of science is divided into as many fields, including the philosophy of biology, the philosophy of economics, the philosophy of psychology, etc.., as the types of systems about which a branch of science is devoted to their investigation. The philosophy of computer science is involved in methodological problems arising within the investigation of software systems. Many of those problems characterize inquiries upon several natural systems, as being coessential with any scientific activity; others are typical of software systems as being human-made systems. In the following, some of them, which are under the interest of the present workshop, are concisely put forward.

The invited keynote speaker of this workshop is Raymond Turner (page 108).

Call for papers

We encourage submitting papers possibly addressing one of more of the following questions:

- Among the aims of scientific investigations on natural systems is achieving to some *theory* systematizing and justifying the attained knowledge on the studied system. Can theories of software systems be defined? Are they mathematical or empirical theories? Which role is played by computational models in the discovery of those theories?

- Common scientific theories enable one to define *law*-like statements expressing regular behaviours of the studied systems. Is it feasible to isolate law-like statements concerning executions of studied computational systems? Under which condition are they supposed to hold? How are those regularities justified, that is, falsified and corroborated? Is *probability* involved in the confirmation of software law-like statements?

- *Abstractions and idealizations* are widely used in science to simplify and modify theoretical constructs in order to be able of deriving, from the abstracted and idealized theories, desired consequences, theorems, or laws. Abstraction is, on the other hand, a key concept in the design, specification and verification of programs. How are abstraction techniques developed in computer science related to the problems of abstracting and idealizing mathematical or empirical theories and models in science?

- Models, theories, and empirical regularities are sources of *scientific explanations* of empirical phenomena. Is explanation a significant philosophical issue in computer science? What are good explanations of software systems' executions? How are *causal process* arising at the physical implementing level of a computing system involved in the explanation of observed executions, especially malfunctions, of such system?

- In the light of the potential answers to the previous questions, what is the *epistemological status* of computer science? Should it be conceived as an applied mathematical discipline, a scientific discipline, or a technological discipline? What is the relation, from an epistemological and methodological viewpoint, between computer science and software engineering?

- Beside typical methodological issues, other philosophical topics characterize the philosophy of computer science and concern the *ontology of programs* and the *ontology of computational processes*. What are programs? What are software specifications? What is the relation, from on ontological point of view, between software and hardware? What are the computing process prescribed by programs and specifications? Are they the continuum physical processes, or the discrete procedures described by computational models?

Long abstracts (up to 1000 words) should be sent before 30th of January to petros@math.ntua.gr.

Software and causality

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Suppose you are sitting at your computer and press the "e" key. An "e" appears on the screen and in the document. Does the key-press *cause* the "e" to appear on the screen and to be inserted into the text? This seems to fit the Interventionist template change which key is pressed and the inserted letter will be different. Nevertheless my inclination would be to say that the key-press does not *cause* these results.

The way most interactive programs work is that they set up what are sometimes called *listeners*. A listener is a chunk of software that is activated whenever a listened-for event occurs. The listener examines the notification and takes one or another series of actions.

An analogy may help. Imagine you hear the phone ring, you pick it up, and you say "hello." Did the ringing *cause* you to perform those actions? I doubt that many people would say it did.

Causation tends not to be a standard frame of reference in computer science. But computer scientists and software developers often talk in terms of causal/mechanistic explanations. One might explain the appearance of the "e" as follows. The user pressed the "e" key, which led to the listener being notified, which led to the program taking certain actions, etc. The construction *led* to *¡something¿ happening*[1] is understood to mean something like *set up a situation in which ¡something¿ happened*.

This way of speaking does not necessarily attribute the ¡something¿ happening directly to whatever created the situation in which it happened. Room is left for disengagement between cause and effect. Perhaps one reason for this disengagement is that there is no physical causation in software. Since software is not physical it cannot be party to a physical interaction.

This sort of disengagement carries over to our everyday experience. I would say that flipping a light switch does not *cause* a light to go on. Flipping the light switch enables an electric current to flow through a circuit, which leads to the light going on. The switch itself doesn't have any direct connection to the light.

An Interventionist might respond in three ways.

1. Computer scientists do talk about causal (i.e., "because"-based) explanations. Interventionist causality is often understood to be a theory of causal explanation rather than a theory of causation.
2. Interventionist causality is about probabilities rather than physical causation. Smoking does indeed raise the probability of cancer occurring. And pressing the "e"-key does indeed raise the probability of an "e" appearing on the screen. This response seems to me to be something of a stretch.
3. If you follow the electrical circuits, pressing the "e"-key is in fact physically connected to the observed results. Pressing the "e"-key sends a signal from the keyboard to

[1]One might even use the term *cause* in place of *led* to, but I don't want that to confuse the situation.

the computer, which triggers the software listener, which triggers the performance of the pre-programmed actions, which insert an "e" on the screen and into the document. At the hardware level, it is all driven in a forward chaining manner. One thing physically triggers another, which triggers another, etc. But most software developers don't think in terms of hardware. A hardware explanation would be like saying that the ringing of the telephone causes you on some physiological level to pick it up and say "hello." And in fact, one might be able to trace the physiological connection: the sound of the phone, triggered vibrations in the ear, which triggered nerve firing to the auditory center of the brain, which triggered, . . . which triggered your picking up the phone and saying "hello." But that's not how we think about our own behavior, and that's not how developers think about software.

Software causality is similar to the mechanism that switches an onrushing railroad train from one track to another. The central element in software causality is the if-statement. An if-statement switches the (similarly) onrushing flow of the program to proceed on one track or the other.

One can trace the functioning of if-statements to hardware multiplexors, which determine which of two address is put into the computer's program counter. But a multiplexor is a fairly straightforward Boolean circuit. Where is the causality and decision making? It all comes down to relays, i.e., devices that switch current flows from one path to another. This is essentially the same situation we saw in the light switch example. In both cases, the switch doesn't cause the effect; switching a flow path enables whatever is flowing along the path to cause the effect.

This raises a question for Interventionist causality. Traditionally one takes an interventionist causal relationship as something like a promise that a physical causal chain connects the cause to the effect. But as in the previous examples, that's not always the case. The causal action may simply change the world in a way that results in the effect through other means.

Recently Baumgartner [1] argued that that if A supervenes on B and A can be shown to be an Interventionist cause of C then it is really B, not A, that is causing Woodward [2] responded by agreeing with Baumgartner's argument but insisting that it still makes sense to say that A stands in a causal relationship to C.

It's likely that an examination and analysis of software causality can help clarify our more general understanding of causality.

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Syntax and Semantics in Evolved Theories

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The paper considers the contribution which syntactic and semantic elements make to genetic program based theory construction. One major philosophical method of analysing scientific theories aims at identifying abstract formal structures which all these theories share. The two principal positions are what are often referred to as the syntactic and semantic approaches. The former is central to logical positivist philosophy of science and stresses inferential patterns within theories [2]. Theories are analysed as deductive axiomatic systems (using quantified first order logic plus various relational extensions) in conjunction with appropriate empirical interpretations of non-logical terms [3]. The syntactic approach was criticised for either ignoring or distorting many aspects of theory construction in science [5] and increasingly fell out of favour as positivism waned. It was gradually replaced by the semantic approach to scientific theories [4]. On this view theories are abstract specifications of a class of models where a model is a structure in which a theory is true. This dispute raises the issue of whether the genetic programs evolved by using the theory language in the pilot work [1] should be approached by a syntactic or a semantic approach or even by a mixture of both. The theory language has a syntactic character (whose properties can be specified in a version of first order logic with suitable extensions) whilst the genetic program operators are probabilistic. It follows that there are substantial grounds for engaging in the study of syntactic features where this includes a central (and perhaps even predominant) role for probabilistically defined ones. More generally, this issue has important implications for whether the current widespread rejection of syntactic analysis of scientific theories is well justified.

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Software Theory Change by resilient near-complete specifications

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Formal specification and formal verification are considered the two essential steps for determining the degree of software reliability, intended as compliance of any execution of a program's instance with respect to its intended behaviour. Despite progress in accuracy in both areas, the phenomenon of software malfunctioning is still one of enormous relevance. One way to express malfunctioning of software systems is by referring to an execution that produces unexpected side-effects, or unexpected postconditions, see Floridi et al. (forthcoming). Consider a computational system's model as defined by the relation:

$$S := < preconditions \Rightarrow [p]postconditions >,$$

which says that given some preconditions, the execution of program variable p necessarily leads to some postconditions. Then, malfunctioning can be expressed as the gap between the expected postconditions in the above formal description and the result of tracing an instance of p execution steps. The latter will give the actual behavior of a running instance of p, to be compared to postconditions in S. When these *diverge* from the execution traces, it could be the result of stronger preconditions being implemented, unexpected execution conditions or weaker postconditions being required by the formal description.[1] To develop a conceptual model describing the dynamics of software change is crucial for the philosophy of computer science and part of the general task of understanding software as theory, see e.g. [8] and chapter 15 of [10]. The complementary task for software engineering and formal methods is to develop formal machineries that describe these processes and their handling precisely. Identified preconditions are meant to guarantee *safety* of program execution. On the other hand, identifying preconditions that (may) lead to execution failure allows to define *completeness*. In [7], the notion of *near-complete* specifications is used to describe approximations to complete specifications, through the use of possible errors classification in weakened post-conditions and an algebra on the possible program states. A similar sense of near-completeness is offered by approaches to requirements engineering where the relevant specification description can be marked up with refinements given by possible, admissible or even acceptable malfunctioning. In this paper, we aim at considering near completeness for specifications as the approximation to safe program states descriptions, analysed in the light of the principles of *theory change*.

[1] The topic of software systems' resistance to change in the environment as graceful degradation is a research topic of its own (see e.g. [2, 3]), including reference to hardware and material execution conditions, and we abstract from it in the present contribution to model specifically the relation of change in specifications as induced from unexpected postconditions.

When understanding software as a theory, a *model* of a correctly functioning computational system S as described above can be taken to mimic the notion of *right theory*. In turn, an *implementation* S_1 of S whose execution generates traces in which the output state diverges from the expected post-conditions

$$S_1 := < preconditions' \Rightarrow [p]postconditions' >$$

is such that any of the pre-conditions (including the input) may differ from those given in S and on that basis one execution of p possibly leads to distinct postconditions than those described by S. We say that S_1 is malfunctioning, and the corresponding abstract specification near-complete, with respect to S if it accommodates possibly faulty states. Such states might (for various reasons) be accommodated in a new specification S_0 which includes them as valid. The process of approximating from the near-complete specification of S_1 to a safe program execution of S_0 corresponds to arriving at the right correct account of a scientific phenomenon. Such analogy strongly relies on:

1. our ability to account for the actual process of software design;
2. a model that accommodates the approximation from near-complete to valid specifications;
3. a conceptual description of how a computational system resists or fails under such stages of approximations.

Resilience for a computational system can be defined as its (graded) ability to perform a computation (i.e. an implementation of a computational system model) that is verified correct in view of the formal specification even under varied or perturbated specifications. This means the implementation reaches an expected, or at least admissible post-condition on a given input, despite a number of initial states of the system are possibly different from those specified or those expected to be necessary. In the present paper we offer a conceptual analysis that aims at including all the three of the steps above. The main contribution consists in offering a notion of resilient software system from near-complete specifications. In the literature on software change, the process we are interested in corresponds to preservation of *behavioral safety* by specification approximation, in order to account for unpredictable or unexpected behavior, see e.g. the theme of system properties and its dimensions in the taxonomy offered in [4]. Moreover, various attempts have been made to tackle this notion of resistance (perseverance of validity) to change. The most common one encountered in this research area is that of system *robustness*. One (older) interpretation of robustness is given in terms of the inability of the system to distinguish between behaviours that are essentially the same, see [9]. More recently, the term resilience has been used to refer to the ability of a system to retain functional and non-functional identity and three of its constituents are given in terms of: the ability to perceive environmental changes; to understand the implications introduced by those changes; and to plan and enact adjustments intended to improve the system-environment fit ([5]).

In the present contribution, we endorse the view that instances of software systems acts under variations of pre-conditions and that a model of computational systems implementing resilience should be given as a function

$$S'' := < preconditions[v_1, \ldots, v_n] \Rightarrow [p]postconditions >,$$

where each v_i can be seen as a variation in the set of preconditions which still allows for an admissible postcondition. The process of theory change that a computational model undergoes is given by the extension or reduction of the set $\mathcal{V}$ of variations that the set of preconditions is able to absorb while preserving an admissible (for the purposes of the computation) state satisfying the (varied) postconditions. An unreachable program state, on the other hand, is given by changing the preconditions set so that one does no longer satisfy the type of specifications assumed and the post-conditions are no longer valid. Additionally, a measure of resilience for computational systems can be offered, based on a metrics of such variations. A resilient system is defined as one that can accommodate such changes, maintaining program state validity. Our aim, is to offer a model that describes admissible variations on the preconditions of a valid program state so as to approximate to an unreachable state, still allowing for a valid output state.

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Philosophical Aspects of Programming Theory Development

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The following three levels of Computing foundations were proposed in [1]: 1) *philosophical*, 2) *scientific*, and 3) *formal (mathematical)*. Here we continue investigations of this three-level scheme applying it to Programming theory development. For this case we study interrelations of the levels concentrating on relations of the philosophical level with the other two levels. It should be noted that the importance of philosophical foundations for information-related disciplines (in particular, for programming) is widely recognized. Different philosophical systems were proposed for this purpose, for example, K. Popper's ontology, philosophy of Kuhn and Peirce, specific epistemology of Hjorland, etc. A short description of philosophical approaches could be found in [2], philosophical aspects of the main mathematical notions were discussed in [3].

Our approach to Programming theory development [4] is based on Hegel's logic [5] presented in the modified form in [6]. Therefore, the main principles of the approach are the principle of *development from abstract to concrete*, the principle of *triadic development* (thesis — antithesis — synthesis), and the principle of *unity of theory and practice*.

The above-mentioned levels identify three types of notions that constitute the basis of each level respectively: philosophical level — *categories* (infinite notions in Hegel's terminology), scientific level — *finite (scientific)* notions, and formal level — *formal finite (mathematical) notions*. Thus, interrelations of the levels are tightly connected with relations between categories, scientific notions, and formal notions respectively. We distinguish two transitions between levels: from categories to scientific notions and from scientific notions to formal notions. The first transition is called *finitization* and the second one — *formalization*. The finitization transition transforms categories presented with the help of Hegel's categories *universal* — *particular* — *singular* into scientific notions described as integrity of their *intensions* — *extensions*. The formalization transition constructs formal intensions and extensions of scientific notions.

At the *philosophical level* we start with the following triad of categories: *subject — goal — means*. Then we enrich this triad with two new triads: *subject — means — means usage* and *goal — means — means construction*, finally obtaining the following pentad: *subject — goal — means — means usage — means construction*.

At the *scientific level* we make finitization of categories of this pentad obtaining the following *pentad of programming notions*: *user — problem — program — program execution — program construction*. We investigate finitization transition between the levels represented by pairs *subjectâ€“user*, *goalâ€“problem*, *meansâ€“program*, *means usageâ€“program execution*, *means constructionâ€“program construction*. We also make further development of the notions of programming pentad, in particular, we specify the notion of program via *program pentad*: *data — function — function name — function composition — function description*.

The second transition — formalization — poses a number of philosophical questions concerning the category of *formal*, its relation to other categories, expressivity and limitations of formal notions, etc. Analysis of these questions permits to explain construction of programming models in integrity of their philosophical, scientific, and formal aspects.

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Heinz Zemanek's almost forgotten Contributions to the Philosophy of Informatics

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Keywords: *Philosophy of Informatics*, *History of the Philosophy of Informatics.*

Whereas philosopher-scientists are and have been well-known throughout the history of ideas, philosopher-*engineers* were always less-known and standing — so-to-say — somewhat in the shadow of history. One of the most famous philosophers of the 20th century, the Austrian *Ludwig Wittgenstein* (1889–1951), is hardly remembered as the engineer and architect which he also was. Vice versa, the German and Austrian computer pioneers *Konrad Zuse* (1910–1995) and *Heinz Zemanek* (1920–2014) are widely recognised as (computer)-engineers, considerably less is publicly known about their (computing)- philosophical thoughts and activities. As far as the public reception of their computer-philosophical legacies is concerned, it seems fair to say that Konrad Zuse is still 'better off' than Heinz Zemanek, because Zuse always had a strong history-political lobby (particularly in Germany) who promoted and defended his pioneering legacy against an Anglo-centric history 'written by the victors' after WWII. For these reasons it was never forgotten that Zuse had not only built the very first electro-magnetic-mechanical computer which was fully and freely programmable, but also laid the foundation of the metaphysical doctrine of *pan-computationalism* with his essay on *Rechnender Raum* (computing cosmos), which he claimed to have conceived mentally already during the 1940s more than twenty years before its printed publication. Pan-computationalism is nowadays a thriving (although i.m.h.o. unscientific) philosophical-metaphysical ideology particularly in the field of 'natural' or 'nature-inspired computing'.

Less known — though philosophically more salient — than Zuse's metaphysical speculations about a computing cosmos are Heinz Zemanek's early contributions to the philosophy of computing, because Zemanek was â€“as far as I knowâ€“ the first philosopher-engineer who had fully grasped the computer-philosophical relevance of the logical language-philosophy designed by his famous compatriot, *Ludwig Wittgenstein*. For this reason, my contribution is focused precisely on Zemanek's *early* philosophy. To this end I will briefly recapitulate his main train of computer-philosophical thought — especially from those of his German-language publications which are hardly available for a wider international audience — and emphasise some interesting points or questions which Zemanek's early computer-philosophical publications left open or un-answered for the future. However, a comprehensive 'history of literature and ideas' about Zemanek's contribution to the philosophy of computing is outside the scope of my contribution and must be left as a 'to-do' for the professional historians from the academic faculty of the humanities.

Zemanek had been a prolific writer until his very old age, such that rigorous selection is needed for the purpose of a short review-contribution with a well-defined focus. In order to 'prove' the often-forgotten point that the philosophy of informatics is actually considerably older than it is widely believed to be, I have chosen the year 1975 as the 'cut-off' date for the recapitulation and discussion of Zemanek's philosophical thoughts. But which ones of Zemanek's many works from before 1976 should be selected for this purpose? In one of his later computer-philosophical essays from the year 1993, Zemanek himself had indicated — in hindsight — which ones of his own early philosophical writings he still regarded as noteworthy and relevant: most of them I was able to retrieve for the purpose of this review.

Those works of Zemanek can be broadly classified into two categories: (A) works in which Zemanek interpreted informatics very specifically in the light of Wittgenstein's philosophy, and (B) works in which Zemanek interpreted informatics more generally in the wider context of a cultural philosophy of technology in the broadest sense.

In my conference talk I shall speak about several further details of what I have briefly sketched in the paragraphs of above. The complete paper, including all the relevant literature references and due acknowledgments, shall be made available after the conference.

A Plea for Explanatory Pluralism in Computer Science

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It has been claimed that explanations of the behavior of computing systems is adequately explicated in terms of the mechanistic model of explanation (Piccinini 2007, Piccinini and Craver 2011). We discuss critically this claim in connection with those computing systems that are characteristically described as physical systems which carry out computer programs. First, we point out that many explanations of behavioral patterns observed in those computing systems do not involve causal accounts of the sort that are required by the mechanistic model of explanation. Moreover, the explanatory power of these explanations does not depend on (and is not increased by providing) detailed causal stories of observed behavioral patterns. Quite to the contrary, explanatory power is actually increased by removing a wide variety of details concerning underlying causal mechanisms. Accordingly, many explanations accounting for those behavioral patterns that are observed across a wide variety of computing systems and are produced by heterogeneous causal processes should be counted as minimal explanations (Batterman and Rice 2014) or functional role explanations (Cummins 1975). Finally, we point out that many model-based explanations of computing system behaviors involve descriptions of computational processes as non-terminating processes, that is, as infinite sequences of states which do admit an initial or start state but no termination state. These model-based explanations do not fit the mechanistic model insofar as mechanistic explanations involve descriptions of mechanisms as sets of entities that are engaged into processes which connect some start condition to a termination condition.

When addressing explanation requests about computing systems behaviors, computer scientists take advantage of a plurality of models of explanation â€“ including functional, minimal, and genuinely causal models. The selection of the more appropriate explanatory strategy for answering an explanation request depends on the pragmatic interpretation of the explanation request (van Fraassen 1980). This pragmatic interpretation hinges, in its turn, on the variety of contexts that are afforded by available descriptions and models of the computing system. These descriptions and models are progressively developed by computer scientists in the multifaceted and interrelated activities of specifying, designing, implementing, testing, and predicting the behaviors of computing systems.

Consider those descriptions that one develops when specifying requirements for computing systems. These specifications express properties that computing systems *must* fulfill, insofar as they reflect desiderata of users, programmers or manufacturers. Accordingly, some explanation requests about computing systems behaviors are fruitfully interpreted in the pragmatic context of the underlying intentions of users. Question
(1) "Why was event x observed?"
can be contextually interpreted as the request of explaining why event x occurred in accordance or else in contrast with those intentions. Similarly, question
(2) "Why do all observed runs of program P in different computing systems fulfill the liveness requirement L?"
can be contextually interpreted in the context of specification requirements.

Explanations requests that are pragmatically interpreted in the context of human intentions typically concern technological artifacts: they do not arise in many scientific domains where the behavior of systems of interest is not shaped by human intentions. But these differences between explanation needs arising in science and technology, respectively, should not blind one to deep commonalities between strategies that one adopts to answer explanation requests across science and technology: model-based explanations play a central role in both cases. In particular, to explain behavioral patterns and events of both technological artifacts and other natural systems whose behaviors are not influenced by human intentions one often relies on representations of these systems as dynamical systems.

Computing systems are often profitably modeled as dynamical systems in which both temporal and state variables are identified with natural numbers. The fact that explanations of computing system behaviors often draw on *discrete* dynamical systems of this sort sheds additional light on the dispensability of a detailed causal story for the purpose of explaining many behavioral events and patterns of computing systems. Indeed, from the viewpoint of classical physics, everything moves continuously, including the physical systems which carry out computer programs. However, to explain behavioral patterns of computing systems this sort of representational accuracy is not needed. Actually, it is the lack of representational accuracy characterizing discrete dynamical systems that enables one to achieve greater explanatory power when one wants to explain why a wide variety of systems, differing from each other in the way of physical components and processes, exhibit the same behavioral patterns *qua* computing systems.

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Connexive Logics

This workshop is organized by

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Modern connexive logic started in the 1960s with seminal papers by Richard B. Angell and Storrs McCall. Connexive logics are orthogonal to classical logic insofar as they validate certain non-theorems of classical logic, namely

- Aristotle's Theses: $\sim(\sim A \rightarrow A)$, $\sim(A \rightarrow \sim A)$
- Boethius' Theses: $(A \rightarrow B) \rightarrow \sim(A \rightarrow \sim B)$, $(A \rightarrow \sim B) \rightarrow \sim(A \rightarrow B)$

Systems of connexive logic have been motivated by considerations on a content connection between the antecedent and succedent of valid implications and by applications that range from Aristotle's syllogistic to Categorial Grammar and the study of causal implications. Surveys of connexive logic can be found in:

1. S. McCall, "A History of Connexivity", in *Handbook of the History of Logic*, edited by D.M. Gabbay et al., vol. **11**: *Logic: A History of its Central Concepts*, Amsterdam, Elsevier, 2012, pp. 415–449.

2. H. Wansing, "Connexive Logic", in *The Stanford Encyclopedia of Philosophy*, Fall 2014 Edition, edited by Edward N. Zalta, forthcoming in `http://plato.stanford.edu/archives/fall2014/entries/logic-connexive`.

Recently, connexive logics have received new attention. This workshop is meant to present current work on connexive logic and to stimulate future research.

The invited keynote speaker of this workshop is Storrs McCall (page 101).

Call for papers

Any papers related to connexive logics are welcome. Topics of interest include (but are not limited to) the following:

- Historical considerations of the notion of connexivity
- Arguments for or against connexive logics
- Examinations of existing systems of connexive logics
- non-explosiveness of logical consequence

Submissions of extended abstracts (up to five pages) should be sent to both organizers as a pdf file at hitoshiomori@gmail.com and Heinrich.Wansing@rub.de.

Deadline for submission: December 1st, 2014.

Notification of acceptance: December 31st, 2014.

On Arithmetic Formulated Connexively

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One of the richest and most tantalizing applications of a non-classical logic is the matter of how mathematics operates within its scope. The *contraclassicality* of *connexive logics* — systems endorsing Aristotle's Thesis (*i.e.*, $\sim(A \rightarrow \sim A)$) and Boethius' Thesis (*i.e.*, $(A \rightarrow B) \rightarrow \sim(A \rightarrow \sim B)$) as theorems — entails that the development of connexive mathematics will be more complex — and, arguably, more intriguing — than intuitionistic or relevant accounts. For example, although formally undecidable sentences in classical Peano Arithmetic remain so with respect to its intuitionistic and relevant fragments, the situation is much more complicated connexively. Let T be a classical theory of arithmetic extending Robinson Arithmetic and let $\mathbf{G}_T$ be its Gödel sentence; then although $\sim(\mathbf{G}_T \rightarrow \sim\mathbf{G}_T)$ is also undecidable in T, this sentence—as an instance of Aristotle's Thesis—will be provable in any connexive arithmetic.

In this talk, we will make a few observations concerning the formalization of such a connexive account of arithmetic. Historically, we will consider Łukasiewicz' number-theoretic argument against Aristotle's Thesis, namely, that counterexamples to Aristotle's Thesis are implied by Euclid's Lemma. Furthermore, we will consider some of the Kneales' remarks concerning Aristotle's Thesis, in which Aristotle's endorsement of connexive principles is interpreted as a rejection of the Zenonian account of provability and the validity of *reductiones ad absurdum*. Suggestively, the very concerns the Kneales attribute to Aristotle form the basis for the hyper-constructive approach to mathematics endorsed by David Nelson. Hence, the prospects for importing Nelson's concerns as a foundation for connexive mathematics and their relevance to Heinrich Wansing's Nelson-like connexive logic C will be considered.

From a formal perspective, we will consider quantifier-free extensions of Richard Angell's connexive logics $\mathsf{PA1}$ and $\mathsf{PA2}$ and examine weak quantifier-free fragments of arithmetic ("protoarithmetical" theories with a successor axiom) formulated in these logics. These formalizations reveal that in any reasonable $\mathsf{PA1}$ theory of arithmetic, bounded quantification—expressible in these quantifier-free theories—will behave pathologically. Furthermore, given the unary truth operator T of $\mathsf{PA2}$, we may observe that any complete $\mathsf{PA2}$ theory of arithmetic either proves the sentence

$$n = n \leftrightarrow \mathsf{T}(n = n)$$

for *every* natural number n or proves its negation, *i.e.*,

$$\sim(n = n \leftrightarrow \mathsf{T}(n = n))$$

for every natural number n. Complete $\mathsf{PA2}$ theories of arithmetic will thus divide into those in which identity is always *literal* (in the sense that every sentence $n = n$ is strictly true) and those in which identity is always *allegorical* (in the sense that no such sentence is strictly true).

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A simple connexive extension of the basic relevant logic BD

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The name 'Connexive logic' suggests that connexive logic shares a certain motivation with relevant logic. And some attempts are known in the literature by relevant logicians such as Richard Routley, Chris Mortensen and Ross Brady, at realizing the connexive theses in relevant logic. The present paper goes in the same direction using a different approach. The main motivation behind the paper involves a problem formulated by Graham Priest and Richard Sylvan, and considered further by Greg Restall. In brief, the problem is to find a proof theory for extensions of the basic relevant logic **BD** in which the negation is interpreted in terms of a four-valued semantics (i.e. the so-called American plan). The difficulty lies in finding the appropriate axioms and/or rules of inference, to capture the corresponding falsity condition for the conditional. Priest and Sylvan suggested two falsity conditions for the conditional, but the corresponding axioms and/or rules of inference remain unknown. The aim of the paper is to show that for a certain falsity condition, inspired by the work on connexive logic by Heinrich Wansing, it is possible to find the corresponding proof theory. The paper also presents two other non-connexive falsity conditions for which the corresponding proof theories are available.

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The Strange Status of The Principle of Conditional Non-Contradiction

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The Principle of Conditional Non-Contradiction (CNC) asserts that any pair of conditionals of the form $\alpha \mathrel{\Box\!\!\rightarrow} \beta$ and $\alpha \mathrel{\Box\!\!\rightarrow} \neg\beta$ is inconsistent. Two variants of CNC are the following:

$\neg(\alpha \mathrel{\Box\!\!\rightarrow} \neg\alpha)$	Aristotle's Thesis (AT)
$\neg((\alpha \mathrel{\Box\!\!\rightarrow} \beta) \wedge (\alpha \mathrel{\Box\!\!\rightarrow} \neg\beta))$	Boethius's Thesis (BT)

The status of principles CNC, AT, and BT and its consequence are in need of clarification for connexive logics as well as standard conditional logics, where the latter include (a) Lewis's modal sphere semantics and (b) Adams's probabilistic semantics for conditionals. Whereas the logical status of CNC and its variants is perfectly clear for connexive logics – connexive logics take principles AT and BT as the cornerstone of any axiomatization of a logic for conditionals — the status of CNC does not seem to be clear for standard conditional logics. For example, Bennet and Gibbard take CNC and its variants as valid principles of both (a) and (b) and argue on that basis for a probabilistic semantics of conditionals in line with Adams.

However, neither CNC nor AT nor BT are valid in these systems. In fact, they cannot consistently be added to either system. Moreover, the addition of weakened principles changes Lewis's modal system in essential ways thereby reducing the appeal and the flexibility of the original formalism. In contrast, in Adam's probabilistic semantics the validity of a weakened version of CNC depends on a small point – excluding conditionals which have an antecedent with zero probability – making the basis on which CNC is valid rather arbitrary.

On the other hand, proponents of CNC and its variants, such as connexive logicians, face a challenge. If — as it is customary — $\Box\alpha$ ("α is necessary") is defined by $\neg\alpha \mathrel{\Box\!\!\rightarrow} \alpha$, under very weak assumptions AT implies $\neg\Box\alpha$ for any α. That means that no state of affairs can obtain with necessity. Thus, the proponent of CNC is faced with two "bad" options: either abandoning the latter definition of necessity or sticking with the fact that the formalism cannot describe any necessary state of affairs in a consistent way.

Natural deduction for bi-connexive logic

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Both bi-intuitionistic logic and connexive logic have received considerable attention recently; see, for example, [1, 2, 4]. A bi-intuitionistic system, 2Int, different form the bi-intuitionistic logic BiInt that is also know as Heyting-Brouwer logic, has been introduced in [3]. In this talk I will present a natural deduction proof system for a connexive version of 2Int. It combines the use of proofs as well as dual roofs with a connexive interpretation of the implication and co-implication connectives of 2Int. Moreover, a formulas-as-types notion of construction is presented for the co-negation, implication, and co-implication fragment of 2Int. This construction makes use of a two-sorted typed lambda calculus.

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Logic and Information

In collaboration with the Society for the Philosophy of Information, this workshop is organized by

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The juxtaposition of 'logic' and 'information' is popular as well as controversial: It is clear that there must be a connection between both, but there is hardly any agreement about the precise nature of the connection. When we focus on how information can clarify what logic is about, it is natural to say that valid arguments are just those arguments where the content of the conclusion does not exceed the combined content of the premises. Yet, such explanations do not have the same status as more entrenched truth-conditional and inferential conceptions of logical consequence, which suggests that information-talk about logic is simply redundant. When, by contrast, we focus on how logic can clarify the nature and dynamics of information, we turn our attention to specific developments in philosophical logic, like logics of knowledge and belief and their many dynamic extensions. While this opens up an entirely new field of formal investigations — often dubbed the dynamic and interactive turn in logic — it is less clear whether such developments establish a special connection between logic and information (after all, there are plenty of logics of X whose existence and usefulness does not imply a special connection between logic and what it is used for).

At least since Carnap and Bar-Hillel's theory of semantic information, many closer connections between logic and information have been developed by, amongst others, Barwise & Perry, Corcoran, and Hintikka. More recently, the simultaneous rise of the philosophy of information and the dynamic and interactive turn in logic has led to a revival of the question of how information and logic can be related. In this workshop we want to approach the subject from the perspective of the philosophy of information, as well as from a logical perspective, and draw attention to a number of questions that have received more attention, or have only been individuated in recent years. These include the possibility of a genuine informational conception of logical consequence, the relation between informational and computational approaches, the relation between information and logics of questions, and the difference between (what van Benthem calls) implicit informational stances in logic like that of intuitionist logic and explicit stances like that of epistemic logic.

The invited keynote speaker of this workshop is Luciano Floridi (page 91).

Towards a More Realistic Theory of Semantic Information

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According to the theory of "semantic information" [3, 2], the information content of a sentence φ is, roughly speaking, characterized by the set of all relevant states of the world that it excludes. In his *Logic of Scientific Discovery* [17], Karl Popper had put forward a similar idea to characterize the empirical content of a theory, in order to support his central claim, namely that the most informative scientific theories are those that are highly falsifiable, while unfalsifiable theories are devoid of any empirical content:

> The amount of positive information about the world which is conveyed by a scientific statement is the greater the more likely it is to clash, because of its logical character, with possible singular statements. (Not for nothing do we call the laws of nature "laws": the more they prohibit the more they say.) ([18], p. 19).

A first straightforward consequence of Bar-Hillel and Carnap's notion of "semantic information" is that contradictions, like "tomorrow it will rain and it will not rain", carry the maximum amount of information, since they exclude all possible states. A second inevitable consequence of the theory is that *all* logical truths are equally uninformative (they exclude no possible state), which justifies their being labelled as "tautologies". Moreover, in classical logic a sentence φ is deducible from a finite set of premises $\psi_1, \ldots, \psi_n$ if and only if the conditional $(\psi_1 \wedge \ldots \wedge \psi_n) \to \varphi$ is a tautology. Accordingly, since tautologies carry no information at all, no logical inference can yield an increase of information. Therefore, if we identify the semantic information carried by a sentence with the set of all possible states it excludes, we must conclude that, in any valid deduction, the information carried by the conclusion is contained in the information carried by the (conjunction of) the premises. While this theory of semantic information appears to provide a convincing theoretical justification of the persistent empiricist dogma that deduction is not informative, both these consequences appear to be at odds with our intuition and clash with the commonsense notion of information, to the extent that they have been often described as genuine "paradoxes" of the theory.

As for the first paradox, Bar-Hillel and Carnap were well aware that their theory sounded counterintuitive in connection with contradictory (sets of) sentences, as shown by the near-apologetic remark they included in their [3]:

> It might perhaps, at first, seem strange that a self-contradictory sentence, hence one which no ideal receiver would accept, is regarded as carrying with it the most inclusive information. It should, however, be emphasized that semantic information is here not meant as implying truth. A false sentence which happens to say much is thereby highly informative in our sense. Whether the information it carries is true or false, scientifically valuable or not, and so forth, does not concern us. A self-contradictory sentence asserts too much; it is too informative to be true ([3], p. 229).

Popper had also noticed that his notion of empirical content of a theory worked reasonably well only for *consistent* theories. For, *all* basic statements are potential falsifiers of all inconsistent theories, which would therefore, without this requirement, turn out to be the most scientific of all. So, for him, "the requirement of consistency plays a special rôle among the various requirements which a theoretical system, or an axiomatic system, must satisfy" and "can be regarded as the first of the requirements to be satisfied by every theoretical system, be it empirical or non-empirical" ([18], p. 72). So, "whilst tautologies, purely existential statements and other nonfalsifiable statements assert, as it were, *too little* about the class of possible basic statements, self-contradictory statements assert *too much*. From a self-contradictory statement, any statement whatsoever can be validly deduced" ([18], p. 71). In fact, what Popper claimed was that the information content of inconsistent theories is null, and so his definition of information content as monotonically related to the set of potential falsifiers was intended only for consistent ones:

> But the importance of the requirement of consistency will be appreciated if one realizes that a self-contradictory system is uninformative. It is so because any conclusion we please can be derived from it. Thus no statement is singled out, either as incompatible or as derivable, since all are derivable. A consistent system, on the other hand, divides the set of all possible statements into two: those which it contradicts and those with which it is compatible. (Among the latter are the conclusions which can be derived from it.) This is why consistency is the most general requirement for a system, whether empirical or non-empirical, if it is to be of any use at all ([18], p. 72).

The second paradox is concisely expressed in the following famous quotation from Cohen and Nagel:

> If in an inference the conclusion is not contained in the premises, it cannot be valid; and if the conclusion is not different from the premises, it is useless; but the conclusion cannot be contained in the premises and also possess novelty; hence inferences cannot be both valid and useful ([4], p. 173).

A few decades later Jaakko Hintikka described this paradox as a true "scandal of deduction":

> C.D. Broad has called the unsolved problems concerning induction a scandal of philosophy. It seems to me that in addition to this scandal of induction there is an equally disquieting scandal of deduction. Its urgency can be brought home to each of us by any clever freshman who asks, upon being told that deductive reasoning is "tautological" or "analytical" and that logical truths have no "empirical content" and cannot be used to make "factual assertions": in what other sense, then, does deductive reasoning give us new information? Is it not perfectly obvious there is some such sense, for what point would there otherwise be to logic and mathematics? ([15], p. 222).

The standard answer to this question has a strong psychologistic flavour. According to Hempel: "a mathematical theorem, such as the Pythagorean theorem in geometry, asserts nothing that is objectively or theoretically new as compared with the postulates from which it is derived, although its content may well be *psychologically new* in the sense that we were not aware of its being implicitly contained in the postulates" ([13], p. 9). This implies that there is no objective (non-psychological) sense in which deductive inference yield new information. This view was severely criticized by Jaakko Hintikka [14, 15]. As a consequence of the undecidability of first-order logic there is no algorithm to check whether the information carried by the conclusion is actually contained in the information carried by the premises. Hence:

> What realistic use can there be for measures of information which are such that we in principle cannot always know (and cannot have a method of finding out) how much information we possess? One of the purposes the concept of information is calculated to serve is surely to enable us to review what we know (have information about) and what we do not know. Such a review is in principle impossible, however, if our measures of information are non-recursive ([15], p. 228).

Hintikka's positive proposal consists in distinguishing between two objective and non-psychological notions of information content: "surface information", which may be increased by deductive reasoning, and "depth information" (equivalent to Bar-Hillel and Carnap's "semantic information"), which may not. While the latter is a sort of (non-computable) "potential information" and justifies the traditional claim that logical reasoning is tautological, the former is an effective notion and vindicates the intuition underlying the opposite claim. In his view, *first-order* deductive reasoning does not increase depth-information, but may increase surface information. Without going into details,[1] we observe here that Hintikka's proposal classifies as non-tautological only some inferences of the polyadic predicate calculus so leaves the "scandal of deduction" unsettled in the domain of propositional logic:

> The truths of propositional logic are [...] tautologies, they do not carry any new information. Similarly, it is easily seen that in the logically valid inferences

[1]For a critical exposition of Hintikka's approach see (21).

> of propositional logic the information carried by the conclusion is smaller or at most equal to the information carried by the premisses. The term "tautology" thus characterizes very aptly the truths and inferences of propositional logic. One reason for its one-time appeal to philosophers was undoubtedly its success in this limited area" ([15], p. 154).

Hence, in Hintikka's view, for every finite set of Boolean sentences Γ and every Boolean sentence φ:

> If $\Gamma \vdash \varphi$, the information carried by φ is included in the information carried by Γ.

This is highly unsatisfactory, especially since the theory of computational complexity has revealed that the decision problem for Boolean logic is co-NP-complete [5], that is, among the hardest problems in co-NP. Although not a proved theorem, it is a widely accepted conjecture that Boolean logic is *practically undecidable*, i.e., admits of no feasible decision procedure.[1]

Thus, some degree of uncertainty about whether or not a certain conclusion follows from given premises cannot be, in general, completely eliminated even in the restricted and "simple" domain of propositional logic. If we take seriously the time-honoured and common-sense concept of information, according to which information consists in reducing uncertainty, we should conclude that in some cases deductive reasoning *does* reduce our uncertainty, and therefore increases our information, even at the propositional level. The scandal of deduction has recently received renewed attention leading to a number of original contributions (e.g., ([19], Ch. 2), and [21, 22, 7, 9, 16, 1].

In this paper we elaborate on ideas put forward in [7, 8, 6] and propose a new more realistic theory of semantic information that classifies as tautological only a very restricted class of propositional inferences and (in one of its versions) partially complies with Popper's view on inconsistency, in that it implies that the information content of an *explicitly inconsistent* theory is null. In our view, the scandal of deduction and the Bar-Hillel-Carnap Paradox are nothing but symptoms of a fundamental difficulty. This can be described as *the mismatch between the central semantic notions in terms of which the (classical) meaning of the logical operators is defined and the ordinary notion of information.* The classical meaning of the logical operators, as defined by the standard truth-tables, is specified in terms of alethic notions of truth and falsity that are obviously information-transcendent. But this is the meaning in terms of which the notion of semantic information is defined. What we need is a meaning-theory whose central semantic notions are *themselves* of an informational nature.

In this vein, we depart from classical semantics and investigate an *informational meaning* of the logical operators whereby the meaning of a complex sentence for an

[1] This means that every decision procedure for Boolean logic is bound to be superpolynomial in the *worst case*. On the other hand, there are very efficient decision algorithms around that work quite efficiently on average. In [10] Finger and Reis present a very interesting empirical analysis of the runtime distribution of a variety of decision methods on randomly generated formulas.

arbitrary agent a is not specified in terms of the alethic notions of truth and falsity, but solely in terms of the information that the agent *actually holds*. (For a general discussion of informational semantics see [1]).) We argue that this informational meaning is captured by a non-deterministic 3-valued semantics — whose values 1, 0 and $\perp$ (undefined) can be naturally described as "yes", "no" and "I don't know" — that was partially anticipated by W.v.O. Quine in his *The Roots of Reference* [20] and is essentially different from Kleene's (deterministic) 3-valued semantics. We show how this semantics allows us to define a notion of *actual information* which is not only effective, but also *tractable*. Tautological inferences, in the strict sense of this semantics, are only those that do not increase actual information. Informative inferences are those that essentially require the introduction and manipulation of *virtual information*, namely information that is not actually contained (even implicitly) in the premises or in the conclusion. The depth at which the nested use of virtual information is required to justify the validity of an inference provides a useful and natural measure of informativity for a propositional inference. For every natural k, and every sentence φ we characterize the k-depth information content $\mathrm{INF}_k^{\Delta}(\varphi)$ carried by φ with respect to a fixed domain Δ of formulae syntactically related to φ and including at least all its subformulae. More precisely, we offer two different characterizations.

First, we define $\mathrm{INF}_k^{\Delta}(\varphi)$ as the set of k-depth *information states* about Δ (*not* possible worlds) which are ruled out by φ according to the 3-valued non-deterministic semantics. By this definition, and for *every* given Δ, Bar-Hillel and Carnap's semantic information is shown to be the limit of INF_k^{Δ} as k approaches infinity. By contrast, for every fixed k, there are valid propositional inferences that do *increase* k-depth information. This notion of k-depth semantic information solves the "scandal of deduction" for propositional logic, but does not solve the Bar-Hillel and Carnap Paradox, for all k-depth information states are ruled out by a k-depth inconsistency. On the other hand, Floridi's approach to this paradox, based on the "veridicality thesis" according to which "information encapsulates truth" ([11] and [12], chapters 4–5), is no use in this context in which the information we are dealing with is *weakly* semantic (no reference to the alethic notions of truth and falsity). Therefore, we investigate a second approach that defines $\mathrm{INF}_k^{\Delta}(\varphi)$ as the smallest k-depth information state over Δ that verifies φ. When there is no k-depth information state that verifies φ, that is, φ is k-depth inconsistent, $\mathrm{INF}_k^{\Delta}(\varphi)$ is undefined. (This situation is conceptually distinct from that in which φ is a k-depth tautology and its information content is the *empty* information state, namely the information state in which all the formulas in Δ are undefined.) Finally, we discuss quantitative measures of information that can be possibly associated with these qualitative notions.

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A quantitative-informational approach to logical consequence

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We propose a definition of logical consequence based on the quantity of information present in the set of premises and in the conclusion. As a starting point, we use the usual languages of classical propositional logic (CPL). Our approach is based on the quantitative concept of information, developed in the Mathematical Theory of Communication. In this sense, information is a measure of one's freedom of choice when one selects a message. The *quantity of information of the i-th message* of a source F, denoted $I_{i(F)}$, is the numerical value defined by: $I_{i(F)} =_{df} -log_2 p_{i(F)}$, where "$p_{i(F)}$" denotes the probability of occurrence of the i-th message of F. The quantity of information in a source F with n elements, denoted by H_F, is defined by: $H_F =_{df} \Sigma_{i=1}^{n} p_{i(F)} I_{i(F)}$. Thus, if H_C and H_D denote the quantities of information in the throw of the coin and dice, respectively, then $H_C = 1$ and $H_D \approx 2.58$. For developed our approach, initially, we consider some elements of a usual axiomatic theory of probabilities, indicating some of its definitions and basic results, which will be used later; these concepts include the notions of random experiment sample space, and event. A *random experiment* is one that, repeated various times, presents different results or occurrences; the *sample space* of a random experiment Σ is the set of all possible results of Σ; an *event* of a random experiment Σ is any subset of the sample space of Σ. We define the probabilistic value of an event as usual in literature. Then, we construct a probabilistic semantics for CPL; we establish a functional relationship, named *situation*, which consists of an association between the formulae of a CPL language and the events of a random experiment, from which we define a probability value for each formula of a given language. The probabilistic value of a formula in a situation $f(\Sigma)$ is a numerical value defined from probabilistic value of the event associated to that formula in $f(\Sigma)$. Next we introduce the definition of probabilistic logical consequence: a formula φ is *probabilistic logical consequence* of a set Γ of formulae, which is denoted by "$\Gamma \models_{\mathbf{P}} \varphi$", if, for every situation $f(\Sigma)$, $P(f(\Sigma))(\Gamma) \leq P(f(\Sigma))(\varphi)$, where $P(f(\Sigma))(\Gamma)$ denotes the probabilistic value of Γ in $f(\Sigma)$. In the next step we discuss the notion of quantity of information present in a formula of a CPL language, such as defined for messages above. The quantity of information of a formula in a situation $f(\Sigma)$ is a numerical value that depends of the informational value of the event associated to that formula in $f(\Sigma)$. Finally, we propose a quantitative-informational definition of logical consequence, which we call informational logical consequence: a formula φ is *informational logical consequence* of a set Γ of sentences, which is denoted by "$\Gamma \Vert\!\models \varphi$", if, for every situation $f(\Sigma)$, $I(f(\Sigma))(\Gamma) \geq I(f(\Sigma))(\varphi)$, where "$I(f(\Sigma))(\Gamma)$" denotes the informational value of Γ in $f(\Sigma)$. We demonstrate some of the results and properties that follow from that definition of informational logical consequence. In particular, we show the existence of arguments which are considered valid according to the classical perspective, true-functional, but which are invalid from the informational perspective. For example, *modus ponens* is informationally invalid, given the possibility that the conclusion of this argument could possess a greater quantity of information than its set of premises. Furthermore, we show that the logic underlying informational logical consequence is not classical, but is, at the least, paraconsistent *sensu lato*. In addition, we demonstrate that although it might satisfy the property of transitivity, informational logical consequence is neither reflexive nor monotonic; in other words, it is not a Tarskian logical consequence.

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Logic Informed

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Do logically valid arguments necessarily preserve truth? Certain inferences involving informational modal operators and indicative conditionals suggest that truth preservation and good deductive argument come apart. Given this split, I recommend an alternative to the standard truth preservation view of logic on which validity and good deductive argument coincide: logic is a descriptive science that is fundamentally concerned not with the preservation of truth, but with the preservation of structural features of information.

Types of Information Pluralism

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In this paper we consider the relationship between various kinds of pluralism (alethic, metaphysical and logical) and their relationship to informational pluralism. In particular, we focus in [1] on this topic. We argue that Allo is committed to an entailment between Informational Pluralism (*IP*) and a particular kind of Logical Pluralism (*LP*). This being the Logical Pluralism of Beall and Restall. We suggest that while a pluralism about semantic information (Semantic Information Pluralism (*SIP*)) is sensible, to base this on Beall and Restall's LP is a mistake, as it commits us to an untenable metaphysics for semantic information. For this reason we consider other motivations for IP, and settle upon a contextualist account of informational content, which retains the core elements of Allo's account but drops the commitment to 'cases' and a fixed schema (GIRP). What form this contextualism will take will be discussed at the end of the paper.

The Logical Pluralism of Beall and Restall suggests that our pre-theoretic Logical Consequence relation is somehow imprecise or vague (or indeterminate) — it is unable to be captured by a single logic. Rather, while there is a core component of our pre-theoretical notion of logical consequence, the Logical Consequence relation is somehow imprecise or vague (or indeterminate) – it is unable to be captured by a single logic. While we can define the core of validity as follows:

GTT An argument is valid$_x$ if, and only if, in every case$_x$ in which the premises are true so is the conclusion.

This is not a full account of validity. In addition, we must also specify *which* cases we are referring to (possible worlds, situations, constructions etc.). Each legitimate specification of a case will result in an equally correct 'precisification' of our pre-theoretic notion of 'Logical Consequence'. As there is a plurality of distinct types of legitimate case that complete this schema, we therefore find ourselves with a plurality of logics.

In his paper, Allo "aims to show that logical pluralism finds a natural and useful application in a theory of semantic information". Moreover, a pluralism about logic *entails* a pluralism about semantic information. Hence, we find ourselves with a schema akin to that of Beall and Restall's (GTT), the Generalised Inverse Relationship Principle:

GIRP The informational content of a piece of information is given by the set of cases it excludes.

The principle as stated above is incomplete. In the same way that there are multiple logics, each with a different specification of cases, so too are there many formal accounts of Informational Content, each resting upon a different specification of cases. Thus, there is a plurality of informational content, and so too, a plurality of semantic information.

We suggest that this is a mistake. Underpinning Beall and Restall's Logical Pluralism is independent argument for the imprecise or vague (or indeterminate) nature of pre-theoretic logical consequence. It is this imprecision or vagueness which *forces* us to adopt a schema-plus-cases approach to representing the pluralism. The (GTT) schema is apt just because it captures the *determinate* features of validity, and leaves room for different cases which in turn offer inadequate, but precise completions of the schema. While this is plausible in the case of logic, we suggest that this approach breaks down when it is applied to informational content. Informational content is simply not imprecise or vague (or indeterminate) in the same way as validity or logical consequence. As such, any pluralism discovered in semantic information must have an alternative metaphysical foundation. Moreover, we should not be surprised to find that this pluralism will not be best outlined using the schema-plus-cases approach of Beall and Restall.

For this reason, we then turn to consider a number of alternative sources of pluralism for semantic information. To do this, we follow Allo's lead, and turn to consider pluralisms in the related areas of logic and truth.

One alternative to Allo's proposal is to adopt Pedersen's approach he presents in [4], where he argues for a close fit between the varieties of logical, alethic, and metaphysical pluralism. We argue that if (IP) entails (LP) then he is committed to Metaphysical Pluralism (MP).

Pedersen's claims that alethic pluralism understood as different ways of being true and his logical pluralism as different ways of being valid. Pedersen argues for an intimate connection between Alethic Pluralism (AP), (LP), and (MP). We suggest that Pedersen's pluralism provides a more plausible framework underpinning Allo's (SIP).

Pedersen's variety of metaphysical pluralism is not the only option open to Allo. An alternative is Price's metaphysical pluralism. We explore whether Price's metaphysical pluralism is compatible with Allo's information pluralism, with a focus on his concern with naturalistic theory of language use ([5], p. 399), and the relationship between truth and factuality.

Thence, on the basis of the commitment to (LP), (IP), and (MP), and Pedersen's distinction between domains of information and domains of reality, we argue that an informational pluralism akin to the logical pluralism of Beall and Restall is inconsistent with the fact that their Logical Pluralism is context invariant in the respects that matter. However, if the information pluralist adopts a context-variance account, then this inconsistency can be avoided.

If we wish to commit to (SIP), in the manner outlined in Allo's article, we suggest that there is an alternative — one that avoids commitment to either the metaphysical assumptions of Beall and Restall, and the controversial relativism of Pedersen. We can dub this a kind of 'methodological pluralism' and this is in the spirit of Floridi's invocation of levels of abstraction. In brief; it is not the real entities that our formal tools are representing that entail a pluralism. Rather, it is the way we go about their representation – different degrees of abstraction garner different perspectives on the same phenomenon. We finish with a brief consideration of how the resultant formulation of IP coheres with the claim that information cannot be defined.

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Procedural theory of analytic information

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The classical theory of semantic information (*ESI*), as formulated by Bar-Hillel and Carnap in 1952, does not give a satisfactory account of the problem of what information, if any, analytically and/or logically true sentences have to offer. According to *ESI*, analytically true sentences lack informational content, and any two analytically equivalent sentences convey the same piece of information. Does this mean that they are equally informative? Moreover, mathematical sentences are true or false independently of worlds and times. Does this mean that mathematics is not informative? *ESI* predicts them to be. So much the worse for ESI as a general theory of information, as many others, for instance Allo (2007) and Sequoiah-Grayson (2006) have pointed out.

These problems are connected with Cohen and Nagel's *paradox of inference.* This paradox arises because of the tension between (a) the *validity* (legitimacy) of an inference, and (b) the *utility* of an inference. One can reformulate the problem posed by the paradox thus: How can (deductive) logic function as a useful epistemological tool? For an inference to be legitimate, the recognition of the premises as true must already have accomplished what is needed for the recognition of the truth of the conclusion; but if the conclusion is to be useful the recognition of its truth should not take place already when the truth of the premises is ascertained. The conclusion of a valid argument is true in a superset of the set of possible worlds in which all the premises are true. Equivalently, the set of possible worlds excluded by the conclusion is a subset of the set of possible worlds excluded by the premises. In this sense it is true that the *empirical semantic content* of the conclusion of a valid argument is *contained* in the premises, which explains why we do not gain any novel piece of *empirical information* by validly inferring the conclusion of a sound argument.

The paradox of inference is an instance of the broader problem of the usefulness of analytically true sentences, because every deductively valid argument with premises $P_1, \ldots, P_n$ and conclusion P corresponds to an analytically true conditional sentence of the form, if P_1 and ... and P_n, then P. The narrow aim of this paper is to offer a principled solution to the paradox of inference. The challenge is to explain how the validity and the utility of a deductive argument do not cancel one another out. I will present a solution based on a distinction between two kinds of information: *empirical (factual)* and *analytic (procedural).* The broad scope of this paper, however, is to offer a no less principled account of why analytic information is far from being trivial. This is to say that the semantic framework within which the paradox of inference is solved is not tailored to that paradox only: the framework is of a much wider scope than that. The scope extends to proposing criteria for comparing the yield of analytic information of analytically true sentences and of equivalent sentences involving empirical expressions.

My starting-point is the characterization of information as being objective and semantic, as found in Floridi (2004). I am going to investigate information that encapsulates its *truthfulness*, and is independent of any informee. Thus I adopt a *realist* view of meaning and information. In particular, I am not going to deal with the counterpart of the 'scandal of deduction' put forward in Hintikka (1970), which is Bar-Hillel-Carnap's paradox of contradictory sentences. Since a contradictory sentence denotes a proposition that excludes *all* possible worlds, it should be the most informative one possible. Yet, since knowledge presupposes truth, I presume that the sentence has to be true in order to provide useful information.

Declarative sentences are informative due to their meanings. I construe meanings as structured hyperintensions, modeled in my background theory Transparent Intensional Logic as so-called constructions. These are abstract, algorithmically structured procedures whose constituents are executed subprocedures. My main thesis is that procedures are the vehicles of information. Hence, although analytically true sentences provide no empirical information about the state of the world, they convey analytic procedural information, in the shape of constructions prescribing how to arrive at the truths in question. Moreover, even though analytically equivalent sentences have equal empirical content, their analytic content may be different. Finally, though the empirical content of the conclusion of a valid argument is contained in the premises, its analytic content may be different from the analytic content of the premises and thus convey a new piece of information.

The paradox of inference arises if only the propositions, or truth-values (in the case of mathematical arguments), denoted by the premises and conclusion are considered, and evaporates if considering instead the procedures expressed by the premises and the conclusion. I will show what one learns when validly inferring a conclusion from true premises. While the product of the procedure assigned to the conclusion as its meaning is informationally contained in the premises, the procedure itself need not be (namely, whenever the argument is non-circular). Provided it is not so contained, then what learnt when drawing the conclusion is a new procedure producing the relevant proposition/truth-value. This procedural approach also maps out how to solve the broader problems why and how analytically true sentences are informative, and why and how analytically equivalent sentences can yield different information. Not only that; based on the notion of (literal) procedural meaning, it is smooth sailing defining the difference between the analytic and the logical validity of sentences and arguments, and specifying the analytic content of sentences. Given the added complexity involved in the fine-grained individuation of analytic information, it is sensible to further investigate the informational aspects of algorithmically structured procedural meanings, and the aspects that affect the epistemic utility of valid arguments. I will examine a criterion for comparing information yield based on the notion of meaning refinement and a criterion based on the set-theoretic inclusion of analytic contents. Yet, other criteria involving analytic content remain still an open issue. The complexity of the work going into building such a procedural theory of information is almost certain to guarantee that complications we are currently unaware of will crop up. A sensible approach will be to further develop the theory by including provability logic and the theory of complexity.

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Depth-bounded Probability Logic: A preliminary investigation

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Probability is traditionally the tool of choice for the quantification of uncertainty.

Yet, over the past few decades, a number of arguments have been put forward to the effect that probability fails to capture intuitively reasonable patterns of belief. One such argument, which has recently been revived, is due to Ellsberg, who in turn reworks previous examples by Keynes and Knight. The gist of the argument is the observation that whilst probability is suitable for the quantification of uncertainty (i.e. lack of knowledge), it fails to represent *ignorance* (i.e. lack of reliable information on which one should base their probabilistic assessment).

Unfortunately however, the contribution of this research strand to the foundations of uncertain reasoning is heavily hindered by the lack of rigour which characterises the key part of the criticism. In particular, it is often said that in cases of interest agents do not possess "enough information" to define a probability for their decision problem. Whilst this conveys interesting intuitions, it is certainly far from being clear enough for theoretical purposes.

In response to this, the first goal of our investigation is to put the question concerning the adequacy of probabilistic reasoning on a rigorous logical footing. Not only logic provides a suitably precise language to achieve this, but also provides us with a sharp diagnostic tool for identifying the root of the inadequacy of the probability norm. To grasp the idea, recall that a probability function over the (finite) propositional language $\mathcal{L}$ is a map $P : \mathcal{SL} \to [0,1]$ satisfying:

(P1) if $\models \theta$ then $P(\theta) = 1$,

(P2) if $\models \neg(\theta \wedge \phi)$ then $P(\theta \vee \phi) = P(\theta) + P(\phi)$,

where $\mathcal{SL}$ is the set of sentences built from $\mathcal{L}$ as usual, $\models \subseteq \mathcal{SL}^2$ is the classical notion of consequence and $\theta, \phi \in \mathcal{SL}$. It is apparent that probability is constrained by logical consequence, and in particular, tautologies must get probability 1. This conflicts with the basic results in computational complexity, and in particular with the general

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intractability of the SAT problem. This means that probability (via **P1**) imposes to agent i a requirement that i may not be in a position to actually satisfy. If our concern is to quantify uncertainty for decision-making purposes this is a most serious drawback.

This diagnosis, however, suggests promptly that we can insist on the "rational belief" component of probability whilst adapting the underlying consequence relation to the inferences we can expect realistic agents to be able to do in practice. A natural candidate for the formal counterpart of the intuitive idea of *realistic agents*, is to consider a plurality of consequence relations $\models_1, \models_2, \ldots$ each representing the inferential ability of one particular *type of agent*, namely the type of agent which is characterised by the relation $\models_k$. This matches the idea that in the real world, not all agents are equally capable when it comes to making inferences.

Over the past years D'Agostino and co-authors have put forward the framework of depth-bounded Boolean logics to capture precisely this intuition. The idea is to define a hierarchy of consequence relations which approximate, and asymptotically coincide with, classical logic. The key property of interest is that each element in the hierarchy is polynomial, with complexity increasing at each subsequent step in the hierarchy.

The goal of our investigation then is to flesh out the framework of depth-bounded probability logic by identifying a hierarchy of "rationality conditions" that we impose on the belief measure adopted by an agent whose reasoning is bounded by a given $\models_k$.

20 Years of Inconsistent Mathematics

This workshop is organized by

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2015 marks the 20th anniversary of the publication of Chris Mortensen's book *Inconsistent Mathematics*. *Inconsistent Mathematics* has been a very important and influential book, and contributed to the most recent wave of systematic studies on inconsistent mathematics in particular and non-classical mathematics in general.

Besides his technical contribution in studying and developing several inconsistent mathematical theories, Mortensen contributed to the philosophy of mathematics in several ways, for example by advancing arguments for the idea that mathematicality lies deeper than consistency, completeness or primeness, and he favored (at least partial) functionality as closer to the essence of mathematics.

Moreover, 2015 marks the 70th birthday of Chris Mortensen, which makes 2015 doubly significative and a great opportunity to discuss in this workshop Mortensen's main contributions to logic, mathematics and philosophy.

The invited keynote speaker is this workshop is Maarten M^{c}Kubre-Jordens (102).

Call for papers

Any contribution on inconsistent mathematics, whether on their technical or foundational aspects, are welcome. Particular topics of interest include, but are not limited to, the following:

- Technical work on inconsistent arithmetics, paraconsistent set theories, inconsistent calculi of infinitesimals, paraconsistent categorial logic, inconsistent geometry, and other branches of inconsistent mathematics.
- Relative interpretability between inconsistent mathematical theories and classical or other non-classical mathematical theories; in general, comparisons between inconsistent mathematics and other kinds of mathematics.
- Metamathematics of inconsistent mathematics.
- Combinations of inconsistent and other kinds of mathematics.
- Paraconsistent approaches to foundational, ontological, and epistemological problems in the philosophy of mathematics.
- Scientific, technological, and philosophical applications of inconsistent mathematics.
- Chris Mortensen's contributions to formal philosophy.

Abstracts of about 500 words should be sent via e-mail as a pdf file no later than January 31th 2015 to Inconsistent.Maths.UNILOG2015@gmail.com.

Authors will be notified by February 28th.

Round Table: Past, Present and Future of Inconsistent Mathematics

Coordinator: Thomas Macaulay Ferguson

Participants: Diderik Batens, Jc Beall, Ross T. Brady, Itala M. Loffredo D'Ottaviano and Graham Priest

On the limitations of naïve set theory with non-classical logics

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Morgan Thomas, in [1], has recently shown that a family of naïve set theories based on LP and other related logics suffer from serious expressive limitations. At first sight these results could seem a serious drawback for the prospects of inconsistent mathematics, since these naïve set theories either lack even the most elementary concepts needed to express basic notions of classical mathematics or are nearly trivial. Thomas takes care not to draw such strong conclusions on the possibility of inconsistent mathematics, but some readers might still be tempted. I will argue that these results suggest instead a different, non-foundation role of set theory in non-classical mathematics.

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Hermeneutical and genetic-epistemological glances at paraconsistent category theory

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There are various ways of building paraconsistent logics. Dualizations of intuitionistic logical systems is a now well-known strategy to achieve this goal. For several reasons, spelling out this process in category theoretical terms seems to be one of the most common ways to go. Nevertheless, it is not quite clear how enlightening or epistemologically fruitful this technique is.

Under what constraints shall we proceed for a given case and how might be the best way to interpret the system obtained and the process itself? Does this shed light on the purported foundational role of categorial logics or some sort of broad mathematical pluralism?

To address these issues I will try to assess an example of dualization following Russian philosopher Andrei Rodin's "hermeneutic" insights about the differences between the formal and categorial methods of theory building and drawing on Jean Piaget's epistemological reconstruction of Category Theory.

Frege's Puzzle

This workshop is organized by

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The purpose of this workshop is to bring together people working on several aspects and implications of the so-called *Frege's Puzzle* (i.e., the puzzle about cognitive differences between true identity statements that Frege classically presents as the main motivation for the recognition of senses as semantic values besides references). There are problems of all sorts both in the formulation of the puzzle and in Frege's solution of the puzzle. But if Frege's solution is rejected (i.e., if we eliminate senses) then a new problem arises for Millianism in explaining cognitive differences between co-referential expressions. The workshop should discuss all these many problematic aspects and the main alternative solutions that have been proposed.

The invited keynote speaker of this workshop is Eros Corazza (page 86).

Frege's Puzzle: Much Ado about Nothing?

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In her paper "Can Frege Pose Frege's Puzzle", Stavroula Glezakos argues for the claim that, unless one presupposes the theoretical notion of sense, there is no in-principle epistemic divide between sentences of the form "$a = a$" and "$a = b$". It would follow that Frege's puzzle cannot be used to argue in favour of senses, as Frege has done, on pain of circularity. Here I argue that a criterion of name identity based on the notion of explicit co-reference can be specified that does not presuppose the notion of sense. I show how such criterion is plausibly implicitly at work in setting up the puzzle, and that it can be deployed to rescue Frege from the accusation of circularity.

Frege's Puzzle: Can we Pose it on Frege's behalf?

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In this paper I first review the main elements of the so-called Frege's Puzzle, and argue that there is something odd in the argument that Frege builds based on it: Frege first rejects a possible hypothesis for the nature of identity in order to make plausible the distinction between sense and reference. But, after the distinction is made, the rejected hypothesis is the only one compatible with it after all. Next, I discuss Glezakos' (2009) position regarding the Puzzle. I argue that, although she does point out something quite important, we do not have to accept her conclusion that there is no puzzle that can be formulated in neutral terms.

In What Sense (Statement) of the Puzzle is Problematic?

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In this paper I take issue with Glezakos's (2009) account of why Frege's puzzle is unpuzzling. On her view, Frege's statement of the puzzle — how can sentences of the form $a = a$ and $a = b$, if true, differ in cognitive value if they express the same semantic content/are made true by the same object's self-identity? — should not be considered any puzzling either because it is question-begging, or because, suitably posed, it does not even arise in the first place. I argue that if, as she takes it, Frege's statement is "problematic" it is not because it is on the whole question-begging, but because it rests upon a couple of unsupported assumptions; the assumptions that i) there is no type-token ambiguity on the proper way of stating the puzzle, and ii) it is of the (sentence) forms themselves one may sensibly say they differ in cognitive value.

Wettstein on Frege's Puzzle

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In this talk I focus my attention on the proposal given by Howard Wettstein in 1980 to the cognitive phenomenon stated by Gottlob Frege in his paper "On sense and reference". I offer three arguments in order to show that his answer does not resolve this phenomenon. Particularly, I defend three ideas: first, it is legitimate that philosophical semantics, in contrast with what Wettstein defends, provides an answer to the cognitive phenomenon; second, Wettstein does not conceive Frege's argument correctly by considering it generates a semantic theory from a purely cognitive phenomenon; third, the dissolution supplied by Wettstein is assumed by Frege when he states the phenomenon.

9 – Sessions

Universal

The invited keynote speaker of this session is Melvin Fitting (page 90).

Homotopy theoretical aspects of abstract logic

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Categories whose objects are logics and whose morphisms are translations typically come with one or several natural notions of when a translation should be called an equivalence. This datum of a category with a distinguished class of morphisms, also called a *homotopical category*, is all one needs to form an $(\infty, 1)$-category and import a wealth of notions and techniques from homotopy theory.

We will start the talk by presenting the homotopy theoretical point of view on Tarski style logics. On a category of Tarski style logics and translations there are two natural notions of weak equivalence:

Definition. One can call a translation $f : (S, \vdash) \to (S', \vdash')$ a *weak equivalence*, if $\Gamma \vdash \varphi \Leftrightarrow \hat{f}(\Gamma) \vdash \hat{f}(\varphi)$ (i.e. it is a "conservative translation") and if for every formula φ in the target there exists a formula in the image of $\hat{f}$ which is logically equivalent to φ (it has "dense image").

Definition. One can call a translation $f\colon (S, \vdash) \to (S', \vdash')$ an *equivalence*, if there exists a translation $g\colon (S', \vdash') \to (S, \vdash)$ such that $\varphi \dashv\vdash f(g(\varphi))$ and $\psi \dashv'\vdash' g(f(\psi))$.

We will the introduce the notions of homotopical categories and $(\infty, 1)$-categories and show that taking any of the above notions of equivalence, the corresponding $(\infty, 1)$-category is complete and cocomplete and that one is a reflexive sub-$(\infty, 1)$-category of the other. The $(\infty, 1)$-category of logics and weak equivalences is 0-truncated. By a result of Mariano/Mendes [2] the subcategory of congruential logics is locally presentable in the sense of $(\infty, 1)$-categories.

We will then present a variety of approaches to, and perspectives on, abstract logic suggested by the homotopy theoretical view point:

A fixed $(\infty, 1)$-category of logics can be presented by different models, e.g. different categories with classes of weak equivalences, and possibly additional structures. There are several natural candidates for models of Tarskian logics.

The fact that the categories presented before are 0-truncated is due to the fact that we only asked for provability and did not distinguish different proofs. Once we do this, we get a refined structure of an $(\infty, 2)$-category. Further refinements to (∞, n)-categories might be attained by considering relations between proofs, e.g. via type theory.

One can further try to devise homotopical invariants of logics and try to compare the various approaches to abstract logic through Tannaka theoretic or other $(\infty, 1)$-categorical view points.

Most of this talk will cover selected topics from the article [1], additionally we will answer some questions posed in this article.

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Applied Ontology, Logical Pluralism and the Logical Constants

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We discuss the interplay between applied ontology and philosophical logic in the study of constants in logical pluralism. Logical pluralism and the role and meaning of logical constants have been discussed in recent years from many philosophical angles. We propose to pursue a different line of analysis, where logical constants are selected and motivated in the light of their contribution to ontological needs as clarified in Applied Ontology (AO). This leads to some further considerations on logical pluralism.

Generally speaking, AO aims to make clear the assumptions on which a modelling approach or the interpretation of a collection of data rely. On the one hand, AO focuses on frameworks to represent, ontologically analyse, and logically reason about (possibly complex) systems from a given perspective (task or application domain). On the other hand, AO focuses on frameworks that an agent uses to understand what is and what is not to be represented. This latter perspective, called foundational, aims to be general

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and independent from tasks and domains. AO is strongly intertwined with logic since it relies on logic to improve conceptual clarity, robustness of the analysis as well as consistency of the result.

Our starting observation is that the AO perspective and its methodology can be fruitfully applied to logic itself. In particular, we focus on the analysis, from the AO viewpoint, of the role of the logical constants in different logical systems. In this context, a certain view on logical pluralism follows from the adopted AO perspective. The ontological analysis of logical languages and of their use in ontological modelling leads to the identification of a framework that homogeneously motivates the co-existence of logics (pluralism) and guides the distinction between logical and non-logical constants.

Expanding non-classical logics

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We deal with connectives considered by some authors as Skolem, Moisil, Smetanich, Rauszer and Kuznetsov. We also consider a connective similar to the Delta connective used in fuzzy logic.

We mostly work in the context of intuitionistic logic. However, we also consider some substructural logics.

We work at the propositional, first order, and propositionally quantified level. In all those levels, we particularly consider whether the addition of a given connective constitutes a conservative expansion.

We consider both syntactic and semantic aspects. Syntactically, it is often not enough to add axioms, even if the corresponding class of algebras is a variety. A rule is sometimes necessary and there are two options: a local or a global rule. The behavior of the corresponding logics is different, for example, concerning the deduction theorem. From the semantic point of view, these two options correspond to two different ways of defining the notion of consequence: a truth-preserving or a truth-degree-preserving way, which, in general, do not coincide.

From a historical point of view, there will be some remarks concerning an old paper by Skolem.

Logical Structures as Formal Objects

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Bourbaki claimed that axiomatic method in mathematics leads to investigation of mathematical structures. One could define a *mathematical structure* as an ordered m-tuple $\mathfrak{A} = <A_1, \ldots, A_p; R_1, \ldots, R_q>$, where $m = p + q$, $A_1, \ldots, A_p$ are the *basic sets* of the structure $\mathfrak{A}$, and $R_1, \ldots, R_q$ are the *main relations* of $\mathfrak{A}$, which belongs as elements to some elements of the *scale of sets* $S = S(A_1, \ldots, A_p)$, i. e., the least set of sets such that 0) $A_1, \ldots, A_p \in S$; 1) if $M \in S$, then power-set $\mathcal{P}(M) \in S$; 2) if $M_1, \ldots, M_n \in S$, then $M_1 \times \ldots \times M_n \in S$ for any $n > 0$.

So every mathematical structure is a system of relations on some sets of initial objects. The examples are the models and the propositional algebra. But many logical systems are sets of rules rather than relations: the formal languages, calculi, algorithms, and formal systems are such systems. We could take a *rule* as a separate theoretic object, different from a relation, because some rules (such as algorithm commands and impelings from logical pragmatics) are not relations themselves but stimulae. The general form of a *rule, defined on a cartesian product* $M_1 \times \ldots \times M_n \times M_{n+1}$ is one of the following two: i) $A_1, \ldots, A_n \therefore B_1, \ldots, B_m$, ii) $F_1(a_1), \ldots, F_n(a_n) \therefore G_1(b_1), \ldots, G_m(b_m)$, where $A_i/a_i \in M_i$, $B_j/b_j \in M_{n+1} (i \leqslant n, j \leqslant m)$ and '$\therefore$' denotes the transition from *premisses* $A_1, \ldots, A_n$ or $F_1(a_1), \ldots, F_n(a_n)$ of the rule to one of its *conclusions* $B_1, \ldots, B_m$ or $G_1(b_1), \ldots, G_m(b_m)$.

Now we can define a *logical structure* as an ordered m-tuple $\mathfrak{L} = <A_1, \ldots, A_p; \mathfrak{r}_1, \ldots, \mathfrak{r}_q>$, where $m = p + q$, $A_1, \ldots, A_p$ are the basic sets of the structure $\mathfrak{L}$, and $\mathfrak{r}_1, \ldots, \mathfrak{r}_q$ are the *main rules* of $\mathfrak{L}$, which are defined on some elements of the scale of sets $S = S(A_1, \ldots, A_p)$. The most typical kind of a logical structure is a calculus. Let's define a *calculus* as a logical structure such that its every main rule is defined on some cartesian product of its certain basis' elements. Let $\varnothing$ be empty set, K be an arbitrary alphabet, and ϕ be a grammar, i. e., set of formation rules; then calculus $L = <\varnothing; \phi, K>$ is *a formal language* (here we take elements of K as 0-ary formation rules). Let then $\mathcal{A}$ be a set of axioms such that $\mathcal{A} \subseteq \phi(\mathrm{K})$, and $\mathfrak{R}^t$ be a set of transformation rules; then $\mathcal{C} = <\varnothing; \phi, \mathfrak{R}^t, K, \mathcal{A}>$ is a logical structure of that kind they usually call in logical syntax a calculus itself; we could name it *a deductive system*. Markovian normal and Turing's machine algorithms are also logical structures but not calculi in our general sense. Post's canonical calculi and formal systems in the sense of Curry and of Smullyan are calculi according to the above definition.

Logical consequence and measuring of semantic information via distributive normal forms

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In this presentation we will analyze different attempts to describe logical consequence via a theory of semantic information. As one of its central goals, a theory of semantic information has to provide the ways of measuring both the amount of information expressed in a sentence as well as the amount preserved in valid implications. In this presentation we are concerned with the works by Bar-Hillel and Carnap (1952, 1953) and Hintikka (1965, 1970) on the field. These authors suggest that the measure of semantic information of a sentence φ is a function of the informational measures of the clauses composing the distributive normal form of φ. Further, they define information of φ in terms of the probability of $\neg\varphi$. As we can define at least two different notion of probability, they suggest both the notions of depth and surface information, the former dealing with the "absolute" probability of the sentence and the latter with an epistemologically relativized notion of probability. Although these attempts are highly suggestive, we claim they need reformulation. Firstly, as far as we are concerned with the classical consequence relation, an adequate theory needs to be committed with Bar-Hillel-Carnap's "paradox" of semantic information. Furthermore, for an adequate description of the processes of achieving new information through proofs, an adequate theory needs to block the thesis that valid arguments have null information. None of those proposals jointly achieve both the expected results. In order to improve Bar-Hillel and Carnap's and Hintikka's proposals, we claim that an adequate informational characterization of logical consequence needs to look also for other logical properties associated with the distributive normal forms. Secondly, we claim that Rantala and Tselishchev's (1987) objection that Hintikka's surface information only applies to a specific method of deduction needs to be taken seriously.

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Logic as Universal Language and its Limit: Solution to the Problems of Quantification over Types and Orders in Type Theory

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Russell's doctrine that logic is all-encompassing (1903, 1910–13) faced a challenge when Russell met Russell's paradox and then Russell-Myhill paradox. Russell was thus led to impose serious restriction on his logic. He firstly exposed a sort of simple theory of types (1903) and, after abandoning it, he developed his ramified theory of types (RTT; 1908, 1910–13). The hierarchy of propositions and propositional functions in RTT was justified by Vicious Circle Principle (VCP) which says that no proposition or propositional function can be in the range of its own variables. Consequently, his logic ceased to be a genuine 'universal language'.

After proving his Undefinability theorem, a result of his considerations related to the Liar paradox, Tarski (1933/1956) suggested another hierarchy, that of languages and T-predicates. Tarski's abandonment of the idea of universal logical language became a part of the conventional wisdom of modern logic: we do use meta-languages, our logic is thus limited.

Recent philosophical logic attempts to dismiss hierarchies. For a clear exposition of a criticism of hierarchies see Kripke (1975), Priest (1987/2006). The criticism involves also an idea I am going to examine in this paper, namely that a type theorist should quantify over type-levels (i.e. types / orders). Some (false) method of quantification over type-levels has been already used for a criticism of RTT and its applications (cf., e.g., Priest 1987/2006, Hart 2009).

But the idea goes back to Gödel's review of Russell's logic (1944). Gödel in fact claimed that the very formulation of RTT (cf., e.g., Russell's 'No proposition or propositional function can quantify over the type it belongs to') violates its own rules – RTT is thus self-refuting. The very same point was elaborated by Fitch (1946) as a form of an '*ad hominem* argument'; Mostowski (1946) read it as an unconvincing attack on RTT, but Fitch (1947) did not confirm having such an aim. However, Fitch (1964) patently condemned Russell's and Tarski's hierarchies for their incapability to provide a logic which could serve as a universal language. Priest (1987/2006) elaborated such criticism further, rejecting all hierarchical approaches as expressively incomplete. He endeavours to avoid any meta-language (for a certain recent investigation see Weber 2014).

After providing an overview of the crucial motives in the debate, I offer solutions to the problems. For that purpose I utilize my modification of Tichý's RTT (1988) as a framework. The main problems are concerned with: i. the proper method (if any) of quantifying over orders, ii. the proper method (if any) of quantifying over types, iii. the method (if any) of quantifying over all entities, iv. formulation of RTT without violating its own rules, v. the sense in which RTT can be a universal logical language.

Finally, we briefly compare our results with similar attempts to solve some of the problems within type theories which are developed and utilized in computer science. This point is more important than it may seem: type theories are, *inter alia*, implemented in various theorem provers, which are contemporary realizations of the logical *calculus rationator* dreamt by Leibniz.

A Metalogical Exploration of Logical Structures and their Cognitive Relevance

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Logical systems are the expression of the consequences of logical structures. In such structures, some are order structures and others are algebraic structures. For example, classical propositional logic is the set of the consequences of an integration of the Boolean lattice (as an order structure on conditionals) and of the Boolean algebra (on disjunctions and conjunctions).

This paper holds that order structures make us order statements and so, are epistemically relevant for the expression of succession, inference, correlation or causality. On the other hand, logical algebras are tools that we use for representing systems of categories of entities. The psychology of reasoning has shown, in the last decades, that laypersons tend to make systematic fallacies on conditionals (namely, the affirmation of the consequent and the denial of the antecedent fallacies), treating conditionals ($\supset$) as biconditionals ($\equiv$), and on inclusive disjunctions ($\vee$), treating them as exclusive disjunctions ($\veebar$). We also hold that in inferences on incompatibilies ($|$), people tend also to make fallacies, treating them as exclusive disjunction on negated propositions ($\neg P \veebar \neg Q$). We are now, by the way, conducting experiments on such fallacies on incompatibilities. We have also already shown that all these previous fallacies can be modeled as invalid crushes of Klein groups. We will now show that such crushes imply an invalid use of the Boolean lattice, by an inversion of its irreversible order, and of the Boolean algebra, by a reduction of its tree-like structure to pairs of dichotomous oppositions. So, the learning of classical propositional logic, through its adequate use of the two Boolean structures, makes people cognitively more performant: the Boolean lattice allows the discovery of multiple antecedents in conditionals and so, of multiple possible causes to an effect; the Boolean algebra allows tree-like categorizations that avoid too simple oppositional dichotomous categorizations.

Given that conditionals, disjunctions and incompatibilities are all translatable one into the other, we show that the Boolean algebra and the Boolean lattice are each other representable by the other structure, so that all the previous fallacies can be viewed as various forms of reductions of the antisymmetric structure of classical logic to a symmetric structure. Doing so, the fallacies are cognitive shortcuts that put an overload of information in the premises of inferences.

The structural approach to logic is a metalogical standpoint that makes of non-classical logics variations on order structures or on algebraic structures. From this standpoint, our study of the relation between the Boolean structures, the fallacies as simplifications of these structures and their cognitive consequences, will allow us to explore some non-classical structures, in which some Boolean properties are not respected, like in non-reflexive or non-contrapositive order structures, or in non-idempotent or non-distributive algebras. Through this structural exploration, we can identify some of the cognitive functions of these non-classical structures, their possible fallacious use and some cognitive effects of these fallacies. From this structural approach, we draw important pedagogical consequences on the teaching of logic.

From Deductive Systems of Logic to Logic of Information

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Deductive systems of logic are built with the use of logical consequence, which in the axiomatic form introduced by Tarski in the early 1930's is a finite character closure operator on the set of sentential formulas associated with the underlying it algebraic form of logical calculus (e.g. sentential calculus). Axioms building the association depend on particular choice of the rules of inference. For instance, one of the additional axioms imposed on the consequence operator makes the subsets closed with respect to the consequence closure operator (Cn-closed subsets) also closed with respect to the rule of modus ponens. In the algebraic form this restricts the Cn-closed subsets to filters in the Boolean algebra generated by the logical operations.

Language, natural or formal, artificial is a natural context for the study of information, which was conceived in the analysis of communication. However, the concept of information becomes a powerful tool for interdisciplinary inquiry in multiple contexts when it is defined in a more general way without any restrictions associated with languages. The author in his earlier publications introduced and elaborated on a very general definition of information. Information was defined in terms of the categorical opposition of the one and many, as that which makes one out of many either by a selection or by a structure binding the variety into a whole. The definition was subsequently used to formulate a theory for the concept of information in terms of closure spaces, i.e. sets equipped with a closure operator on its subsets.

The fundamental role is played in this theory by the complete lattice of closed subsets of a set (the variety in which unity is identified). The level of (direct product) irreducibility of this lattice is associated with the level of integration of information [1]. The lattice of closed subsets defining an information system can be associated with the algebraic structure of syllogistic and therefore was considered a candidate for the logic of information in the algebraic sense under some restricting conditions on the closure operator [2]. However, this restriction imposed on the closure operator (existence of an orthocomplementation on the lattice of closed subsets) was quite strong, excluding some of most interesting instances of information systems.

In this paper a generalization of logic from its traditional linguistic context to arbitrary information systems is achieved by using an extension of Tarski's concept of consequence operator to the case when the place of the Boolean algebra structure generated by the logical functors of sentential calculus is taken by the complete lattice of closed subsets for the closure operator defining an information system. The fact whether this lattice admits an orthocomplementation or not is here irrelevant, therefore there is no restriction for information systems at all. The approach presented in this paper can be used to analyze the logic of computation in its traditional Turing machine understanding, as well as in its generalized formulations, for instance in terms of geometric constructions [3].

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Universal Logical Hermeneutics

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According to Bogusław Wolniewicz [2, p. 254–255] there are two kinds of hermeneutics: intuitive and logical one. Intuitive hermeneutics consists in just guessing what the author in question had in mind and tried to convey that to the reader. Such way of interpreting philosophical texts does not differ essentially from a philological interpretation of the texts of pure poetry. Another way of interpretation is represented by logical hermeneutics being the set of rules and of criteria to govern the logical interpretation of philosophical systems. Here the system means a set of pronouncements stemming either from a definite author or from a particular book and its unity consists in the fact that it is the expression of the beliefs either of one man or one group of men and is the collective work of a philosophical school on record.

The method of logical interpretation of a philosophical system S is aimed to its axiomatization, i.e. to such transformation of a system that it becomes one in which everything (except the axioms themselves) is semantically determinate and deductively complete. To make this we need firstly to point out a definite theory T dealing with the same subject-matter as the system in question and secondly to provide a definite rules of translation (a dictionary) to map the propositions of the systems in to formulas of the language $L(T)$ of the theory T and thus to eliminate the ambiguities present in the system S. Indeed, different theories may be chosen to that purpose and different rules of translation would be adopted. The crucial moment is that as an instrument for interpreting the system S a given theory T may be more appropriate than another T' and a particular set of rules better than another one.

Besides the many troubles on that way there is one more problem concerning the notion of the theory T. Any theory presupposes a logical system laying in its foundation and Wolniewicz by default supposes that it is a classical one. But from the history of philosophy one perfectly knows the troubles occurring while we trying to interpret contradictions in particular philosophical theories, e.g. in M.Heidegger's works. As is generally known such troubles we sometimes would overcome by employing paraconsistent logical systems. Hence, maybe it is worth in such cases employ (in the framework of logical hermeneutic interpretation) some theory T based on paraconsistent logic.

Moreover, it seems that in any particular case the question should arises concerning the exploitation of the particular logical system underlying the definite theory T which we choose for logical interpretation. Any logical system is a theory of some subject area and thus if we construe a philosophical theory trying to obtain its logical interpretation then we from the very beginning owe to take into account specificity of the universe of discourse. And before we start the process of logical interpretation it is worth to ponder over this issue as the first thing.

Anyway, taking into account that universal logic regards the common issues of logical systems it seems that we can speak of *universal logic hermeneutics* (modifying Wolniewicz's term of logical hermeneutics) when analyzing the aspects of the appropriateness of one or another logical system for the aims of logical interpretation of philosophical theories. This is especially important in case of logical hermeneutic evaluation of the same philosophical system because they may have different validity because two different interpretations be really of different hermeneutic value if our choice against the background of logical system will be wrong: it would simply lead to triviality.

Nevertheless, sometimes just the choice of non-classical logical system for interpretation of a philosophical theory gives us a side benefit. For example, according to Wolniewicz there are at least four parameters to be taken into account where the first is the relative size of the set A of the propositions interpreted to the whole set of the propositions of the system S. This parameter may be represented by the fraction A/S which is called the *reach* of an interpretation. An interpretation A_2 will be better than interpretation A_1 if it has a greater reach, i.e. if $A_1/S < A_2/S$. And if we will use for interpretation the paraconsistent system of da Costa paraconsistent first-order logic $C_1^=$ (cf. [1]) which, in essence, contains the classical first-order predicate calculus $C_0^=$, then a paraconsistent interpretation A_2 will be better than a classical interpretation A_1 because we will have $A_1/S < A_2/S$ due to the immediate inclusion of classical propositions into the set of paraconsistent propositions.

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Geometrical formulation of a class of consequence structures

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Keywords: *consequence structures, generalized closure structures, invariants, orbits.*

A *consequence structure* is a pair (X, Cn) such that Cn is an operation on $\wp(X)$, the power set of X. The *consequence operator* Cn satisfies the following conditions: (i) *inclusion*: $A \subset Cn(A)$; (ii) *idempotency*: $Cn(Cn(A)) = Cn(A)$ and (iii) *monotonicity*: if $A \subset B$, then $Cn(A) \subset Cn(B)$.

We consider a class of consequence structures such that the consequence operator satisfies also: (iv) *emptyness*: $Cn(\varnothing) = \varnothing$ and (v) *union property*: if $\{A_i\}_{i \in I}$ is a class of subsets of X, then $Cn(\bigcup_{i \in I} A_i) = \bigcup_{i \in I} Cn(A_i)$. Structures (X, Cn) of this kind will be called *generalized closure structures*.

Consider a group $(G, \cdot, e)$ acting on a set X. We denote by $g(x)$ the result of the action of $g \in G$ in $x \in X$. Therefore, we have $(g \cdot g')(x) = g(g'(x))$ and $e(x) = x$. If $x \in X$, we define the G-*orbit* of x as the set $G(x) = \{g(x) : g \in G\}$. If $A \subset X$, we define $G(A) = \{G(a) : a \in A\}$. We call A a G-*invariant* if $A = G(A)$. It is easy to see that $G(A)$ is G-invariant, and it is the least G-invariant that contains A. Notice that all G-invariants is an union of G-orbits. In fact:

$$G(A) = \bigcup_{a \in A} G(a).$$

Moreover, the set of G-orbits is a partition of X. We can see $G(A)$ as an operation on $\wp(X)$:

$$\hat{G} : \wp(X) \to \wp(X), A \mapsto \hat{G}(A) = G(A).$$

So, we have the following results.

Proposition. If a group $(G, \cdot, e)$ acts on a set X, then $(X, \hat{G})$ is a generalized closure structure.

We are going to prove the converse of the proposition above.

Proposition. If (X, Cn) is a generalized consequence structure, there exists a group $(G, \cdot, e)$ acting on X such that $Cn = \hat{G}$.

A surprising consequence of pluralism about logical consequence

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Generally, in case of a disagreement between two or more theorists about the correct analysis of a specific subject area, there seem to be at least three basic options: exactly one of the theorists is right, none of them is right, or — in some way or other — more than one of them are right. The philosophy of logic can be seen as an instance of this general case as each of these options has been defended regarding the factual plurality of logical systems. Following Susan Haack's typology ([2], p. 221), *logical monists* claim that there is just one correct system of logic, *instrumentalists* hold that there is no correct logic because the notion of extra-systematic correctness is inappropriate for logic, while, finally, according to *logical pluralists* there is more than one correct system of logic. Jc Beall's and Greg Restall's [1] recent formulation of logical pluralism in terms of pluralism about logical consequence (PLC) started a lively discussion about the scope, the viability, and the aims of this view. In my talk, I focus on an argument against PLC brought up by Graham Priest [3] and also discussed by Stephen Read [4].

Beall and Restall make room for different instances of "validity" resulting from different specifications of "cases" in their *Generalized Tarski Thesis* ([1], p. 29): "An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion." Given different classes of cases, an argument that is valid in one class of cases K_1 may fail to be so in a different class of cases K_2. The idea of Priest's argument against PLC (see [3], p. 203) is that there might be a case or situation s about which we are reasoning and that is both in K_1 and K_2. The question that arises is whether one should reason according to the notion of validity appropriate for K_1 or for K_2. Apparently, we cannot use both as there will be some inference $\alpha \vdash \beta$ valid in K_1 but not K_2. Now, Priest asks, "suppose that we know (or assume) α holds in s; are we, or are we not entitled to accept that β does?" ([3], p. 203) Read's [4] challenge takes the argument further: what if there is a genuine conflict between the notions of validity resulting from K_1 and K_2 insofar as $\alpha \vdash \beta$ is valid in K_1, while $\alpha \vdash \neg\beta$ is valid in K_2? In response to similar arguments, Beall and Restall [1] admit that, in some cases of this kind, they are committed to rejecting one of the logics in question (see, e.g., [1], p. 117). I argue that, given further assumptions explicitly made Beall and Restall themselves, Priest's and Read's arguments can be made more general. It seems that, from an epistemological point of view, PLC leads to the surprising consequence that one cannot be pluralist about logical consequence.

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Cognition

The invited keynote speaker is this session is Vinod Goel (page 92).

Theories of Questions and Contemporary Direct Democracy

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The aim of the talk is to analyze basic conceptions concerning the structure and classifications of questions formulated within institutions of direct democracy. Of course, in referenda of all kinds we deal exclusively with simple yes-no questions. However, these questions can also be divided in different types following various criteria, including, for instance, the intentions or aims of a given referendum (i.e. decision-making, public consultations, surveys), the structures of the questions asked, and their presuppositions. There are also cases in which numerous yes-no questions are posed in a single referendum. Obviously, such a situation requires a different interpretation of the answers.

Another problem that we will examine is the structure and status of answers. It is worth emphasizing that within the institutions of direct democracy, the questions posed can have four possible answers — yes, no, none of the above (NOTA) and/or a refusal to vote. To address this issue, the usefulness of certain non-reductionistic conceptions of questions elaborated by Polish School of Logic is briefly discussed. A few significant examples of questions and the interpretations of results from real referenda are presented in order to highlight some important issues concerning the institutions of modern democracy. Finally, the problem of the systematic reconstruction of presupposed civic competencies required in a model democratic society is briefly analyzed.

Logical Aspects of Computational Creativity in the Music Domain

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Human creativity can be found in many contexts of human life, i.e. we find creativity not only in science and art, but also in all sorts of daily problem solving. Nevertheless, computational approaches towards creativity are considered to be one of the hard problems in computer science and artificial intelligence. This presentation uses conceptual blending (Fauconnier and Turner, 2003) in order to show that many interesting aspects of creativity in music can be modeled using this psychologically motivated approach. Conceptual blending is a theory that merges two input domains in a non-trivial way in a blend space that inherits properties from both domains.

A computational approach of concept blending requires a formal foundation. We propose a computational variant of a further development of Gougen's interpretation of concept blending as a colimit construction (Goguen, 1999). Based on this framework we will examine the following two application scenarios in the music domain:

- Given two harmonic styles (idioms) of music in form of sets of cadences (or more general: chord progressions), blending the two styles yields a new harmonic style which allows the creative harmonization of melodies.
- Given a particular harmonic style (idiom) of music in form of a small set of cadences (or more general: chord progressions), blending chord progressions produces an extension of the given style that is novel and interesting.

For the underlying representation of chords and chord progressions a simple feature logic is used. This allows the computation of the blend space in a two-stage process (given two musical input styles): in order to compute Goguen's generic space (i.e. a generalization), Heuristic-Driven Theory Projection (HDTP) computes anti-instances of the input conceptualizations by restricted higher-order anti-unification (Schwering et al., 2009). Candidates for the generic space are used to compute in a second step a blend space via a colimit construction. The blended space combines features of the input spaces and results in a novel and interesting musical style.

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On the Role of Universal Logic in Concept Invention

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In this talk, we will discuss the role of universal logic in concept invention. We will in particular motivate conceptual blending theory as one seminal approach, originating in cognitive science, to model concept invention formally.

Conceptual blending has been employed very successfully to understand the process of concept invention, studied particularly within cognitive psychology and linguistics. However, despite this influential research, within computational creativity little effort has been devoted to fully formalise these ideas and to make them amenable to computational techniques. Unlike other combination techniques, *blending* aims at creatively generating (new) concepts on the basis of input theories whose domains are thematically distinct but whose specifications share structural similarity based on a relation of analogy, identified in a generic space, called base ontology.

The creative and imaginative aspects of blending are summarised by [1] as follows:

> [...]the two inputs have different (and often clashing) organising frames, and the blend has an organising frame that receives projections from each of those organising frames. The blend also has emergent structure on its own that cannot be found in any of the inputs. Sharp differences between the organising frames of the inputs offer the possibility of rich clashes. Far from blocking the construction of the network, such clashes offer challenges to the imagination. The resulting blends can turn out to be highly imaginative.

Our approach to concept invention via conceptual blending is inspired by methods rooted in cognitive science (e.g., analogical reasoning), ontological engineering, and algebraic specification. Specifically, we will introduce the basic formalisation of conceptual blending as given by [2] and discuss some of the aspects of universal logic that go into this framework, including a pluralistic approach to logic.

We will moreover illustrate how the distributed ontology language DOL (see [5] for a sketch of the language and [4] for the theoretical background) can be used to declaratively specify blending diagrams and to compute colimits as a basis for novel concepts. Finally, we will illustrate the potential of formalised conceptual blending for computational creativity as outlined in [3].

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Toward Logical Models of Understanding: Axiology, Logical Pragmatics, Pragmatic Logics, What Else?

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Savoir c'est bien. Comprendre, c'est encore mieux. (P. Langevin)

Understanding is a trans-disciplinary area (understanding of language expressions, understanding of behavior, understanding of situation) that deals with cognitive processes concerning the assimilation of new content and its inclusion into the system of existing ideas and concepts. Up to now logical structure of understanding is not clear, and the main advances are related to text interpretation and understanding in hermeneutics. Here we outline a logical approach to general understanding systems based on pragmatic logics, theory of values and a variety of evaluations.

In this context we take a special interest in Ch.S.Peirce's ideas concerning relationships between logics, information and semiotics [1] which bring about the arrival and intensive development of both Logical Pragmatics and Pragmatic Logics. The former is associated with the pragmatic truth theory, whereas the latter supposes an axiological consideration of logical concepts, the specification of pragmatic truth values and the application of effectiveness principle in the form of pragmatic maxim.

An important contribution of the "Father of Pragmatism" consists in considering logic as a normative science and defining truth as the good of logic. On the one hand, pragmatic approach gives us a functional (or axiological) interpretation of truth where some proposition or belief is true, if it has some utility (enables the attainment of useful practical result). To differ from descriptive correspondence theory, here the nature of truth is attributed to the reason of truth and supposes the transition from prescriptive proposition (norm) to reality (see Figure 1). Here the opposition "Description-Prescription" clarifies the meaning of the opposition "Truth-Value".

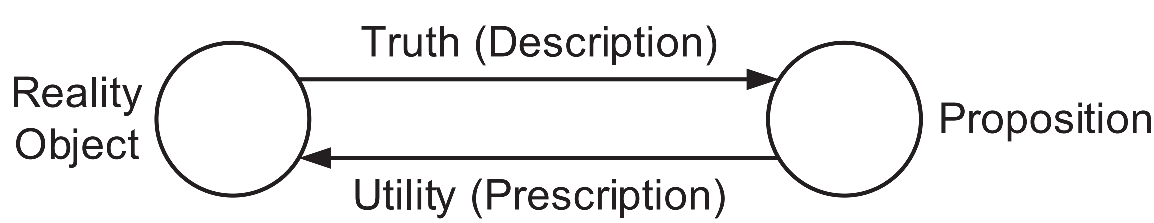

Figure 1: The Opposite Status of Classical Truth and Utility. Truth is the correspondence between reality object and proposition giving its description; inversely, Utility is the correspondence between prescription and reality object (the usefulness of norm).

On the other hand, a well-known Peirce's definition of truth as "the concordance of an abstract statement with the ideal limit towards which endless investigation would tend..." [1] or even more radical sentence by W. James [2] that "truth is the expedient in the way of our thinking" anticipated modern theories of approximated, incomplete, partial, gradual truth.

The next step on the way from logical semantics to logical pragmatics was made by Polish scientists K. Ajdukiewich, a founder of Pragmatic Logic [3] and T. Kotarbinsky, the author of Praxiology, as well as by Russian logician A.A. Iwin [4], who constructed the logics of values and evaluations.

Another Russian logician, B. Pyatnitsyn, who specified the class of pragmatic logics, is worth mentioning. Typical cases of pragmatic logics are inductive and probabilistic logics; more recent examples encompass various modal logics such as epistemic, doxastic, deontic, preference, decision, communication logics. All these logics express the relationships between some standards given by modalities and their use in practice.

The treatment of understanding in hermeneutics associated understanding with values (already W. Dilthey told about the close relation between understanding and evaluation).

So we develop a logical approach to understanding based on logical pragmatics, pragmatic logics and evaluation primitives such as "goal", "action", "norm", "rule", "agreement", etc. Since understanding is an evaluation on the basis of some standard (or norm), a necessary condition for understanding is the existence of such standard. The interpretation as a stage of understanding is mainly reduced to the search for the standard of evaluation and justification of its application to real-world situation. In their turn, explanations are reasoning processes enabling understanding facility. Moreover, von Wright's teleological explanation [5] is the procedure of understanding itself. Here, from an evaluation related to agent's goal and a proposition describing causal relation between this goal and required resources, a new proposition about agent's normative actions is inferred.

A variant of unified lattice-based modeling of logical pragmatics for modal evaluations is presented in [6]. Our future work will focus on granular logical pragmatics to model understanding levels.

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Revision and Logical Neutrality (or: a plea for ecumenical reasons)

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What grounds are there for revising our logic? One natural suggestion is that we should move from a logical theory L to L′ when L′ does better than L in terms of "simplicity, ontological leanness (Occam's razor), explanatory power, a low degree of ad hocness, unity, [and] fruitfulness." ([6], p. 147) This is to apply relatively familiar standards of theory choice to the case of logic.[1]

There are special difficulties in the case of logic, however, since our choice of a logical theory impacts the background theory in which we carry out our theory choice.[2] It is by no means obvious that we can have a logically neutral account of how L and L′ meet these criteria. Certain ways of developing these criteria are not logically neutral as I sketch below.

I will address (a) whether there are any general logic-neutral criteria for the adequacy of a logic and (b) whether there are any adequacy-based limits to logical revision. I will argue that there is a type of answer to (a) on the basis of a substantive and plausible answer to (b). In other work, I have argued that a logic adequate to meet Feferman's demand that a logic support "sustained ordinary reasoning" (in the sense of sustained ordinary mathematical reasoning) must obey all three standard structural rules. It must be monotonic, reflexive, and transitive and possess a substantive version of the deduction theorem. This gives the guts of an answer to (b) which would allow for a hedged answer to (a) — that there are logically neutral criteria on the assumption that all "reason- able logics" share a certain set of basic laws.

As an example of how criteria might fail to be logically neutral, consider the dispute between Neil Tennant and John Burgess about the explanatory power of Tennant's rel-

[1] This sort of view is also argued for in [7, 2].

[2] This objection, framed as an objection to Quinean and Goodmanian reactive equilibrium accounts of justication of logic, was first developed in [9] and followed up on by [8].

evantistic logic CR . We have two competing logics, CR and classical, and two criteria of theory goodness: "explanatory power", here cashed out as the ability to recapture ordinary non-pathological mathematical reasoning and "fruitfulness", here cashed out in terms of how much information about a theorem a proof provides. By the lights of classical logic , classical logic and CR are more or less on a par with respect to explanatory power and CR dominates classical logic with respect to fruitfulness.[1] However, by the lights of CR , classical logic dominates CR with respect to explanatory power and, let's say, they are more or less on a par with respect to fruitfulness. So as far as we've cashed out "explanatory power" and "fruitfulness", they are not logic neutral since it is possible to construct a case where changing the background logic yields different results on how well the logics under consideration meet them.

If we are classical logicians, what should we do? Given that by classical logic's lights, CR is an improvement, it would seem we have a strong reason to revise down from CR . However, we know in advance that once we are working with CR , we will no longer be able to generate the reason that got us to re- vise down from classical logic. In fact, it is not beyond the pale that once we have revised down, we have a strong reason to readopt classical logic. Call the evaluation of two logics unstable if by the lights of one logic L, another logic L′ is more adequate, but conversely by the lights of L′, L is superior. When we have an unstable evaluation, we cannot simply apply the standard "chose the more adequate logic" since this standard fluctuates with the proposed logic chosen. The problem is familiar from [4], but even more dramatic in the case of logic since logic is such a fundamental part of our evaluative apparatus.

There are a cluster of important issues here. In a prima facie reasonable dispute between two logics, do we have any logic neutral criteria by which to evaluate proposed revisions of our logic? If so, do we have enough to arbitrate the dispute? If not, is there a sense in which we could work with partisan criteria? Perhaps we can proceed by working within the "intersection" of the two logics.[2] It would then seem that there is a lower bar on how far down we could revise since extreme sublogics of, say, classical logic will be virtually guaranteed to be inferior to classical logic by their own lights. For example, since CR is a sublogic of classical, the resulting appraisal would be that we ought not to abandon classical logic. This seems too quick, but it is not clear that there is a more reasonable way to proceed in evaluating a proposed change of logic. Should we proceed the same in all cases of logical revision or do some differences — one logic being a sublogic of another, for example —- demand a different method of evaluation?

[1] The reason for this has to do with a "cut-elimination" theorem which guarantees that the only classical mathematical proofs we lose are pathological. This theorem only has an actual classical proof, i.e. which only has a proof which utilizes cut-elimination. If it's right, there is a CR proof. But the CR theorist is not allowed to presume this since they do not accept the reasoning that guarantees its existence. From the standpoint of justifying a cut-elimination theorem from "below", we must use CR. But there is not and may never be an actual CR proof.

[2] There's an interesting issue about what this is. I will assume for now it means the intersection of their derivability relations.

In my view, there is currently much confusion on these issues. For example, [1] gives a classical proof that we can develop bivalent model theory in a weak metatheory. Interesting as this is, it is entirely irrelevant to the sort of worries mooted here. And still others have attempted to show that in the special context of (set-theoretic) model theory, they can appeal to non-logical principles (such as the law of excluded middle, suitably interpreted) so as to prove the adequacy of their account in non-special contexts [3]. But the justication of this claim is still given in classical logic, which is only adequate justication on the presumption that Field is right. So this justication is by no means suasive. At best, these responses reveal the pervasiveness and inevitability of near classical thinking, even in non-classical contexts. If this is right, then there is good reason to think that the vast preponderance of work advocating adopting a deviant logic simply misses the point. So shall I argue.[1]

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[1]Some have argued that the demand that we develop the metatheory of a weak logic in a logic no stronger than the proposed logic is misplaced [5]. If right, this would show that Bacon and Field's work [1, 3] is largely unnecessary.

Modal

The invited keynote speaker is this session is Una Stojnić.

A recipe for safe detachment

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Let the unconditional obligation $\mathsf{O}B$ denote 'It ought to be that B; and let the conditional obligation $\mathsf{O}(B \mid A)$ denote 'If A, then it ought to be that B'. Unconditional obligations $\mathsf{O}B$ can be rewritten as conditional obligations the condition of which vacuously holds, i.e. $\mathsf{O}(B \mid \top)$.

For $n \geq 2$ the following detachment rule permits the derivation of an unconditional obligation from one or more conditional obligations:

$$A_1, \mathsf{O}(A_2 \mid A_1), \mathsf{O}(A_3 \mid A_2), \ldots, \mathsf{O}(A_n \mid A_{n-1}) \mathrel{|\!\sim} \mathsf{O}A_n \tag{D}$$

(D) is intended as a rule for safe detachment. That is, an agent deriving a number of unconditional obligations via (D) is meant to be able to jointly fulfill these obligations given the circumstances. Note that applications of the more common rules of factual detachment (from A and $\mathsf{O}(B \mid A)$ to infer $\mathsf{O}B$) and deontic detachment (from $\mathsf{O}A$ and $\mathsf{O}(B \mid A)$ to infer $\mathsf{O}B$) are special instances of (D). (To see how (D) encompasses the deontic detachment rule, note that the latter is equivalent to 'from $\mathsf{O}(A \mid \top)$ and $\mathsf{O}(B \mid A)$ to infer $\mathsf{O}B$'.)

The special turnstile '$\mathrel{|\!\sim}$' is there to warn the reader that (D) is a defeasible rule. Despite its intuitive appeal, it has been argued that (D) fails in a number of cases, including (but not restricted to) the following:

(i) Violations. If the obligation $\mathsf{O}p$ is violated, i.e. if $\neg p$ is the case, then we do not want to infer $\mathsf{O}p$ from $\mathsf{O}(p \mid q)$ and q.
(ii) Specificity cases. Of the two obligations $\mathsf{O}(p \mid q)$ and $\mathsf{O}(\neg p \mid q \wedge r)$, the latter is more specific: whereas the former is triggered whenever q is the case, the latter is triggered only in the more specific context $q \wedge r$. In such more specific contexts, we want to detach only the more specific obligation $\mathsf{O}\neg p$, and not the less specific obligation $\mathsf{O}p$.
(iii) Irresolvable conflicts. Consider the obligations $\mathsf{O}(p \mid q)$ and $\mathsf{O}(\neg p \mid r)$, none of which is more specific than the other. Then if both q and r are the case, we wish to detach neither $\mathsf{O}p$ nor $\mathsf{O}\neg p$, since we cannot possibly fulfill both of these obligations.

Although cases like (i)-(iii) have been well-studied in isolation, what is lacking is a good general account which tells us when exactly it is safe to detach an obligation in the possible presence of violations and conflicting (possibly more specific) obligations.

In such a more general setting, a number of new and interesting problems arise, giving rise to different strategies for the defeasible application of (D). Consider, for instance, the set $\{\neg p, r, s, \mathsf{O}(p \wedge q \mid r \wedge s), \mathsf{O}(\neg q \mid r)\}$. The two obligations in this set are in conflict, as we cannot jointly fulfill both. $\mathsf{O}(p \wedge q \mid r \wedge s)$ is more specific than $\mathsf{O}(\neg q \mid r)$, but the former obligation is violated in view of $\neg p$. Given this set of formulas, one way to proceed is to detach neither of these obligations. A slightly less cautious strategy is to first remove violated obligations, and next to apply (D) in view of the remaining obligations (in the absence of further conflicts). The latter strategy would permit the detachment of $\mathsf{O}\neg q$, whereas the former would not.

Taking into consideration a wide number of existing and new examples, I present a number of strategies for applying (D) in such a way that all and only unproblematic obligations are detached, so that all detached obligations can be jointly fulfilled by the agent.

Transworld Identity: Some Questions and Some Answers

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Even though the roots of modal logic go back to Aristotle's philosophy, various problems has emerged concerning the metaphysics of modality as its semantics has been formally founded in the second half of the 20th century. Among these problems, the problem of transworld identity has been one of the most significant ones. The problem in a nutshell is that as properties of an object changes from one world to another, whether it is possible for that object to retain its identity; and if it does, whether we can have a criterion that allows us to say that it is still the same object.

The problem is obviously related to the problem of essentialism. One can relate the problem of transworld identity — strictly or remotely — to many logical principles, but the principle of indiscernibility of identicals is generally the one the problem has been discussed with.

This study aims at giving a brief introduction to the problem of transworld identity as it is stated by Chisholm in his article "Identity Through Possible Worlds: Some Questions", and some answers to these questions given by Hintikka, and some more follow-up questions by Quine this time in the transtemporal realm in "Worlds Away", redesigned as a reply to Hintikka's theory.

Counterfactuals within Scientific Theories

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The language of our scientific theories is rife with alethically modal statements. The truth of counterfactual conditionals concerning matters that scientific theories describe, however, is not adequately given by the application of standard possible world semantics. As developed by Lewis and others, this semantics depends on entertaining possible worlds with miracles, worlds in which laws of nature, as described by science, are violated. This is clearly unacceptable if one is interested in evaluating certain counterfactuals not as sentences broadly of natural language, but more narrowly as propositions concerning only the connections between possibilities warranted by particular scientific theories.

It is clear that many scientific theories *do* describe with mathematical precision the possibilities they warrant, and the practice of science itself often involves introducing additional structure on these possibilities to represent relevant similarities among them. These structures include so-called uniformities, which are used to introduce the concept of a uniformly continuous variation. Any uniform space — a collection with a uniformity — turns out to be a model of Lewis' system of spheres (equivalently, his similarity measures), in particular his modal logic **VWU**. If the uniformity is separating — the uniform-structure analog of the Hausdorff condition from topology — then the corresponding system of spheres (similarity measure) yields Lewis' modal logic **VCU**. (For both cases in general, the so-called Limit Assumption does not hold.) The possible worlds, however, are all consistent with the scientific theory of interest, so evaluating counterfactuals using them does not require entertaining miracles.

The analysis here is in a sense the reverse of that often found in presentations of modal logic: instead of providing a system of axioms or inference rules for sentences with various modal operators, and then proceeding to find mathematical models thereof, my approach is instead to look to the practice of the mathematical sciences, identifying the kinds of structures placed on the models of a scientific theory that are used, if only unsystematically, in alethetically modal scientific reasoning, and then point out that these structures allow one to define counterfactual conditions satisfying familiar axioms.

The advantage of this approach is that it provides the means to answer (at least in part) one of the difficult questions about possible worlds semantics: whence the similarity measure? Even in discourse internal to a scientific theory, there will typically be no canonical notion of similarity amongst the models of that theory. Nevertheless, the context of investigation can often determine which features of these models are relevant for answering a given question, and a similarity measure can then be constructed to respect these relevant features.

As an example of application, I consider the possibilities described by the theory of general relativity — relativistic spacetimes — and the context of empirically adequate approximation and idealization, e.g., evaluating counterfactuals such as, "If our universe were to have the (idealized) properties $\{P_i\}$, then our cosmological measurements would not be too different than they are." In such cases, the relevant notion of similarity can be determined by approximation of classes of certain observable quantities for certain observers described within these cosmological models.

Reflexive Insensitive Modal Logics

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This talk deals with modal logics that are rendered insensitive to the presence or absence of reflexivity in the accessibility relation by a suitable modification of the standard semantics. In [1] a sound and compete axiomatization of the minimal logic for such a semantics was provided. This result was improved by [2], accounting for the analogs of T, S4 and S4.3. In this paper, we show how to associate a normal modal logic L with its reflexive insensitive counterpart, which we call L°, and give general theorems describing the conditions under which characterization results for L° follow from the analogs for L.

We will show that different normal modal logics can be associated to the same reflexive insensitive logic and that the translation process is inextricably linked to the admissibility of the rule $\vdash \Box\alpha \Rightarrow \vdash \alpha$. These facts will give rise to a general framework explaining the previous results in [1, 2], and allow us to extend them in full generality. The first theorem in this direction is the following.

Theorem. Let $K + \Gamma$ be a normal modal logic axiomatized by the addition of Γ to K. Furthermore, assume $K + \Gamma$ admits the rule $\vdash \Box\alpha \Rightarrow \vdash \alpha$ and it is sound and complete with respect to some class of frames $\mathbb{C}$. Then $K^\circ + \Gamma^\circ$ is sound and complete with respect to all $\mathbb{C}'$, such that $\mathbb{C}' \leftrightsquigarrow \mathbb{C}$ (where $\mathbb{C}' \leftrightsquigarrow \mathbb{C}$ if and only if any frame in one class is obtained from a frame in the other class by adding and/or removing reflexive arrows).

We will also present some results in the case where the rule $\vdash \Box\alpha \Rightarrow \vdash \alpha$ is not admissible, shedding light on the connection between modifications of the standard semantics and the admissibility of rules.

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Questions de Valeur et de Concept ('Matters of Value and of Concept')

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Keywords: *Philosophy and History of Logic*, *modal logic*, *many-valued logics*, $\mathcal{G}_n$ *hierarchies*, *philosophical logic*.

Inspired by A. Tarski's works, in the late 1970's G. Priest proposed a new technique to build a hierarchy of logics, called $\mathcal{G}_n$, which he considered a re-foundation of normal modal logic. That was before G. Priest himself adhered to some form of paraconsistent logic, and his views at the time reflected a belief in the primacy of contemporary classic logic. By a brief retrospective of the questions of both modal and many-valued logics, we argue, on historical, philosophical and technical grounds, that his previous classicist proposals deeply diverge from what Lewis had in mind and yield an entirely different and non-modal logic. We treat normal modal logics as infinite-valued logics and suggest that their finite-valued extensions are simpler and more promising alternatives to Priest's re-foundation, like Beziau's basic logics.

Completeness for some Beziau logics

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In [1], J.-Y. Beziau defined a paraconsistent logic **Z** by a translation into the language of the modal logic **S5**, where negation is understood as "it is possible that not". Equivalently, such a negation can be understood as "it is not necessary", and was used by K. Gödel in [2].

In [5, 6, 7, 8, 9] some extensions of Beziau's result were given. In particular, completeness results for selected cases of Beziau logics were given in [8]. In [9] a connection between two classes of logics, considered respectively in [6, 7] was proposed, this result corresponds to a Segerberg theorem determining a connection between normal and regular modal logics.

In the present paper some further completeness results for Beziau style logics, obtained by non-normal worlds semantics, will be proposed.

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Modal Logics of Partial Quasiary Predicates

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Modal logics recently have found many applications in various fields, including theoretical and applied computer science, philosophy, linguistics. Traditional modal logics are usually based on classical predicate logic. However, classical logic has some fundamental restrictions which don't allow taking into account sufficiently incompleteness, partiality and uncertainty of information. This leads to a problem of construction of new program-oriented logical formalisms based on wide classes of partial mappings over nominative data. *Composition-nominative logics* (CNL) [1] could be such a formalism. Composition-nominative modal logics (CNML) combine traditional modal logics and CNL of quasiary predicates. Their important variant, modal transitional logics (MTL), can adequately represent changes and development of subject-domains. In this paper we consider pure first-order MTL of partial quasiary predicates without monotonicity restriction. (MTL of equitone (monotone) quasiary predicates were studied in e.g. [2].)

Transitional modal system (TMS) is a central concept of MTL: it is an object $((S, R, Pr, C), Fm, Jm)$, where S is a set of states of the universe, R is a set of relations on states $\rho \subseteq S \times S$ (i.e. transition relations), Pr is a set of predicates over state data, C is a set of compositions on predicates; Fm is a set of formulas of the language, Jm is an interpretation mapping of formulas on states. We distinguish multimodal (MMS) and temporal (TmMS) transitional modal systems, among MMS we specify general and epistemic TMS. For MMS, we have a number of basic modalities K_i with corresponding transition relations $\rhd_i$, therefore $R = \{\rhd_i | i \in I\}$. For general TMS, there is one basic modality $\Box$ ($\Diamond$ can be defined in terms of $\Box$) and $R = \{\rhd\}$. For TmMS, we consider $\Box\!\uparrow$ and $\Box\!\downarrow$ as basic compositions, and $R = \{\rhd\}$. Along with modalities, basic compositions of pure first-order MTS contain those of pure first-order CNL (see [1]): logical connectives $\neg$ and $\vee$, renominations $R^{v_1,\dots,v_n}_{x_1,\dots,x_n}$ and quantification $\exists x$.

We define languages and semantic models of MTL and investigate their semantic properties; it is shown that modalities can be carried over renominations, and interaction between modalities and quantifiers is described. Significant difference between MTL of monotone and non-monotone predicates is demonstrated: as an example, the converse Barcan formula (i.e. formula $\Box\forall x\Phi \to \forall x \Box \Phi$) is not valid in the case of non-monotone predicates, however (see [2]), this formula is valid in the case of monotone predicates. Properties of logical consequence relations for sets of formulas, specified with states, are considered. Basing on these properties, corresponding sequent calculi can be constructed.

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Infinitary Modal Logic for Convergence in Distance Spaces

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Some infinitary modalities, such as the common knowledge operator of epistemic logic and the "always" operator of temporal logic are well known. In [1], an infinitary diamond operator and Kripke structures equipped with distance functions have been put to work.

To develop a modal logic on these distance-structures (or, d-structures), we make use of an infinitary modal propositional language, $L^{\downarrow}$, with infinitary disjunctions and conjunctions; unary possibility operators $\Diamond^{r}$ for each $r \in \mathbb{R}$; and an essentially infinitary modal "propositional convergence" operator $\Diamond^{\downarrow}$. Given a d-structure:

- $\Diamond^{r}\alpha$ is true at w iff α is true at a world w' with a distance less than r from w, and
- $\Diamond^{\downarrow}((\alpha_i)_{i\in\mathbb{N}}, \beta)$ is true at a world w iff there is a sequence of α_i-worlds with finite distances from w and converging to a β-world w' with some finite distance from w.

Let $K^{\downarrow}_{\omega_1}$ be the logic semantically characterized as the set of all $L^{\downarrow}$-propositions valid in the class of all Kripke d-structures. We prove some properties of the logics based on $K^{\downarrow}_{\omega_1}$.

On an intuitive interpretation of $\Diamond^{\downarrow}$, the proposition $\Diamond^{\downarrow}((\alpha_i)_{i\in\mathbb{N}}, \beta)$ means that the infinite sequence of propositions $(\alpha_i)_{i\in\mathbb{N}}$ denotes an infinite number of tasks that can be accomplished (from the point of view of the inhabitants of w) in a period of time ending possibly with a state of affairs including β. By having such interpretations, $\Diamond^{\downarrow}$ becomes also a useful notion of possibility.

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Paraconsistency

The invited keynote speaker os this session is Juliana Bueno-Soler (page 83).

Paraconsistent Dynamic Epistemology

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Dynamic epistemic logics study knowledge updates and changes within the context of modal epistemic logic. The underlying logic in these cases is almost always classical, and the model is updated to eliminate the inconsistencies. The inconsistencies are eliminated as they render the theory trivial in classical logic. However, if the underlying logic can allow non-trivial inconsistencies without collapsing the model, the classical methodology to update the epistemic model may not work. Because, some inconsistencies may not lead to trivialities, some inconsistencies may not trivialize the epistemology of the agents.

In this paper, we introduce a modal paraconsistent logic of public announcements. In public announcement logic, an external and a truthful announcement is made. Then, the model is updated by removing the states that contradict the announcement. In classical logic, "keeping the states that agree with the announcement" and "removing the states that contradict the announcement" are identical. However, in paraconsistent logic (and in many other non-classical logics), they are not. In this paper, we stipulate a methodology of dynamic updates where the agents keep the states that agree with the announcement. Some of those states might as well satisfy the negation of the announcement (hence causing inconsistencies). We model this situation by using a dynamic modal logic with a very versatile semantics, i.e. the topological semantics.

First, by using a variety of tools from topology, we define homeomorphic and homotopic models: the models that preserve the truth under some certain public announcements. Then, we generalize these models, and construct paraconsistent models in which public announcements and updated models are defined. Finally, we reduce the paraconsistent public announcement logic to paraconsistent modal logic by giving some reduction schemes which are familiar from public announcement logic.

This work relates to rationality of paraconsistent players where two strategies that were once unified in the case of classical logic become more visible and distinct. Moreover, by extending an earlier work, we observe that topological semantics provide a broader framework in which a variety of dynamic epistemic notions surface. Also, this work presents an alternative to the well-known "belief revision" paradigm.

Paraconsistent dynamic epistemology finds a wide variety of applications in computer science where it can be possible to allow imperfect but rational agents, and the security protocols of the agents are required to work under inconsistencies.

A Panoramic View over the Dualization of Intuitionistic Logic

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When someone wants to study an object, from a scientific point of view, one tries to find the most innovative tools to do so. If the object does not show at first sight its charms, then we put our glasses on for look better and we even look for more sensitive and fine tools to dissect it and study it in detail. In the study process, we discover what the object is but also what the object is not. In fact, through the study of an object, we discover related objects and we even invent new objects.

In this moment, the object in my mind is intuitionistic logic. Since the first formal systems for this logic appeared around 1930, it became the engine of much of the research in the area of non-classical logics. We strive to find different ways to characterize it and nowadays there are several proof theory systems and semantics for it. In the process of studying this logic, a myriad of systems that resemble it have been discovered; yet an even larger number of systems that look like opposing systems has been revealed, somehow these are dual systems.

The main objective of this work is to identify the state of the art of the dualization of intuitionistic logic and the role of paraconsistent logics in it. But as Brunner and Carnielli states in [1], the question of the purported duality between intuitionistic and paraconsistent ways of thinking arises from time to time, but the notion of duality involved in the discussion is far from clear, and thus the question could hardly be considered as solvable.

However, I have read many others interested in the same issue. Different researchers around the world have shown their efforts in order to clarify these notions and to create not only a dual intuitionist logic but also a whole hierarchy of them. Let see the approach taken by Brunner and Carnielli [1], Urbas [5], Queiroz [3], Goré [2], Shramko [4], Kamide [6], just to mention some examples.

The question now is how do we compare their results. Some of them have proceeded in a syntactical way some others in a semantical one; some focused in an interesting fragment of the language some others took the full language, etc. Variants are many, we identify the way they present intuitionistic logic, the notions of duality they use and finally we try to present them in an organized manner to get a good picture of them.

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Quasi hybrid logic: Semantics and Proof Theory

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The aim of this talk is to present a paraconsistent version of hybrid logic, which we call quasi-hybrid logic. It combines two perspectives: hybrid logic [2] and paraconsistent logic [6]. The world's semantics with nominals for this logic allows (local) inconsistencies without "explosion". In order to obtain that, a model is defined with two different valuations: one for positive literals, and another for negative literals. The existence of Robinson diagrams in hybrid logic is crucial, since it enables the representation of models as sets of hybrid formulas and then it is possible to evaluate a model with regard to its number of inconsistencies.

We will also discuss proof-theoretical aspects of quasi-hybrid logic. There are tableaux systems and Hilbert systems for quasi-classic logic and for hybrid logic (see [4, 5] and [1], respectively). The challenge is how to combine the proof systems features of both of them.

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Belnap's logic as a logic of experts

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Belnap's logic (also called first-degree entailment) is the most influential four-valued logic. Its operators generalize the strong Kleene truth tables (we take $\{0, 1, \bot, \top\}$ as the set of truth values):

	$\neg$
0	1
1	0
$\bot$	$\bot$
$\top$	$\top$

$\wedge$	0	1	$\bot$	$\top$
0	0	0	0	0
1	0	1	$\bot$	$\top$
$\bot$	0	$\bot$	$\bot$	0
$\top$	0	$\top$	0	$\top$

The semantic values in this logic have usually been read either as alethic truth values or as epistemic ones, giving rise to different interpretations and applications of the logic. We will focus on the epistemic readings, where the values represent the quality of the information about the status of the sentence: when the sources of evidence all speak in favor of a sentence, its value is 0; when all sources of evidence are against the truth of a sentence, the value assigned is 1; when there is no information about a sentence, the value assigned is $\bot$, and $\top$ is reserved for sentences which have some sources of information giving evidence in favor of them and some sources of information providing evidence against them.

Even though Belnap's logic gives an important set of valid arguments and has very interesting formal properties, when one looks at the definition of the operators from a philosophical perspective some of the assignations of truth values look *prima facie* unjustified. Consider $\bot \wedge \top = 0$. On the epistemic interpretation, if there is no information about p and contradictory information about q, one should draw the conclusion that all the evidence is against $p \wedge q$. We want to argue that this assignment of truth values (as well as the dual $\bot \vee \top = 1$) is an anomaly (as was originally recognized by Belnap himself and later on by Camp in *Confusion*, Harvard U. P., 2002, pp. 154–157).

In the talk we will consider the specific case where the sources of information are experts that give their opinion on the truth value of sentences. The aim of the talk is to analyze in detail how the logic of each expert can be combined into a logic for the group of experts. Some of the well-motivated combinations will take us to Belnap's logic and will give a natural interpretation for the (in those cases apparent) anomaly, other combinations will create logics different from Belnap's. As an example, if each expert uses a strong Kleene logic with semantic values $E = \{0, 1, \perp\}$, then a natural combination of the logics into a group logic will use as semantic values non-empty subsets of E. Once the details of the semantics are given, it can be proved that the seven-valued logic that is generated coincides with Belnap's logic and has no anomalous assignation of semantic values.

Some Results on 3-valued Paraconsistent Logic Programming

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In the XX century, with the development of several different types of logical reasoning and formalization of a wide variety of non-classical logics, an important area of Logic, namely Logic Programming, was intensively developed both theoretically and in concrete applications in different branches of Artificial Intelligence. The study of Logic Programming based on paraconsistent logics is more delicate than it seems, and several important theorems cannot be direct translated from Classical First-Order Logic to paraconsistent logic, as sometimes is assumed. Thus, based on the studies initiated in [4] and continued in [2] about the foundations of Paraconsistent Logic Programming based on different paraconsistent logics in the hierarchy of the *Logics of Formal Inconsistency* (LFIs, see [1]), we show in this talk some results on the theory of clausal resolution of a system of Logic Programming defined over the three-valued paraconsistent first-order logic LPT1 (see [3]). A suitable definition of Herbrand models for this logic is also proposed, and a useful version of the Herbrand Theorem is also obtained. It is worth observing that LPT1 is equivalent to the first-order version of da Costa-D'Ottaviano's logic J3, as well as to the first order LFI known as **LFI1*** (see [3]).

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Vasiliev's ideas for non-Aristotelian logics: insight towards paraconsistency

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As is well-known, Nicolai Alexandrovich Vasiliev[1] (1880–1940), professor of philosophy at the Imperial University of Kazan and also educated as a medical doctor, was a distinguished logician and philosopher noted for defending a bold non-classical logico-theoretical project. His contributions to the field may be related to the many-valued, intensional and paraconsistent approaches. Furthermore, Vasiliev describes a methodology which anticipates not only the most salient aspects of the metalogical methods dear to present day logic, but also aspects of what is today known as universal logic.[2]

In the first years of the last century, Vasiliev published four papers in Russian (Vasiliev 1910, 1911, 1912 [2003], 1913), in which he argues not only for the derogation of the Principle of the Excluded Middle — *the Law of the Excluded Third* — and of the Principle of Non-Contradiction, but also proposes a complete revision of classical traditional logic. Although Vasiliev never completely developed his ideas by making

[1]Sometimes also translated as Vasil'ev, Vassilieff and even Wassilieff.

[2]Nowadays universal logic can be seen as a general theory of logics, but also can be conceived as an actual field of theoretical inquiry. Its objective is understand the common substratum to all known particular logics through the analysis of the notion of logical consequence and the minimum requisites to the completeness of these systems. Beziau and Costa-Leite (2005, p. 5) argue: "In the same way that universal algebra is a general theory of algebraic structures, universal logic is a general theory of logical structures. During the 20th century, numerous logics have been created intuitionistic logic, modal logic, many-valued logic, relevant logic, paraconsistent logic, non monotonic logic, etc. Universal logic is not a new logic, it is a way of unifying this multiplicity of logics by developing general tools and concepts that can be applied to all logics."

clear all the logical features of the systems of logic resulting from his suggestions, we believe that his ideas outline an ingeniously conceived paraconsistent approach.

Vasiliev delivered a summary of his theories at the 5th International Congress of Philosophy, held in Naples in 1924, in whose proceedings a three-page abstract by Vasiliev appears. However, in this publication (Vasiliev, 1925) Vasiliev's ideas do not have the same clarity as they do his papers from a decade earlier.

The restricted circulation of Vasiliev's works seems to explain why his ideas had so little influence on the foundational debates in logic at the time he was writing and, specifically, on the emergence of non-classical logics. His first paper, Vasiliev (1910), was positively reviewed by S.I. Hessen the year it appeared (Hessen 1910), and a year afterward it was discussed in a negative review by K.A. Smirnov (Smirnov, 1911). Although Vasiliev's works were included by Church in his celebrated *A Bibliography of Symbolic Logic* (Church, 1936), it was only after 1962 that Vasiliev's ideas began to become known in the international philosophical community (Smirnov, 1962; Comey 1965; Arruda 1977; D'Ottaviano *apud* Arruda, 1990, p. xiii; Vasiliev, 2003).

At the end of the 1970s, Vasiliev's works attracted the attention of da Costa's followers in Brazil. Ayda I. Arruda formalized Vasiliev's ideas for an imaginary logic and prepared a Brazilian translation of some of his works (specifically, Vasiliev 1910, 1912, 1913), which was finally published under the editorship of Itala M. Loffredo D'Ottaviano (Arruda, 1990). This publication allowed Vasiliev's ideas to be studied and debated in the logical-philosophical community in Brazil. In other studies, Arruda introduces systems of imaginary logic — V1, V2, and V3 — which are constructed and presented formally, resulting in a careful interpretation of Vasiliev's statements and suggestions (Arruda, 1977 and 1980; Vasiliev, 2003).[1]

In this paper, we intend to analyze some of Vasiliev's main theses on non-Aristotelian logic, in order to show that many of his ideas match those of present-day paraconsistent positions. Considered from a historical perspective, Vasiliev's contributions to the history of logic have great intentional value, and contain many original ideas and insights that can be developed in various directions.

Despite the derogation of the Principle of Non-Contradiction being part of the setting in which a paraconsistent logico-theoretical posture can be established, this aspect is not in itself conclusive. Therefore, we emphasize that the overpassing of the ex falso is a sufficient condition for declaring a logico-theoretical system paraconsistent. In this regard, the postulate of the absolute difference between the true and the false introduced by Vasiliev forestalls his imaginary logic from trivialization. The imaginary logic described by Vasiliev lacks the Principle of Non-Contradiction and is thus non-trivial, but it is also consistent due to the Principle of Non-Self-Contradiction. Therefore it clearly fulfills the conditions for being one of the first and richest outlines of the paraconsistent approach to have appeared before the working out of the first paraconsistent logic systems, in the strict sense, with the works of Jaśkowski (1948 [1949] and da Costa

[1]It is not our objective in this work to present the technical details of the imaginary systems of logic proposed by Arruda.

(1963, 1974).

Vasiliev thus figures among the great scholars who envisioned a program of development, of concepts, of questions, and of methods, by means of which the progress of logic in the 20th century would take its route.

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A Paraconsistent Formalization of Nicolaus Cusanus's Logical-Philosophical Method in *De Docta Ignorantia*

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Cusanus' thought has received much scholarly attention in the past decades; yet, although Cusanus' 'paradox-centered' philosophy is relevant for the historical development of paraconsistent logic (as Priest argued), his method has not been examined in light of these recent logical developments (except by Ursic — I argue, unsatisfactorily). Accordingly, I show that Cusanus' method developed in *De Docta Ignorantia* (1438–1440) is rigorous enough to allow for formal treatment, and I carry out a formalization of its main part.

Cusanus' fundamental insight around which he builds his philosophical method of 'learned ignorance' in *De Docta Ignorantia* can be explained by considering an object X that is by definition unnameable and unthinkable — our language and thought cannot grasp it. Yet, 'unnameable' is also a name, and thus both applies and does not apply to X; we can conjoin 'X is unnameable' (D1) with its negation: 'X is unnameable *and* X is nameable' (D2), which is true; still, 'unnameable and nameable' is again only a name, and thus both applies and does not apply.

We can conjoin D2 with its negation to form D3, then add its own negation, etc. *ad infinitum*: X can be 'named' by means of this infinite series. In general terms, if we cannot think of, or grasp, X, then it will also break any 'rule' that says we cannot think of it or grasp it; thus, paradoxically, we can; yet any fixed formulation, positive or negative, is only true as part of an infinite series in which at each step the negation of the content of the previous step is added. Cusanus calls this 'paradoxical object' X 'God', building his philosophical theology on its properties; he employs a neoplatonic 'hierarchy of perfection' to rank the infinite series of 'names of God' which results.

My formalization uses and extends Priest's 3-valued logic LP ($\mathcal{V} = \{1, i, 0\}$, $\mathcal{D} = \{1, i\}$). Take any infinite set S and an asymmetric binary relation R so that $\forall x \exists y \in S, \neg(x = y), x\ R\ y$. We define $G_S x = \neg(x \in S)$. This, in Cusan language, corresponds to 'God is x'; by means of it we generate an infinite set of 'names of God' from S: $N_S = \{x \in S : v(G_S x) = i\}$ (equivalent to $N_S = \{x : (x \in S) \wedge (x \notin S)\}$). We can generate paradoxical elements in S by Priest's 'Inclosure Schema': e.g. $X_{\max}$: $x \in S$, $X_{\max}\ R\ x$, and $X_{\min}$: $\forall x \in S, x\ R\ X_{\min}$. These are 'names for God': $v(G_S X_{\max}) = v(G_S X_{\min}) = i$. Further, they can be proven to coincide: $X_{\max} = X_{\min}$. An examination of all elements in will reveal the same properties as Cusanus describes at each stage of his argument (the *maximum* coincides with *minimum*, God is and is beyond the 'coincidence of opposites', etc.)

In conclusion, such applications show that paraconsistent logics are a crucial tool for investigating paradoxical constructions such as Cusanus proposes, valuable not only in terms of the inherent philosophical interest of such questions but also for proper accounts of the methods of some past philosophers: Cusanus, and likely other more 'unconventional' logicians and philosophers, in Western and non-Western traditions.

Logics of trial and error mathematics: dialectical and quasi-dialectical systems

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Formal systems represent mathematical theories in a somewhat static way, in which axioms of the represented theory have to be defined from the beginning, and no further modification is permitted. As is clear, this representation is not comprehensive of all aspects of real mathematical theories. In particular, these latter — as often argued, starting from the seminal work of Lakatos (see [2]) — are frequently the outcome of a much more dynamic process than the one captured by formal systems. For instance, in defining a new theory, axioms can be chosen through a trial and error process, instead of being initially selected.

Dialectical systems, introduced by Roberto Magari in [4], are apt to characterize this dynamic feature of mathematical theories (see [3] for a similar, yet non equivalent, characterization). The basic ingredients of these systems are a number c, encoding a contradiction; a computable function h, that tells us how to derive consequences from a finite set of statements D; and a proposing function f, that proposes statements to be accepted or rejected as provisional theses of the system. Call final theses those theses that are eventually accepted by the system.

In this paper, we prove several results concerning dialectical systems, mostly by using recursion-theoretic tools. In particular, we offer a degree theoretic characterization of dialectical sets, i.e. those sets that are the sets of final thesis for some dialectical system. We prove that all dialectical sets are Turing equivalent to some computable enumerable set.

Then, in order to better analyze the intended semantic of dialectical systems, that is to say, to study how Magari's proposal fits the idea of trial and error processes in mathematics, we introduce a more general class of systems, that of quasi-dialectical systems. These are systems that naturally embeds a certain notion of "revision". We prove that quasi-dialectical sets lie in the same Turing-degrees of dialectical sets, hence showing that — in some sense — they display the same computational power. Nonetheless, we conclude by proving that quasi-dialectical sets and dialectical sets are different, and by showing their respective place in the Ershov hierarchy (see [1]).

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Studies on da Costa's paraconsistent differential calculus

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When the interpretation of what is now known as Integral Differential Calculus was first introduced by Leibnitz and Newton in 1684 and 1687, respectively, the notion of infinitesimal permeated its fundaments and this fundamental notion brought severe critics to the fundaments of the then new-born Infinitesimal Calculus. In 1872 with the rigorous definition of real number given by Karl W.T. Weierstrass (1815–1897) it owed a precise definition of the concept of limit, which became a fundament of the presentation of Integral Differential Calculus. The concept of infinitesimal remained latent until 1966, when A. Robinson presented the fundaments of what would be known as Non-standard Analysis, which, with methods from the Modern Mathematical Logic, constructed a convenient structure to the development of Integral Differential Calculus by departing, as originally conceived by Newton and Leibnitz, from the infinitesimal. In the year 2000, Newton C. A. da Costa presented the paraconsistent differential calculus, whose underlying set theory and logic are, respectively, da Costa's paraconsistent set theory CHU_1 and paraconsistent predicate calculus with equality $\mathrm{C}_1^=$. Its structure consists in a hyperring $\mathbb{A}$ and the quasi-ring $\mathbb{A}^*$, that extend the set $\mathbb{R}$ of the real numbers. In 2004, T.F. Carvalho presents, under the orientation of I.M.L. D'Ottaviano, his PhD thesis which studies and improves the calculus proposed by da Costa, he presents da Costa's definitions for the basic concepts, proves some new theorem that generalize important classical result and presents some applications of these results, particularly in which relates to differential calculus. In the present work, we intend to give a new version to the paraconsistent differential calculus in order to prepare the environment to a paraconsistent integral calculus.

Clarifying some rationality criteria of AGM-like Paraconsistent Belief Revision

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Belief revision is the process of changing beliefs to take into account a new piece of information. The AGM system, most influential work in this area of study, adopts the following rationality criteria (Gärdenfors and Rott, 1995):

(i) where possible, epistemic states should remain consistent;
(ii) any sentence logically entailed by beliefs in an epistemic state should be included in it;
(iii) when changing epistemic states, loss of information should be kept to a minimum;
(iv) beliefs held in higher regard should be retained in favour of those held in lower regard.

The strong relation among those criteria will be discussed — the focus is to set the opposition between the first and the third criteria (consistency and minimality), specially rose in systems of AGM-like Paraconsistent Belief Revision (developed by Testa, Coniglio and Ribeiro). The point to be elucidated is that AGM imposes unnecessarily strong criteria for revision according to an economic standard of rationality, and paraconsistency can model a more interesting system. The exposition is designed to be of interest to researchers in diverse related fields.

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The inapplicability of (selected) paraconsistent logics

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In some cases one is provided with inconsistent information and has to reason about various consistent scenarios contained in that information, assuming no inconsistency is actually true. Our goal is to argue that the so-called filtered paraconsistent logics are not the right tool to handle such cases and that the problems generalize to a large class of paraconsistent logics.

A wide class of paraconsistent (inconsistency-tolerant) logics is obtained by filtration: adding conditions on the classical consequence operation (one example is weak Rescher-Manor consequence: ϕ is such consequence of Γ just in case ϕ follows classically from at least one maximally consistent subset of Γ). We start with surveying the most promising candidates and comparing their strength. Then we discuss the mainstream views on how non-classical logics should be chosen for an application and argue that none of these allows us to chose any of the filtered logics for action-guiding reasoning with inconsistent information, roughly because such a reasoning has to start with selecting possible scenarios and such a process does not correspond to any of the mathematical models offered by filtered paraconsistent logics. Finally, we criticize a recent attempt to defend explorative hypothetical reasoning by means of weak Rescher-Manor consequence operation by Meheus et al.

Argumentation

The invited keynote speaker of this session is Christian Straßer (page 106).

Fallacy and virtue argumentation

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The text of your abstract here. In 1970, Charles Leonard Hamblin, with his book, Fallacies, re-appraised fallacy theory and signaled the need of going beyond what he called the 'standard treatment'. In more recent studies on argumentation theory there has been an interest in approaching themes in argumentation from the perspective of virtue theory (Cohen, 2009; Aberdein, 2010), following and building on similar approaches in virtue ethics (Anscombe, 1958, etc.) or virtue epistemology (Sosa, 1991; Zagzebski, 1996, etc.).

In this paper I plan to propose a way of modeling a theory of fallacy that draws its inspiration from both the recent work on the virtue theoretic approach in argumentation and the work of Aristotle (mainly the Nicomachean Ethics, but also the Topics and the Sophistical Refutations). At first, I will discuss the way fallacy can be seen as argumentational vice in the view of Andrew Aberdein (Aberdein, 2014). Following this I will argue for the need of clarification regarding what exactly is virtue/vice defined as a disposition. Then, I will pass on and propose that with argumentational virtues modeled after Aristotle's concept of 'practical wisdom' (Nicomachean Ethics VI, 5 etc.), one can use, in addition to the concept of 'vice' proposed by Aberdein, the concepts of 'practical syllogism' (Nicomachean Ethics VI, 5; VII, 3-4 etc.) and 'incontinence/akrasia' (Nicomachean Ethics VII, 1–3 etc.) to draw up the general lines of a theory of fallacy specific to the virtue theoretic approach in argumentation: the concept of 'fallacy' being better described in certain cases in terms of 'incontinence' rather than 'vice' (some Aristotelian observations on the difference between the two may be relevant in this case — Nicomachean Ethics VII, 1 etc.). This will be illustrated along the way with examples of practical syllogisms and how the major premises in them are superseded as a result of a process of decision making. This process may be rational (continence) or irrational (incontinence) when it comes to the selection and usage of argumentative means. Also, the process may have a positive/good/moral (virtuous) purpose or the opposite (vicious). Along these lines two topics shall be addressed:

1. the distinction between sophisms and paralogisms;
2. whether incontinence is by necessity conducive to vice.

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Epistemic dialogical logic with possibility of revision

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In the present article I try to find logical systems and tools which allow construction of epistemic logic which can integrate the belief revision theory and, eventually, resolve some problems of belief revision theory itself. Classically, in *epistemic logics* [1], knowledge is understood as list of true propositions that, once obtained, could not be revised any more. The set of epistemic alternatives shrinks from the moment the new knowledge is obtained. In information theory such understanding of knowledge corresponds to *hard information*. Despite this fact, in less formal epistemology the variety of understandings of knowledge persists and informs in different theories. In particular, *Epistemic Contextualism* claims that every knowledge-claim could change depending on context.And, consequently, what we considered as an instance of knowledge, could be reconsidered as non belonging to knowledge the next moment of evaluation.

Belief Revision Theory displays better than classical epistemic logics the possibility of revision. I try to introduce the variant of logic for belief revision, which combines some results of AGM with dialogical logic and does a bit more, namely:

(i) By the fact of being developed in dialogical frame, the system accounts for the interactive character of belief change.

(ii) Dialogical logic is a sort of game-theoretical semantic, where every step in logical reasoning is represented as the result of dialogue between *Opponent* and *Proponent*. Dialogical logic claims integrate the pragmatic aspect of interaction into logical analysis.

(iii) By the fact of using the *Constructive Type Theory*, the system decreases the gap between object-language and metalanguage.
(iv) *Constructive Type Theory* is a system for logical and linguistic analysis which does not have syntax-semantic distinction, and, respectively, does not have metalanguage, being self-explanatory in its proper object language.
(v) By the fact of using the instrument of bracketing of previously acquired beliefs, the system allows to account for the history of belief change in every concrete case.

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A Unified Framework for Different Types of Normative Conflicts

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It is commonly known that Standard Deontic Logic leads to triviality when applied to normative conflicts (sets of obligations and/or permissions that cannot be jointly satisfied). Over the past three decades, a large variety of logics have been developed that can handle conflicts of the form $OA \wedge O\neg A$ (see, for instance, the items in the list of references). Other forms of normative conflicts have been largely ignored or are reduced to conflicts of the form $OA \wedge O\neg A$.

The aim of this paper is threefold. First, I shall argue that, in order to do justice to the nature of normative conflicts, one should distinguish between different types of conflicts and that some of these should not be reduced to conflicts of the form $OA \wedge O\neg A$. Next, I shall show that, even if one allows for multiple forms of normative conflicts, it is possible to handle them within a unified framework. The framework that I shall present will be based on rather simple Kripke models. The two main characteristics of the semantics will be that the models allow for gluts and/or gaps for one or more connectives and that, for each premise set, the semantic consequence relation is defined with respect to a specific selection of the models of the premise set. Finally, I shall discuss the advantages of this type of semantics as compared to other kinds of semantics that have been used for this purpose, such as neighbourhood semantics.

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What does The Slingshot needs to shoot?: Slingshots Arguments and Plural Logics

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The family of arguments called "Slingshots Arguments" receive this name in (Barwise and Perry, 1981) due to the minimal machinery used and its cogent consequence. Usually seen as a kind of collapsing argument, the argument consists in proving that, once you suppose that there are some items that are references of sentences (as facts or situations, for example), these items collapse into just two items: The True and The False. In this talk we're going to focus in what is this machinery, i.e., in what are the essential ingredients that the argument use to prove this conclusion.

In (Dunn, 1988) the author shows how to characterize the Slingshot by way of three devices. As he points out the argument makes use of: (i) Indiscernibility of Identicals, (ii) a certain notion of Replacement and (iii) a term forming operator. If this is correct then term forming operators and terms play an important role in the construction of the argument. In (Oliver and Smiley, 2013) the authors defend the thesis that there is such thing as Plural Phenomena in the language and, particularly, a term may denote plurally several things at the same time. The main point of this presentation is to analyze if the argument can also be recovered in the context of Plural Logics if we change the concept of terms to a slightly different notion such as plural terms. It is expected that this investigation leads us to an answer to the question: What does the Slingshot needs to shoot?

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Dia-Logics and Dia-Semantics: a Bilattice-Based Approach

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About one hundred years ago Russian scientist N.A. Vasiliev introduced a two-leveled logical structure: ***Logic*** and ***Meta-Logic*** (interior logic or event logic together with exterior logic or affirmation logic) [1]. According to Vasiliev, we ought to make difference between two levels of knowledge: 1) an empirical level based on real-world's events; 2) a conceptual level depending on our thinking (see [2]). In other words, the structure of Logic is influenced by selected ontology (in modern Computer Science and Artificial Intelligence sense), whereas Meta-Logic is often seen as Truth Logic. Following this tradition, we take a two-leveled structure including ***Logic*** and ***Dia-Logic***.

The sources of dialogue paradigm in logic rise to ancient Greek thinkers, specifically to the representatives of sophist school. For instance, the invention of such fields as *eristic* — the art of contest, disputation and polemics — and *rhetoric* together with eloquence techniques and argumentation systems — is due to Protagor and Prodick.

Later on, Socrates and Plato introduced *dialectics* — the art of conversation in order to find truth by opposing and coordinating individual beliefs. In his turn, Aristotle developed the fundamentals of disputation theory; in this context, such his logical writings as Topics, Rhetorics, on Sophistical Refutations are of special concern. These deep dialogue traditions were supported by Middle-Aged schools of rhetoric and disputation, hermeneutics and argumentation, and so on.

Unfortunately in the course of several centuries dialogue paradigm in science got weaker and weaker. In the logic of New Age monologue took the place of dialogue. The reflection of lonely thinker substituted beliefs exchange in the dialogue, and the foundations of logical theories were related to the laws of individual thinking and reasoning. In this context, Hegel's dialectics based on self-evolution of ideas is quite opposite to Socrates' and Plato's dialectics.

The revival of dialogue tradition in XXth century was initiated by Russian philosopher and linguist M.M. Bakhtin who introduced the term ***"Dialogic"*** [3]. In his opinion, Dialogic is based on dialogical relations and/or takes into account dialogical context.

Nowadays the term "Dialogic" is used at least in two different senses. By Dialogics in a wide sense we denote a multi-disciplinary area aimed at creating the general dialogue theory that is based on the principle of considering dialogue as a universal communication unit and necessary prerequisite for cooperation and mutual understanding between agents.

By Dia-Logic in a narrow sense we mean a branch of modern logic based on dialogical representations or related to logical analysis of dialogues. Here two main ways of constructing dialogical models are possible: from dialogues to logics and vice versa from logics to dialogues.

On the one hand, the subject of Dia-Logic is the development and use of dialogue concepts and models in modern logics and logical semantics, in particular the creation of dialogue logics, dialogue games, game-theoretic semantics and so on. Here the key issue was the renewal of dialogical (or dialectical) viewpoint in logic by P. Lorenzen (see [4,5]), the author of seminal paper Logik und Agon (the Greek word *agon* means *game*, *contest*), as well as the introduction of game-theoretical semantics by J.Hintikka [6]. Recently such trend as computational dialectics has appeared to simulate and implement formal dialogue structures.

On the other hand, Dia-Logic deals with logical modeling of communicative acts, collective beliefs, co-ordination processes, argumentation procedures, negotiations, commitments, etc. In this context, various dialogue logics, argumentation models, illocutive logics are constructed. Besides, Dia-Logic is closely related to such trends as informal logic, Toulmin's collective logic, Tard's social logic, social semantics of agents communication language, etc.

To specify dialogic semantics (dia-semantics) we use the analogy with Belnap's approach [7] and introduce two basic dia-logical lattices: negotiation (search for consensus) lattice C and its dual — disputation lattice D. Minimal dialogic semantics is easily obtained as Cartesian product of two-valued semantics.

Let us denote by 1 and 2 two agents taking part in the dialogue (in argumentation theory these agents are called the proponent and the opponent respectively). We shall represent the appropriate truth values sets as primitive lattices $V_1 = \{T_1, F_1\}$ for the agent 1 and $V_2 = \{T_2, F_2\}$ for the agent two. It is easy to construct the new product lattice $V_1 \times V_2$ viewed as shared semantic space. Thus, we obtain a primitive dia-logical semantics as the direct product of semantics for agent 1 and agent 2.

If the dialogue is considered as agents negotiation unit to obtain their consensus or make compromise decision, then the order relation on $V_1 \times V_2$ may be seen

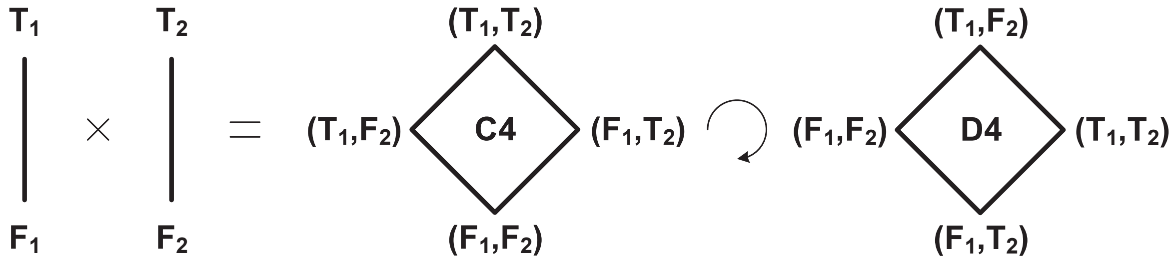

Figure 1: Construction of primitive consensus lattice C4 and transition to disputation lattice D4 by 90^{o} rotation

as the consensus order $\leq_C$. The semantics of basic consensus (negotiation) logic can be specified by the consensus lattice C4 given by the following Hasse diagram in figure 1.

Let us give the explanation of basic dia-semantics. Here we denote $(T_1, T_2) = T, (T_1, F_2) = I, (F_1, T_2) = E, (F_1, F_2) = F$. Let $(T_1, T_2) = T$ be concerted truth (the truth for both agents that means the elaboration of their agreement), $(T_1, F_2) = I$ — internal truth (the agreement is based on the belief of the first agent), $(F_1, T_2) = E$ — external truth (the agreement is based on the belief of the second agent), $(F_1, F_2) = F$ — concerted falsity or falsity for both agents (any agreement between agents is not possible or mutual refusal from beliefs co-ordination is fixed). In other words, here the values T and F are viewed as consensus points in agents negotiation, and the values I and E are referred as contrariety points in agents disputation.

Here $(F_1, F_2) \leq_C (T_1, F_2) \leq_C (T_1, T_2)$; thus, the concerted (collective) truth for both agents is better than internal (individual) truth, and the latter is better than concerted falsity. The designated value of the dialogic C_4 is $(T_1, T_2) = T$. If we rotate the lattice C4 by 90^{o}, then we obtain the disputation lattice D4 (Figure1) with the order relation $\leq_D$ (the win-loss ordering).

In this case the argumentation semantics may be used, for instance, T_1 — "argument found", and F_2 — "no counter-argument". Then (T_1, F_2) is interpreted as "win of the first agent — lost of the second agent", because the first agent has found an irrefutable argument; (F_1, T_2) — inverse situation ("loss of the first agent — win of the second agent"); (T_1, T_2) — as draw (arguments of both agents are mutually refutable); (F_1, F_2) — as mutual refusal from the disputation.

Now let us develop a bilattice-based approach to dia-semantics. The concepts of bilattice and prebilattice were proposed by M. Ginsburg [8] and M. Fitting [9] respectively.

Primarily, we introduce the triple $\langle [0,1]^2, \leq_C, \leq_D \rangle$ called a *bi − ordered dia − logical set* (BODS). If the components of BODS $\langle [0,1], \leq_C \rangle$ and $\langle [0,1], \leq_D \rangle$ form complete lattices, then this bi-ordered set becomes a *dia − logical prebilattice*. Finally, if two different order relations are linked by Fitting's negation operation $\neg_f$, that satisfies the conditions:

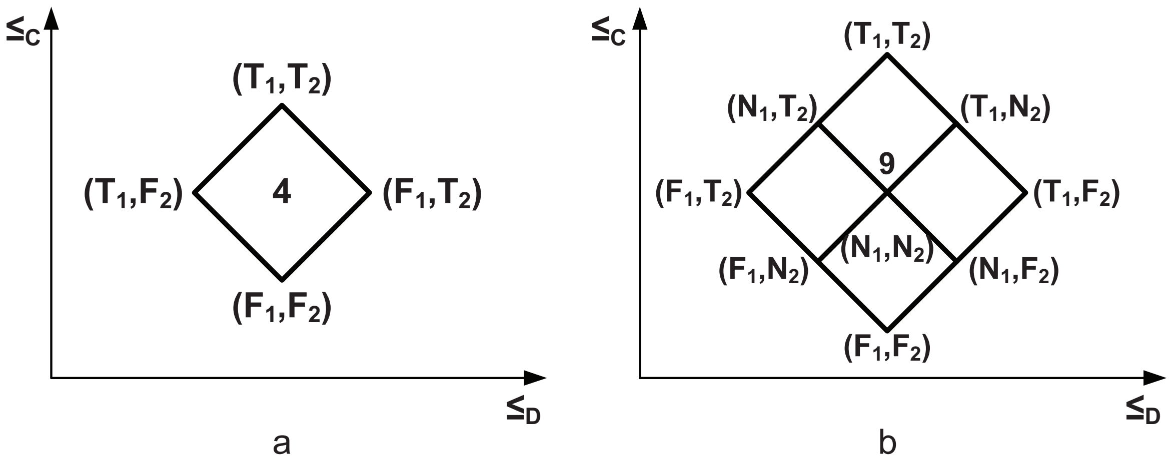

Figure 2: Examples of dia-logical bilattices: a) simple minimal dia-logical bilattice **4**; b) bilattice **9**

1) $\forall v, w \in [0,1]$, $v \leq_C w \Rightarrow \neg_f v \leq_C \neg_f w$ (monotonicity by $\leq_C$),
2) $\forall v, w \in [0,1]$, $v \leq_D w \Rightarrow \neg_f v \geq_D \neg_f w$ (anti-monotonicity by $\leq_D$),
3) $\forall v \in [0,1]$, $\neg(\neg v) = v$ (involution),

then we obtain a *dia – logical bilattice* (DBL). Some examples of DBL are shown by double Hasse diagrams in figure 2.

A minimal dia-logical bilattice to model the dialogues under certainty is **4**, and a bilattice **9** allows us to take into account the case of uncertainty (or doubt). So the truth values in 9 form the set $V_9 = \{(T_1,T_2), (T_1,N_2), (T_1,F_2), (N_1,T_2), (N_1,N_2), (N_1,F_2), (F_1,T_2), (F_1,N_2), (F_1,F_2)\}$, where (T_1,N_2) — "truth for the first agent, doubt for the second agent"; (N_1,T_2) — "doubt for the first agent, truth for the second agent"; (N_1,N_2) — "doubt for both agents", (N_1,F_2) — "doubt for the first agent, falsity for the second agent", (F_1,N_2) — "falsity for the first agent, doubt for the second agent". In this case we get three consensus points $(T_1,T_2),(N_1,N_2),(F_1,F_2)$ and six opposition points. The preference structure by $\leq_C$ has the form

$$(F_1,F_2) \leq_C (N_1,F_2) \leq_C (T_1,F_2) \leq_C (T_1,N_2) \leq_C (T_1,T_2).$$

Basic operations over dia-logic bilattices were specified, and their properties were studied. Our future work will be related to granular and fuzzy structures (see [10,11]) in dia-logics.

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Application of Argumentation in Generalization Problems

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Intelligent decision support systems (IDSS) often contain inconsistent and conflicting information [1]. The methods of the classical logics cannot be applied for inconsistent knowledge bases as they do not provide mechanisms of 'revision' of previously made conclusions. We propose to use the argumentation for dealing with conflicts and inconsistency in data and to apply it for enhancing the quality of classification models of generalization algorithms.

Argumentation gives us much more instruments for modelling plausible reasoning. From well known formalisms of the argumentation theory, we use the theory of defeasible reasoning proposed by J. Pollock [2] and apply the first order logic. In the classical argumentation theory only qualitative answer "pro et con" is possible, i.e. whether the argument is acceptable or not. For solving this problem, it is proposed to use the mechanism of justification degrees [3]. A justification degree is a numerical assessment of argument plausibility.

As the practical application of the argumentation theory, the modified algorithm for calculating justification degrees to improve the quality of classification models, obtained by generalization algorithms, is proposed [4]. There is a number of machine learning algorithms that are able to solve the problem of inductive concept formation on the basis of analyses of real data presented in the form of database tables. Thereby the machine learning algorithms based on learning sets build classification rules that can be further used to identify a class to which an object belongs.

Let $O = \{o_1, o_2, ..., o_N\}$ be a set of N objects that can be represented in an IDSS. Each object is characterized by K attributes: $a_1, a_2, ..., a_K$. Quantitative, qualitative, or scaled attributes can be used[1]. Among a set O of all objects represented in a certain IDSS, separate a set V of positive objects related to some concept (a class) and W is a set of negative objects not concerned with this concept (a class). We will consider the case where $O = V \cup W, V \cap W = \emptyset$. Let a learning set $U = \{x_1, x_2, ..., x_n\}$ be a non-empty subset of objects O such that $U \subseteq V \cup W$.

Thus, the concept was formed if one manages to build a decision rule that for any example from a learning set U indicates whether this example belongs to the concept or not. The algorithms that we study form a decision in the form of production rules. The decision rule is correct if, in further operation, it successfully recognizes the objects that originally did not belong to a learning set. The generalization algorithms build a generalized concept as a set of decision rules $\mathbf{R}$. It is known that the main criterion of the quality for a built generalized concept (i.e. a decision rule set $\mathbf{R}$) is a successful classification of a test set of examples (examples not entering into a learning set U) by the given decision rules. It is proposed to use the argumentation methods for obtaining an improved set $\mathbf{R^*}$, that is able to classify test examples with a greater accuracy than the original set $\mathbf{R}$.

The quality of a decision rule set $\mathbf{R}$ depends, primarily, on the representativeness and consistency of a learning set U. The basic idea is to divide the learning set of examples U into two subsets U_1 and U_2, such that $U_1 \cup U_2 = U$, $U_1 \cap U_2 = \emptyset$, and to produce separate learning on each of these subsets using any generalization algorithm that generates classification rules of the form of production rules. It it is proposed to use the methods of argumentation for obtaining an improved classification model, combining the results of a separate learning on U_1 and U_2. Let decision rule sets $\mathbf{R_1}$ and $\mathbf{R_2}$ using some generalization algorithm(in particular, algorithms C.4.5(Quinlan) [5], CN2(Clark and Boswell) [6] and GIRS(Vagin, Fomina and Kulikov) [7] can be used) be built for learning sets U_1 and U_2. Our task is to form a consistent set $\mathbf{R^*}$ that combines rules from both sets $\mathbf{R_1}$ and $\mathbf{R_2}$. The method of combining multiple sets of decision rules in a conflict-free set of rules by defining justification degrees for all defeasible rules in such a way that all conflicts arising in a learning set becomes solvable was proposed. This method using the GIRS algorithm as a basic generalization algorithm was successfully implemented and tested on MONK's Problems from UCI Repository of Machine Learning Datasets (Information and Computer Science University of California)[8]. The use of argumentation allowed to enhance the classification accuracy on 7.86% (from 74.31% to 82.17%) for the given problem.

Application of argumentation methods for the generalization problem allowed to enhance the classification accuracy for test problems. Furthermore, it was analyzed the influence of noise on the classification accuracy. The use of argumentation for noisy data as well significantly improved classification results.

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Language

The invited keynote speaker of this session is Ernest Lepore (page 94).

A quest for new quantum words: reasons

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With the help of the two preliminary surveys carried out with CERN[1] physicists, which indicated that natural language is inadequate to think, communicate and educate about quantum reality across disciplines, we want to investigate which concepts and corresponding new words would make spoken language better suited for this purpose, by, for example, reducing the information loss when explaining the quantum formalism with ordinary language. An effort in this direction was already initiated by David Bohm with a language called 'Rheomode'. We will present pedagogical arguments for such an enlarged language in order to deal with quantum realm.

On Assigning

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The relation of words to objects is a key issue in the history of logic, especially after the analytic turn as initiated by Frege (*Sinn und Bedeutung*). It can be said that almost all the great philosophers of logic from Frege to Kripke had to deal with the issue of the sense / reference relation. A reference is basically the relation of one linguistic expression to one extra-linguistic reality, while the sense is the way one linguistic expression relates to one extra-linguistic reality (like in Frege's distinction between the 'Morning Star' and the 'Evening Star'). For the sake of clarity, a reference can be divided into a specific function of designation (identifying or indicating one thing that an expression relates to) and a generic function of denotation (designating one thing and predicating something of that thing). For instance, in the sentence 'The snow is white', 'The snow' designates one real entity while 'white' does not designate but actually denotes the whiteness. The relation of one expression to its reference remains a complex problem that splits into several logical options in semantics as well as in pragmatics. From the semantic side, the reference is something that can be described in view of an interpretation ('denoting' in Russell's terms); from the pragmatic side, it

[1] European Organization for Nuclear Research.

is something that can be produced by an action ('referring' in Strawson's terms). In this paper, I assume that the issue of the sense/reference relations inherited from Frege can be shaped in another way in the light of an approach of general logic as opposed to special logic (Lavelle (2015) 'Elements of Special and General Logic'). Special logic focuses respectively on the semantic and pragmatic stakes of the True or the Non-True while general logic, as an example of non-classical logic, broadens the spectrum to some other values than the True, such as the Good, the Beautiful or the Useful. This does not mean that a proposition or a statement is true, good, beautiful or useful as such, but its content can be interpreted in terms of true or false (Verus / Falsus, V/F), good or bad (Bonus / Malus, B/M), beautiful or ugly (Pulcher / Deformis, P/D), or useful of free (Utilise / Gratis, U/G). This equals making the difference between a multi-valued logic and an alter-valued logic that is grounded on this broader spectrum of values. In this respect, like in Frege's, the reference is something that a linguistic expression refers to, while the sense is the way the expression refers to the reference. But unlike in Frege's, the way the expression refers is not only the sense as such, but a kind of mediate relation that I propose to term 'assignation', to be distinguished from the assignment in model theory. An assignation is the semantic or pragmatic operation of selecting one specific relation of the sense to the reference that allows giving one specific category of interpretation to a proposition or a statement (epistemic, ethical, aesthetical, or technical). Assignation is a kind of relation that, so to speak, is located in between the sense and the reference in that it conditions the interpretation of one sentence. For instance, in 'The snow is white', one can make an epistemic interpretation in terms of True or False, but one can also make an aesthetical interpretation in terms of Beautiful or Ugly. There is nothing in the sentence as such, be it its nature, its form or its content, that indicates what kind of interpretation should prevail, but the operation of assignation. This relation of assigning could be named differently depending on the logical framework that one adopts: 'annotation' in Mill's lexicon (between connotation and denotation), or 'attension' in Carnap's (between intension and extension). The notion of assignation also implies that the difference between semantics and pragmatics is relative since the operation of assigning merges interpretation with action.

Natural language and proof-theoretic semantics: denotational ghosts in inferential machine

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In the domain of formal semantics the general "meaning as use" approach is almost exclusively limited to artificial languages (i.e., non-empirical discourse). Lately, however, this began slowly to change and attempts of providing proof-theoretic semantics for natural languages (i.e., empirical discourse) started to appear as well. In this paper we review some of the state of the art approaches to natural language analysis from the proof-theoretic semantics point of view.

We are using the term proof-theoretic semantics very loosely here to encompass not only the traditional Dummett-Prawitz paradigm, but in general all approaches that are shunning away from the orthodox model-theoretic semantics and adhering to the general idea that meaning is rather something to be done (constructed, executed, computed…) and not just some inert entity awaiting to be denoted. In other words, we will focus on those theories that see meaning as a certain procedure.

More specifically, we review Ranta's Type-Theoretical Grammar [4] and Sundholm's approach [5] (both based on Martin-Löf's Constructive Type Theory (CTT) [3]), Wieckowski's approach [7, 8] (based on CTT and subatomic systems for natural deduction), Francez and Dyckhoff's approach [1, 2] (based on natural deduction) and our own approach (based on modified version of Tichý's Transparent Intensional Logic (TIL) [6]).

However, as we will show, some of these approaches still suffer from denotational pollution in various degrees (as was already pointed out by Francez and Dyckhoff in [2]). Our task will be to locate and examine these denotational "ghosts" in otherwise inferential machine and, hopefully, get rid of them.

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Linguistic Crisp and Symbolic Logics in Mathematics and Derivations

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Unfortunately, mathematics is regarded as one of the most difficult aspects in any education system. Today, in scientific and technological developments and innovative thinking elements are all dependent on the logical aspects two of which are the most significant important in mathematics, namely, crisp and symbolic logic principles. Mathematics cannot be thought without logic and especially formal logic for applications in various disciplines such as physics, engineering and any quantitative science. Today, the basis of computers and specifically software cannot be achieved if the principles of logic are not well digested either during formal educational system or as practical rational and approximate reasoning by careful people even though they may not have systematic education. In the rational logical thinking the three most important sequential steps are first linguistic expressive power, deductions and finally the formal proof of the set forth hypotheses. The scientific researches start with critical review of previous articles in search and identification of logical statements (propositions with antecedent and consequent parts), their discussion, improvement, modification or complete denial with innovative suggestions and finally the applications and verification for final proof, which may be achieved through mind, laboratory or field observations.

Prior to the mathematical logical initiation in the history its proper foundation has developed along the arithmetic (theoretical and rational relationships in the natural numbers) and geometry, which had the fundamental logical deductions. This is the main reason why the geometry has long historical development than the mathematical formulations, axioms and deductions, which have started in concentrative manner after the 19th century. Formal arithmetic references started with the pioneering work of Peano (1889) with a set of axioms and based on the logical system of Boole. Katz (1998) stated the flaws in Euclid geometrical axioms. In the scientific arena for the relativistic theory of Einstein the Euclid geometry could not represent the physical facts, and therefore, a new geometrical perspective is sought and then Riemann geometry came to play role for description of the physical natural events.

Provided that the meaning of each word that refers to objects is known, it is then possible to convert it to a set presentation either in crisp or fuzzy logical domains (Şen, 2013). Conversion of the words and their linguistic interrelationships through the grammatical structure of each language appear in the form of logical statements, which are called propositions. These statements form the foundation of the mathematics and whoever understands the content of the statements s/he is able to translate it into mathematical symbols as symbolic logic that has been first suggested by the pioneering work of Al-Khwarizmi (780-850) the father of algebra. Contemporary work in the foundations of mathematics often focuses on establishing which parts of mathematics can be formalized in particular formal logic.

This article provides the basic systematic steps for mathematical formula derivation based on crisp logical principles, which are translated to symbolic logic leading to mathematical equations that are used in science and technology.

Communication is Possible according to a New Interpretation of Solipsism

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Solipsism in its traditional form says that communication is impossible. It is indeed true that we cannot communicate, if communication is taken in its traditional form. New interpretation of solipsism could give us some new opportunities for understanding language, physical things and communication.

Illocutionary Logic, Discourse Pragmatics and Universal Grammar

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Until now philosophers and logicians have overall studied illocutionary acts that individual speakers attempt to perform at a single moment of utterance. Could we enrich logic so as to deal with entire discourse? Wittgenstein and Searle pointed out difficulties. Most conversations lack a proper conversational purpose; their background is indefinitely open. However the ability to converse is part of linguistic competence. In my view there are four universal conversational goals: the descriptive, the deliberative, the declaratory and the expressive goals that correspond to the four possible directions of fit between words and things and that are achieved in the conduct of any discourse. Illocutionary logic can study the formal structure and dynamics of such language games because a system of constitutive rules underlies their conduct. I will compare my approach to others as regards methodology and issues.

Like Montague, I believe that pragmatics should use the resources of formalisms and philosophical logic in order to construct a theory of meaning and use. I will explain how to further develop intensional and illocutionary logics, the logic of attitudes and of action in order to characterize our ability to converse. One important issue is to analyze the logical form of intentional actions and to explicate the minimal rationality of speakers and the generation of individual and collective speech act tokens in discourse.

Are there universal transcendent features that any natural language must possess in order to provide for its human speakers adequate means of expression and of communication of their conceptual thoughts? As Frege, Austin and Searle pointed out, illocutionary acts are the primary units of meaning and communication in the use of language. In my view discourse protagonists do not only mean to perform at the moment of their utterances individual illocutionary acts directed towards facts of the world like assertions, promises, directives, votes and thanks. They also mean to pursue language games with a proper conversational goal like explanations, theoretical and practical debates, negotiations, elections and exchange of salutations. Such language games are second level illocutionary acts that are most often collective and last during an interval of time. It is in the very attempted performance of illocutionary acts that human agents express and communicate their conceptual thoughts. Whoever attempts to perform an illocutionary act represents its felicity conditions. For this reason, speech act theory and illocutionary logic contribute to the theory of linguistic universals in formulating the necessary and universal laws governing felicity conditions. I will argue that the logical form of illocutionary acts imposes certain formal constraints on the logical structure of a possible natural language as well as on the mind of competent speakers. Certain logical, ontological, syntactic, semantic and pragmatic features are universal because they are indispensable. Moreover, in order to perform and understand illocutionary acts, competent interlocutors must have certain abilities which are traditionally related to the faculty of reason. The theory of felicity of illocutions fixes limits and imposes a logical order to possible human thoughts, actions and experiences.

Logic and Sense

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Each logic, natural or scientific, uses appropriate language, more strictly — its *sensible expressions* — expressions having *sense*, a *logical sense*. The contemporary logic, logic of language, can define the sense strictly with regard to some general aspects of development of cognition of the world and, at the same time, contributing to an explication of one of the most important traditional philosophical problems: *Language adequacy of our knowledge in relation to cognition*. For any language L the adequacy could be achieved first of all if some general conditions of logical meaningfulness of the language L are satisfied. They are immediately connected with the logical sense of its expressions.

In logic we can distinguish three kinds of sense of expressions of language L:

- *syntactic sense*, when expressions of L are well-formed; it is defined in syntax of L,

and, in accordance with Carnap's and Frege's distinction, two kinds of *semantic sense*:

- *intensional sense*, when expressions of L have a meaning, *intension*; it is defined in intensional semantics of L,
- *extensional sense*, when expressions of L have a denotation, *extension*; it is defined in extensional semantics of L.

In the paper, formal-logical considerations relate to syntax and bi-level intensional and extensional semantics of language L characterized categorially in the spirit of some ideas of Husserl (1900–1901), Leśniewski (1929, 1930), Ajdukiewicz (1935, 1960) and in accordance with Frege's ontological canons (1892), Bocheński's motto: *syntax mirrors ontology* and some ideas of Suszko (1958, 1960, 1964, 1968): *language should be a linguistic scheme of ontological reality and simultaneously a tool of its cognition.* In the logical conception of language L outlined in the paper, expressions of L have syntactic, intensional and extensional senses and satisfy some general conditions of language adequacy. The adequacy ensures their unambiguous syntactic and semantic senses and mutual, syntactic and semantic compatibility, correspondence guaranteed by the acceptance of a *postulate of categorial compliance.* From the postulate three principles of compositionality follow: one syntactic and two semantic already known to Frege. In the paper, they are applied to some expressions with quantifiers.

Language adequacy connected with the logical sense described in the logical conception of language is, of course, an idealization, but only expressions with high degrees of precision and adequacy, after due justification, may become theorems of science.

Paradox

The invited keynote speaker of this session is Ekaterina Kubyshkina (page 93).

Paraconsistent intuitionistic logic for future contingents

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I put forward some models of intuitionistic logic (I call them S-models), such that they can capture the fundamental anti-realist intuition, according to which propositions do not exist "out there", but need to be accounted for by what occasionally is or is not the case. Consequently, no sentence in these models can be either true or false, unless there is a matter of fact accounting either for its truth or for its falsity. Indeterministic events, in the same models, are events, for which there is actually no matter of fact accounting either for the sentence describing them, or for the negation of that sentence.

I test the way these models behave with respect to indeterministic universes by testing how they behave with respect to Aristotle's Sea-Battle's paradox in *De Interpretatione*, vol. **9**.

The model is a septuple $\langle W, f, R, D, V, \Vdash, now\rangle$, whereon:

- W is a non-empty set of worlds.
- f assigns a positive or negative integer to the elements of W.
- R orders W into a partial order.
- D assigns a non-constant domain of elements to each world.

- V assigns a set of atomic sentences to each world. [All worlds are complete as far as atomic sentences are concerned. Atomic sentences and truth-tables are supposed to completely determine whatever can be described without any reference to moments other than the present.]
- $\Vdash$ settles the truth-value of sentences about whether a sentence is true in a world of the model; it reads, "*w forces p*" and is equivalent to, "*p* is *true* in *w*". [NB. Sentences with this connective make part of the worlds of the model itself. So the model contains its own metalanguage.]
- *now* is a higher-order function picking up the world that stands for the actual present.

The models heavily depend on Beth's condition for disjunction:[1]

- w forces $p \vee q$ if, and only if, there is a bar B for w, and, in each world of B, either p is forced or q is forced.

The solution in a nutshell: Maximality of worlds with respect to the present makes either "There is a Sea-Battle" or "There is no Sea-Battle" belonging to every possible tomorrow. Thus the disjunction "Either there is a Sea-Battle or not" belongs to every possible tomorrow. By applying the Beth condition for disjunction, "There will be a Sea-Battle tomorrow or not", unlike its disjuncts, becomes true at present.

Notice that the underlying logic is not classical. The Excluded Middle is falsified, whenever there is no future bar, where an event is either established or made impossible. Consider, "There will be a Sea-Battle in the future or not". The disjunction is truth-valueless in case there is a maximal linear path passing through the actual present, and upon which no Sea-Battle ever happens, but all the worlds of the path have access to future worlds where some Sea-Battle happens.

The Role of Paradoxes in Belles-Lettres

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The founder of structural linguistics F. Saussure noted that language is a "peculiar algebra". His followers used to develop this idea by borrowing of elements from formal logical and mathematical languages. In this way they made attempts to reveal the deep structures of any text regardless of its genre.

One of disadvantages of the structural paradigm is the examination of the text as a closed system. However, from it does not follow that formal logical analyses are inapplicable to certain works of art. They are especially compelling in cases when some

[1]As in E. Beth, "Semantic Construction of Intuitionistic Logic", *Kon. Nederlandse Ac. Wetenschappen afd. Letteren: Mededelingen*, vol. **19(11)**, 1956, pp. 357–388.

authors put into their texts paradoxes in order to give the reader a sign to interpret the meanings, encoded in the vertical structure of the text. As G. Deleuze noted, there is a "logic of sense", but it is yet not sufficiently studied.

Using of paradoxes in belles-lettres is motivated by the writers' desire to denote something without its direct naming. As P. Abelard noted, when someone says: "There is no rose", in the mind of the addressee does not arise the idea of nothing, but of a rose. Contemporary French philosopher J. Derrida is engaged with the same problem: "How to say the things without saying them".

For illustration of the role of paradoxes in literary text can be pointed out the tragedy Hamlet. In explicit form Shakespeare introduced them in the conversation between his main character and the clown (gravedigger). The first misunderstanding between them stems from Hamlet's question: "Whose grave's this?" The clown answers: "Mine". After that he explicates his statement: "You lie out on't (grave), sir, and therefore 'tis not yours. For my part, I do not lie in't, yet it is mine". The sophisticated gravedigger uses two meanings of the verb to lie: 1) to be in a horizontal position; 2) to say something which is not true, in order to avoid the liar's paradox. Hamlet exposes his manoeuvre: "Thou dost lie in't, to be in't and say it is thine. 'Tis for the dead, not for the quick; therefore thou liest". At the beginning of his play, Shakespeare also resorted to paradox, but in an encoded form that requires interpretation. In *Logique du sens* G. Deleuze points out the paradoxical nature of the beginning of communication, but he does not spot this problem in Shakespeare's play. Horatio tries to engage the Ghost in conversation by four commands:

1) "If thou hast any sound, or use of voice, speak to me!";
2) "If there be any good thing to be done... speak to me!";
3) "If thou art privy to thy country's fate... speak!";
4) "Or if thou hast uphoarded in thy life extorted treasure... speak of it!"

The Ghost's silence is a negative answer. After it every other attempt to start a conversation with him is senseless from the position of propositional two-valued logic. From the three-valued logic of action, developed by G.H. von Wright, the situation is different. The Ghost's silence could be interpreted as an abstention from speech act, not as an inability to speech. On this basis the reader could formulate the ban that probably was imposed on the Ghost in the realm of the dead: "You are allowed to appear before the people to whom you are not allowed to speak, but you are not allowed to appear before your son to whom you are allowed to speak". Resolution of this problem requires an intervention of a third person, who can convey the unspoken hypothetical message to the absent person.

Other examples of paradoxes can be discovered in works of R. Musil, Th. Gautier, A. Schnitzler, S. Beckett, etc.

Condorcet Paradox and Program Logics

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The theory of social choice is based on several ideas from formal logic. On the other hand, applications of results from the former to the later do not seem to be largely prevalent till now. There seem to be many interesting interconnections between results in the theory of social choice and formal logic in the context of reasoning in programming language semantics. We examine results from voting theory, which studies individual and collective social choice and aggregation of the choices. In particular we demonstrate an interesting correspondence between the Condorcet Paradox from voting theory and formal program logics in theoretical computer science.

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On How Kelsenian Jurisprudence and Intuitionistic Logic help to avoid Contrary-to-Duty paradoxes in Legal Ontologies

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Classical Logic has been widely used as a basis for ontology creation and reasoning in many knowledge specific domains. These specific domains naturally include Legal Knowledge and Jurisprudence. As in any other domain, consistency is an important issue for legal ontologies. However, due to their inherently normative feature, coherence (consistency) in legal ontologies is more subtle than in other domains. Consistency, or absence of logical contradictions, seems more difficult to maintain when more than one law system can judge a case. This is called a conflict of laws. There are some legal mechanisms to solve these conflicts, some of them stating privileged fori, other ruling jurisdiction, etc. In most of the cases, the conflict is solved by admitting a law hierarchy or a law precedence. Even using these mechanisms, coherence is still a major issue in legal systems. Each layer in this legal hierarchy has to be consistent. Since consistency is a direct consequence of how one deals with logical negation, negation is also a main concern of legal systems. Deontic Logic, here considered as an extension of Classical Logic, has been widely used to formalize the normative aspects of the legal knowledge. There is some disagreement on using deontic logic, and any of its variants, to this task. Since a seminal paper by Alchourron, the propositional aspect has being under discussion. In this case, laws are not to be considered as propositions. This is in fully agreement with Hans Kelsen jurisprudence. On a Kelsenian approach to Legal Ontologies, the term "Ontologies on laws" is more appropriate than "Law ontology". In previous works we showed that Classical logic is not adequate to cope with a Kelsenian based Legal Ontology. Because of the ubiquitous use of Description Logic for expressing ontologies nowadays, we developed an Intuitionistic version of Description Logic particularly devised to express Leagal Ontologies. This logic is called iALC.

In this work we show how the iALC avoids some Contrary-to-duty paradoxes, as Chisholm's paradoxes and its variants. For each of these paradoxes we provide an iALC model. Finally we discuss the main role of the intuitionistic negation in this issue, finding out that its success may be a consequence of its paracomplete logical aspect. An investigation on the use of other paracomplete logics in accomplish a logical basis for Kelsenian legal ontologies is highly motivated.

On paradoxical and non-paradoxical systems of propositions referring to each other

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As well-known, all classical paradoxes involve a kind of self-reference. A paradox without any self-reference was proposed by Yablo twenty years ago in [1] (for a subsequent discussion see [2–6]). This new paradox can be considered as an unfolding of the paradigmatic Liar Paradox: it consists of propositions indexed by natural numbers such that each of the propositions states "all propositions with greater indices are wrong". Our purpose is to investigate arbitrary systems of propositions some of which state that some others are wrong, and to learn which of these systems are paradoxical and which are not. For this, we introduce a first-order finitely axiomatized theory of a language with one unary and one binary predicates, T and U. Heuristically, variables mean propositions, Tx means "x is true", and Uxy means "x states that y is wrong". The theory is Π^0_2 but not Σ^0_2. We study which model-theoretic operations preserve or do not preserve the theory, and provide a natural classification of its models. Furthermore, we say that a model (X, U) is non-paradoxical iff it can be enriched to some model (X, T, U) of this theory, and paradoxical otherwise. E.g. a model of the Liar Paradox consists of one reflexive point, a model of the Yablo Paradox is isomorphic to natural numbers with their usual ordering, and both are paradoxical. Generalizing these two instances, we note that any model with a transitive U without maximal elements is paradoxical. On the other hand, any model with a well-founded U^{-1} is not. We show that the paradoxicality (and hence non-paradoxicality) is a Δ^1_1 but not elementary property, and provide a classification of non-paradoxical models.

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Sorites Paradox and the Need for Many-Valued Logics

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Sorites paradoxes are a class of paradoxical arguments which arise as a result of using vague terms such as “heap” or “bald”. While precise terms have sharp boundaries of application, vague terms lack such precise boundaries.

With vague terms there are objects to which:

a) the vague term applies,
b) the vague term doesn't apply, and
c) it is uncertain whether vague term applies or not (so called borderline cases).

In borderline cases it is uncertain whether the vague term in question applies to them or not. Moreover, this uncertainty cannot be resolved by any enquiry.

Since there are three aforementioned classes into which we can divide objects in a range of significance of any vague term, it might be tempting to use three-valued logic to deal with sorites paradoxes. This way we can ascribe exactly one truth value to all sentences of sorites paradox and we don't need resort to either supervaluationism or subvaluationism.

Another approach to solving sorites paradoxes is based on an intuition that vagueness is a matter of degree and logic of vagueness should reflect that with different degrees of truth. If A measures 190 cm and B 195 cm then sentence “B is tall” seems to be truer than “A is tall”. Fuzzy logic therefore takes advantage of its infinitely many truth values.

In my talk I will critically assess many-valued logics and fuzzy logic. My goal is to show that these approaches to sorites paradoxes either have presuppositions that their proponents wouldn't assent to or that they generate more problems than they claim to resolve.

Towards non-classical approaches to circular definitions

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The standard theory of definitions says that definitions must be eliminable and conservative. Anil Gupta and Nuel Belnap have developed a theory of circular and interdependent definitions that rejects the eliminability requirement. Their theory is a generalization of the revision theory of truth, and as with truth, the theory of definitions has focused on the classical scheme. Gupta and Belnap remark that the moves that have been made in response to the semantic paradoxes can be mirrored with circular definitions, which includes adopting a non-classical logic. There are has not been much investigation of non-classical approaches to circular and interdependent definitions.

In this talk, we will motivate the study of circular definitions in non-classical settings, starting to fill the theoretical gap. We see three main payoffs to such non-classical theories of definitions. The first is that they may provide additional material with which to compare theories of truth. Second, the consequences of various theories can be explored in a setting removed from the particular philosophical debates surrounding truth. Third, it will illuminate general logical features of circular definitions, showing which features of circular definitions are dependent on their circularity and which on the choice of semantic scheme. We then motivate some desiderata for non-classical theories of definitions. We will close by presenting some results for three-valued schemes, focusing on a strong Kleene theory.

An Analysis of Bach-Peters Sentences

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Most analyses of quantifiers and determiners have serious difficulties in dealing with Bach-Peters sentences (Bach, [1]; McCawley, [5]), such as

(1) The pilot$_i$ who shot at it$_j$ hit the Mig$_j$ that chased him$_i$.

(1) involves the so-called crossing coreference: the two definite descriptions in (1) involve pronominal reference (*it* and *him*) to each other. So neither of them can be taken to have a wider scope under which the other falls. In this paper, I present analyses of Bach-Peters sentences that respect the crossing coreference by adopting (and extending) the liberal notion of *quantifier* formulated by Lindström [4].

I formulate two equivalent analyses of (1). On both analyses, (1) involves a single quantifier (or determiner) amounting to the combination of the two occurrences of *the*. On the first analysis, which results from generalizing Russell's analysis of *the* for double *the* constructions, (1) involves a triadic quantifier, '$Q^{\text{the}-\text{the}}$', that combine with three 2-place predicates:

(2) a. $Q^{\text{the}-\text{the}}(R, S, T)$,

where 'R', 'S' and 'T' are 2-place predicates abbreviating '$\lambda x, y$ [is.a.pilot$(x) \wedge$ shot.at(x, y)]', '$\lambda y, x$ [is.a.Mig$(y) \wedge$ chased(x, y)]' and '$\lambda x, y$ hit(x, y)', respectively. On the second analysis, (1) involves a binary determiner, '$D^{\text{the}-\text{the}}$', a functor that combines with two predicates to yield a 2$^{\text{nd}}$-order predicate:

(2) b. $D^{\text{the}-\text{the}}(R, S)(T)$.

This analysis results from liberalizing generalized quantifier theory by allowing polyadic determiners, determiners that can combine with multiple predicates. Now, we can analyze the quantifier and determiner as follows:

Definition.
a. $Q^{\text{the-the}} =_{\text{df}} \lambda X, Y, Z\ \exists x \exists y[\forall z \forall w(Xzw \wedge Ywz \leftrightarrow z = x \wedge w = y) \wedge Zxy]$.
b. $D^{\text{the-the}} =_{\text{df}} \lambda X, Y \lambda Z\ \exists x \exists y[\forall z \forall w(Xzw \wedge Ywz \leftrightarrow z = x \wedge w = y) \wedge Zxy]$.
(Here 'X', 'Y' and 'Z' are 2$^{\text{nd}}$-order 2-place variables.) On both analyses, (1) is true if and only if (a) a pilot shot at a Mig that chased him, (b) no other pilot and Mig were in the same relation and (c) the pilot hit the Mig.

These analyses generalize the usual, Russellian analyses of *the* to double *the* constructions without reducing them to single *the* constructions. And they agree with the Russellian analyses on degenerate double *the* constructions, those that do not involve genuine crossing, such as (3a) and (3b):

(3) a. *The pilot$_i$ who shot at* ***the Mig that chased him$_i$*** hit the Mig that chased him$_i$.
b. The pilot who shot at it$_j$ hit *the Mig$_j$ that chased* ***the pilot who shot at it$_j$***.

Karttunen [3] takes (1) to abbreviate either (3a) or (3b). But I think (1) has a reading on which it is not equivalent to either. On this reading, which is analyzed in (2a) and (2b), (1) implies (3a) and (3b), but not vice versa. My analyses also differ from that of Hintikka and Saarinen [2], on which (1) is true as long as there are a pilot and a Mig such that (*i*) the Mig is the only one that chased the pilot, (*ii*) the pilot is the only one who shot at the Mig and (*iii*) the pilot hit the Mig. On this analysis, (1) might be true while there are *multiple* pilot and Mig pairs that satisfy these three conditions.

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Tools and Results

The invited keynote speaker of this session is Rodrigo Freire (page 92).

Fixing Truth Values for Arithmetical Sentences

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While the subject matter of ring theory is usually interpreted in any structure that satisfies the axioms of rings, others mathematical contents are in general presented with some privileged interpretation in mind. As a paradigmatic case of the latter we have the arithmetic, in which we agree with Meadows [2] that " There appears to be an almost universal belief amongst mathematicians and philosophers that the language and practice of arithmetic does refer to a unique structure". Assuming that arithmetic has unique interpretation, a question that naturally arises is: How can we justify our access to this interpretation? Certainly no such justification lies in the formal system itself.

As a consequence of this scenario, we have that the validity of sentences of the rings theory can be identified with the derivability of these sentences in the formal system, while for the sentences of arithmetic this identification does not hold. Moreover, fixing an interpretation for the arithmetic implies that every sentence of arithmetic must assume a truth value in this interpretation. Therefore, the subsequent question is: How can we justify the *possibility* that every sentence of arithmetic has a truth value?

We treat this issue by considering the approach of Freire [1] in the context of set theory, and adapted them to arithmetic: The truth values of arithmetic propositions are fixed by the principles that govern the behavior of the most basic arithmetical components, the numbers and operations over these numbers. We defend that such principles are associated with the practice of elementary arithmetic, and they must be correct with respect to the history of this discipline, and are themselves neither a formal system nor a substitute for the formal apparatus, among others desirable characteristics.

With this approach, arithmetic has a privileged interpretation because the truth values of its sentences are based on principles governing the discipline, which does not occur with the theory of rings and other formal systems. Furthermore, to provide an answer to the initial question, this approach also provides a means for conducting investigations regarding the truth values of sentences of arithmetic that are formally undecidable.

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A Contextual Definition of an Abstraction Operator in Second-Order Logic

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The method of contextual definitions is a very important tool for improving the expressiveness of particular logistic systems while at the same time maintaining the syntax of the relevant systems unchanged. It was employed, for instance, by Russell in his theory of definite descriptions and it is at the core of Martin-Quine's theory of virtual classes and relations — which explains how a considerable amount of class talk can be recast in first-order logic and some of its extensions without assuming any specific set-theorical principle.

Contextual definitions can also serve important philosophical purposes related to ontological reductionism: They can be used to show that, where ostensible references are made to a specific kind of entity by a particular theory T, such references can be paraphrased away by contextual means in order to reveal that T has in fact no commitment to the existence of those entities.

In this talk I shall first consider the significance of the method of contextual definitions in general for both technical and philosophical purposes. I will then give a developed example of how it can employed to increase the expressive power of a classical deductive system of second-order logic with an unrestricted comprehension axiom-schema. It will be shown that a contextual definition can be given for an abstraction operator '[$\ldots$: __]' that allows us to build second-order terms out of formulas of the relevant language, and that the expected axioms and rules of inference governing these terms can be simulated in the deductive system on the basis of the proposed definition.

Updates in Logic

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We generalize the framework of substructural logics based on the ternary semantics introduced by Routley and Meyer in 1973. The aim is to capture various logic-based formalisms dealing with common sense reasoning and logical dynamics introduced in artificial intelligence. With this objective in mind, we interpret the ternary relation of this ternary semantics as a sort of update: the first argument of the ternary relation is interpreted as an initial situation, the second as an informative event and the third as the resulting situation after the occurrence of the informative event. Within this framework, we define two update logics, the second only playing a technical role. The generality of our framework allows us to extend and generalize existing results about decidability and correspondence theory for substructural logics.

Our first update logic generalizes the usual substructural language with new structural and logical connectives that can express more properties of the ternary relation (viewed as an update). In that respect, we introduce three structural connectives $,_1$, $,_2$ and $,_3$ that are evaluated at the three different points of the ternary relation and we introduce their corresponding logical connectives $\circ_i, \supset_i, \subset_i$ for $i \in \{1, 2, 3\}$. We briefly introduce formally below a simplified version of our logical framework without modalities.

If P is a given set of propositional letters, *formulas* ϕ, ψ are built inductively from P and the connectives $\circ_i, \supset_i, \subset_i$ for $i = 1, 2, 3$, and *structures* X, Y are built inductively from formulas and the structural connectives $,_1$, $,_2$ and $,_3$. For example, $(X ,_1 Y) ,_3 Z$ is a structure. Then, the semantics for $\circ_1, \supset_1, \subset_1$ and $,_1$ is defined as follows. A *model* $\mathcal{M} = (W, R, I)$ is a triple where W is a non-empty set, R is a ternary relation on W, and $I : W \to 2^P$ is a valuation mapping. The truth conditions are defined as follows. Let $\mathcal{M}$ be a model with $x \in W$ and let $p \in P$. Then,

$$
\begin{array}{lll}
\mathcal{M}, x \vDash p & \text{iff} & p \in I(x) \\
\mathcal{M}, x \vDash \phi \circ_1 \psi & \text{iff} & \text{there are } y, z \in W \text{ such that } Rxyz, \\
 & & \mathcal{M}, y \vDash \phi \text{ and } \mathcal{M}, z \vDash \psi \\
\mathcal{M}, x \vDash \phi \supset_1 \psi & \text{iff} & \text{for all } y, z \in W \text{ such that } Rxyz, \\
 & & \text{if } \mathcal{M}, y \vDash \phi \text{ then } \mathcal{M}, z \vDash \psi \\
\mathcal{M}, x \vDash \phi \subset_1 \psi & \text{iff} & \text{for all } y, z \in W \text{ such that } Rxyz, \\
 & & \text{if } \mathcal{M}, z \vDash \psi \text{ then } \mathcal{M}, y \vDash \phi \\
\mathcal{M}, x \vDash X ,_1 Y & \text{iff} & \text{there are } y, z \in W \text{ such that } Rxyz, \\
 & & \mathcal{M}, y \vDash X \text{ and } \mathcal{M}, z \vDash Y
\end{array}
$$

The truth conditions of the connectives $\circ_i, \supset_i, \subset_i$ for $i = 2, 3$ are defined similarly, but are evaluated w.r.t. the second and the third argument of the ternary relation R respectively. As one can easily notice, the usual connectives of substructural logic $\to, \leftarrow$ and $\circ$ correspond to our connectives $\supset_1, \subset_2$ and $\circ_3$.

For this first update logic, we give a sequent calculus, a natural deduction calculus and a display calculus. We show that cut elimination holds for the display calculus. It turns out that our sequent calculus and our natural deduction calculus are generalizations of the non-associative Lambek calculus and our display calculus is a generalization of the modal display calculus (introduced by Wansing in 1994). For example, the inference rules of our sequent calculus for the substructural connectives $\{\circ_{\mathrm{i}}, \supset_{\mathrm{i}}, \subset_{\mathrm{i}} : i = 1, 2, 3\}$ are the following:

$$\frac{X \vdash \phi \quad Y \vdash \psi}{(X \,,_{\mathrm{i}}\, Y) \vdash \phi \circ_{\mathrm{i}} \psi} \qquad \frac{\Gamma[(\phi \,,_{\mathrm{i}}\, \psi)] \vdash X}{\Gamma[\phi \circ_{\mathrm{i}} \psi] \vdash X}$$

$$\frac{(X \,,_{\mathrm{i}}\, \phi) \vdash \psi}{X \vdash \phi \supset_{\mathrm{j}} \psi} \qquad \frac{Y \vdash \psi \quad \Gamma[\phi] \vdash X}{\Gamma[(\psi \supset_{\mathrm{j}} \phi \,,_{\mathrm{i}}\, Y)] \vdash X}$$

$$\frac{(\phi \,,_{\mathrm{i}}\, X) \vdash \psi}{X \vdash \psi \subset_{\mathrm{k}} \phi} \qquad \frac{X \vdash \phi \quad \Gamma[\psi] \vdash Y}{\Gamma[(X \,,_{\mathrm{i}}\, \psi \subset_{\mathrm{k}} \phi)] \vdash Y}$$

where $(i, j, k) \in \{(1, 2, 3), (2, 3, 1), (3, 1, 2)\}$ and where $\Gamma[X]$ (or $\Gamma[\phi]$) is a structure which contains the structure X (respectively the formula ϕ) as a substructure. As one can easily notice, if we omit the subscripts i, j, k in these inference rules, we obtain the non-associative Lambek calculus.

Our method to prove the completeness of our calculi is new, we use a specific Henkin construction. This new method combined with the generality of our framework allows us to prove a Sahlqvist-like correspondence result for substructural logics. To obtain this result, we define a second update logic more expressive than our first update logic. It is a temporal logic evaluated on models where the ternary relation is split into two binary relations. The (first-order) frame correspondents of axioms and inference rules (of a specific form) are obtained in two steps. First, we translate them into the temporal language of our second update logic by adapting techniques introduced by Kracht for display calculi. Second, if these translations are Sahlqvist formulas, we translate them into their (first-order) frame correspondents thanks to the standard Sahlqvist algorithm for temporal logic. Doing so, we contribute to developing a basic correspondence theory for substructural logics.

Computability of unification and admissibility in contact logics

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The unification problem in a logical system L can be formulated as follows: given a formula $\phi(X_1, \ldots, X_n)$, determine whether there exists formulas ψ_1, …, ψ_n such that $\phi(\psi_1, \ldots, \psi_n) \in L$.

The research on unification was motivated by the admissibility problem: given a rule "from $\phi(X_1, \ldots, X_n)$, infer $\psi(X_1, \ldots, X_n)$", determine whether for all formulas χ_1, …, χ_n, if $\{\phi(\chi_1, \ldots, \chi_n) \in L$, then $\psi(\chi_1, \ldots, \chi_n) \in L$.

In [9], Rybakov proved that there exists a decision procedure for determining whether a given rule is admissible in intuitionistic propositional logic. Later on, Ghilardi in [7, 8] proved that intuitionistic propositional logic has a finitary unification type and extended this result to $K4$. See also [5, 6].

Contact logics are logics for reasoning about the contact relation between regular subsets in a topological space. See [2]. In contact logics, formulas are built from simple formulas of the form $C(a, b)$ and $a \equiv b$ — where a and b are terms in a Boolean language — using the Boolean constructs $\perp$, $\neg$ and $\vee$, the intuitive reading of $C(a, b)$ and $a \equiv b$ being "the regular regions denoted by a and b are in contact" and "the regular regions denoted by a and b are equal".

The main semantics of contact logics are the contact algebras of the regular subsets in a topological space (see [3, 4]). But contact logics have also received a relational semantics that allow to use methods from modal logic for studying them. See [1].

In this setting, one important issue is the mechanization of reasoning in contact logics. Since admissible rules can be used to improve the performance of any algorithm that handles provability, it becomes natural to consider admissibility and unification within the context of contact logics.

In this talk, we will examine variants of contact logics. The central results in this talk are the following: the admissibility problem and the unification problem are decidable in contact logics; contact logics have a unitary unification type.

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Librationist Motives and Perspectives

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Keywords: *Sedationism, Bialethism, Mathematicalism, Denumerabilism, Expressionism.*

Librationism is officially most fully published in [2], but the arxived [3] — which was lectured upon by invitation at the Steklov Institute of Mathematics in Moscow on September the 4th and at the Mathematics Department of the Universidade Federal da Bahia in Salvador, Brazil, on October the 16th — develops £ more precisely with consequences for how it accounts for fundamental, i.e. important, mathematical and philosophical matters.

We relate how £ plus the *Skolem-Fraenkel Postulation* (SFP) interprets ZFC by extending an interpretation of ZF by [4] in a system weaker than ZF with collection minus extensionality; this sharpens a result of Dana Scott. £ does not commit to the consistency of something as strong as ZFC, and maybe we should rest with £ as a much weaker system in the spirit of Feferman's attitude that classical set theory is as medieval theology. Notice that by [2] £ is fully impredicative. On the other hand, £ + SFP^X, where SFP^X is an arguably plausible extension of SFP, may even interpret ZFC + 'there are X inaccessible cardinals' (if such systems are consistent), in an essential countable framework — on the last point compare [1] and superseded publications dating back to 2004 as well as [7]; recent results show that X may be replaced by arbitrary Mahlo-cardinals.

Distinguish between theorems *about* a system and theorems *in* a system by using 'thesis' for the latter usage and maintain the previous usage. Formula A is an *antithesis* of a system iff the negjunction (negation) $\neg A$ is a thesis of it, and A is a *nonthesis* of a system iff it is not a thesis of it. S is an extension of T iff all theses of T are theses of S, and a *sedation* of T iff no thesis of S is an antithesis of T. S is a *sedate extension* of T iff it is a sedation and an extension of T. *Sedationism* is the view that we should only accept sedate, proper extensions of classical logic. The alphabet of £ is | and ., and *symbols* of £ just those strings of these that count as powers of two if | is taken as '1' and . as '0' in the binary numerical system; combining the Peirce arrow (dualized later by Sheffer) and Łukasiewicz' notation strategy we understand the formulas £ posits by the appropriate strings of symbols (as by the definitions and formation rules and elementary number theoretic concatenation definition) of £ to **be** the finite von Neumann ordinals of $\mathbf{L}_\varsigma$ so denoted where ς is the level of Gödel's constructible hierarchy needed for our semantic construction à la [6] ([5] and descendents merit comparison); it is sufficient for ς to be a Σ_3-admissible ordinal. £ thinks it has the language of set theory minus identity plus truth predicate T and enumerator sign €. The latter pins down an enumeration from the finite von Neumann ordinals of £ to any universal set (*denumerabilism*), and the former connects to a semantic predicate; these both use the *internal* Gödel coding $\ulcorner\ \urcorner$ which mirrors the *external* coding invoked above in a manner so that all sets have names (*expressionism*). £ is a sedate, proper extension of classical predicate logic, and deals with paradoxes in a novel manner. The *mathematicalist* point of view supported is that mathematics is more fundamental than logic. A is a *maxim* of £ iff A is a thesis of £ and $\neg A$ is a nonthesis of £, and a *minor* iff a thesis of £ and an antithesis of £. £ has novel *regulations* and *prescriptions*; unlike classical inference rules the regulations of £ are sensitive to whether antecedent theses are maxims or minors, and prescriptions are similarly unlike traditional axioms. We consider £ a *super-formal* and *contentual* system; the former is on account of its countenance of infinitary regulations and the latter on account of its expressionistic treatment of "free variables" (*noemata*) as names.

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Lambda Theory: to a Zero-Order Logic with Quantifiers

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Keywords: *nothing*, *void*, *empty set*, *null-class*, *zero-order logic.*

In this talk, we present an important consequence of the introduction of a new constant called Lambda in order to represent the object "nothing" or "void" into a standard set theory: the possibility of the existence of a zero-order logic with quantifiers.

The Lambda theory is expressed in the standard first-order logic. It means that the quantified variables $x, y, z \dots$ must be instantiated by first-order objects.

Second-order logic, in addition to individuals, quantifies variables that range over relations (properties) and functions too.

Second-order logic is extended by higher-order logics and type theory.

In the other direction, we have the zero-order logic. It is often assimilated to propositional calculus because quantification is not possible on variables of propositions. But zero-order logic is sometimes also presented as a first-order logic without quantifiers. A finitely axiomatizable zero-order logic is isomorphic to the propositional logic. With axiom schema, it is a more expressive system than propositional logic (cf. the system of primitive recursive arithmetic).

As a consequence of the introduction of Lambda in the language of a standard first-order logic, we think that we can consider the possibility of the existence of a zero-order logic with quantifiers that range over *pre-elements* only. As pre- element, Λ is a zero-order entity. Indeed, Λ appears to be the smallest constituent that can be added to a set theory.

In this case, propositional calculus becomes the fundamental structure of any logic.

Duality, self-duality and generalized quantifiers

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We make some observations on the theory of generalized quantifiers; in particular we observe that, for any n, there are no self-dual $n \to 0$ quantifiers.

Foundations of semantic and syntactic proofs in the context of metatheories

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This presentation aims to put into perspective the semantic and syntactic proofs of meta-theoretical results. On the one hand, we seek to clarify which means a result grounded in model theoretical techniques and, on the other hand, we develop the argument that there is a real gain in the exposure of a syntactic proof. More specifically, we have based our discussion on the two proofs of relative consistency between ZFC and NBG: first, the widely used semantic proof, second, the syntactic proof given by Schoenfield in 1954.

Cosmic Logic. On the Conway-Kochen Free Will Theorem

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In two papers (see 1 and 2), Conway and Kochen propose the idea that the free will of experimenters in Quantum Mechanics is a reflection of an objective free will of the quantum-mechanical world (which they call Nature!). I discuss the cosmological implications of their view and introduce a general no-go theorem for a deterministic interpretation of the multiverse. The radical indeterminist standpoint prohibits cloning or reduplication not only at the quantum-mechanical level, but also on the cosmological scale of an infinite multiverse. The general no-cloning theorem (see 3) states that there are no homeomorphic images of macro-or microphysical objects in a multidimensional universe with the cardinality of the continuum. The internal logic of the proof uses set-theoretic combinatorial arguments revisited by Brouwer's and Bishop's topological arguments. The constructivization is achieved by introducing the notion of a local complementation which operates in the interaction of a local observer, the fixed-point observer, and any local or non-local observable system. In that context, measurement from a cosmo-logical point of view is essentially finitistic to the extent that experimental and computational procedures must produce finite results even in an infinite multiverse landscape. Those results are naturally expressed by polynomials with their internal logic designed to account for the content of mathematical, physical and physico-mathematical theories.

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Normalisation in substructural term calculi

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The resource control lambda calculus, proposed in [3], is a term calculus with explicit control of resources obtained by introducing operators for duplication and erasure of variables, which encode the structural rules of contraction and thinning in the typed term calculus. Substructural term calculi are obtained by imposing restrictions on the syntax of the resource control lambda calculus along with appropriately modified operational semantics and the type assignment rules. These substructural term calculi are computational interpretations of substructural logics [1], namely of relevant and affine logic.

In this paper, we study normalisation properties of the proposed substructural term calculi. We prove the strong normalisation of the simply typed calculi. Further, we prove the characterisation of strong normalisation by means of strict types, a particular subset of intersection types.

We also discuss connections with other term calculi that extend Curry-Howard correspondence to substructural logics [2, 5]. This paper represents a continuation of the investigation reported at UNILOG 2013 [4].

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Transreal Proof of the Existence of Universal Possible Worlds

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Transreal arithmetic is *total*, in the sense that the fundamental operations of addition, subtraction, multiplication and division can be applied to any transreal numbers with the result being a transreal number [1]. In particular division by zero is allowed. It is proved, in [3], that transreal arithmetic is consistent and contains real arithmetic. The entire set of transreal numbers is a *total semantics* that models all of the semantic values, that is truth values, commonly used in logics, such as the classical, dialectic, fuzzy and gap values [2]. By virtue of the totality of transreal arithmetic, these logics can be implemented using total, arithmetical functions, specifically operators, whose domain and counterdomain is the entire set of transreal numbers.

Taking Wittgenstein's comments on logical space as a starting point, we develop a mathematically well defined notion of logical space. We begin by defining a Cartesian co-ordinate frame with a countable infinitude of transreal axes. We notionally tie each axis to a distinct, atomic proposition. With this arrangement every point is a distinct possible world whose co-ordinates are the semantic values of its propositions. Furthermore the points composing the whole of this space bijectively map the set of all possible worlds. In other words, each one of all possible worlds is a unique point in this *world space*. This allows us to rigorously apply topology to problems involving all possible worlds, including all logics because these appear in some possible worlds. Thus we provide both a universal metalogic and a foundation for particular universal logics.

We then introduce a more abstract space by taking each point in world space, that is we take each possible world, and use it as an axis in a very high dimensional space of functions. We call a particular subset of this space *proposition space*. In proposition space a given point, that is a given proposition, has, as co-ordinates, its semantic value in each possible world. When we apply mathematical or logical operations in this proposition space we are operating on all possible worlds at the same time.

We use linear transformations to define accessibility relations in world space and to define logical transformations in proposition space. In proposition space we define necessity and possibility as appear in modal logics and we establish a criterion to distinguish whether a proposition is or is not classical. In world space we establish our main result.

We extend standard results of topology to transreal spaces and prove that, in world space, there is a dense set of, at least countably many, hypercyclic, possible worlds that approximate every possible world, arbitrarily closely, by repeated application of a single, linear operator — the *backward shift*. In other words we prove the existence of a countable infinitude of worlds which approximate every possible world by repeated application of a single operator. That is we prove the existence of universal, possible worlds.

Proving existence is useful but it leaves many questions open. We mention just two. Are there any classical, hypercyclic worlds? Is there a construction for any hypercyclic world?

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Modelling choice sequences of high types

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Choice sequences play an important role in intuitionistic mathematics, where a function is regarded as a construction process rather than a completed object. The value of a choice sequence, or a freely growing sequence, can be generated effectively for each input, though the generating algorithm might be unknown to the creating subject. A lawless sequence is a choice sequence such that at any moment of its construction only its initial sequence is known and nothing is known about its future values. The theory of lawless sequences is inconsistent with classical logic.

Most research on choice sequences is done in intuitionistic analysis FIM, for sequences of natural numbers. The consistency proofs for theories with choice sequences use specific intuitionistic models such as topological, Kripke and Beth models (see, for example, [1]). A Beth model well represents the idea of a freely growing sequence because in this model a choice sequence is incomplete at any moment and can grow differently along different paths.

Bernini [2] introduced intuitionistic n-functionals, which are choice sequences of high types (where the type $n = 1, 2, 3, \ldots$). Thus, a 1-functional is a usual choice sequence of natural numbers and an $(n+1)$-functional is a freely growing sequence of n-functionals.

Here we consider the following main properties of choice sequences of high types (in these formulas $F^n, G^n, \ldots$ are variables for n-functionals and $\mathcal{F}^n, \mathcal{G}^n, \mathcal{H}^n, \ldots$ are variables for lawless n-functionals):

- the axiom of density: $\forall x \forall F^n \exists \mathcal{F}^n (\forall y < x)(\mathcal{F}(y) = F(y))$;
- the principle of open data: $\varphi(\mathcal{H}^n) \supset \exists x \forall \mathcal{G}^n [(\forall y < x)(\mathcal{G}(y) = \mathcal{H}(y)) \supset \varphi(\mathcal{G})]$, where formula φ has no parameters with types $> n$ and $\mathcal{H}^n$ is the only parameter of φ with type n;
- the axiom of choice with uniqueness: $\forall x \exists ! G^n \varphi(x, G) \supset \exists F^m \forall x \varphi(x, F(x)(0)^{m-n-1})$, where $n < m$ and formula φ has no parameters with types $> m$.

We create a model for intuitionistic n-functionals to prove the consistency of the theory with the aforementioned axioms. Beth model $\mathcal{B}_s$ ($s \geq 1$) is constructed recursively for all functionals of types $\leq s$; nodes at each level of the model's tree are made of finite sequences of functionals from previous levels. This model also proves the consistency of the theory of the creating subject in the language of n-functionals.

The soundness proof for the model $\mathcal{B}_s$ is carried out in the classical typed theory.

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Decidable Fragments of First Order Logic Gödel Incompleteness Property

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The founding of the theory of cylindric algebras, by Tarski, was a conscious effort to create algebras out of first order predicate calculus. Let $n \in \omega$. The classes of non-commutative cylindric algebras (NCA_n) and weakened cylindric algebras (WCA_n), of dimension n, were introduced by Németi as examples of decidable fragments of first order logic with n variables. In this article, we give new proofs for the known facts, due to Németi, that the classes NCA_n and WCA_n are both decidable and have the finite model property. We also prove that the free algebras $\mathfrak{Fr}_m NCA_n$ and $\mathfrak{Fr}_m WCA_n$ are not atomic for every finite $m \geq 0$. In other words, the corresponding fragments of first order logic have (weak) Gödel incompleteness property.

A Modified Quasi-Set Theory without Identity

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Various philosophers have argued, for a variety of reasons, that the relation of identity, that is absolute identity — the relation that holds between a thing and itself and a thing and nothing other than itself, either does not exist or does not apply to certain classes of objects. The latter view has recently found proponents in the philosophy of science (Krause 1992, Krause and French 2006, 2010). Following on a suggestion from Schrodinger (1952), Décio Krause and Steven French have argued that identity does not apply to the fundamental particles of quantum physics. Krause and French go into great technical detail in support of their case. They develop a formal axiomatized system, which they call 'quasi-set theory'. In so doing, they show that an alternative to ZF set theory can be provided which does not presuppose that every entity in a domain of discourse is identical with itself. This is an important achievement, as quasi-set theory can be used to show that notions such as that of a cardinal number can be made sense of without appealing to identity (see also Domenech and Holik 2007). In what follows I will show how Krause and French's quasi-set theory might be further developed in order to provide support for a stronger thesis, that the relation of absolute identity does not exist at all. This thesis is most famously associated with the late Peter Geach (1972, 1973, 1980, 1991), and I shall call it 'GT', for 'Geach's Thesis', in his honour. The central objection to GT is that it is incompatible with classical semantics. By modifying quasi-set theory I will show that a non-classical semantics can be developed which is compatible with GT, and therefore answer the objections against this thesis.

A Generalization of Kuznetsov's Theorem and Its Consequences

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Let $\mathcal{L}$ be a propositional language with the connectives $\wedge$, $\vee$, $\rightarrow$, $\neg$ and a countable set *Var* of propositional variables. The metavariables for $\mathcal{L}$-formulas are A, B, etc. Expanding $\mathcal{L}$ by modality $\bigcirc$, we obtain language $\mathcal{L}_{\bigcirc}$, and expanding $\mathcal{L}$ by modality $\Box$, language $\mathcal{L}_{\Box}$. The set of $\mathcal{L}$-formulas, $\mathcal{L}_{\Box}$-formulas and $\mathcal{L}_{\bigcirc}$-formulas are denoted respectively by $\boldsymbol{Fm}$, $\boldsymbol{Fm}_{\Box}$, and $\boldsymbol{Fm}_{\bigcirc}$. Also, we will need an axillary language, $\mathcal{L}_{\Box\bigcirc}$, which is obtained by adding the modalities $\Box$ and $\bigcirc$ to $\mathcal{L}$. Accordingly, the set of $\mathcal{L}_{\Box\bigcirc}$-formulas is denoted by $\boldsymbol{Fm}_{\Box\bigcirc}$. The letters α, β, etc., are used as metavariables for formulas of $\mathcal{L}_{\bigcirc}$, $\mathcal{L}_{\Box}$, and $\mathcal{L}_{\Box\bigcirc}$. (Since formulas of these languages will not be used in one and the same context, confusion is unlikely.)

The following logics are in focus: **Int** (formulated in $\mathcal{L}$), **KM** and **K4.Grz** (both formulated in $\mathcal{L}_{\Box}$), as well as **Grz** (defined in $\mathcal{L}_{\bigcirc}$); see [5]. Also, if L is one of these logics, we consider the lattice $\mathrm{NExt}L$ of its normal extensions. We remind that L' is said to be a normal extension of L, if both are defined in the same language, $L \subseteq L'$, and L' is closed under the inference rules postulated in L. We assume that substitution and modus ponens are among the postulated inference rules of the logics under consideration. In addition to these rules, the rule $\alpha/\bigcirc\alpha$ (necessitation rule) is postulated for **K4.Grz** and **Grz**.

We define $\mathbf{S}$ (in $\mathcal{L}_{\Box}$) to be a **KM**-*sublogic* if $\mathbf{S} \subseteq \mathbf{KM}$ and $\mathbf{S}$ is closed under substitution and modus ponens. A numerous examples of **KM**-sublogics can be found in [4]. For instance, one of the **KM**-sublogics is **mHC**; cf. [1, 4]. The following theorem is a slight generalization of Kuznetsov's theorem of the equipollence of **Int** and **KM**; cf. [2, 5].

Theorem 1. Let $\mathbf{S}$ be a **KM**-sublogic. Then, for any set $\Gamma \cup \{A\}$ of $\mathcal{L}$-formulas,

$$\mathbf{S} + \Gamma \vdash A \Longleftrightarrow \mathbf{Int} + \Gamma \vdash A.$$

Now we introduce the following translations:

$\boldsymbol{t}\colon \boldsymbol{Fm} \rightarrow \boldsymbol{Fm}_{\bigcirc}$ (*Gödel-McKinsey-Tarski translation*; see [5], p. 160.)
$\boldsymbol{s}\colon \boldsymbol{Fm}_{\Box\bigcirc} \rightarrow \boldsymbol{Fm}_{\Box}$ (*splitting*; see [5], p. 165.)
$\boldsymbol{T}\colon \boldsymbol{Fm}_{\Box} \rightarrow \boldsymbol{Fm}_{\Box\bigcirc}$ (*translation* T; see [5], p. 178.)
$\boldsymbol{tr}\colon \boldsymbol{Fm}_{\Box} \rightarrow \boldsymbol{Fm}_{\Box}$, where $\boldsymbol{tr} = \boldsymbol{s} \circ \boldsymbol{T}$.

Briefly, the first three of these translations can be described as follows:

$\boldsymbol{t}$: $\bigcirc p \mid \boldsymbol{t}(\alpha) \wedge \boldsymbol{t}(\beta) \mid \boldsymbol{t}(\alpha) \vee \boldsymbol{t}(\beta) \mid \bigcirc(\boldsymbol{t}(\alpha) \rightarrow \boldsymbol{t}(\beta)) \mid \bigcirc\neg\boldsymbol{t}(\alpha)$
$\boldsymbol{s}$: $\boldsymbol{s}(A) = A \mid \boldsymbol{s}(\bigcirc\alpha) = (\alpha \wedge \boldsymbol{s}(\alpha)) \mid \boldsymbol{s}(\Box\alpha) = \Box\boldsymbol{s}(\alpha)$
$\boldsymbol{T}$: $\bigcirc p \mid \boldsymbol{T}(\alpha) \wedge \boldsymbol{T}(\beta) \mid \boldsymbol{T}(\alpha) \vee \boldsymbol{T}(\beta) \mid \bigcirc(\boldsymbol{T}(\alpha) \rightarrow \boldsymbol{T}(\beta)) \mid \bigcirc\neg\boldsymbol{T}(\alpha) \mid \Box\boldsymbol{T}(\alpha)$

Accordingly, for any $\nu \in \{\boldsymbol{t}, \boldsymbol{s}, \boldsymbol{T}, \boldsymbol{tr}\}$ and a suitable set Δ of formulas, we define $\nu(\Delta) = \{\nu(\alpha) \mid \alpha \in \Delta\}$.

Further, we define the mappings:

$$\begin{aligned}
&\sigma\colon \mathbf{Int} + \Gamma \mapsto \mathbf{Grz} + \boldsymbol{t}(\Gamma)\\
&\tau\colon \mathbf{mHC} + \Delta \mapsto \mathbf{K4.Grz} + \boldsymbol{tr}(\Delta)\\
&\lambda\colon \mathbf{mHC} + \Delta \mapsto \{A \mid \mathbf{mHC} + \Delta \vdash A\}\\
&\mu\colon \mathbf{K4.Grz} + \Delta \mapsto \{\alpha \in \boldsymbol{Fm}_{\bigcirc} \mid \mathbf{K4.Grz} + \Delta \vdash \boldsymbol{s}(\alpha)\}.
\end{aligned}$$

It is well known that σ is an isomorphism; see e.g. [3], Proposition 1. Also, it was announced in [1], Section 3, and proved in [4], Corollary 6, that τ is an isomorphism. Using Theorem 1 and that σ and τ are isomorphisms, we obtain the following.

Theorem 2. In the diagram below mappings λ and μ are join epimorphisms and the diagram is commutative.

$$\begin{array}{ccc}
\mathit{NExt}\boldsymbol{mHC} & \xrightarrow{\tau} & \mathit{NExt}\boldsymbol{K4.Grz}\\
{\scriptstyle\lambda}\downarrow & & {\scriptstyle\mu}\downarrow\\
\mathit{NExt}\boldsymbol{Int} & \xrightarrow{\sigma} & \mathit{NExt}\boldsymbol{Grz}
\end{array}$$

Theorem 2 resembles Theorem of [3], which also states commutative of the diagram of the normal extensions of the logics **Int**, **Grz**, **KM** and **GL** (formulated in $\mathcal{L}_{\Box}$). Indeed, the proof of Theorem 2 implements the same idea which the proof of Theorem of [3] does.

Since **KM** is a normal extension of **mHC**, in view of Theorem of [3] and Theorem 1, $[\mathbf{mHC}, \mathbf{KM}] \subseteq \lambda^{-1}(\mathbf{Int})$ and $[\mathbf{K4.Grz}, \mathbf{GL}] \subseteq \mu^{-1}(\mathbf{Grz})$.

We conclude formulating the following problems:

Problem 1. Is it true that $\lambda^{-1}(\mathbf{Int}) = [\mathbf{mHC}, \mathbf{KM}]$?

Problem 2. Is it true that, for any $L \in \mathrm{NExt}\mathbf{Int}$, $\lambda^{-1}(L) = [\mathbf{mHC} + L, \mathbf{KM} + L]$?

Problem 3. Is it true that $\mu^{-1}(\mathbf{Grz}) = [\mathbf{K4.Grz}, \mathbf{GL}]$?

Problem 4. Is it true that for any $M \in \mathrm{NExt}\mathbf{Grz}$, $\mu^{-1}(M) = [\mathbf{K4.Grz} + M, \mathbf{GL} + M]$?

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Logics of Non-Deterministic Quasiary Predicates

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Logics of quasiary predicates naturally arise when partial predicates over partial variable assignments are considered. Such logics are generalizations of traditional logics of n-ary predicates. In [1, 2] main properties of logics over partial single-valued (deterministic) predicates were studied. Here we investigate properties of many-valued (non-deterministic) quasiary predicates and their special cases.

Logics of such predicates are defined in the following way. Let V be a set of *names* (variables) and A be a set of *basic values* (urelements). Partial mappings from V to A are called *nominative sets*; their class is denoted ${}^{V}\!A$ (in traditional terms nominative sets are *partial variable assignments*). The class of non-deterministic (many-valued) predicates over ${}^{V}\!A$ is denoted $PrND^{V,A}$. Then an algebra

$$APrND^{V,A} = \langle PrND^{V,A}, \vee, \neg, R^{v_1,\ldots,v_n}_{x_1,\ldots,x_n}, \exists^x \rangle,$$

with such operations (called compositions) as *disjunction* $\vee$, *negation* $\neg$, *renomination* $R^{v_1,\ldots,v_n}_{x_1,\ldots,x_n}$, and existential *quantification* $\exists^x$ is defined. The compositions are defined in the style of Kleene's strong connectives using technique described in [3]; parameters $x, x_1, \ldots, x_n, v_1, \ldots, v_n$ belong to V, $n \geq 0$. Such algebras (for various A) form a semantic base of a logic of non-deterministic quasiary predicates. Formulas of the logic are terms of these algebras. Interpretations are defined in a usual way.

For a predicate $p \in PrND^{V,A}$ its *truth* and *falsity domains* are denoted $T(p)$ and $F(p)$ respectively; the *image* of $d \in {}^{V}A$ under p is denoted $p[d]$.

A predicate $p \in PrND^{V,A}$ is

- *single-valued* (has gaps) if $T(p) \cap F(p) = \varnothing$;
- *glut* (dual to single-valued) if $T(p) \cup F(p) = {}^{V}A$;
- *monotone* if $p[d_1] \subseteq p[d_2]$ for any $d_1, d_2 \in {}^{V}A$ such that $d_1 \subseteq d_2$;
- *antitone* (dual to monotone) if $p[d_2] \subseteq p[d_1]$ for any $d_1, d_2 \in {}^{V}A$ such that $d_1 \subseteq d_2$.

Obtained predicate classes are denoted $Pr^{V,A}$, $PrG^{V,A}$, $PrM^{V,A}$, and $PrA^{V,A}$ respectively. These classes (for various A) form a *semantic base* for corresponding logics of non-deterministic quasiary predicates.

The following *consequence relations* are required to catch adequately specifics of these classes:

- logical consequence $\models$;
- truth domains consequence $\models_{\mathrm{T}}$;
- falsity domains consequence $\models_{\mathrm{F}}$;
- truth-falsity domains consequence $\models_{\mathrm{TF}}$.

The main obtained results for the defined logics of non-deterministic quasiary predicates are the following:

- it is proved that predicate classes $Pr^{V,A}$, $PrG^{V,A}$, $PrM^{V,A}$, and $PrA^{V,A}$ form sub-algebras of $APrND^{V,A}$;
- properties of these sub-algebras are studied;
- properties of consequence relations $\models$, $\models_{\mathrm{T}}$, $\models_{\mathrm{F}}$, and $\models_{\mathrm{TF}}$ are investigated;
- sound and complete calculi of a sequent type are constructed for the considered logics.

The constructed logics are more adequate for specification of software systems because they are semantics-based logics that reflect such system properties as quasiarity, partiality, and non-determinism.

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Predictive Competitive Model Game Trees

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Competitive game tree modeling and predictive game logic is briefed. The techniques are developed on descriptive game models and compatibility is characterized. Specific game models are presented to illustrate the techniques. Minimal prediction is a technique defined since the authors model- theoretic planning project. It is a cumulative nonmonotonic approximations attained with completing model diagrams on what might be true in a model or knowledge base. A predictive diagram for a theory T is a diagram $D(M)$, where M is a model for T, and for any formula q in M, either the function $f\colon q \to \{0,1\}$ is defined, or there exists a formula p in $D(M)$, such that $T \cup p$ proves q; or that T proves q by minimal prediction. A generalized predictive diagram is a predictive diagram with $D(M)$ defined from a minimal set of functions. The predictive diagram could be minimally represented by a set of functions $f_1, \ldots, f_n$ that inductively define the model. The free trees we had defined by the notion of provability implied by the definition, could consist of some extra Skolem functions $g_1, \ldots, g_l$ that appear at free trees. The f terms and g terms, tree congruences, and predictive diagrams then characterize fragment deduction with free trees. The predictive diagrams are applied to discover models for game trees. The techniques are developed on a descriptive game logic where model compatibility is characterized on von Neumann, Morgenstern, Kuhn VMK game descriptions model embedding and game goal satisfiability.

Definition 1. A VMK game function situation consists of a domain $\mathbb{N} \times H$, where $\mathbb{N}$ is the set of natural numbers, H a set of game history sequences, and a mapping pair $\langle g, f \rangle$. f maps function letters to (agent) functions and g maps pairs from $\mathbb{N} \times H$ to $\{\mathrm{t}, \mathrm{f}\}$.

We can state preliminary theorems on VMK sequent action games. Basic agent logic soundness and completeness areas were examined by the first author on e.g. (Nourani 1996,1999). Making preliminary assumptions on VMK game situations let us examine soundness and completeness questions. Let us consider stratification as the process whereby generic diagrams are characterized with recursive computations on action sequent functions on game trees. Example conclusions are:

Theorem 1. A set of first order definable game tree goals G is attainable iff there exists an predictive diagram for the logical consequences to G on the game tree model.

Theorem 2. There is a sound and complete action sequent logic on VMK game situations provided

(i) VMK action sequent language is a countable infinitary language fragment.
(ii) The information partitions are definable on a countable generic diagram.
(iii) Game tree node ordinal is definable with a countable conjunction on the generic diagrams.

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Harmonizing involutive and constructive negations

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It is well known that in linear logic it is possible to define two distinct forms of consistency on the basis of the two constants for false: $\perp$ and 0. *Strong consistency* is defined as $\perp$ is not provable, while *weak consistency* as 0 is not provable. It can be easily shown that the first entails the second, but the reverse does not hold. So for example, the set $\{a, a^{\perp}\}$ is not $\perp$-consistent, but it is 0-consistent. In classical logic, 0 is equivalent to $\perp$, so the distinction does not hold. In intuitionistic logic, it is even impossible to express $\perp$-inconsistency.

The significance of these two forms of consistency is here investigated by exploiting the two kinds of negations arising in this setting: $A^{\perp} := A \to \perp$ and $A^{0} := A \to 0$. The aim of this work is twofold. First, we investigate the possibility of making involutive (thus respecting classical principles) and constructive (thus respecting intuitionistic principles) negations co-exist in a natural deduction framework. One of the interesting features of such a setting is that it allows to avoid the well-known collapse obtained when classical and intuitionistic negations are mixed in a single system. Secondly, we justify our framework along proof theoretic lines.

Fuzzy Syllogisms

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Traditionally, the connectives in Aristotelian logic (**AL**) are denoted by ***a***, ***e***, ***i***, ***o*** and mean inclusion, disjointedness, non-void intersection, and non-inclusion of sets, i.e. we have for any interpretation "**.**":
"P ***a*** Q" $\Leftrightarrow \forall x[\sim(x \in$ "P" $\wedge \sim(x \in$ "Q"$))]$,
"P ***e*** Q" $\Leftrightarrow \forall x[\sim(x \in$ "P" $\wedge\ x \in$"Q"$)]$,
"P ***i*** Q" $\Leftrightarrow \exists x[x \in$"$P$" $\wedge\ x \in$"Q"$]$,
"P ***o*** Q" $\Leftrightarrow \exists x[x \in$"$P$" $\wedge \sim(x \in$"Q"$)]$.

In *propositional* Aristotelian logic (**PAL**) set variables P and Q are replaced with propositional variables p and q.Then in **PAL** we define for any interpretation "**.**":
"p ***a*** q" $= \sim($"p" $\wedge \sim$"q"$)$,
"p ***e*** q" $= \sim($"p" $\wedge$ "q"$)$,
"p ***i*** q" $=$ "p" $\wedge$ "q",
"p ***o*** q" $=$ "p" $\wedge \sim$"q".

It is easy to prove that the problem of logical consequence for **AL** is reduced to the same problem for **PAL**. Let us introduce *fuzzy propositional* Aristotelian logic (**FPAL**). It has the same syntax as for **PAL** and has the following semantics:
"p ***a*** q" $= 1 - \min\{$"p"$, 1 -$ "q"$\}$,
"p ***e*** q" $= 1 - \min\{$"p"$,$"q"$\}$,
"p ***i*** q" $= \min\{$"p"$,$"q"$\}$,
"p ***o*** q" $= \min\{$"p"$, 1 -$ "q"$\}$.

Let $s, t \in [0,1]$, $s \leq t$ and α be a **FPAL** statement.Then $s \leq \alpha \leq t$ is called an *estimate* (for the statement α). Estimates can be considered as statements of a crisp logic with fuzzy interpretation. By definition, "$s \leq \alpha \leq t$" is true if and only if the double inequality $s \leq$ "α" $\leq t$ takes place. The logic **eFPAL** of estimates has the relation '|=' of logical consequence: for a set E of estimates and an estimate ε, E $|=\varepsilon$ if and only if there is no interpretation such that all $\delta \in$E are true and ε is false. But **eFPAL** has also the relation '$|=^s$' of *strong* logical consequence: $E \models^s s \le \alpha \le t$ if and only if $E \models s \le \alpha \le t$ but it is not true that $E \models s' \le \alpha \le t'$ when $s < s'$ or $t' < t$. It is clear that s and t are defined uniquely.

For example, let us consider the strong logical consequence

$a \le p$ ***i*** $q \le c,\ b \le q$ ***a*** $r \le d \mid=^s g \le p$ ***i*** $r \le h.$

Then g and h functionally depend on a, b, c and d. It turns out that $g = \min\{a, b\}$ if $a + b > 1$ otherwise $= 0$, and $h = \max\{c, d\}$ if $c + d < 1$ otherwise $= d$. One can consider $a \le p$ ***i*** $q \le c,\ b \le q$ ***a*** $r \le d \mid- g \le p$ ***i*** $r \le h$ as a sound rule of inference in **eFPAN**. We say that this rule is the *fuzzy syllogism* with the pattern ***iai*** and the parameters a, b, c, d. In general, let λ, μ, $\nu \in\{$***a***, ***e***, ***i***, ***o***, ***a****, ***o****$\}$ where * denotes the inversion of binary relation. Then the rule $a \le p\ \lambda\ q \le c,\ b \le q\ \mu\ r \le d \mid- g_{\lambda\mu\nu} \le p\ \nu\ r \le h_{\lambda\mu\nu}$is the fuzzy syllogism with the pattern $\lambda\mu\nu$ and the parameters a, b, c, d. The task consists in finding of $g_{\lambda\mu\nu}$ and$h_{\lambda\mu\nu}$ for all fuzzy algorithms. Clearly, there are exactly $6^3 = 216$ of functions $g_{\lambda\mu\nu}$ (and$h_{\lambda\mu\nu}$). However, it is enough to find only 6 of them. In fact, there are simple dependences between them.

Clearly that $a \le p$ ***a*** $q \le b \Leftrightarrow 1- b \le p$ ***o*** $q \le 1- a$ and $a \le p$ ***e*** $q \le b \Leftrightarrow 1- b \le p$ ***i*** $q \le 1- a$.Therefore, we can exclude from consideration the syllogisms with patterns containing the symbols ***a*** and ***e***; thus, only $3^3 = 27$ syllogisms remain. It easy to reduce them to syllogisms with 6 patterns: ***aai***, ***aaa****, ***iaa***, ***iai***, ***iii***, ***a*ai***. Here are the functions g and h for these syllogisms:

$g_{aai} = \min\{1- c, d\}$;

$h_{aai} = 1$;

$g_{aaa*} = 1- d$;

$h_{aaa*} = 1$;

$g_{iaa} = b$ if $a + b > 1$, or $1-c$ if $a + b \le 1$ and $c + d < 1$, or 0 if $a + b \le 1$ and $c + d \ge 1$;

$h_{iaa} = 1-a$ if $a + b > 1$, or $\max\{d, 1-a\}$ if $a + b \le 1$;

$g_{iai} = \min\{a, b\}$ if $a + b > 1$, or 0 if $a + b \le 1$;

$h_{iaa} = \max\{c, d\}$ if $c + d < 1$, or d if $c + d \ge 1$;

$g_{iii} = \min\{a, b\}$;

$h_{iii} = \max\{c, d\}$ if $b > c$ and $a > d$, or c if $b \le c$ and $a > d$, or d if $b > c$ and $a \le d$, or 0 if $b \le c$ and $a \le d$;

$h_{a*ai} = 0$;

$h_{a*ai} = \min\{a, b\}$ if $a + b > 1$, or 0 if $a + b \le 1$.

We have found the functions g and h using the method of analytical tableaux for inference in **eFPAN**.

On various concepts of ultrafilter extensions of first-order models

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There exist two known concepts of ultrafilter extensions of first-order models. One comes from modal logic, where it has firstly appeared for binary relations in [1], was used to characterize modal definability [2], and later was generalized to n-ary relations and studied in [3, 4]. Another one comes from algebra of ultrafilters, a technique of extending semigroups by ultrafilters over them, which was used in various Ramsey-theoretic results of number theory, algebra, and dynamics, see [5]. Recently this approach was generalized to arbitrary first-order models in [6], where it was shown that the extended model relates to the original one, roughly, in the same manner as the Stone–Čech compactification to the underlying space.

Here we firstly observe that the operation of ultrafilter extension itself can be extended, in the same way, to ultrafilters over the underlying model-theoretic structure. This leads to the concept of generalized models consisting not of relations and operations but rather of ultrafilters over them. Furthermore, the proposed construction allows to describe both aforementioned concepts of ultrafilter extension uniformly and, moreover, observe a spectrum of possible types of ultrafilter extensions. Finally, we establish new facts on formulas preserved under ultrafilter extensions, provide topological characterizations of the extensions, and describe their interplay with operations of relation algebra, generalizing results of [6, 7].

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Periodicity vs reflexivity in revision theories

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Revision sequences were introduced in 1982 by Herzberger and Gupta (independently) as a set-theoretical tool in formalising their respective theories of self-referential truth. Since then, revision has developed in a general method for the analysis of theoretical concepts, with several applications in many areas of logic and philosophy, like rational belief, circular definitions, theories of truth and meaning, and others (Löewe, 2006, provides a survey).

Revision sequences are understood as iterations of some fixed function, called the *revision operator*, and are usually formalised in set theory as ordinal-length sequences. This move from finite iterations to transfinite ones has to face with the problem of what to do at limit stages. The process of revision is no longer determined by the iterated application of the revision operator from some starting point: we have to add the limit rule as a further parameter in order to specify a revision process, and this addition affects the entire revision theory by an element of (usually unwelcomed) arbitrariness.

A common idea of revision process is shared by all revision theories, but specific proposals can differ in the limit rule they suggest and in the philosophical analysis of the introduced arbitrariness. Two prominent proposals in the literature on revision are Herzberger's (1982) and Gupta-Belnap's (1993): they are representative of two quite opposite ways of handling arbitrariness in revision sequences (deterministic vs non-deterministic limit rules) and show different mathematical features, which we can resume under the properties of *periodicity* and *reflexivity*, respectively.

In this talk we will concentrate on the role played by periodicity and reflexivity in connection with the appraisal of both deterministic and non-deterministic limit rules. This study aims for a critical use of the method of revision in philosophy and, in particular it aims to avoid the risk, in performing a revision-theoretic analysis, of ascribing to the object of the analysis, for instance the concept of truth, properties which depend on the mathematical tool.

We will isolate a notion of *cofinally dependent* limit rule, encompassing both Herzberger's and Belnap's ones, to study periodicity and reflexivity in a common framework and to contrast them both from a philosophical and from a mathematical point of view. We establish the equivalence of weak versions of these properties with the revision-theoretic notion of recurrent hypothesis and draw from this fact some observations about the problem of choosing the "right" limit rule when performing a revision-theoretic analysis.

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Embedding of First-Order Nominal Logic into Higher-Order Logic

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Nominal logic, also referred to as hybrid logic, is a general term for extensions of ordinary modal logics that introduce a new sort of atomic formulae, the so called *nominals*. These nominals are only true at one possible world and false at every other world. The *shifter*, denoted @, can be used to evaluate the truth of a formula at a given world corresponding to nominal i as in $@_{i\phi}$. The simplest of those nominal extensions including the above ingredients is often denoted $\mathcal{H}$ [4]. Early nominal logics originated from Arthur Prior's research on *tense logics* [7] and were, since then, heavily researched, most notably by Robert Bull, Robert Blackburn [4] and Valentin Goranko [9].

Classical higher-order logic (HOL) (also known as Church's Simple Theory of Types [5]) is an expressive formal system that allows quantification over arbitrary sets and functions. Its semantics is meanwhile well-understood [2] and several sophisticated automated theorem provers for HOL (with respect to Henkin semantics) exist (e.g. *LEO-II*, *Satallax*, *Isabelle*).

We employ an embedding for quantified (first-order) ordinary modal logic into HOL (see [1]) and appropriately augment it for the sound and complete embedding of first-order nominal logic (FONL). The embedding approach suggests that FONL can be regarded a fragment of classical higher-order logic and thus allows the out-of-the-box automation of the here discussed nominal logic using common HOL automated theorem provers.

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As a proof of concept, we implemented the above embedding for *first-order nominal tense logic*, a quantified version of the nominal tense logic used by Blackburn [4], as a theory in Isabelle/HOL and a TPTP problem file in THF format [8]. The embedding implementation was successfully applied to automatically prove certain correspondence theorems between frame conditions and axiom schemes, including those of regular modal logic and those which can only be expressed within the extended expressivity of nominal logic [3]. Further experiments could be expanded to more complex applications, e.g. for formalizations from the field of linguistics and philosophy.

The self-imposed restriction to first-order quantification within the embedding is somewhat artificial: Second-order or even higher-order quantification could easily be reduced to the quantification of the HOL meta-logic. But since the semantics of higher-order notions of nominal logics is not immediately clear, the formal treatment of higher-order quantification in the nominal logic context is further work [6].

Already existing automated theorem provers, such as *hylotab*, *htab* and *spartacus* are restricted to propositional nominal logic and subjacent modal logic K. In contrast, our embedding allows the flexible adjustment of the underlying modal logic (by imposing frame conditions at meta-logic level) and the addition of further modal operators.

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Philosophy

The invited keynote speaker of this session is Jc Beall (page 80).

Between triviality and informativeness — Identity: Its logic and its puzzles

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D. Lewis (*Logic Matters*, 1972, pp. 192–193) famously affirmed that there is never “any problem about *identity*. We never have. Identity is utterly simple and unproblematic.” However, philosophers and logicians seem to be very far from giving up thinking about identity and the various persisting puzzles it raises.

This talk will address the logical nature of the identity relation in model-theoretical terms. I shall introduce a simple generalization of the binary identity relation, and starting from the unary case we will be able to see in a clearer way the trivial flavor of identity as observed by Lewis. Specifically, the present treatment will show why standard definitions of identity are typically circular: (generalized) identities are (given some model $\mathcal{M}$ for the underlying language) mere echoes of the domain M of $\mathcal{M}$.

The unary case (i.e., self-identity) seems to give room to no puzzle at all. As Lewis himself puts it, “[e]verything is identical to itself; nothing is ever identical to anything except itself” (*ibid.*). Lewis’ view could thus be taken as the view that identity is really self-identity. It will be shown, however, that there are formal reasons to consider higher arities as well. One reason is that M may be echoed not only through the ‘repetition’ of its elements, as in the case of generalized identity whose members are tuples such as (a), (a,a), (a,a,a), etc. (for $a \in M$), but also through function applications such as $f{:}\,M \to M$, $g{:}\,M^2 \to M$, $h{:}\,M^3 \to M$, etc. The unary case is simply the special case in which the function is the zero-ary identity function, whereas the standard binary relation is the unary identity function, and so on.

This kind of function application is not necessarily problematic, though, if we keep the functional notation in the language as well. For instance, in the case of Frege’s identity puzzle (cf. ‘*Sinn und Bedeutung*’, 1892), we could argue that there can only be any cognitive difference between $a = a$ and $a = b$, if b is not a constant but rather some function application such as $f(c)$, where c may then be a constant. This requires, of course, that different constants denote *different* individuals of M. I shall briefly discuss §5.53 of Wittgenstein's *Tractatus* (1922) ("identity of object [should be expressed] by identity of sign") in connection with a recent paper by B. Rogers and K.F. Wehmeier; as well as Frege's own initial conception (in his *Begriffsscrift*, 1879) of identity as a relation between symbols.

It is thus when one allows different constants to denote the same object that identity begins (at best) to have informative content, and (at worst) to generate persistent philosophical puzzles. I shall discuss the relation between this point and the role of identity in natural languages, still bearing on Frege (1892). I shall suggest that the shift from the trivial logic of identity to the actual functioning of identity statements in natural language should be accompanied by a corresponding shift in our traditional conception of an individual in the first place. In brief, we should search for a notion of individual *occurrence* as the basic notion on top of which individuals *simpliciter* are built up. This in turn would allow us to distinguish identity as a relation holding between identical occurrences, and identity as a relation holding between (possibly different) occurrences of the same individual *simpliciter*. I shall finally discuss how this treatment could lead to a general method of solving standard identity puzzles found in the literature.

Logic and Metaphysics

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From Plato's *Sophist* and Kant's *Critique of Pure Reason* to Frege's *Foundations of Arithmetic* and Wittgenstein's *Tractatus* the nature of the relation between what is and what can be stated or represented in formal-logical system has always been one of the most fundamental problems for both logicians and metaphysicians. Within the last couple of decades we witness a revitalization of this issue because a wide variety of philosophical projects which somehow rest on some sort of a logical foundation tends to reach some universal result about reality. To give a few examples, these projects range from Alain Badiou's theory of subject which takes set theory as ontology to Timothy Williamson's argument concerning necessary existents which takes modal logic as metaphysics, or from John Searle's social ontology which takes the semantics of speech acts as its starting point to Graham Priest's real contradictions as limits of thought which takes non-classical (dialethic) logic as the right logic. At this presentation we try to investigate the possibility of putting any limitations to such transitions from what is logical to what is ontological relying on the ontological foundations of the formal-logical objects themselves. To this end we focus on what a formal-logical object is and try to uncover if they rest on anything non-formal. We defend the view that a formal-logical object is constructed whenever a finite set of signs is ordered in a finite sequence by means of the order of natural numbers within the topos (Tr. mekân) of pure constructions which as well rest on the intuitive senses (Ger. Sinne) of One, Two and Three. Formal-logical objects rest ontologically on the natural numbers which should be taken to be a priori objects existing independently of and prior to any formal-logical construction. We conclude the paper with some philosophical consequences of the results we have reached in our investigation.

Multidimensional Questions in Knowledge Dynamics: A Study in Diachronic Logic

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"The primary and pervasive significance of knowledge lies in its guidance of action: knowing is for the sake of doing. And action, obviously, is rooted in evaluation."
— C. I. Lewis

Diachronic logic was introduced by Roman Suszko in [11] as a systematic study of transformations in *epistemological oppositions* — *E-oppositions* for short — in standard set-theoretic settings. An *E-opposition* is therefore an ordered pair of the form $\langle S, O\rangle$, where S is called *the subject* (*the mind*) and O is *the object* (*the world for the mind*). The main component of the subject is a formalized language. The object (model) correlated with the subject is a model of the language. The model-theoretic semantics allows one to define the relation of consequence in a given language; for example, two players (namely — a machine and its environment) within the framework of game semantics and/or computational logic form of an E-opposition. Two extensions of diachronic logic are presented from the point of view of the practical turn in logic [2]. The first is *axiological.* Following the famous formula by C.I. Lewis [8], the concepts of *change*, *development* and *progress* are explicated by means of hierarchical knowledge systems, i.e. structures of the form: $S = \langle T, R, h\rangle$, where T is a set of transformations of E-*oppositions*, h is a hierarchy of values in S, and R is a relation over T. The second extension — an *erotetic* one — carries with it a lesson from the methodological analysis of evolutionary developmental biology. The so-called *single-interrogative* (or *individual-interrogative*) *approach* to scientific problems appears to be somewhat inadequate. Hence E-oppositions with multidimensional questions (interrelated interrogatives) should be consistently elaborated. Relevant concepts related to *erotetic non-rationality* (i.e. irrationality and counterrationality) are briefly sketched out.

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Is 'self-evidence' evident?

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This talk focuses on the notion of 'self-evidence'. It is usually understood that a self-evident proposition is the one that does not need any explanation or argument. One, a priori knows a self-evident propositions' truth and this proposition is not derived from any other propositions. Thus it may said that a self-evident proposition is 'evident' to everyone. But we also know that some self-evident propositions are thought to be so, while others evaluate those propositions not self-evident. Then what is to be a proposition to be self-evident? Does self-evidence depends on the agent's theoretical inclinations? Are there types of self-evident propositions? Are there fixed conditions that makes a proposition self-evident? Is it more appropriate to use 'being taken self-evident' instead of 'being self-evident'? Is 'self-evidence' a pragmatic consideration? I will deal those questions in order to argue that 'self-evidence' is a certain degree subjective notion that plays a necessary role in the logical deductions.

Rational Agents with-in Logic and its Semantics

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In this presentation, I will argue some philosophical points of Logics. It seems that there are many topics worth to be argued, however, I will mentioned only with the issues which are relevant to the semantics, especially algebraic semantics.

It would be argued with Philosophy of Logic that there is different opinion concerning the 'correct' exposition of logical principle P between supporters of logical system S and supporters of one S'. The word 'correct' should be paid attention. This implies some presupposition: only one particular logical system has the correct exposition of logical principle, in some sense. This is compared with the problem concerned with Continuum Hypothesis in mathematics, some people have argued with the genuine continuums before Cohen.

I will diagnose this presupposition disease. It is easy to be seen in algebraic-semantics and criticism of it. Algebraic-semantics is made by being only transformed from axioms of logics into equations or inequalities, therefore, I think, we cannot understand the meaning of logic by algebraic- semantics. I will talk about it from the result of McKinsey-Tarski (1944). However, this criticism is based on the disease of having a lust for the ultimate first principle. Moreover, it is ill to wish for significance and meaning in logical system and in semantics.

The meaning and significance of logical system and of its semantics, I think, do not exist in the object-language, but can be found in meta-languages (reality) of logical systems and ontic (Wirklichkeit) or ontological space which are beyond the meta-language. It is, therefore, natural and better to argue with them without the word 'correct', 'genuine' and 'right', in this sense my position is different from logical pluralism. And to argue philosophically, it is important to mention to Rational Agents who are user of object- and meta-language and connect to reality of logical systems and the Wirklichkeit.

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The Inseparability of Lingua Universalis from Calculus Ratiocinator

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Historians of logic have made great use of Leibniz's distinction between a *lingua universalis* (*lu*) and a *calculus ratiocinator* (*cr*). In [4] Hintikka states: 'On the one hand, Leibniz proposed to develop a [...] *lingua characteristica* which was to be a universal language of human thought [...] On the other hand, [...] a *calculus ratiocinator* [...] as a method of symbolic calculation which would mirror the process of human reasoning' (p. ix). Hintikka describes two parallel streams of the development of modern logic: Boole, Peirce and Schröder developed *calculus ratiocinator* and Frege concentrated on *lingua characteristica* (p. ix). Heijenoort (in [7]) defends Frege's *Begriffsschrift* ([3]) against the criticism of Schröder: 'unlike Boole, his logic is [...] not merely a *calculus ratiocinator*, but a *lingua characterica*. If we come to understand what Frege means by this opposition, we shall gain a useful insight into the history of logic' ([7], p. 324); and 'However, the opposition between *calculus ratiocinator* and *lingua characterica* goes much beyond the distinction between the propositional calculus and quantification theory' (ibid., p. 325). {All underlining is mine for emphasis.}

In [6] Peckhaus questions the opposition of these two as indicated by the question mark in the title. I argue that the historians of logic are somewhat misled and that

Aristotle, Leibniz, Boole and Frege, employed the integrated inseparability of *cr* and *lu* rather than an opposition in their works.

Aristotle begins [1] with: 'First we must state the subject of the enquiry [...] it is about demonstrative understanding. Next, we must determine what a proposition is, what a term is, and what a deduction is [...]' (24[a]). This is a perfect introduction to the integration of *cr* and *lu*. The purpose of logic is to lay down the rules for valid deduction, but to understand these rules we must first construct a *lu* through which we will formulate the types of deduction and the rules for valid deduction. Aristotle provides the rules for determining validity of syllogisms, this is *cr*, and these are sustained until today. It is also clear to Aristotle that without the ideography, without knowing what is the major term, minor term and the middle term, these rules cannot even be stated; hence, *lu* and *cr* are inseparably linked.

Leibniz in [5] stated this inseparable link clearly in a letter to Remond: 'I should still hope to create a *universal symbolistic* [*spécieuse générale*] in which all truths of reason would be reduced to a kind of calculus. At the same time this could be a kind of universal language or writing [...]' (1715). A *lu* then is a necessary condition for *cr*.

Boole stated in [2]:

> That which renders Logic possible, is the existence in our minds of general notions, [...] The theory of Logic is thus intimately connected with that of Language. A successful attempt to express logical propositions by symbols, the laws of whose combinations should be founded upon the laws of the mental processes which they represent, would, so far, be a step toward a philosophical language (1847, pp. 4–5).

This integration of laws of thought with language is the inseparability of *cr* from *lu*. Frege's in [3] has a beginning reminiscent of Aristotle:

> In apprehending a scientific truth we pass, as a rule, through various degrees of certitude. [...] a general proposition comes to be more and more securely established [...] through chains of inferences [...] The most reliable way to carry out a proof [...] is to follow pure logic. My initial step was to attempt to reduce the concept of ordering in a sequence to that of *logical* consequence [...] I had to bend every effort to keep the chain of inferences free of gaps. [...] I found the inadequacy of language to be an obstacle; [...] This deficiency led me to the idea of the present ideography (1879, pp. 5–6).

Frege begins proof theory as a component of *cr*, but in developing a *cr* for sequences he finds the inadequacy of the language used by the mathematicians at his time; so he moves to his notational ideography which is the *lu* that became necessary for him to carry out the *cr*.

In conclusion, contrary to the claims of some historians of logic that *cr* and *lu* are in opposition, we find that as far as the greatest logicians Aristotle, Leibniz, Boole and Frege are concerned, they all, amazingly, in synchronicity, begin with *cr* as the core of logic, but then quickly realize that attaining any adequate *cr* is impossible without a well worked out ideography provided through the construction of an *lu*. An *lu* then is a necessary condition for *cr*, hence *lu* and *cr* are never in opposition.

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Universal-Particular Relationship in Solipsist Logic

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Şafak Ural's solipsist ontology allows us to understand the way our consciousness and language work, as well as their relations with physical things. One of main points in this redefinition is the "space conception". Space conceptions make it possible both to define physical things in terms of my consciousness and re-evaluate the subject-object relationship from a new perspective.

Şafak Ural's solipsist ontology gives us a chance to investigate universal and particular relationship from a different point of view. With solipsist ontology we can construct an inferential system based on language. Our goal is to indicate the relation of language and logic from point of solipsist ontology.

A meta-theoretical interpretation of the logical square and hexagon of opposition

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Below I submit novel interpreting the square and hexagon as meta-theoretical ones organizing logical interconnections among the notions "inconsistent", "consistent", "incomplete", "complete", "a-priori", and "a-posteriori" in a system of meta-theoretical knowledge.

Let the symbol "t" stand for a theory based on classical logic. Let the symbol INCONS(t) stand for the meta-theoretical statement "t is logically inconsistent". The symbol CONS(t) stands for the meta-theoretical statement "t is logically consistent". COMP(t) stands for the statement "t is logically (semantically) complete". INCOMP(t) stands for the statement "t is logically (semantically) incomplete". APOSTERIOR(t) stands for the statement "t is a-posteriori (empirical) one", i.e. "t is either logically inconsistent or logically (semantically) incomplete". APRIOR(t) stands for the statement "t is non-empirical (a-priori) one", i.e. "t is logically consistent and logically (semantically) complete". The paper submits quite a new thesis that the above mentioned six meta-theoretical statements make up the following logical square and hexagon of opposition.

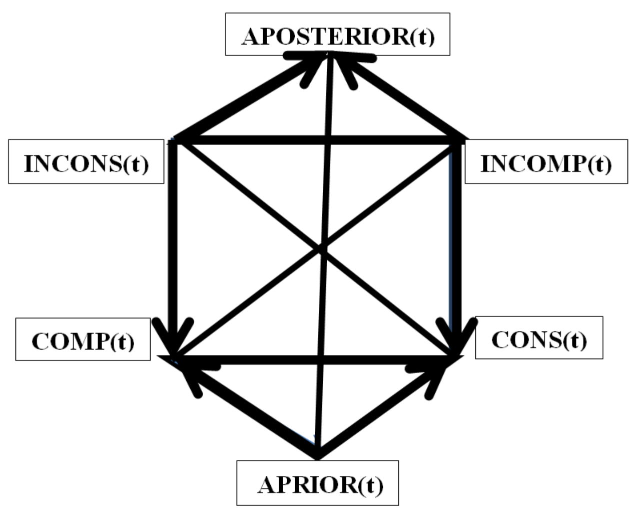

Figure 1: The square-and-hexagon-of-opposition of the meta-theoretical statements

In this graphic model the relations of logical contradiction (contradictory-ness) are represented by the lines crossing the square. The relations of logical subordination (logical consequence) are represented in the picture 4 by the arrows. The relation of logical contrariety (contrary-ness) is represented by the upper horizontal line of the square. The relation of logical sub-contrariety (sub-contrary-ness) is represented by the lower horizontal line of the square.

This square graphically represents the knowledge of several famous meta-theorems as a logically organized system. According to this graphical model, the first-order-predicate-logic and the propositional one belong to the set of a priori theories. On the contrary the formal arithmetic theory (investigated by Gödel) belongs to the set of a posteriori ones.

Obviously, in the times of Aristotle, Leibniz, and Cant it was impossible to create the above-presented meta-theoretical interpretation of the square of opposition as the empirical basis for such creation did not exist in their times: the relevant meta-theorems had not been proved yet. The set of proofs of meta-theorems necessary and sufficient for making up the empirical basis of the above-submitted meta-theoretical interpretation of the square appeared only in XX century.

The logic of David Hume's Dialogues concerning natural religion

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The topic discussed in Parts 10 and 11 of David Hume's *Dialogues Concerning Natural Religion* is whether the design of the world supports the claim that the designer of the world is a benevolent being. The one who defends this position is Cleanthes; the one who opposes it is Philo. Throughout Philo's critical comments, at no point does he claim that he has now refuted the benevolence claim. By applying formal logic to their respective positions, I am able to show the precise point at which the benevolence claim has been refuted, and why the remainder of Part 11 continues as it does.

These two Parts of the *Dialogues*, therefore, can be seen to have a pentimento — like character, wherein the full appreciation of the Hume's position requires that we go beyond the words on the page to the underlying logic.

Philosophical Significance of Title of Lindenbaum's Maximalization Theorem

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The Lindenbaum maximalization theorem (LMT) says:
(*) Every consistent set of sentences has its maximally consistent extension.

This theorem has numerous applications in metalogic and metamathematics, for example, in proving the completeness theorem for first-order logic by the Henkin method or as the main formal background for the theory of degrees of completeness. Other, recently even more important, use of LMT consists in its comparison with other maximalization principles or the axiom of choice. From this perspective, (LMT) is interesting for reverse mathematics. On the other hand, (LMT) seems to have an importance for some problems of the philosophy of logic. Tarski and Lindenbaum proved in the 1930s that classical propositional logic (CPL) is the only maximal consistent extension of intuitionistic propositional logic IPL (and, a fortiori, so-called intermediate logics located between IPL and CPL). If one considers CPL as an extension of IPL and not claim that the latter is the logic, the former can be considered as the absolute logic.

Since (LMT) goes via the consequence operation it is syntactic in its character. This situation does not change in predicate logic (PL). On the other hand, the analysis of propositional calculus cannot be automatically extended to PL, because it is not Post-complete. Hence, a semantic aspect becomes important in PL also in the case of (LMT). In general, the collection of truths in a given model M is always maximally consistent. If we take Peano arithmetic as the critical theory, the existence of non-standard models as associated with incompleteness, shows that the pragmatic aspect cannot be eliminated from the concept of the standardness. Thus, the concepts of the standard model and the intentional model are equivalent (or functions at least as homonyms). Consequently, this fact justifies Tarski's observation that the concept of truth is meaningless for purely formal (non-interpreted) systems. Although (LMT) assures that we have many possible Lindenbaum's extension in arithmetic, the choice between alternatives cannot be fully justified even by semantics.

Computation

The invited keynote speaker of this session is Ahmet Çevik (page 85).

NP system and Mimp-graph association

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Since reasoners/provers logic-based carry with them the production of exponentially size proofs, which can hamper the presentation of proofs in feasible time, it is important the use of methods that reduce the size of the generated proofs. It is well know that the use of techniques based on graphs can deal with the compression of logical proofs which can be employed during the construction of proofs. Mimp-graph is a new structure to represent proofs through references rather than copy. This structure was initially developed for minimal propositional logic but the results have been extended to first-order logic. Mimp-graph preserves the ability to represent any Natural Deduction proof and its minimal formula representation is a key feature of the mimp-graph structure, it is easy to distinguish maximal formulas and an upper bound in the length of the reduction sequence to obtain a normal proof. Thus a normalization theorem can be proved by counting the number of maximal formulas in the original derivation. The strong normalization follow as a direct consequence of such normalization, since that any reduction decreases the corresponding measures of derivation complexity. Sharing for inference rules is performed during the process of construction of the graph. This feature is very important in order to be used in automatic theorem provers. In [1] and [2], it is presented a new deductive system, denominated Np, that shows to be more effective to be implemented in automatic theorem proofers. The major difference in this system, compared to the traditional ones, is that it makes use of Peirce rule instead of the classical absurd rule. A worth of mentioning case is the Glivenko theorem for the minimal implicational system Np. It was proved that the implicational fragment with the Peirce rule is normalizable, therefor has a sub-formula principle, and besides that all classical proof can be viewed as a intuitionistic proof except for the final part, that has only Peirce's rules. In this paper we will presented a association between Np system and mimp-graph as well the main properties that this association has.

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Philosophical Aspects of Programming Theory Development

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The following three levels of Computing foundations were proposed in [1]: 1) *philosophical*, 2) *scientific*, and 3) *formal (mathematical)*. Here we continue investigations of this three-level scheme applying it to Programming theory development. For this case we study interrelations of the levels concentrating on relations of the philosophical level with the other two levels. It should be noted that the importance of philosophical foundations for information-related disciplines (in particular, for programming) is widely recognized. Different philosophical systems were proposed for this purpose, for example, K. Popper's ontology, philosophy of Kuhn and Peirce, specific epistemology of Hjorland, etc. A short description of philosophical approaches could be found in [2], philosophical aspects of the main mathematical notions were discussed in [3].

Our approach to Programming theory development [4] is based on Hegel's logic [5] presented in the modified form in [6]. Therefore, the main principles of the approach are the principle of *development from abstract to concrete*, the principle of *triadic development* (thesis — antithesis — synthesis), and the principle of *unity of theory and practice*.

The above-mentioned levels identify three types of notions that constitute the basis of each level respectively: philosophical level — *categories* (infinite notions in Hegel's terminology), scientific level — *finite (scientific) notions*, and formal level — *formal finite (mathematical) notions*. Thus, interrelations of the levels are tightly connected with relations between categories, scientific notions, and formal notions respectively. We distinguish two transitions between levels: from categories to scientific notions and from scientific notions to formal notions. The first transition is called *finitization* and the second one — *formalization*. The finitization transition transforms categories presented with the help of Hegel's categories *universal — particular — singular* into scientific notions described as integrity of their *intensions — extensions*. The formalization transition constructs formal intensions and extensions of scientific notions.

At the *philosophical level* we start with the following triad of categories: *subject — goal — means*. Then we enrich this triad with two new triads: *subject — means — means usage* and *goal — means — means construction*, finally obtaining the following pentad: *subject — goal — means — means usage — means construction*.

At the *scientific level* we make finitization of categories of this pentad obtaining the following *pentad of programming notions*: *user — problem — program — program execution — program construction*. We investigate finitization transition between the levels represented by pairs *subject–user, goal–problem, means–program, means usage–program execution, means construction–program construction*. We also make further development of the notions of programming pentad, in particular, we specify the notion of program via *program pentad*: *data — function — function name — function composition — function description*.

The second transition — formalization — poses a number of philosophical questions concerning the category of *formal*, its relation to other categories, expressivity and limitations of formal notions, etc. Analysis of these questions permits to explain construction of programming models in integrity of their philosophical, scientific, and formal aspects.

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On non-standard finite computational models

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In 1924, Tarski proved that the many existing and well-known definitions of finite set are equivalent. Tarksi mentioned Peano-finiteness, Dedekind-finiteness and some inductive definitions due to Russell, Sierpiński and Kuratowski. A first fact to be noted is that he had to use the Axiom of Choice in his proof. Another fact is that, due to the duality between finite and infinite, when defining one of these concepts the respective dual is obtained by means of negations.This role played by the negation adds a logical dimension to this discussion. Thus, besides the Axiom of Choice, the fact that we are inside Intuitionistic or Classical Set Theory has consequences on the relationship among the many mathematical definitions of finiteness. We have to conclude that finiteness is a relative notion in mathematics. Relative is used to denote when the variation in the position of an observer implies variation in properties or measures on the observed object. We know from Skolem's theorem that some notions are relative. For example, we have some models where R is countable, some where it is not. This fact depends on the position of the observer. In this specific case it may depend on whether he/she is inside the model or not. This internal/external point-of-view is the simplest case analysis. Based on the fact that finiteness is a relative concept in mathematics, we intend to start a discussion on the role of finiteness in the Theory of Computation by fixing the finiteness concept used in a Turing-machine definition. For example, by choosing the Dedekind definition of finite we have Turing-Dedekind machines and hence, Turing-Dedekind computable "functions". Some Turing-Dedekind machines when observed outside the model have infinite set of states and/or infinitely long transition-tables. However, they are finite when observed from inside the model. We show many other non-standard finite computational models.

We use Topos Theory to provide the models and Local Set Theory to express the properties that are internally and externally interpreted. There is a slight analogy with Malament-Hog(g)art space. In General Relativity, different observers have different notions of finite time, in LST, different models have different notions of finiteness. In both cases, the notion of computability is different. Finally we remark the strong consequence that the existence of Natural Numbers object (NNO) in the Topos has. When the model has a NNO the finiteness notions collapse into Peano-finite and the computational models collapse to standard ones.

New Insights into Minimum Satisfiability

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Minimum Satisfiability (MinSAT) is the problem of finding an interpretation that minimizes (maximizes) the number of satisfied (falsified) clauses in a multiset of Boolean clauses, where a clause is a disjunction of literals, and a literal is either a propositional variable or a negated propositional variable [2]. MinSAT is an optimization variant of the Propositional Satisfiability (SAT) problem, and is also considered to be the dual problem of the Maximum Satisfiability (MaxSAT) problem [3].

It is well-known that SAT offers a generic problem solving paradigm for solving *decision problems*, and nowadays it is often applied in fields as diverse as hardware verification, bioinformatics and planning. Given the success of SAT-based problem solving, the Artificial Intelligence community has recently investigated whether MaxSAT and MinSAT can be used as generic problem solving paradigms for solving *optimization problems*. As a result, several works have shown that both MaxSAT and MinSAT are suitable formalisms for modeling and solving certain combinatorial optimization problems [1,5]. Nevertheless, the solving techniques and encodings to be used in MinSAT are usually different from the ones used in MaxSAT.

In this talk we focus on recent advances in MinSAT solving. In particular, we concentrate on modeling and inference results that we have investigated, and we compare our solutions with the solutions proposed for MaxSAT.

Regarding modeling, we first present how some relevant optimization problems are encoded to MinSAT, and we show that the space complexity of the resulting MinSAT encodings is smaller than the complexity of the corresponding MaxSAT encodings. Then, we report on an empirical investigation that provides evidence that MaxSAT and MinSAT are complementary generic problem solving paradigms for optimization.

Regarding inference, we describe how to compute a MinSAT solution with an inference system [4]. In other words, our aim is to define an inference rule that when applied to a multiset of clauses ψ is able to derive as many empty clauses (i.e.; contradictions) as the maximum number of clauses that can be falsified in ψ. Unfortunately, the resolution rule —the inference rule commonly used to detect contradictions in SAT— is unsound for MinSAT because it preserves satisfiability but not the number of falsified clauses.

We first define a resolution-style inference rule that preserves the number of falsified clauses when the premises of the rule are replaced by its conclusions. Then, we prove that such a rule provides a complete inference system for MinSAT; i.e., the application

of the rule a finite number of times, following a given strategy, derives as many empty clauses as the maximum number of clauses that can be falsified in a MinSAT instance.
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Behavioral equivalence of equivalent hidden logics

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This work advances a research agenda which has as its main aim the application of Abstract Algebraic Logic (AAL) methods and tools to the specification and verification of software systems.

It is based on a generalization of the notion of abstract deductive system to handle multi-sorted deductive systems which differentiate visible and hidden sorts. The main results are obtained by generalizing properties of the Leibniz congruence — the central notion in AAL.

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In this talk we discuss the relationship between the behavioral equivalence of equivalent hidden logics. We also present a necessary and sufficient intrinsic condition for two hidden logics to be equivalent.

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Theorems of Tarski and Gödel's Second Incompleteness — Computationally

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The (first and second) Incompleteness Theorems of Kurt Gödel are among the most significant mathematical results in the twentieth century; these theorems not only concern (and are about some properties of reasoning in) mathematics, but also affect philosophy and computer science. In [2] the author argued that the first incompleteness theorem is the un-effective version of Gödel's Completeness Theorem, in the sense that the completeness theorem is equivalent to the existence of a consistent completion of every consistent theory and the incompleteness theorem is equivalent to the existence of a computable enumerable and consistent theory whose no consistent completion can be computable enumerable. Let us note that the completion procedure preserves decidability, i.e., any consistent and decidable theory can be extended to a complete, consistent and decidable theory (see [4] or [2]).

In this talk we will argue that Tarski's theorem on the undefinability of `Truth` in sufficiently expressive languages is equivalent to Gödel's (semantic form of the) first incompleteness theorem relativized to definable oracles. Then we will move forward to proving Gödel's Second Incompleteness Theorem by elementary methods using the concepts from computability theory. Since Gödel's second theorem (inability of sufficiently strong theories to prove [a statement of] their own consistency) is not robust with respect to the notion of consistency, its proof is much more delicate and elegant than the proof of the first theorem; indeed the proof appears in very few places (see [1] a review of the first edition of [3]). Though, some BOOK proofs (in the words of Paul Erdős) for the first incompleteness theorem exist in the literature, a nice and neat proof (understandable to the undergraduates or amateur mathematicians) for the second theorem is missing. Here, we will present a proof for this theorem from computational viewpoint which will be based on finitizing the theory presented in [2] used for proving the first theorem.

To be more precise, in [2] there was introduced an undecidable (and consistent) theory $\mathcal{T}$ which can be completed to a computable enumerable and consistent theory; a consistent and computable enumerable extension of it, called $\mathcal{S}$, is "essentially undecidable" (cf. [4]) in the sense that no consistent and computable enumerable extension of $\mathcal{S}$ could be complete. This is essentially the Gödel-Rosser Incompleteness Theorem. Relativizing this theorem to definable oracles we get Tarski's theorem on the undefinability of truth, which on its face value has nothing to do with (oracle) computations. As another new result we will see that a finitely axiomatized theory $\mathcal{Q}$ can interpret $\mathcal{S}$, but at a high cost and that is accepting Church's Thesis as to the equivalence of the informal notion of computability with the formal notion of recursivity. A couple of gains are the ability to prove

(1) Church's Theorem on the undecidability of (provability in) first-order logic, and

(2) Gödel's Second Incompleteness Theorem for sufficiently strong and sufficiently expressive theories.

In the latter a new proof for this great theorem is provided by elementary (computation theoretic) methods. The main idea is that by formalizing Gödel's first incompleteness theorem for computable enumerable (and consistent) theories that contain (the finite theory) $\mathcal{Q}$ we can prove that the theories which prove the statement "no consistent and computable enumerable extension of $\mathcal{Q}$ is complete" cannot prove their own (standard) consistency.

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Algebraic and Logical Operations on Operators: One Application to Semantic Computation

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Different domains use operators without giving a clear definition. For instance: operators in vector spaces and in Hilbert spaces; in the theory of partial equations: the *nabla* operator ∇; in analysis, the *laplacian* operator Δ, where $\Delta = \nabla^2$, etc. In chemistry, "the Fock's matrix approximates the operator energy of a quantum system". Logic uses different operators: connectors, modal operators, quantifiers. The psychology uses also operators (Piaget, Frey...). In linguistics, the notion of operator is important but it is not always well defined; for instance, in the categorial grammars, the instances of categories are operators with functional types (Church's type); the linguist S.K. Shaumyan systematically used the abstract operators (combinators) of Curry's Combinatory Logic.

The concept of operator is not exactly the same as that of a function in set theory. From now on, let us reserve the word 'operator' to denote function-as-operation-process concept, and 'function' and 'map' for the set-of-ordered-pairs concept (cf. J.R. Hindley and J.P. Seldin, *Lambda-Calculus and Combinators: An Introduction*, 2008).

In computer science and formal systems, we have at least three different approaches: Church's λ-calculus, Curry's Combinatory Logic, and an algebraic approach with Cartesian Closed Categories. In Curry's approach, complex operators are built by abstract operators, called *combinators*, used to compose different operators, by intrinsic ways, that is: compositions and transformations of operators are independent of any interpretation given to these operators by specific domains. Our approach starts from the Cartesian Closed Categories. We generalize the algebraic work of Lawvere, by an introduction of different types and different sorts of objects. In this paper, we present a study about close connection between typed combinatory logic (CL) and an algebraic approach of operators.

We introduce types generated from a set S of sorts (or basic types). The symbol 'n' designates a natural integer; (i) a function 'μ' (or a word on S) from an ordered set '$[\mathrm{n}] = \{1, 2, \ldots, \mathrm{n}\} = [\mathrm{l}(\mu)]$', into S, represents a (cartesian) type; (ii) if 'μ' and 'ν' are types, then '$\mu \to \nu$' is also a type (*i.e.* a functional type). All the types defined on S are generated by the rules (i) and (ii). A S-coprojectif f^{μ}_{ν} is a map from the set '$[\mathrm{l}(\mu)]$' to the set '$[\mathrm{l}(\nu)]$', such that '$f^{\mu}_{\nu}\, o\, \nu = \mu$' (where '$o$' designates the functional composition in set theory). $\mathrm{T}[K_0]$ is the set of all S-coprojectifs. A space of complex operators $\mathrm{T}[\Sigma]$ is defined from a given set Σ of basic operators. The set of complex operators $\mathrm{T}[\Sigma]$ is algebraically structured by two more abstract T-operations, called "*greffe*",

[1]Sens, Texte, Informatique, Histoire.
[2]Langues, Logiques, Informatique, Cognition.

noted O, and "*intrication*", noted ⊗. The symbol 's' designates an element of S and 'ν' designates a type on S; by X we consider a set (*i.e.* a domain of objects) on that the operators act. If the map ν is identified to the sequence of images: '$s_1, \ldots, s_n$', then: $X^\nu = X^{s_1}x...xX^{s_n}$, where X^{s_i} is a set of objects of the same sort 's_i'. An operator 'σ^s_ν' with the type '$s \to \nu$' is an element of T[Σ]; it is such that, to this operator is associated a family of function '$(\sigma\bullet)^\nu_s$' (*i.e.* contravariant operation) from X^ν to X^s for a given domain X. A multioperator 'σ^μ_ν' is an element (or an arrow) of T[Σ]; it is such that it generates functions '$(\sigma\bullet)^\nu_\mu$' (*i.e.* contravariant operation) from the set X^ν to X^μ. This multioperator 'σ^μ_ν' is built up by the T-operations of "*greffe*" and "*intrication*", from different operators '$\sigma^{s_i}_\nu$' (components of the multioperator 'σ^μ_ν'), where '$\mu = s_1 s_2 \ldots s_n$'. We proved the following proposition: the set T[Σ] of multioperators, built up from a set Σ of operators, is the smallest set such that:

(i) $\Sigma \subset \mathrm{T}[\Sigma]$;
(ii) $\mathrm{T}[K_0] \subset \mathrm{T}[\Sigma]$, where $\mathrm{T}[K_0]$ is the set of all S-coprojectifs;
(iii) T[Σ] is closed under O and ⊗, which are associative;
(iv) each multioperator of T[Σ] is built by O and ⊗ from Σ and the S-coprojectifs of $\mathrm{T}[K_0]$.

To the T-operations O and ⊗ correspond combinators of Combinatory Logic (CL). The forms of these combinators will be given in the paper. Specific graph structures, called "*treilles*", are associated to the constructions of multioperators; these structures are not trees, but they are analog to DOAGs (Directed Oriented Acyclic Graphs) in computing theory.

A semantic study of natural languages leads to a formal analysis of lexical units (verbs and prepositions) by means of operators, multioperators and operands, with different types, and composition of theses operators and multioperators, by combinators or "*greffe*" and "*intrication*". These analyses are presented by means of "*treilles*" structures. We will give some examples of such formal computation.

Completeness

The invited keynote speaker of this session is María Manzano (page 94).

Metatheory of Tableau Systems

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Tableau proofs have a number of advantages in comparison to other proof methods. They can often be conducted automatically and countermodels are often delivered by failed proofs. The advantages are most evident in comparison to standard axiomatic proofs. The chief disadvantage of the tableau method is its intuitiveness, which is extremely problematic in proving soundness and completeness of tableau consequence systems with respect to some semantic consequence relation.

In our talk a perfectly formal account is presented of the question of the tableaux as well as tableau proofs. The approach we propose turns out to be quite successful in dealing with such metalogical problems as soundness and completeness, which will be demonstrated. The account we present extends ideas described in such works as [1, 2, 3]. And we especially extrapolate the tableau method for modal logic, delivered in the work [2] on other kinds of sentential calculi as well as calculi of names.

We begin with a logic, which is to be identified with a particular consequence relation, described semantically. The outcome is a collection of tableau rules that determine together with a concept of tableau proof a tableau consequence relation. Such a collection is called a tableau system. Hence, tableau proofs are regarded a syntactical concept, even if the tableau procedure requires some extensions of the formal language in question. All the tableau concepts we construct are set-theoretical, the graph concept of tableau proof turns out merely didactic presentation of purely formal concepts. And we define generally formal concepts: (a) tableau rule, (b) open, closed and maximal branch, (c) open, closed and complete tableau and (d) branch consequence relation.

By means of such general, formal concepts we are in a position to deliver exact conditions to be satisfied by collections of tableau rules defining tableau systems. In the general metatheory of tableaux we deliver the proofs of metatheorems are included to the effect that equality of the semantical consequence relation and the tableau consequence relation follows from those conditions to be satisfied.

The above mentioned theorem is to be applied to constructions of tableau systems, if the systems are to be sound and complete with respect to a semantical structure. When describing tableau systems we simply apply general concepts and make sure the rules we formulate meet the formal conditions. If it is the case we immediately obtain a sound and complete calculus.

The theory we deliver covers sentential calculi as well as calculi of names. In our talk we present main metatheoretical concepts, the chief metatheoretical theorem and show some instructive examples of application.

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On the Source of the Notion of Semantic Completeness

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Keywords: *Completeness*, *Natural Deduction*, *Sequent Calculi*.

In this talk we examine how the notion of semantic completeness emerged from the entanglement of model theory and proof theory which characterized logical investigations in the first Thirties of the last century. We emphasize the fundamental role played in this process by K. Gödel, who basically introduced that notion in the form then become dominant, carving it out from the clear distinction between syntactic and semantic analysis of a formal system. In his thesis, in fact, he was able to perform the main step from what was a simple operative distinction between two different points of view to a theoretical distinction between two different approaches to the study of formal theories.

We emphasize that other metatheorical notions — such as, for instance, consistency, categoricity, syntactic completeness — focus, each in their own specific way, on the relationship between a formal deductive theory and the field of scientific investigation (concerning geometry, number theory, analysis, etc.) of which it was expected to become the theory. We stress the role played by what can be called the "descriptive" completeness of an axiomatic formal theory, which basically requires that the informal theory be what afterwards will be named the "intended" model of the formal theory. When confronted with semantic completeness, on the other hand, it is not immediately clear what the "corresponding field of scientific investigation" consists of, beyond an elusive reference to the realm of logic. We note that this fact can help to understand why Gödel's work of 1930 remained by and large disregarded within the logic community.

In the second part of the talk we maintain that the link to the pre-gödelian notion of completeness which we referred to as "descriptive" completeness is at the root of the introduction of the natural deduction calculi by G. Gentzen. Both in his thesis and in the 1936 consistency proof for the elementary number theory, the completeness question for the formal system is resumed in the question: "Is anything missing?".

To this end, in the latter paper he conducted a detailed analysis of the way in which in his formal system it was possible to formalize the proof of Euclid's classic theorem on prime numbers. On this basis he maintained that he had succeeded in providing "the most possible complete" list of the inference forms and methods of conceptual definitions usually employed in the elementary number theory. No analogous adequacy or completeness question is asked by Gentzen with respect to the formal systems (the natural deduction and the sequent calculi) he provided to formalize the quantificational theory. We argue that this gap is motivated by his conviction of the non-existence of a realm of first-order logical truths to be captured by his calculi. There is no informal area of investigation that his formal calculi are intended to model. His reference, in this case, was constituted by an informal set of valid quantificational inference schemata determined by the patterns of reasoning actually employed by mathematicians. The point of building the natural deduction calculi was precisely to show that it was possible to formalize the valid quantificational inference schemata which are more or less implicit in mathematical practice. This was the kind of completeness he was aiming to achieve by means of his new calculi, which was a long way from the question of semantic completeness. However, the way the rules of the natural deduction calculi analyse the correct inferences associated with each individual logical operator was not completely analytical. In fact, Gentzen's inferential approach mixed the formalization of the meanings of the logical operators with an account of the consequence relation. Moving from natural deduction calculi to sequent calculi, Gentzen aimed to separate the task of determining the meaning of logical constants from the question of accounting for the inferential features of the deductive system.

Lastly, we emphasize that by devising the sequent calculi Gentzen introduced still another notion of completeness, which pertains the relationship among the rules of the system. By proving the eliminability of the Cut rule, the Hauptsatz proves that the activity of logical analysis suffices to demonstrate the truth of all logical consequences of previously analysed truths. In other words, it proves that the Cut-free fragment of sequent calculi is able to emulate any mathematical reasoning that does not depend on the principles of any specific mathematical theory.

About Syntactic Analogues to Proof of the Completeness Theorem

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Syntactic analogues to proof of semantic completeness meta-theorems for some logics are proposed. They are based on the replacement of semantic assertions of the form 'A has value v^k', for the corresponding formulas $J_k(A)$ with J_k-operator. The J_k-operators (introduced by J. Rosser and A. Turquette) are exploited in the proofs of semantic completeness theorems of a number of truth-complete logics.

Proof of the completeness due to Kalmar is preceded with the proof of lemma 1 [1], which is generalized to the n-valued case.

Lemma 1. Let A be a wff, $S_1, S_2, ..., S_m$ be pairwise distinct variables in A, and, for $S_1, S_2, \ldots, S_m$, a distribution of logical values is given.
Suppose that for every wff B: B' is $J_i(B)$, if B has value v^i.
Then $S'_1, S'_2, ..., S'_m \vdash A'$.

In the proof of this lemma the semantic statements of the form 'B has value v^k' are used, which in turn correspond to the formula $J_k(B)$, with J_k-operator belonging to the syntax of logic L. Thus the phrase 'for $S_1, S_2, ..., S_m$ a distribution of logical values is given' corresponds to 'for $S_1, S_2, ..., S_m$ a distribution of J_k-formulas $(J_{k_1}(S_1), J_{k_2}(S_2), ..., J_{k_m}(S_m))$ is given', which will replace the previous one. We get the following reformulation of Lemma 1.

Lemma 1*. Let A be a wff, $S_1, S_2, ..., S_m$ be pairwise distinct variables in A, and, for $S_1, S_2, ..., S_m$ a distribution of J_k-formulas
$(J_{k_1}(S_1), J_{k_2}(S_2), ..., J_{k_m}(S_m))$ is given. Suppose that for every wff B:
B' is $J_k(B)$, if for B formula $J_k(B)$ is accepted.
Then $S'_1, S'_2, ..., S'_m \vdash A'$.

Let's define the syntactic analogue of a valid formula.

Definition 1. A wff A is called the W-formula iff for all possible distributions of J_k-formulas $(J_{k1}(S_1), J_{k2}(S_2), \ldots, J_{km}(S_m))$ for variables $S_1, S_2, ..., S_m$, that occurred in A, there are the following inferences in the logic L:
$(J_{k1}(S_1), J_{k2}(S_2), \ldots, J_{km}(S_m)) \vdash J_1(A)$.

Theorem 1. If a wff A is a W-formula of L, then A is a theorem of L.

Let's consider a class of logics whose language have some unary operators O_i, where $(1 \leq i \leq n)$:
1) Suppose we have a language of logics L_n, with unary c^1_a and binary c^2_b connectives, with metavariables A, B for wff's and let the classic logic CL $(\neg, \wedge, \supset)$ hold for formulas $O_i(A)$.

The next necessary conditions for obtaining the desired results will be determined by the following below lemmas which we need to prove in the logic L_n:
2) For each i $(1 \leq i \leq n)$ there exists m $(1 \leq m \leq n)$, such that
$O_i(A) \supset O_m(c_a^1 A)$.
3) For each i, k $(1 \leq i, k \leq n)$ there exists m $(1 \leq m \leq n)$, such that
$(O_i(A) \wedge O_k(B)) \supset O_m(A \, c_b^2 \, B)$.
4) $((O_1(B) \supset O_1(A)) \supset \ldots \supset ((O_n(B) \supset O_1(A)) \supset O_1(A)) \ldots)$.
5) There are rules of inference: $O_1(A)/A$ and $A/O_1(A)$, in which O_1 is the designated operator.

Let's replace J_i-formulas for formulas $O_i(A)$ in Lemma 1* and Definition 1.

As the result we have syntactic analogues to proof of the completeness theorem.

Theorem 2. If a wff A is a W-formula of L_n, then A is a theorem of L_n.

Let's define a class of logic in which conditions 1–5 are proved.

Definition 2. A logic submitted in an axiomatized form that meets the above conditions 1–5 will be called an AF-logic.

Finally, let's note that, for example, the axiomatizations of Łukasiewicz logic L_3 and Bochvar logic B_3 belong to the class of AF-logics, in contrast to Heyting logic H_3.

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Atomic systems in proof-theoretic semantics and the problem of completeness

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Within proof-theoretic semantics [2] in the tradition of Prawitz and Dummett the validity of atomic formulas is usually defined in terms of derivability of these formulas in atomic systems. Such systems can be sets of atomic formulas or sets of atomic rules like production rules. One can also allow for atomic rules which can discharge atomic assumptions, or for higher-level atomic rules which can discharge assumed atomic rules. The validity of complex formulas is then explained with respect to atomic systems. Further use of atomic systems is made in explaining logical consequence and the logical constant of implication. An implication $A \to B$ is valid with respect to an atomic system if and only if for all extensions of that system it holds that whenever A is valid with respect to an extension then also B is valid with respect to it. The dependence on extensions guarantees that validity is monotone with respect to atomic systems. It has been conjectured that intuitionistic logic is complete for certain notions of validity of this kind.

In our talk we will present negative as well as positive completeness results (cf. [1]) for some of these notions of validity. First, we consider validity based on atomic systems of production rules. We show by a counterexample that intuitionistic propositional logic is not complete for this notion of validity. In addition, the counterexample shows that validity is not closed under substitution. To be on a par with intuitionistic derivability, which is closed under substitution, we then consider a strengthened notion of validity which is closed under substitution by definition. Failure of completeness of intuitionistic logic can in this case be shown by an indirect proof of the existence of a counterexample. Second, we consider validity with respect to atomic systems of arbitrary higher-level atomic rules. Completeness holds for the two fragments of disjunction-free and negative ($\neg A$) formulas of intuitionistic propositional logic. As our main result we show that full intuitionistic logic is, however, *not* complete for validity based on higher-level rules.

The validity-based approach in proof-theoretic semantics will be opposed to a view of atomic systems as (inductive) *definitions* of atomic formulas. In this case, the situation is very different, since extending a definition changes in general what has been defined originally. When atomic systems are considered as definitions, one does not have monotonicity of derivability with respect to extensions. Furthermore, derivability in atomic systems is no longer transitive. This is in contradistinction to validity, which is a monotone consequence relation with respect to non-definitional atomic systems. Our talk concludes with a proposal for a hybridization of validity-based semantics with the definitional approach to (higher-level) atomic systems.

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History

The invited keynote speaker of this session is Samet Büyükada (page 84).

Adorno and Logic

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The dichotomy of Continental and Analytical philosophy proved to be less than fruitful when it comes to the formers ability to contribute to the discussion about Logic. Often, continental philosophy's understanding of logic seems superficial or redundant to their philosophical goal. At times, a different kind of logic arises out of the philosophical analysis, such in the case of G.W.F. Hegel and his "dialectical logic": another form of logic which differs from "formal logic" as a study of form, nonetheless an intriguing one since it incorporates form and content in its analysis (2010: 751). At first, it might seem that Hegel rejects formal logic as the basic formation of thinking; however an examination of one of his later epigones might reveal that things are not as simple as they seem. In this line of argument I shall focus mainly on the thoughts of the Jewish-German Philosopher, Theodor W. Adorno, one of the leading members of what is known as the "Frankfurt School", the twentieth century most influential traditions in continental philosophy, which could be helpful in adding depth and color to our conception of logic.

It is a consensus that "Logic is concerned with the principles of valid inference" (1971: 1), and that the two branches of logic: formal logic and philosophical logic, although somewhat different, aim at articulating "what makes arguments consistent or inconsistent, valid or invalid, sound or unsound" (1989: 1). However, One can wonder whether a continental philosopher, such as Adorno, can be put to the scrutinizing test of the above principles of logic. For example, can we even judge a statement of a continental philosopher and ascribe a truth value to it? The problem arises, first, out of what is deemed as continental philosophy use of ambiguous terms, fragmented writings, non-linear narratives (or argumentations), and the reliance on non-discursive elements, which analytical philosophy tries to avoid, for instance: Adorno (and Horkheimer's) Dialectic of Enlightenment (1947) [non-linear arguments] or Adorno's Minima Moralia (1951) [fragmented writings]. And second, it seems as though Adorno rejects the principle of non-contradiction, deductive and systematic thought, as he adopts a "dialectical logic" [the acceptance of two contradictory statements or concepts] which might condemn him to irrationality, as argued by Hans Albert (1977c: 286-7).

This lecture argues that Adorno did not repudiate Logic as such, but on the contrary, he thought greatly of it. Nonetheless, Adorno had several reservations to the extent one can actually rely solely on logic as its unexamined axiom. I examine Adorno's understanding of logic in his Introduction (1977a) to The Positivist Dispute in German Sociology (1977) and his essay from the same volume On the Logic of the Social Sciences (1977b). From these articles we can delineate the reason why Adorno was accused in rejecting the principle of non-contradiction: first, Adorno asserts there is no method devoid of content, no intellect without the sensuous, therefore: logic is inseparable from knowledge; second, knowledge is in fact an object of inquiry which is contradictory [Adorno based this on his analysis of modern society (an object according to Adorno) as antagonistic]; and thus if we assume that thought and object are identical, then we must accept two contradictory statements about the same object (1977a: 4).

Actually, Adorno isn't dispensing logic, but logic as a facade, a semblance for equating different theories since they all share the same method of logic. Instead of merely demarcating Adorno as professing a type of irrationalism as Hans Albert does (1977c: 287), since Adorno's account presumably entails the adoption of two contradictory statements, I would claim that what Adorno is offering us is an open logical method of thinking. This method of thinking pays respect to the material which holds contradictions in it, and which prevents us in adopting an identificatory logic [object and subject are identifiable]. Yet, it points to a different time when no contradiction in the object might mean a logic that is rational in the sense that the proposition A=A will be true. Perhaps then we will arrive at utopia: a non-antagonistic society [thus: a non-contradictory object].

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Intensionality: uncomfortable but necessary in the history of logic

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In classical first-order logic intension plays no role. It is extensional by design since it models the reasoning needed in mathematics. But formalizing aspects of natural language or everyday reasoning needs something richer, something intensional; namely, a semantics accepting the idea of multiple reference for terms. Formal systems in which intensional features can be represented are generally referred to as intensional. The first steps to work out these questions were made purely syntatically by Barcan, and Carnap was the first one to give a semantics for the extension/intension method. With Kripke (1963), the now standard possible world became established. This framework was a major event in the history of logic, and one that has always been an area of close contact between logic and metaphysics, something that continues to this day (see, for example, Kripke (2013)). An important constraint on intensional logic has been the desire to have intensional systems which retain as much extensionality as possible, the traditional ideal behind first-order predicate logic. The pioneering semantics of Kripke endows each world with its own domain of individuals. Extensionalism comes in here with some difficult consequences:

- Although constants are given intensions, variables are given only extensions, which amounts to non-uniformly taking variables to be rigid designators.
- Predication is extensional, even when its argument is a constant bearing an intension.

Extensionalism is also a driving force behind the counterpart-theoretic approach to quantified modal logic by Lewis (1968), who considers the nonextensionality of modal logic ”a historical accident” that can be overcome. In Lewis’s approach, the need for a tracing principle is denied, as individuals are strictly world-bound. Tracing individuals across different worlds is catered for a context-dependent counterpart relation between inhabitants of different worlds. Afterwards, in the system of Montague, a typed lambda-calculus, constants may have non-constant intensions, but variables are stil rigid designators. Fitting (2004) gives a readable account of the difficulties facing the most common frameworks of quantified modal logic and related intensional issues, criticizing both the idea of variables as rigid designators and counterpart theory. Against this background he introduces his first-order intensional logic. But as in many other intensional systems, object variables are distinguished from intension variables, again reducing ease of use. To sum up, both intensional logic and quantified modal logic have been affected by a number of technical issues. The purpose of this work will be to systematically study and classify intensional logic systems and the logical problems they solve and they still maintain. Both philosophical and technical issues will be considered. The final goal is to produce a conceptual map representing the different elements, issues, and problems related to the lack of expressivity in the logical history of intensionality.

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The First Studies on Algebraic Logic in Turkey

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The studies of Sâlih Zeki are important in the introduction of algebraic logic in Turkey. He published his lectures he gave on logic in Dârülfünun under the title “Mîzân-ı Tefekkür”, and here he introduced in detail the algebraic logic which was developed by George Boole. According to Sâlih Zeki, there are three points of view in logic: Mantık-ı sûrî (formal logic), mantık-ı musavver (quantificational logic), mantık-ı işârî (algebraic logic). Among these, he adopted algebraic logic and while explaining it, he entirely adhered to George Boole's system. Those who study algebraic logic accept that the functioning of the human mind is based on mathematical grounds, therefore thinking is essentially a mathematical process, and they argue that algebraic symbols and operations should be used in analyzing logical expressions.

Various conceptions of science in the light of the distinction between characteristica universalis and calculus ratiotinator

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In our opinion Leibniz's distinction between characteristica universalis and calculus ratiotinator corresponds to Berkeley's distinction between two uses of our language, communicative and instrumental. We use language communicatively, if we have a certain object in mind and due to this fact our propositions have an appropriate content. We use it instrumentally, on the other hand, if we use our symbols "blindly", i.e. no content is being connected with them and manipulating them we want only to reach some goal.

Berkeley's distinction enables us to characterize various concepts of science. In an old Aristotelian conception is scientific discourse understood communicatively. Science has its object and its propositions have a real content. Deep changes in a mathematical praxis in the modern times led to a gradual revision of this "semantic monism". These changes are closely connected with emergence of algebra in which our language started to be used not only communicatively. Symbols used in algebraic operations (first of all symbols of imaginary numbers) evoke in us no idea and that is why they seem to have no content. in a different Mathematicians thus started to believe that language has not only communicative nature, but also instrumental or calculative function. We can meet a strict application of this distinction among the Cambridge algebraics (namely in the work of G. Peacock). According to them we use language instrumentally in algebra only, a communicatively in the subordinated sciences (arithmetic, geometry, dynamics, mechanics). This "semantic dualism" can be found in the work of D. Hilbert who distinguishes two kinds of formulae (finitary and infinitary) or in the works of logical positivists who distinguish between analytical and synthetical judgments. Problems connected with this "semantic dualism" are overcome by W.V. Quine whose pragmatism is paradoxically in a way a return to the old Aristotelian "semantic monism". However, it is not a restoration of old theoretical ideal of science but installation of a new scientific paradigm. According to Quine language is not used communicatively (we do not contemplate anything) but in all contexts instrumentally only. Modern science is according to Quine an instrument by means of which we implant a manageable structure into flux of experience.

Jerzy Łoś and the Origin of Temporal Logic

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A significant number of books and papers, concerning the origin of temporal logic, have been published by prominent publishing houses and prestigious journals for twenty five years. In vast majority of those works Arthur Norman Prior has been considered the inventor or the discoverer of temporal logic, whereas Jerzy Łoś has not even been mentioned (e.g. [5, 6, 7, 8]). However, having recognized Prior's contribution be crucial and irreplaceable, one should admit fairly that it is Łoś who invented the logic of time. That means particularly that (a) Łoś constructed, described and examined the first mature calculus of temporal logic, and (b) Prior was aware of and inspired by Łoś ideas when beginning his own works in the field.

The objective of this paper is to justify those claims. Among others, in the paper we reconstruct the original temporal system of logic, presented by Łoś. It is a well known fact that, when beginning his tense logic manifesto, Arthur Norman Prior ([10, 11]) was inspired by John Findlay. Findlay's text is only a small footnote on some vague possibility to incorporate tense conventions into modal logic ([1]). On the other hand Łoś' pioneering work is not usually even being mentioned within the context.

It has been well established that Prior's work on temporal logic began in 1953, whereas Hiż's review was published as early as 1951 ([2]).

Finally Prior himself admitted that, when beginning his programme and when preparing his *Time and Modality* ([9]), he had known Hiż's review and was influenced by Łoś' ideas ([10], pp. 212–213). And in his late work Łoś' paper has even been placed in the bibliography, even if nowhere else ([11], p. 161). The more Łoś' influence over Prior and the whole temporal logic gets obvious, the more mysterious the textbooks are which refuse to mention the true founder of the logic of time.

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Pierre de La Ramée as a logician Pontoneer

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The Meaning of Ramus, a major Logician of the XVIth Century, changes according to the point of view: insignificant in relation to the contemporary formal Logic, but linchpin between the Middle Age and the Modernity, it is the methodical Argumentation, gathering Logic with Rhetoric, which gives his Pontoneer's Signification. The Presentation proposes then to explain the main Shifts and Adaptations of the ramist System. The new Interest for Aristoteles' *Topics* consisted in the thinking of the Probable against that of Certanties, Truth and Arguments of Authorities ; favorable for Diversity and Novelties, the loci allow an analytical Access to Experience, well necessary during the Period of Discoveries. La Ramée makes yet of inventio of the loci, fruit of the new humanist Rhetoric, his logical Battle Horse. Used as *argumenta*, he transforms them in Tools of the *iudicium*; he bounds so the *Topics* to the *Organon*'s

Analytics in a global Logic, unifying both the "natural Logic" with Syllogistic, Induction with Deduction, dialectical *ars disserendi* with rhetorical Figurs and Colours. Without interdisciplinar Barriers more, the Ramism unifies knowledges in a pacifist Encyclopedism, especially thanks wellknown students of the Academy of Herborn, Alsted, Althusius and Comenius.

Algebra and Category

The invited keynote speaker of this workshop is Olivia Caramello (page 85).

Composition-Nominative Logics as Institutions

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Composition-nominative logics (CNL) are program-oriented algebra-based logics. They are constructed according to the principles of *development from abstract to concrete*, *priority of semantics*, *compositionality*, and *nominativity* [1]. Many-sorted algebras of partial mappings form a semantic base of CNL. Mappings are defined over classes of nominative data considered in integrity of their intensional and extensional components [2]. The hierarchy of nominative data induces a hierarchy of CNL. We identify the propositional and nominative levels of CNL. The latter level is subdivided on quasi-ary and hierarchic-ary sublevels. These sublevels are further subdivided on logics of pure predicates, logics of predicates, and program logics [3]. The logics of quasi-ary and hierarchic-ary mappings can be considered generalizations of classical predicate logics.

Theory of institutions presents a powerful formalism for structuring theories over logical systems [4, 5]. Its applications demonstrate importance of this formalism for studying semantic properties of programming and specification languages, databases, ontologies, cognitive linguistics, etc.

Here we present different types of composition-nominative logics as institutions. We study properties of presented formalisms and compare them with institutional presentations of classical logic.

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Algebraizable Logics and a functorial encoding of its morphisms

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This work presents some results about the categorial relation between logics and its categories of structures. A (propositional, finitary) logic is a pair given by a signature and Tarskian consequence relation on its formula algebra. The logics are the objects in our categories of logics; the morphisms are certain signature morphisms that are translations between logics [1, 3]. Morphisms between algebraizable logics [2] are translations that preserves algebraizing pairs [4]. Recall that in the theory of Blok-Pigozzi, to each algebraizable logic $a = (\Sigma, \vdash)$ is canonically associated a unique quasivariety $QV(a)$ in the same signature Σ (its "algebraic codification"). So given $a = (\Sigma, \vdash)$, $a' = (\Sigma', \vdash')$ and $f : a \to a'$ morphism of algebraizable logics, we have the functor $f^\star\colon \Sigma' - Str \to \Sigma - Str$ $(M \mapsto (M)^f)$ such that "commutes over Set" and restricts over its quasivarieties $f^\star\colon QV(a') \to QV(a)$; in this vein we show that morphisms of (Lindenbaum) algebraizable logics can be completely encoded by certain functors defined on the quasivariety canonically associated to the algebraizable logics, more precisely, there is a anti-isomorphism between morphisms of (Lindenbaum) algebraizable logics and functors of quasivarieties associated with theses logics. It is not difficult to see that this kind of functor $f^\star\colon QV(a') \to QV(a)$ has a left adjoint: we suspect that, at least under some conditions, this left adjoint has a connection with a "*Gödel translation generalized*".

The functorial codification of logical morphisms is useful in the development of a categorial approach to the representation theory of general logics [5]. We intend research local-global principle for the logic consequence relatively to the class of Lindenbaum-algebraizable logics and study closure properties by constructions like products and filtered colimits, among others. We intent to use this local-global approach to study meta-logic properties like “Craig interpolation property” and “Beth property” and apply in our representation theory of logics.

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Separator Method for Constructing Canonical Types of Formulas

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We present a new method of finding the canonical types of formulas based on three-valued projection logic functions. The method is focused on separation of all tuples of values for variables into disjoint sets. For every such set we take its own simple canonical type that identifies this set. Combining the results for each set we obtain the required canonical type for a closed class.

Projection Logic and Canonical Forms

This article deals with a *projection logic* $P_{3,2}$ of three-valued logic functions of several variables including single variable case defined on the set $\{0, 1, 2\}$ and taking values in the set $\{0,1\}$ [1]. If we consider a closed class of $P_{3,2}$, and allow all of its functions possess only values from $\{0,1\}$, then we would obtain a *projection* of a closed class of three-valued logic, which itself is a closed class relative to Boolean logic. Two formulas are called *equivalent* if they represent equal functions. If we could assign to every arbitrary formula F some special formula $C(F)$ such that for every two equivalent formulas F_1, F_2 the equality $C(F_1) = C(F_2)$ holds as the graphical coincidence of two symbolic raws, then we may say about existence of a *canonical type of formulas.* The full disjunctive normal form (FDNF) and Zhegalkin's polynomial are well-known examples of canonical types.

Separator Method

The separator method is designed to construct canonical types for formulas based on functions of many-valued projection logic. At first, we separate all the tuples between values on the "projection" consisting of $\{0,1\}^n$ tuples and the "remainder", which contain at least one value '2', combining each part by conjunction with indicators I of the conditions mentioned above. In the FDNF such indicators of tuples were conjunctions of variables and their negations. For $P_{3,2}$ maximal conjunctions of unary functions are $j_a(x_i) = I(x_i = a)$ [2].

Secondly, we combine the projection and the remainder by addition by modulo 2 or disjunction.

$$f^n = \mathrm{pr} f^n \wedge I_{\{\forall i,\, x_i \in \{0,1\}\}} \vee / \oplus\, f^n \wedge I_{\{\exists i,\, x_i = 2\}}$$

The functions of $P_{3,2}$ are determined by tuples on which they are equal to 1. Fixing a tuple we obtain that all indicators except one possess value 0, so we may combine them by non-trivial functions that preserve constants 0 and 1 such as $\oplus$ and $\vee$.

For the "remainder" we separate tuples by some partition and for every part we construct a simple canonical type combining some functions into conjunctions and preserving property to indicate tuples of values for the variables. In such a conjunction variables can be repeated in general functions.

Some classes represent functions such that the equality $f(\alpha^1) = 1$ implies $f(\alpha^i) = 1, i = 1, \cdots, k$ for some tuples α_i. We use indicators for such sets of tuples and Euler's formulas for the union of intersecting sets to preserve the uniqueness of a canonical type. This procedure allows us to associate well-known results from P_2 with the problems of many-valued logic closed classes.

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On non-deterministic algebras

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The conflict between incomplete, vague, imprecise and/or inconsistent real-word information, on the one hand, and the principle of truth-functionality, on the other, motivated A. Avron and I. Lev to introduce in [1] the so-called *non-deterministic matrices* (Nmatrices for short), a generalization of the usual matrix semantics. In an Nmatrix, the truth-value of a complex formula can be chosen *non-deterministically* out of some non-empty set of options. That is, Nmatrices are based on *non-deterministic algebras*, in contrast with the usual logical matrices, which are based on standard algebras.

The notion of non-deterministic algebras was introduced in Computer Science in order to deal with nondeterminism. For instance, non-deterministic algebras were proposed to recognize terms from absolutely free algebras.

Several propositional logics can be semantically characterized by a single logic matrix, but the characteristic matrix of many of them is infinite, and so it is not a good decision procedure for these logics. Nmatrices, by its turn, allow to replace, in many cases, an infinite characteristic matrix by a finite characteristic matrix and thus obtain properties such as decidability.

In this paper we propose the study of Nmatrices (and the underlying non-deterministic algebras) from the point of view of Universal Algebra. Thus, after introducing the category of non-deterministic algebras, some concepts such as product, subalgebra, congruence, quotient algebra and ultra product are studied in the non-deterministic context.

One of the goals of this research is the possibility of obtain a kind of algebrization, by adapting the Blok and Pigozzi's techniques to the non-deterministic algebras, for logics which do not admit an algebraic analysis in this sense, such as (some of the) *Logics of formal inconsistency.*

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The Category TrCx and some results

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In [3], da Silva, D'Ottaviano and Sette proposed, in 1999, a very general definition for the concept of translation between logics.

Coniglio in [2] introduced the concept of meta-translations, which are functions between languages that preserve certain properties of the domain logic. In [1] we find a simplified version of the concept of meta-translation, named contextual translation. The contextual translations are a special class of translations between logics defined as in [3].

In [3] the authors introduced the category **Tr** whose arrows are translations between logics and objects are the logics. The objective of this work was to characterize and study some properties of the category **TrCx**, whose arrows are as contextual translations between logics and objects are the logics. Furthermore, we established a relation between the **TrCx** and the **Tr**.

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Morita-equivalences for MV-algebras

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This talk is based on [2] and [3]. We generalize to a topos-theoretic setting two classical categorical equivalences arising in the context of MV-algebras: Mundici's equivalence [4] between the category of MV-algebras and the category of lattice-ordered abelian groups with strong unit (ℓ-u groups, for short) and Di Nola-Lettieri's equivalence [3] between the category of perfect MV-algebras and the category of lattice-ordered abelian groups (ℓ-groups, for short), not necessarily with strong unit. We show that these generalizations yield respectively a Morita-equivalence between the theory $\mathbb{MV}$ of MV-algebras and the theory $\mathbb{L}_u$ of ℓ-u groups and a Morita-equivalence between the theory of perfect MV-algebras and the theory of ℓ-groups. These Morita-equivalences allow us to apply the 'bridge technique' of [1] to transfer properties and results from one theory to the other, obtaining new insights which are not visible by using classical techniques. Among the results obtained by applying this methodology to Mundici's equivalence, we mention a bijective correspondence between the geometric theory extensions of the theory $\mathbb{MV}$ and those of the theory $\mathbb{L}_u$, a form of completeness and compactness for the infinitary theory $\mathbb{L}_u$, a logical characterization of the finitely presentable ℓ-groups with strong unit and a sheaf-theoretic version of Mundici's equivalence. In the case of Di Nola-Lettieri's equivalence, after observing that the two theories are not bi-interpretable in the classical sense, we identify, by considering appropriate topos-theoretic invariants, three levels of bi-intepretability holding for particular classes of formulas: irreducible formulas, geometric sentences and imaginaries. Lastly, by investigating the classifying topos of the theory of perfect MV-algebras, we obtain various results on its syntax and semantics also in relation to the cartesian theory of the variety generated by Chang's MV-algebra, including a concrete representation for the finitely presentable models of the latter theory as finite products of finitely presentable perfect MV-algebras. Among the results established on the way, we mention a Morita-equivalence between the theory of lattice-ordered abelian groups and that of cancellative lattice-ordered abelian monoids with bottom element.

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Dialectica Categories, Cardinalities of the Continuum and Combinatorics of Ideals

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We propose to revisit some old work of Andreas Blass ([1, 2]) connecting Dialectica categories ([3, 4, 5]), considered as models of Linear Logic, to tools of Vojtáš ([9]) that relate cardinal invariants via inequalities. We sharpen these results in the light of new developments in Set Theory (such as those introduced in [7] and described in [8]).

As a study case, we investigate how certain cardinals defined in terms of ideals (see, e.g., [6], p. 32) are naturally encompassed by this approach — at least in the specific situation where those ideals are defined in terms of pre-orders (i.e., reflexive, transitive binary relations).

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Categorical Logic Approach to Formal Epistemology

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Formal approaches to epistemology (which include logical epistemology, computational epistemology and modal operator epistemology etc.) either proceed axiomatically or concentrate on learning and knowledge acquisition using toolboxes from logic and computability theory.

It seems that we can extend the field of logical epistemology investigations exploiting systems of categorical logics rather than the propositional logics where deductive categories consist of formulas as objects and coded proofs as arrows (cf. [1]). In such categories for any object-formula A there is a special identity arrow $1_A : A \vdash A$ and transitivity of proofs is expressed by means of the composition operation which being applied to arrows $f : A \vdash B$ and $g : B \vdash C$ generates an arrow $g \circ f : A \vdash C$ (in essence, a composition is a form of cut rule). Besides that, in deductive categories the following equations take place: $f \circ 1_A = f, 1_B \circ f = f, (h \circ g) \circ f = h \circ (g \circ f)$ for any $f : A \vdash B, g : B \vdash C, h : C \vdash D$. Operations on arrows are considered as rules of inference and there is given a formula T (truth). For example, we can introduce a binary operation $\supset$ (= if…then) for forming the implication $A \supset B$ of two given formulas A and B simultaneously introducing the following operations on arrows

$$\frac{f : A \vdash B}{\ulcorner f \urcorner : T \vdash A \supset B} \quad \frac{g : T \vdash A \supset B}{g^s : A \vdash B}$$

together with the additional identities $\ulcorner f \urcorner^s = f, \ulcorner g^s \urcorner = g$ for any $\ulcorner f \urcorner : A \vdash B$ and $g : T \vdash C \supset D$.

Modal operation K (= it's known, that) for forming modal formula KA would be introduced the same way in deductive categories. Additional new arrows will be depended on the accepted modal axioms. In particular, introducing an arrow $d_{AB} : K(A \supset B) \vdash KA \supset KB$ and a partial unary operation on arrows:

$$\frac{f : T \vdash A}{nec(f) : T \vdash KA} \quad \frac{f : A \vdash B}{Kf : KA \vdash KB}$$

we obtain (adding respective identities on arrows e.g. $(d_{AB}nec(f))^s = K(f^s)$ for $f : T \vdash A \supset B$) a category equivalent of normal epistemic modal logic – epistemic bicartesian closed deductive category $ECBC$.

But to consider different agents of knowledge we have to introduce operations of the type $K_xA, K_eA, K_zA, \ldots$ and in this case we need a family of knowledge operations of different agents i.e. parameterized epistemic operations. Here helps one more category construction — so-called functors. A functor $\mathbf{F}$ between two deductive categories $\mathbf{C}, \mathbf{D}$ is a function assigning to any formula A of $\mathbf{C}$ a formula $\mathbf{F}(A)$ of $\mathbf{D}$, and any arrow $f\colon A \vdash B$ of $\mathbf{C}$ an arrow $\mathbf{F}(f)\colon \mathbf{F}(A) \vdash \mathbf{F}(B)$, such that $\mathbf{F}(1_A) = 1_{\mathbf{F}(A)}$, $\mathbf{F}(g \circ f) = \mathbf{F}(g) \circ \mathbf{F}(f)$. If now we construe the system of (endo)functors between $\mathbf{C}$ and $\mathbf{C}$ then as formulas K_xA, K_yA, K_zA we will take the result of functors actions $\mathbf{K}_x(A), \mathbf{K}_y(A), \mathbf{K}_z(A)$ with parameters x, y, z. Besides, formulas-objects of the type $\mathbf{K}_x(A) \supset \mathbf{K}_y(B)$ and inferences of the type $\mathbf{K}_x(A) \wedge \mathbf{K}_x(A) \supset \mathbf{K}_y(B) \vdash \mathbf{K}_y(B)$ (i.e. the knowledge situations caused by actions of different agents) might be derived. Moreover, to obtain a category of $\mathbf{K}$-functors we introduce arrows $K_{xy}(A, B)\colon \mathbf{K}_x(A) \vdash \mathbf{K}_y(B)$ which are the compositions of the arrows $K_{xy}(A)\colon \mathbf{K}_x(A) \vdash \mathbf{K}_y(A), \mathbf{K}_y(f)\colon \mathbf{K}_y(A) \vdash \mathbf{K}_y(B)$ as well as arrows $\mathbf{K}_x(f)\colon \mathbf{K}_x(A) \vdash \mathbf{K}_x(B)$, $K_{xy}(B)\colon \mathbf{K}_x(B) \vdash \mathbf{K}_y(B)$ and $\mathbf{K}_y(f) \circ K_{xy}(A) = K_{xy}(B) \circ \mathbf{K}_x f = K_{xy}(A, B)$.

Degrees of belief which Bayesians identifies with the probability also could be introduced in $ECBC$ but transforming it into a 2-category where 2-arrows are parameterized by these degrees of belief. A 2-arrow can be depicted as a "bygon" (cf. [1]).

$$\begin{tikzcd} \mathbf{K}_x(A) \arrow[r, bend left, "\mathbf{K}_x(f)"] \arrow[r, bend right, "\mathbf{K}_x(g)"'] \arrow[r, phantom, "\Downarrow Pr(\mathbf{K}_x(g)|\mathbf{K}_x(f))"] & \mathbf{K}_x(B) \end{tikzcd}$$

i.e. the shape of 2-morphism $Pr(\mathbf{K}_x(g)|\mathbf{K}_x(f)) : \mathbf{K}_x(f) \Longrightarrow \mathbf{K}_x(g)$ between morphisms $\mathbf{K}_x(f), \mathbf{K}_x(g) : \mathbf{K}_x(A) \vdash \mathbf{K}_x(B)$ where $Pr(\mathbf{K}_x(g)|\mathbf{K}_x(f)$ is the the conditional probability. We can "horizontally" compose 2-morphisms

$$\begin{tikzcd} \mathbf{K}_x(A) \arrow[r, bend left, "\mathbf{K}_x(f)"] \arrow[r, bend right, "\mathbf{K}_x(g)"'] \arrow[r, phantom, "\Downarrow Pr(\mathbf{K}_x(g)|\mathbf{K}_x(f))"] & \mathbf{K}_x(B) \arrow[r, bend left, "\mathbf{K}_x(h)"] \arrow[r, bend right, "\mathbf{K}_x(i)"'] \arrow[r, phantom, "\Downarrow Pr(\mathbf{K}_x(i)|\mathbf{K}_x(h))"] & \mathbf{K}_x(C) \end{tikzcd}$$

to obtain a 2-morphism

$$Pr(\mathbf{K}_x(i)|\mathbf{K}_x(h)) \times Pr(\mathbf{K}_x(g)|\mathbf{K}_x(f)) : \mathbf{K}_x(f) \Longrightarrow \mathbf{K}_x(i)$$

and "vertically" compose them

$$\begin{tikzcd}[column sep=huge]
\mathbf{K}_x(A) \arrow[r, bend left=60, "\mathbf{K}_x(f)"{name=F}] \arrow[r, "\mathbf{K}_x(g)"{name=G, description}] \arrow[r, bend right=60, "\mathbf{K}_x(h)"'{name=H}] & \mathbf{K}_x(B)
\arrow[Rightarrow, from=F, to=G, "Pr(\mathbf{K}_x(g)|\mathbf{K}_x(f))"]
\arrow[Rightarrow, from=G, to=H, "Pr(\mathbf{K}_x(h)|\mathbf{K}_x(g))"]
\end{tikzcd}$$

to obtain a 2-morphism

$$Pr(\mathbf{K}_x(i)|\mathbf{K}_x(h)) + Pr(\mathbf{K}_x(g)|\mathbf{K}_x(f)) : \mathbf{K}_x(f) \Longrightarrow \mathbf{K}_x(i).$$

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10 – Contest: the Future of Logic

Modern logic (starting with George Boole at the mid of the XIX century) changed the world, it led to new understanding of reasoning, language, mathematics. It gave new directions in philosophy and gave birth to computation.

After 150 years we may wonder what is the future of so successful a science, nowadays much of the time in the shadow of its multifaceted offsprings

This contest wants to promote a reflexion on what can be the future of logic considering its 150-years history. Here are a few questions:

1) Will or can logic give a better understanding to sciences / fields such as physics, biology, economics, music, information?

2) How will evolve the internal life of logic, its objectives and tools?

3) How will develop the interactions between logic and philosophy, logic and mathematics, logic and computation?

To take part to the contest submit a paper of 10 to 15 pages by March 31st, 2015 to unilog.contest2015@gmail.com.

Previous Winners of UNILOG contest are:

- Carlos Caleiro and Ricardo Gonçalves
  UNILOG'2005 = "Identity between logical structures"
- Till Mossakowski, Razvan Diaconescu and Andrzej Tarlecki
  UNILOG'2007= "Translations between logical systems"
- Vladimir Vasyukov
  UNILOG'2010 = "Combination of logics"
- Nate Ackerman
  UNILOG'2013 = "Logical Theorems"

The best papers will be selected for presentation in a special session during the event and a jury will decide during the event who are the winners: gold, silver and bronze medals.

Members of the Jury are Walter Carnielli (president), Patrick Blackburn and John Corcoran.

The prize will be offered by Anne Mätzener, representative of Birkhäuser.

Investigations into a Polynomial Calculus for Logics

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In this work, ideas going back to Zhegalkin's article *On the Technique of Calculating Propositions in Symbolic Logic* (1927) are presented, along with results broadening their scope. Zhegalkin interprets the Propositional logic (PL) fragment found in Section A, Part I of Whitehead and Russell's *Principia Mathematica* as having truth-values in $\mathbb{Z}/2\mathbb{Z}$, connecting Boolean valuations and proof calculus through an arithmetic of propositions, also designing rudimentary criteria for the satisfiability of given formulae. This approach is reminiscent of the contemporary idea of furnishing satisfiability problems as a problem of finding zeroes for systems of polynomials, representing wffs, in a polynomial ring. Restating such problems as a problem of finding roots provides great flexibility to decision procedures, and all-purpose economy – as shown by W. A. Carnielli in [1, 2, 3]. Thus, the method of finding roots is paramount and lies among our principal concerns in the present work.

As already suggested by W. A. Carnielli in [1], Hilbert's *Nullstellensatz* is a general and powerful result which can be explored for this purpose, also extending such treatment to many-valued logic. Nonetheless, a clear limitation is that the general statement of the theorem assumes an algebraically closed field, while no finite field $GF(p^k)$ can be algebraically closed: let $a_1, a_2, \ldots, a_{p^k} \in GF(p^k)$ be all its elements, then the polynomial $(x - a_1)(x - a_2)\cdots(x - a_{p^k}) + 1$ has no roots in $GF(p^k)$. We then need to consider the *Nullstellensatz* case-by-case, having a criterion for the shape of a polynomial $f = f(x_1, \ldots, x_n)$ provided the existence of simultaneous zeroes with $g_1, \ldots, g_m$ vanishing f or, in contraposition terms, if we want to decide that a given set of polynomials $f, g_1, \ldots, g_m$ has no simultaneous zeroes provided that there is no way to write $f^k = \sum_{i=1}^{n} h_i g_i$ given any h_i polynomials in the same field, what seems to be hard.

A method using Gröbner basis has been also proposed, and here we explore other methodologies giving criteria for the existence of simultaneous zeroes relying on information about the set of polynomials in hand, hence without having to consider a too general expressiveness as above in many cases.

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The Future of Logic as a Geometry of Scientific Thought

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What is Logic? It is not an easy task to answer that question without assuming some perspectives of what logic should be. According to the tradition that goes from Aristotle to Frege, Logic can be seen as the "science of valid argument": logic teach us how to perform a reasoning without committing any fallacies; logic will give us the laws that govern the concept of "logical consequence", though this characterisation seems to contain a vicious circle, namely, by defining "Logic" by means of the adjective "logical", which is used in the expression " logical consequence". But, by considering that logic is the realm of knowledge that offers the concept of "validity" of an argument, Logic can be defined as the branch of human activity in which we are concern with the "laws of thought and their consequences".

Then, by assuming such a conception of logic, we can ask what be the future of logic. In other words, how logic can preserve this status of being the "science of thought" and, at the same time, be expanded so that its concepts and laws can be seen as fruitful to the other sciences. Could a logical concept be interesting to physics, for instance, without being only an instrumental for good or valid reasoning? Our answer to this is to considering logic as "geometry" — which mediates scientific descriptions and problem solving. More precisely, logic can be seen as special "vector algebra" in logical space. With this approach, we see that the future of logic can be glimpsed: logic will be a "science of thought" that can be useful to other sciences, not only as a source of valid arguments, but as a realm that can directly describe their concepts of numbers and vectors.

The Future of Logic (and Ethics)

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Questions about the future of logic, as well as the future of ethics, are asked far less often than questions about the future of philosophy, generally. I think asking more of the former will make some of those concerned about the answers to the latter feel much better. Building off of Corcoran (1989), which claims that "the ethics of the future must accord logic a more central and explicit role", this paper argues that the philosophy of the future must accord the connection between logic and ethics a more central and explicit role. The support for this claim has three main facets:

(1) A history of logic since Boole and De Morgan which places great emphasis on Kripke, Marcus, and Prior. I connect the history to the claim by arguing that Marcus and Prior's views, which directly connect logic and ethics, are much more satisfactory than Kripke's attitude toward these issues.
(2) The content of Corcoran (1989).
(3) Examples drawn from extending our attention to non-deductive logics in addition to the infinitely-more studied deductive systems.

The Future of Logic: Foundation-Independence

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Throughout the 20th century, the automation of formal logics in computers has created unprecedented potential for practical applications of logic — most importantly the mechanical verification of mathematics and software. But the high cost of these applications makes them infeasible but for a few flagship projects, and even those are negligible compared to the ever-rising needs for verification. We hold that the biggest challenge in the future of logic will be to enable applications at much larger scales and simultaneously at much lower costs.

This will require a far more efficient allocation of resources. Wherever possible, theoretical and practical results must be formulated generically so that they can be instantiated to arbitrary logics; this will allow reusing results in the face of today's multitude of diverging logical systems. Moreover, the software engineering problems concerning automation support must be decoupled from the theoretical problems of designing logics and calculi; this will allow researchers outside or at the fringe of logic to contribute scalable logic-independent tools.

Anticipating these needs, the author has developed the MMT framework. It offers a modern approach towards defining, implementing, and applying logics that focuses on modular design and logic-independent results. This paper summarizes the ideas behind and the results about MMT. It focuses on showing how MMT provides a theoretical and practical framework for the future of logic.

Unified Logic, an Alternative for Combining Logics

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The aim of this note is to defend the possibility of unifying logics instead of combining them so that the philosophical insights, which are supposed to be fulfilled, play role from the outset and in the process of originating, not that they come to stage after technical issues and in order to interpret them. I am to argue for the feasibility of this project both from philosophical and technical points of view.

The philosophical method on the basis of which I am to argue that a unified logic is feasible is *phenomenology*. I will benefit from some of phenomenological ideas, which I am going to discuss in the presentation, besides the *dialogical approach* which provides us with a powerful tool to formulate semantics for different systems of logic and to explain the insights behind them.

I begin from the observation that for most of the logical systems it is true that each of them contains some genuine insights about logical relations and, disregard their probably inadequate extension from a strict phenomenological point of view, they formulated some basic notions which would otherwise remain ambiguous. Then the first task of unifying logics is to recognize these insights.

Therefore, it is clear that no pre-given technique to unify logics is possible, for in any case original philosophical considerations are needed. Then, in order to show the feasibility of this project what can be done is to show it in practice. This is the main task of my presentation.

I will formulate a logic which gathers insights, I mean phenomenologically admissible ones namely those obtained through *ideation* and *eidetic variation*, which are so far represented in various logics. I will present an axiomatic version of such a unified logic and I will also introduce a dialogical semantics for it.

In an important way the logic I will articulate should be considered as a proof of concept — not necessarily as an introduction of the true unified logic.

No prediction is possible in the spontaneous activities of science, but as a plausible future I will be going to conclude that to realize the phenomenological idea of a unified logic is in principle possible and some techniques achieved during the past century, above all the dialogical semantics, could help in this respect.

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It is nevertheless worth mentioning that such a project would not be in conflict with developing polarities of logical systems. The point is that if logic and philosophy would be supposed to be tied together it would hardly be by means of plural logical systems, rather a unified logic is required and it is now, having the achievements of the 20th century at hand, not far from possible.

The Future of Logic

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The future of Logic will be one without any self-referential paradoxes. It will even lack the so-called 'paradoxes of material and strict implication'. This will be achieved through the extension of the Lambda Calculus to the formalisation of 'that'-clauses referring to 'propositions'. And that will also allow sentences involving empty or indexical terms to be included in those covered. For such sentences can still express propositions, even if different propositions might not then have different syntactic expressions. As a result Tarski's and Gödel's Theorems will be relegated to academic curiosities. Above all, Logic will cease to be the mathematical study of arrangements of sentences, and return to being a moral science concerned to hold fast the standard meaning of sentences.

Triggering a Copernican Shift in Logic through Sequenced Evaluations

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Over the past fifty years, the fabric of our society has been radically transformed by successful logic-based applications. In today's world, logic chips (i.e., CPUs) and the logic-grounded software that controls them support nearly all socio-economic infrastructure, from banking to defense. These global computing applications are the major arena where logic as a body of knowledge meets, and is tested against reality.

Although classical first- order logic (FOL) appears to work perfectly for truth functional applications such as computer chip design where propositions are guaranteed to have a truth value (i.e., a positive XOR negative charge), inference problems characteristic of widely used logic-grounded software technologies [6], and representation problems characteristic of logic-based research into natural language understanding [3] suggest that there are foundational areas in logic that are not yet completely understood.

In some cases, issues in foundational areas became the source of software restrictions. For example, the expressivity of database languages such as SQL was restricted to avoid the sorts of impredicativity paradoxes to which higher order logics are exposed [1]. In other cases, foundational issues became a source of serious software errors. For example, the conflation of wff's and evaluable propositions led to erroneous aggregates in large data sets such as found at the World Bank and other large organizations where many of their wff's are unevaluable. These real world issues point to existing unresolved foundational issues in FOL including evaluability and identity and substitution.

At the center of logic's foundations is the characterization of the components of a proposition as ***argument*** and ***predicate***. From Aristotle to Boole, Frege, and Russell, the consensus view of both components has been referential. That is to say arguments and predicates refer to entities in their respective domains in the world. Arguments refer to substances or objects; predicates refer to properties or attributes. First order logic (FOL) was built with this referential view as a foundation.

Using examples drawn from Boole's 'The Laws of Thought', this essay proposes a novel view of the notions of argument and predicate, treating them as ***temporal*** differences in evaluation sequencing and not as ***static*** differences in domain (i.e., objects vs. attributes). We show that this new foundational view supports for four main beneficial changes to FOL that would improve FOL's internal consistency, its integration with other abstract disciplines, and its external applicability.

1. Wffs can be disentangled from evaluable propositions so that all and only those wffs that are determined to be evaluable are granted status as propositions for which bi-valence is thereby guaranteed. Moreover, the variety of ways in which a wff may not be evaluable provides a richer notion of negation and context-sensitive meaning than is explicit in FOL. The distinction between evaluable and unevaluable wffs can also be used to resolve the liar in a novel way that meets constraints imposed by [2].

2. Higher order quantification may be recast as first-order logic assertions where the concepts used as predicates in one assertion are used as arguments in another. This simplifies a significant amount of logical apparatus without sacrificing expressivity.

3. By separating typing from use, and by treating logical operators as those operators that apply to any type, FOL and computation can be tightly integrated. And the reason for the existence of mathematical facts such as the Irrationals can actually be explained when grounded in a suitably modified interpretation of FOL.

The relationships between logical propositions and their physical representations can be expanded to include multiple internal physical representations (e.g., different memory locations) for the same internal logical proposition supporting applied research in database optimization and theoretical research in para-consistent (e.g., contraction-free) logic as well as non-verbal and non-linguistic representations that support research in context-sensitive natural language understanding.

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Part IV

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