Universal Logic III
Estoril 2010
Tutorial on Truth Values
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Session 2, Part 2

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2.
A non-Fregean logic: **QAS**

Abstract

In this second part, Frege's question-answer machinery is refined and results in a family of many-valued semantics: Question-Answer Semantics (**QAS**). The logical values are non-Fregean values, i.e. ordered answers to initial questions about a sentence that differ from Suszko's bivalent non-Fregean logic.
A comparison is made with Shramko & Wansing's generalized truth values, and an application is suggested to two ancient Indian “logics”. The variety of non-classical logics is determined by the variety of conditions for truth ascription.

Definition 1. Question-Answer Semantics

A Question-Answer Semantics is a model **QAS** = $\langle \mathfrak{M}, \mathbf{A} \rangle$ upon a sentential language $\mathscr{L}$ and its set of logical connectives f_c. It includes:

a logical matrix $\mathfrak{M} = \langle \mathbf{Q}; V; D \rangle$, with :

- a function $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha),\ldots,\mathbf{q}_n(\alpha) \rangle$ that turns any sentence α of $\mathscr{L}$ into a specific statement (the sense of which is given by *n* appropriate questions about α);
- a set *V* of logical values (where Card(V) = m^n);
- a subset of designated values $D \subseteq V$.

a valuation function **A**, such that $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha),\ldots,\mathbf{a}_n(\alpha) \rangle$ is an element of *V* that affords the meaning of the sentence α by giving an ordered set of *m* sorts of answers to each corresponding question $\mathbf{q}_i$ in $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha),\ldots,\mathbf{q}_n(\alpha) \rangle$. This semantic framework results in a variety of logics $\mathbb{L} = \langle \mathscr{L}; \models_M \rangle$ such that, for every set of premises Γ and every α in $\mathbb{L}$, if $\mathbf{A}(\Gamma) \subseteq D$ then $\mathbf{A}(\alpha) \subseteq D$: $\Gamma \models_M \alpha$.

The logical values in **QAS** and generalized truth-values (GTV):

- Card(**A**) = the number n of elements = $\mathcal{P}(n)$ iff $m = 2$ (yes-no answers)

Example: $n = \mathbf{2} = \{T,F\}$, therefore $\mathcal{P}(n) = \mathcal{P}(2) = 4$

$$m^n = 2^2 = 4$$

- the logical values in $V^{n+1} = \mathcal{P}(n)$ are combined elements from V^n in GTV

ordered answers to questions in **QAS**

Example 1: Belnap's and Dunn's useful logic FDE

$\mathbf{Q}(p) = \langle \mathbf{q}_1(p), \mathbf{q}_2(p) \rangle$

$\mathbf{q}_1(p)$: "Is p told true?" $\mathbf{q}_2(p)$: "Is p told false?"

For every n, $\mathbf{a}_n(p) = 1$ (yes) or 0 (no)

1. $\mathbf{T} = \{T\}$
2. $\mathbf{F} = \{F\}$
3. $\mathbf{B} = \{F,T\}$
4. $\mathbf{N} = \varnothing$

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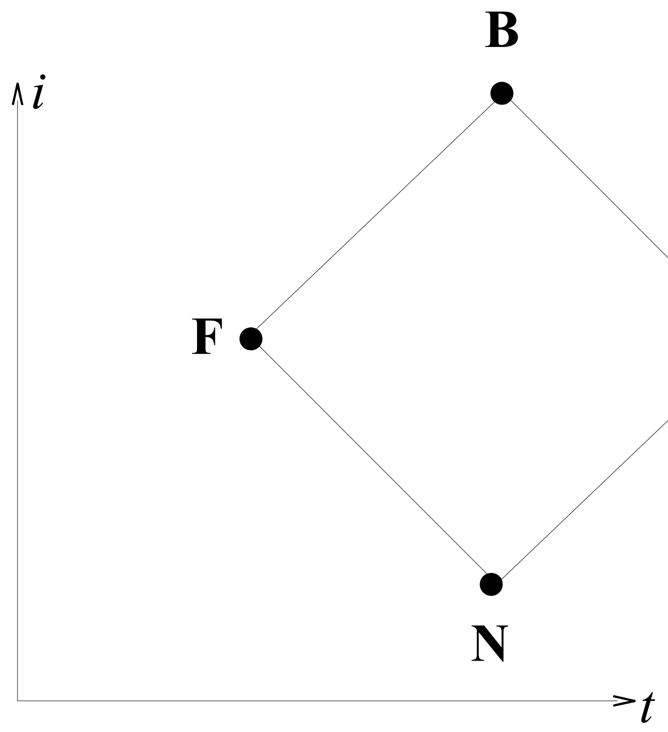
For every n, $\mathbf{a}_n(p) = 1$ (yes) or 0 (no)

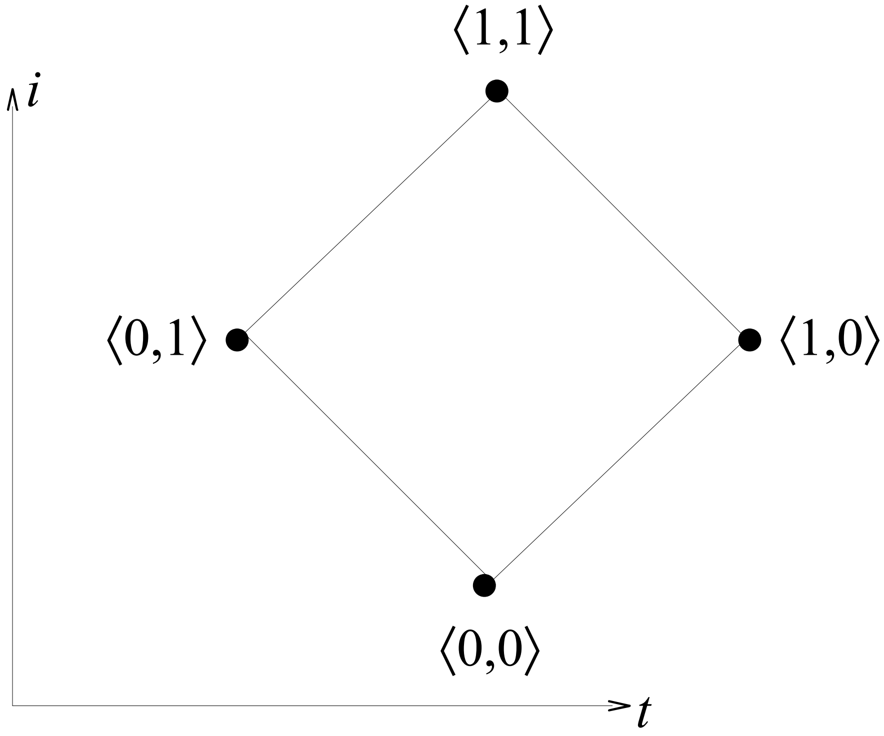
1. $\langle 1,0 \rangle$
2. $\langle 0,1 \rangle$
3. $\langle 1,1 \rangle$
4. $\langle 0,0 \rangle$

$\mathbf{q}_1$ and $\mathbf{q}_2$ have independent answers, unlike Frege's theory of judgment
$\mathbf{a}_1(p) = 1$ (or 0) $\not\Rightarrow$ $\mathbf{a}_2(p) = 0$ (or 1)

GINSBERG (1988): Bilattice *FOUR*$_2$



GINSBERG (1988): Bilattice *FOUR*$_2$



<u>Example 2</u>: Shramko & Wansing's 16-valued logic

$\mathbf{Q}(p) = \langle \mathbf{q}_1(p), \mathbf{q}_2(p), \mathbf{q}_3(p), \mathbf{q}_4(p) \rangle$

$\mathbf{q}_1$(p): "Is p told only-true?"
$\mathbf{q}_2$(p): "Is p told only-false?"
$\mathbf{q}_3$(p): "Is p told both-true-and-false?"
$\mathbf{q}_4$(p): "Is p told neither-true-nor-false?"

For every n, $\mathbf{a}_n(p) = 1$ (yes) or 0 (no)

1. $\mathbf{N} = \varnothing$
2. $\boldsymbol{N} = \{\varnothing\}$
3. $\boldsymbol{F} = \{\{F\}\}$
4. $\boldsymbol{T} = \{\{T\}\}$
5. $\boldsymbol{B} = \{\{F,T\}\}$
6. $\mathbf{NF} = \{\varnothing,\{F\}\}$
7. $\mathbf{NT} = \{\varnothing,\{T\}\}$
8. $\mathbf{NB} = \{\varnothing,\{F,T\}\}$
9. $\mathbf{FT} = \{\{F\},\{T\}\}$
10. $\mathbf{FB} = \{\{F\},\{F,T\}\}$
11. $\mathbf{TB} = \{\{T\},\{F,T\}\}$
12. $\mathbf{NFT} = \{\varnothing,\{F\},\{T\}\}$
13. $\mathbf{NFB} = \{\varnothing,\{F\},\{F,T\}\}$
14. $\mathbf{NTB} = \{\varnothing,\{T\},\{F,T\}\}$
15. $\mathbf{FTB} = \{\{F\},\{T\},\{F,T\}\}$
16. $\mathbf{A} = \{\varnothing,\{F\},\{T\},\{F,T\}\}$

Example 2: Shramko & Wansing's 16-valued logic

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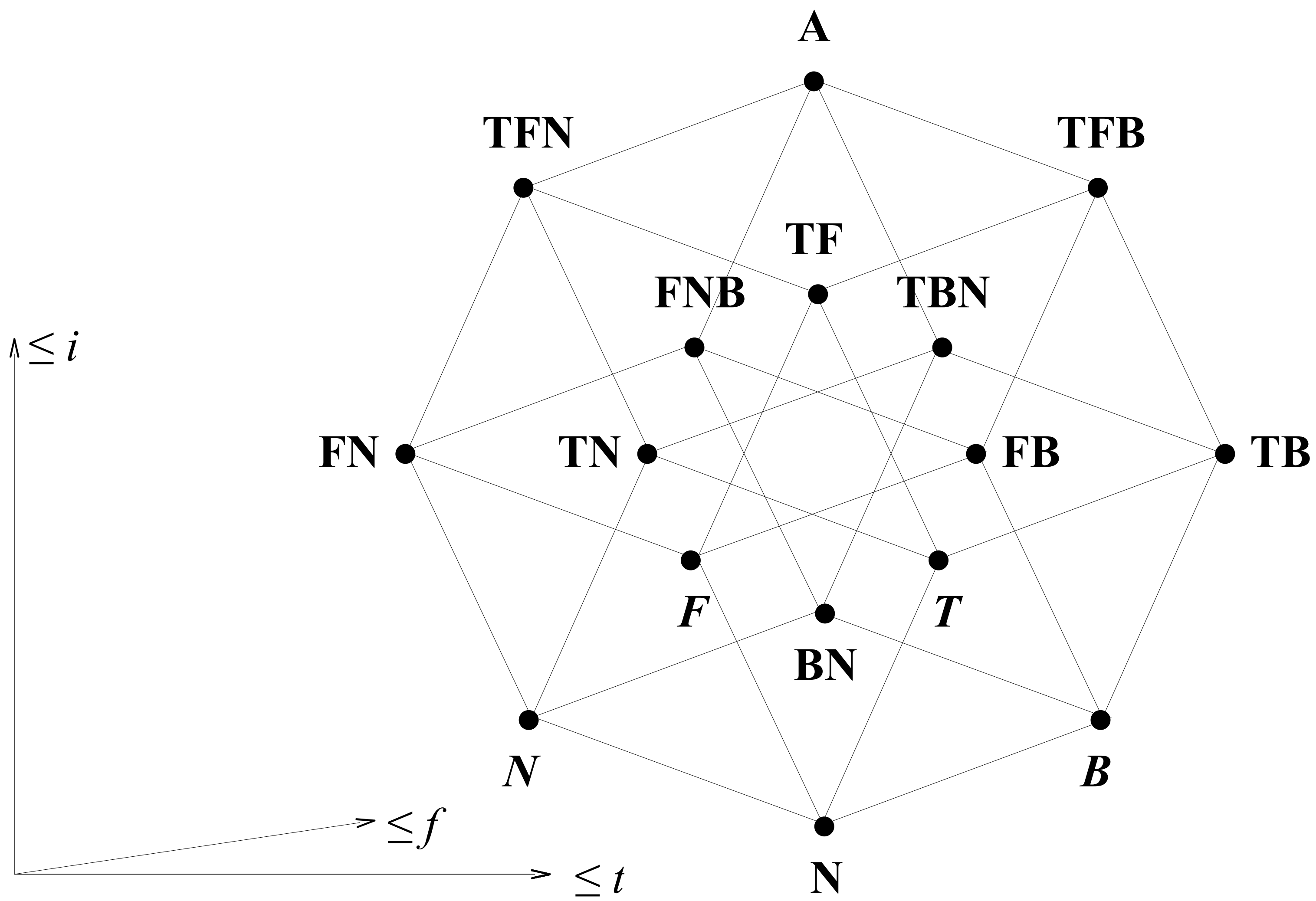
$\mathbf{q}_4(p)$: "Is p told neither-true-nor-false?"

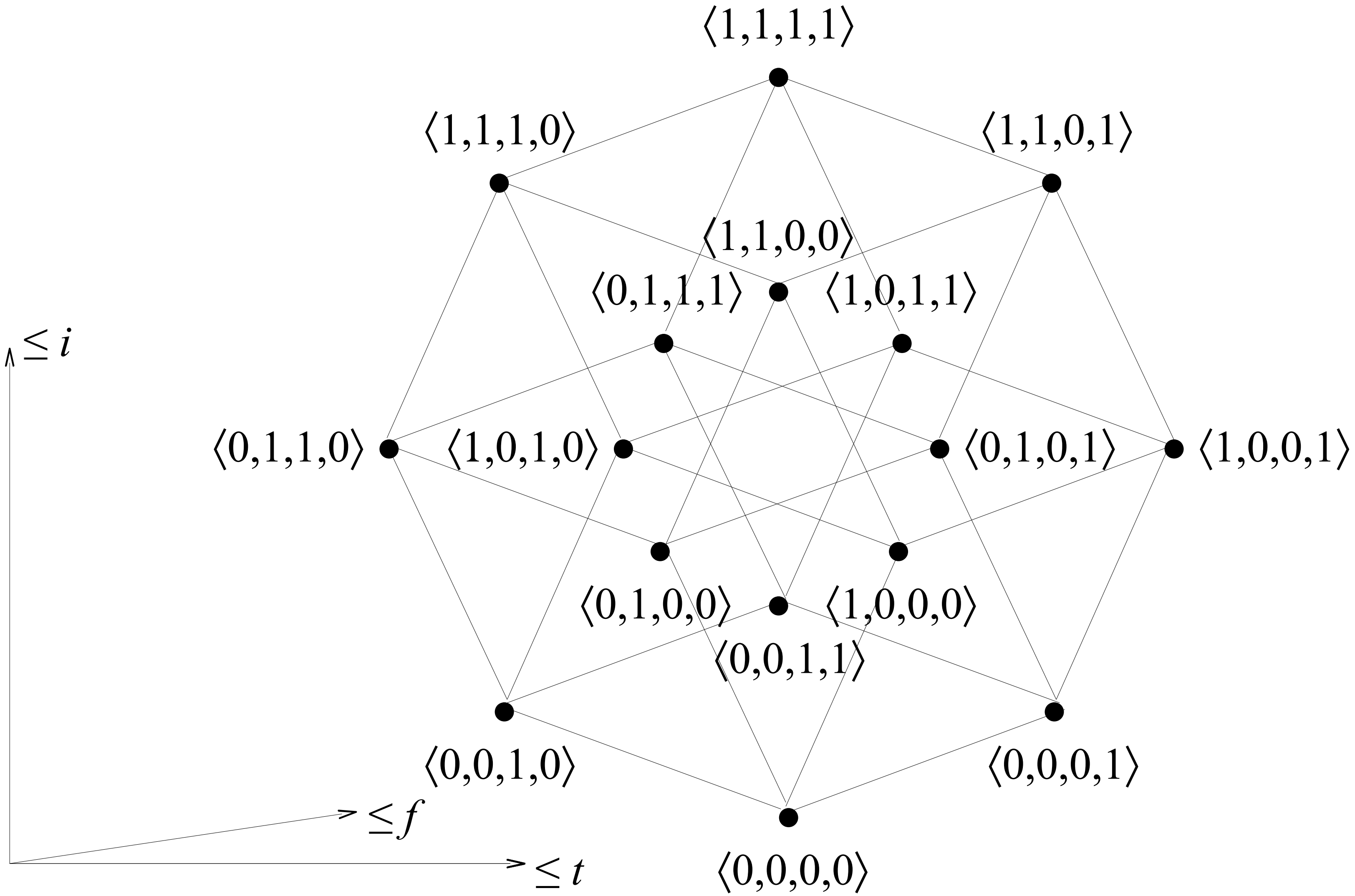
For every n, $\mathbf{a}_n(p) = 1$ (yes) or 0 (no)

1. ⟨0000⟩
2. ⟨0001⟩
3. ⟨0100⟩
4. ⟨1000⟩
5. ⟨0001⟩
6. ⟨0101⟩
7. ⟨1001⟩
8. ⟨0011⟩
9. ⟨1100⟩
10. ⟨0110⟩
11. ⟨1010⟩
12. ⟨1101⟩
13. ⟨0111⟩
14. ⟨1011⟩
15. ⟨1110⟩
16. ⟨1111⟩

SHRAMKO & WANSING (2005): Trilattice *SIXTEEN*$_3$



SHRAMKO & WANSING (2005): Trilattice *SIXTEEN*$_3$



- **QAS** is exemplified by two ancient Indian logics:

  saptabhaṅgī (a pluralist logic)
  catuṣkoṭi (a skeptic logic)

Combined truth-values occur in these logics, with a different interpretation of these combinations (with respect to the question-function **Q**)

- These two logics are not opposite trivial logics (Parsons (1984)):

  saptabhaṅgī is not a fully inconsistent logic ("everything is true")
  For every wffs A,B: $A \models B$ (ultimate eclecticism)

  catuṣkoṭi is not a fully incomplete logic ("nothing is true")
  For every wffs A,B: $A \not\models B$ (complete nihilism)

Application 1: *Saptabhaṅgī* (a logic for pluralism)

The blind men and an elephant

“Blind monks examining an elephant”
Ukiyo-e by Hanabusa Itchō (1888)

Six blind men were asked to determine what an elephant looked like by feeling different parts of the elephant's body.
The blind man who feels a leg says the elephant is like a **pillar**; the one who feels the tail says the elephant is like a **rope**; the one who feels the trunk says the elephant is like a **tree branch**; the one who feels the ear says the elephant is like a hand **fan**; the one who feels the belly says the elephant is like a **wall**; and the one who feels the tusk says the elephant is like a **solid pipe**.

A wise man explains to them:
"All of you are right. The reason every one of you is telling it differently is because each one of you touched the different part of the elephant. So, actually the elephant has all the features you mentioned."

This resolves the conflict, and is used to illustrate the principle of living in harmony with people who have different belief systems, and that truth can be stated in different ways (in Jainist beliefs often said to be seven versions).
A plea for partial truths …

Three sorts of criteria for truth-ascription (Ganeri 2002: 268)

- **Doctrinalism**: *it is always possible, in principle, to discover which of two inconsistent sentences is true, and which is false.*

- **Skepticism**: *the existence both of a reason to assert and a reason to reject a sentence itself constitutes a reason to deny that we can justifiably either assert or deny the sentence.*

- **Pluralism**: *to find some way conditionally to assent to each of the sentences, by recognizing that the justification of a sentence is internal to a standpoint.*
  Anekāntavāda: "doctrine of non-one-sidedness"
  Syādvāda: doctrine of conditionality, Nayavāda: doctrine of standpoints

Vādiveda Suri (1086-1169), *Saptabhaṅgī*: Theory of Seven-Fold Predication

(1) *syād asty eva*: arguably, it (some object) exists **Assertion**

(2) *syān nāsty eva*: arguably, it does not exist **Denial**

(3) *syād asty eva syān nāsty eva*: arguably, it exists; arguably, it does not exist **Successive assertion and denial**

(4) *syād avaktavyam eva*: arguably, it is non-assertible **Simultaneous assertion and denial**

(5) *syād nāsty eva syād avaktavyam eva*: arguably, it exists; arguably, it is non-assertible **Assertion and simultaneous assertion and denial**

(6) *syān nāsty eva syād avaktavyam eva*: arguably, it does not exists, arguably, it is non-assertible **Denial and simultaneous assertion and denial**

(7) *syād asty eva syān nāsty eva syād avaktavyam eva*: arguably, it exists; arguably, it does not exist; arguably, it is non-assertible **Successive assertion and denial and simultaneous assertion and denial**

2.1 What does "syād" mean?

Informal interpretation: "arguably", "maybe", "in some respect (s_1,s_2,…)"
Formal interpretation: truth relative to a theory, possible world, or situation s_x
Metalanguage: ($\models_{s_x}$ p) = "p is true at the situation s_x", and $\models_{s_x} \cong \Diamond$

	JL	CL	ML
(1)	$v(p) = \{\{T\}\}$	$\models_{s_x} p$	$\Diamond p$
(2)	$v(p) = \{\{F\}\}$	$\models_{s_x} \sim p$	$\Diamond\sim p$
(3)	$v(p) = \{\{T\},\{F\}\}$	$\models_{s_x} p$ and $\models_{s_x} \sim p$	$\Diamond p \wedge \Diamond\sim p$
(4)	$v(p) = \{\{\#\}\}$	?	?
(5)	$v(p) = \{\{T\},\{\#\}\}$	$\models_{s_x} p$ and ?	$\Diamond p \wedge \#$
(6)	$v(p) = \{\{F\},\{\#\}\}$	$\models_{s_x} \sim p$ and #	$\Diamond\sim p \wedge \#$
(7)	$v(p) = \{\{T\},\{F\},\{\#\}\}$	$\models_{s_x} p$ and $\models_{s_x} \sim p$ and #	$\Diamond p \wedge \Diamond\sim p \wedge \#$

2.2 What does "avaktavyam" mean?

"non-assertible" = "indescribable", "unsayable", "undescribable" = #

(a) an inconsistent interpretation of # $v(\text{p}) = \boldsymbol{B}$

(b) an incomplete interpretation of # $v(\text{p}) = \boldsymbol{N}$

(a): Bharucha and Kamat (1984), Matilal (1998), Priest (2008)

(b): Ganeri (2002)

- Bharucha and Kamat (1984): $v(\text{p}) = \#$ iff $\models_{S_x} (\text{p} \wedge \sim\text{p})$ $\Rightarrow$ $v(\text{p}) = \text{B}$
- Matilal (1998): $v(\text{p}) = \#$ iff $\models_{S_x}$ p and $\models_{S_x} \sim\text{p}$ $\Rightarrow$ $v(\text{p}) = \text{B}$
- Ganeri (2002): $v(\text{p}) = \#$ iff $\not\models_{S_x}$ p and $\not\models_{S_x} \sim\text{p}$ $\Rightarrow$ $v(\text{p}) = \text{N}$

(A third interpretation (c): "unsayable": # = neither B nor N (S, in Sylvan (?))

We restrict the analysis of # to either N or B, in the following

According to Ganeri (2002), (a) leads to a semantic collapse and (b) holds:

(…) what is the fifth truth-value, [**TB**]*? If Bharucha and Kamat are right then it means that there is some standpoint from which* "p" *can be asserted, and some from which* "$p \wedge \sim p$" *can be asserted. But this is logically equivalent to* [**B**] *itself. The Bharucha and Kamat formulation fails to show how to get a seven-valued logic.*
(Ganeri (2002): 271)

(1) = ***T***
(5) = **TB** = {{T},{T,F}} = {{T}} ∪ {{T,F}} = {{T,F}} = ***B*** = (4)
(5) = (1) ∪ (4) = (4)

Ganeri (2002) unduly conflates two distinct standpoints into a unique one

$$p, p \wedge \sim p \models p \wedge \sim p \qquad \text{(by Simplification)}$$

Therefore: {{T},{T,F}} = {T,F}

*The "oral tradition" supplies the question with one typical reply, arguing to the effect that any combination of Belnap's four truth values would be in a sense superfluous. The argument usually goes as follows. Consider, e.g., the combination **TB*** (= {{T},{F,T}}) *of **T** and **B**. This new truth value would then mean "true and true-and-false". But a repetition of truths gives us no new information (is superfluous)! Thus, the meaning of **TB**, it is claimed, collapses just into "true-and-false", and in this way we simply obtain **B**. An analogues argument reduces **FB** to **B**, and it is not difficult to argue in a similar way that **FT** is, in fact, also **B**.*

*Further, a combination of **N** with any other truth value seems to be superfluous as well, for unifying the empty set with any other set gives just this latter set. As a consequence one might conclude that any attempt to continue generalizing truth values beyond the four values introduced by Belnap should fail due to a collapse of any new truth value into one of the initial four.*

*However, a more careful examination shows that such a conclusion is not justified. First, recall that the proper interpretation of **T** is not simply "true" but "true-only" (and analogously for falsehood). And the combination of "true-only" and "true-and-false", which we get in the new truth value **TB**, is not so trivial and, in any case, is not so easily reducible to "true-and-false" as the above argument seems to suggest. Second, one may notice that this argument works only under the implicit interpretation of the comma between elements in new truth values as set-theoretical union and the identification of a set* x *with the singleton* {x}. *Only then one would be able to conduct the suggested manipulation:* {{T}, {F,T}} = {{T} ∪ {F,T}} = {T,F,T} = {F,T}, *which is obviously incorrect.* {{T}, {F,T}} *is, of course, distinct from **B*** = {T} ∪ {F,T}, *and therefore, it would be more natural to consider the generalized truth value* {{T},{F,T}} *an independent value in its own right. Similarly,* {∅,{F,T}} *is not the same as* {F,T}, *etc.*

(Shramko & Wansing (2005): 124-5)

Definition 2. A Jaina predication is an ordered answer $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha),\mathbf{a}_2(\alpha),\mathbf{a}_3(\alpha)\rangle$ to $n = 3$ basic questions $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha),\mathbf{q}_2(\alpha),\mathbf{q}_3(\alpha)\rangle$, such that $\mathbf{q}_1(\alpha)$: "Is α asserted?", $\mathbf{q}_2(\alpha)$: "Is α negated?", and $\mathbf{q}_3(\alpha)$: "Is α non-assertible?". There are $m = 2$ kinds of exclusive answers $\mathbf{a}_i(\alpha) \mapsto \{0,1\}$ to each ordered question $\mathbf{q}_i$, where 0 is a denial "no" and 1 is an affirmation "yes". This yields the following list of $m^n = 2^3 = 8$ predications and their counterparts in a set **8**:

$(1) = \langle 1,0,0\rangle$	$\{T\}$	$(2) = \langle 0,1,0\rangle$	$\{F\}$
$(3) = \langle 1,1,0\rangle$	$\{\{T\},\{F\}\}$	$(4) = \langle 0,0,1\rangle$	$\{\#\}$
$(5) = \langle 1,0,1\rangle$	$\{\{T\},\{\#\}\}$	$(6) = \langle 0,1,1\rangle$	$\{\{F\},\{\#\}\}$
$(7) = \langle 1,1,1\rangle$	$\{\{T\},\{F\},\{\#\}\}$	$(8) = \langle 0,0,0\rangle$	$\varnothing$

2.3 Why seven?

- The seven predications (*saptabhaṅgī*)

3 main predications (*bhaṅgī*) in Jaina logic:

3 ordered questions **q** about a given sentence p: $\mathbf{Q}(p) = \langle \mathbf{q}_1(p), \mathbf{q}_2(p), \mathbf{q}_3(p) \rangle$

$\mathbf{q}_1(p)$: "is p asserted?", "$v(p) = T$?"

$\mathbf{q}_2(p)$: "is p denied?", "$v(p) = F$?"

$\mathbf{q}_3(p)$: "is p non-assertible?", "$v(p) = \#$?"

- Two possible answers **a**: "yes" = 1, or "no" = 0

Each logical value is a ordered 3-tuple of answers: $\mathbf{A}(p) = \langle \mathbf{a}_1(p), \mathbf{a}_2(p), \mathbf{a}_3(p) \rangle$

The cardinality of **JL** = $2^3 - 1 = 7$

(1) = $\langle 1,0,0 \rangle$ (2) = $\langle 0,1,0 \rangle$ (3) = $\langle 1,1,0 \rangle$ (4) = $\langle 0,0,1 \rangle$

(5) = $\langle 1,0,1 \rangle$ (6) = $\langle 0,1,1 \rangle$ (7) = $\langle 1,1,1 \rangle$ ~~(8) = $\langle 0,0,0 \rangle$~~

2.4 How to define the logical connectives?

What are the semantic values of such compound sentences? Such a question is not one that Jaina logicians thought to ask themselves, as far as I know. So we are on our own here. There are probably several possible answers.

(Priest (2008): 268)

Definition 3. Jaina logic is a model $\mathbf{J}_7 = \langle \mathfrak{M}, \mathbf{A} \rangle$ upon a sentential language $\mathscr{L}$ and its set of logical connectives $f_c = \{\sim, \wedge, \vee, \supset\}$. It includes a logical matrix $\mathfrak{M} = \langle \mathbf{Q}; \mathbf{7}; D \rangle$, with :

- a function $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \mathbf{q}_2(\alpha), \mathbf{q}_3(\alpha) \rangle$;
- a set **7** of logical values;
- a subset of designated values $D \subseteq \mathbf{7}$.

2 plausible Jaina systems:

- where #: neither asserted nor denied (interpretation *à la Ganeri*) $\mathbf{J}_{7G}$
- where #: both asserted and denied (interpretation *à la Matilal*) $\mathbf{J}_{7M}$

A non-value-functional definition: the value of a compound sentence partly depends upon the value of its components

A non-deterministic semantics (Rescher (1962), Avron (2008), Marcos (2009))

For any situation s_x (including s_1,s_2, etc.):

(∧-E)

$$\frac{\models_{s_x} (p \wedge q)}{\models_{s_x} p \qquad \models_{s_x} q} \qquad \frac{v(p \wedge q) = T}{v(p) = v(q) = T} \qquad \frac{\mathbf{a}_1(p \wedge q) = 1}{\mathbf{a}_1(p) = \mathbf{a}_1(q) = 1}$$

(∧-I)

$$\frac{\models_{s_x} p \qquad \models_{s_x} q}{\models_{s} (p \wedge q) \text{ or } \not\models_{s} (p \wedge q)} \qquad \frac{v(p) = v(q) = T}{v(p \wedge q) = T \text{ or } v(p \wedge q) \neq T} \qquad \frac{\mathbf{a}_1(p) = \mathbf{a}_1(q) = 1}{\mathbf{a}_1(p \wedge q) = 1 \text{ or } 0}$$

- The difference between $\mathbf{J}_{7G}$ and $\mathbf{J}_{7M}$: incomplete or inconsistent situations

For any (simple or complex) sentence α in $\mathbf{J}_7$:

$\models_{s_x} \alpha$	$\mathbf{a}_1(\alpha) = 1$
$\models_{s_x} \sim\alpha$	$\mathbf{a}_2(\alpha) = 1$
$\not\models_{s_x} \alpha$ entails $\models_{s_x} \sim\alpha$ or ($\not\models_{s_x} \alpha$ and $\not\models_{s_x} \sim\alpha$)	$\mathbf{a}_1(\alpha) = 0 \Rightarrow (\mathbf{a}_2(\alpha) = 1$ or $\mathbf{a}_{3G}(\alpha) = 1)$

But the converse doesn't hold!

Not ($\models_{s_x} \sim\alpha$ entails $\not\models_{s_x} \alpha$)	not $(\mathbf{a}_2(\alpha) = 1 \Rightarrow \mathbf{a}_1(\alpha) = 0)$

For any (simple or complex) sentence α in $\mathbf{J}_{7M}$:

$\models_{s_x} \alpha$	$\mathbf{a}_1(\alpha) = 1$ or $\mathbf{a}_{3M}(\alpha) = 1$
$\models_{s_x} \sim\alpha$	$\mathbf{a}_2(\alpha) = 1$

Two levels of (in)consistency: external, internal

The degree to which the Jaina system is paraconsistent is, on this interpretation, restricted to the sense in which a proposition can be [***B***], *i.e. both true and false because assertible* ***from one standpoint*** *but deniable* ***from another***. *It does not follow that there are standpoints from which contradictions can be asserted.*
(Ganeri (2002): 272)

Internal consistency was, in classical India, the essential attribute of a philosophical theory, and a universally acknowledged way to undermine the position of one's philosophical opponent was to show that their theory contradicted itself.
(Ganeri (2002): 273)

Strong (internal) paraconsistency: for any $i \in \mathbb{N}$, $\mathbf{a}_i(p) = \{1,0\}$
Weak (external) paraconsistency: for any $i \in \mathbb{N}$, $\mathbf{a}_i(p) = 1 \Longleftrightarrow \mathbf{a}_i(p) \neq 0$

$\mathbf{J}_7$ is an extension of classical logic (CL) in the sense that: for every p, $\mathbf{a}(p) = 1$ if and only iff $\mathbf{a}(p) \neq 0$; and the existential statements that translate the Jaina sentences follow the rules of classical sentential logic, including: double negation, commutativity, and associativity.

$\mathbf{J}_7$ has a *partial* valuation in the sense that, for some x, $x \cap 1 = x$ or 0 whenever $x = 1$. For $\mathbf{a}(\alpha) = \mathbf{a}(\psi) = 1$ means that α is true at some situation and ψ is true at some situation. But those respects needn't be the same: the situation at which α is true need not be the same as the situation at which ψ is true. This entails that $\mathbf{a}(\alpha \cap \psi)$ is partial: $\mathbf{a}(\alpha \cap \psi) = 1$ or 0 whenever $\mathbf{a}(\alpha) = \mathbf{a}(\psi) = 1$.

A characterization of the logical connectives will be followed by their corresponding matrix, where each logical value stands for an ordered combination of answers $\langle \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3 \rangle$. As an example of partial value, (7)-(6) means that the value of the corresponding sentence is either $\langle 1,1,1 \rangle = (7)$ or $\langle 0,1,1 \rangle = (6)$.

NEGATION

Negation 1: $\sim_G$

$\mathbf{a}_1(\sim_G p) = 1$ iff $\models_{s_x} \sim p$, i.e. $\mathbf{a}_2(p) = 1$; $\mathbf{a}_1(\sim p) = 0$, otherwise;

$\mathbf{a}_2(\sim_G p) = 1$ iff $\models_{s_x} \sim(\sim p)$, i.e. $\models_{s_x} p$, i.e. $\mathbf{a}_1(p) = 1$; $\mathbf{a}_2(\sim p) = 0$, otherwise;

$\mathbf{a}_3(\sim_G p) = 1$ iff $\not\models_{s_x} \sim p$ and $\not\models_{s_x} \sim(\sim p)$, i.e. $\not\models_{s_x} \sim p$ and $\not\models_{s_x} p)$, i.e. $\mathbf{a}_3(p) = 1$; $\mathbf{a}_3(\sim p) = 0$, otherwise.

Therefore: $\mathbf{A}(\sim_G(p)) = \langle \mathbf{a}_2(p), \mathbf{a}_1(p), \mathbf{a}_3(p) \rangle$

Negation 2: $\sim_M$

$\mathbf{a}_3(\sim_M(p)) = 1$ iff $\models_{s_x} \sim p$ and $\models_{s_x} \sim(\sim p)$, i.e. $\models_{s_x} \sim p$ and $\models_{s_x} p$, i.e. $\mathbf{a}_3(p) = 1$;

$\mathbf{a}_3(\sim p) = 0$, otherwise.

Therefore: $\mathbf{A}(\sim_M(p)) = \langle \mathbf{a}_2(p), \mathbf{a}_1(p), \mathbf{a}_3(p) \rangle$

Thus, $\sim_G$ and $\sim_M$ are identical. ■

	$f_{\sim}$
(1)	(2)
(2)	(1)
(3)	(3)
(4)	(4)
(5)	(6)
(6)	(5)
(7)	(7)

CONJUNCTION

Conjunction 1: $\wedge_G$

$\mathbf{a}_1(p \wedge_G q) = 1$ iff $\models_{s_x}(p \wedge q)$

By CL, for every sentence α and ψ we have:

$\models_{s_x}(\alpha \wedge \psi)$ entails that $\models_{s_x}\alpha$ and $\models_{s_x}\psi$, but the converse needn't hold

That is: if $\mathbf{a}_1(\alpha) = \mathbf{a}_1(\psi) = 1$, then $\mathbf{a}_1(\alpha \wedge \psi) =$ 1 or 0

Hence $\mathbf{a}_1(p \wedge_G q) =$ 1 or 0 iff $\mathbf{a}_1(p) = \mathbf{a}_1(q) = 1$; $\mathbf{a}_1(p \wedge_G q) = 0$, otherwise.

$\mathbf{a}_2(p \wedge_G q) = 1$ iff $\models_{s_x}\sim(p \wedge q)$, i.e. $\models_{s_x}(\sim p \vee \sim q)$

By CL, we have:

$\models_{s_x}\alpha$ or $\models_{s_x}\psi$ entails that $\models_{s_x}(\alpha \vee \psi)$

That is: $\mathbf{a}_1(\alpha \vee \psi) = 1$ if $\mathbf{a}_1(\alpha) = 1$ or $\mathbf{a}_1(\psi) = 1$; $\mathbf{a}_1(\alpha \vee \psi) = 0$, otherwise.

Thus $\mathbf{a}_2(p \wedge_G q) = 1$ iff $\mathbf{a}_1(\sim p) = 1$ or $\mathbf{a}_1(\sim q) = 1$, i.e. $\mathbf{a}_2(p) = \mathbf{a}_2(q) = 1$; $\mathbf{a}_2(p \wedge_G q) = 0$, otherwise.

$\mathbf{a}_3(p \wedge_G q) = 1$ iff $\not\models_{S_x}(p \wedge q)$ and $\not\models_{S_x}{\sim}(p \wedge q)$, i.e.

$\not\models_{S_x} p$ or $\not\models_{S_x} q$ and $\not\models_{S_x}({\sim}p \vee {\sim}q)$, i.e. ($\not\models_{S_x} p$ or $\not\models_{S_x} q$) and ($\not\models_{S_x}{\sim}p$ and $\not\models_{S_x}{\sim}q$)

By CL, we have:

(Ass) $((\alpha \vee \psi) \wedge \gamma) \Leftrightarrow ((\alpha \wedge \gamma) \vee (\psi \wedge \gamma))$

Hence $\mathbf{a}_3(p \wedge_G q) = 1$ iff ($\not\models_{S_x} p$ and $\not\models_{S_x}({\sim}p \vee {\sim}q)$) or ($\not\models_{S_x} q$ and $\not\models_{S_x}({\sim}p \vee {\sim}q)$), i.e.

($\not\models_{S_x} p$ and $\not\models_{S_x}{\sim}p$ and $\not\models_{S_x}{\sim}q$) or ($\not\models_{S_x} q$ and $\not\models_{S_x}{\sim}p$ and $\not\models_{S_x}{\sim}q$).

For every sentence α: if $\not\models_{S_x}({\sim}\alpha)$, then $\mathbf{a}_2(\alpha) \neq 1$, i.e. $\mathbf{a}_1(\alpha) = 1$ or $\mathbf{a}_3(\alpha) = 1$

Hence $\mathbf{a}_3(p \wedge_G q) = 1$ iff $\mathbf{a}_3(p) = \mathbf{a}_1(q) = 1$, or $\mathbf{a}_3(p) = \mathbf{a}_3(q) = 1$, or $\mathbf{a}_1(q) = \mathbf{a}_3(p) = 1$;
$\mathbf{a}_3(p \wedge_G q) = 0$, otherwise.

Let the partial values be marked in gray, when $\mathbf{a}(\alpha) = 1$ or 0. Therefore:

$\mathbf{A}(p \wedge_G q) =$

$\langle \mathbf{a}_1(p) \cap \mathbf{a}_1(q), \mathbf{a}_2(p) \cup \mathbf{a}_2(q), (\mathbf{a}_3(p) \cap \mathbf{a}_1(q)) - (\mathbf{a}_3(p) \cap \mathbf{a}_3(q)) - (\mathbf{a}_1(p) \cap \mathbf{a}_3(q))\rangle$

Conjunction 2: $\wedge_M$

$\mathbf{a}_3(p \wedge_M q) = 1$ iff $\models_{s_x}(p \wedge q)$ and $\models_{s_x}\sim(p \wedge q)$, i.e. $\models_{s_x} p$ and $\models_{s_x} q$ and $\models_{s_x} (\sim p \vee \sim q)$

By CL, we have: $\models_{s_x}\sim p$ entails $\models_{s_x}(\sim p \vee \sim q)$, and $\models_{s_x}\sim q$ entails $\models_{s_x}(\sim p \vee \sim q)$

$\mathbf{a}_3(p \wedge_M q) = 1$ iff ($\models_{s_x} p$ and $\models_{s_x} q$ and $\models_{s_x}\sim p$) or ($\models_{s_x} p$ or $\models_{s_x} q$ and $\models_{s_x}\sim q$)

Hence $\mathbf{a}_3(p \wedge_M q) = 1$ iff $\mathbf{a}_3(p) = \mathbf{a}_1(q) = 1$, or $\mathbf{a}_3(p) = \mathbf{a}_3(q)$, $\mathbf{a}_1(p) = \mathbf{a}_3(q)$;

$\mathbf{a}_3(p \wedge_M q) = 0$, otherwise.

Therefore: $\mathbf{A}(p \wedge_M q) =$

$\langle \mathbf{a}_1(p) \cap \mathbf{a}_1(q), \mathbf{a}_2(p) \cup \mathbf{a}_2(q), (\mathbf{a}_3(p) \cap \mathbf{a}_1(q)) - (\mathbf{a}_3(p) \cap \mathbf{a}_3(q)) - (\mathbf{a}_1(p) \cap \mathbf{a}_3(q))\rangle$

Thus, $\wedge_G$ and $\wedge_M$ are identical. ■

$f_\wedge$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	(1)	(2)	(3)-(2)	(4)	(5)-(4)	(6)	(7)-(6)
(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)
(3)	(3)-(2)	(2)	(3)-(2)	(6)	(7)-(6)	(6)	(7)-(6)
(4)	(4)	(2)	(6)	(4)	(4)	(6)	(6)
(5)	(5)-(4)	(2)	(7)-(6)	(4)	(5)-(4)	(6)	(7)-(6)
(6)	(6)	(2)	(6)	(6)	(6)	(6)	(6)
(7)	(7)-(6)	(2)	(7)-(6)	(6)	(7)-(6)	(6)	(7)-(6)

In bold red: $\mathbf{A}(p \wedge q) = \langle 1,0,0 \rangle$ or $\langle 0,0,0 \rangle$, hence $\mathbf{A}(p \wedge q) = \langle 1,0,0 \rangle$

DISJUNCTION

Disjunction 1: $\vee_G$

$\mathbf{a}_1(p \vee_G q) = 1$ iff $\models_{s_x}(p \vee q)$

By CL, ($\models_{s_x} p$ or $\models_{s_x} q$ entails $\models_{s_x}(p \vee q)$

Hence $\mathbf{a}_1(p \vee_G q) = 1$ if $\mathbf{a}_1(p) = 1$ or $\mathbf{a}_1(q) = 1$; $\mathbf{a}_1(p \vee_G q) = 0$, otherwise.

$\mathbf{a}_2(p \vee_G q) = 1$ iff $\models_{s_x} \sim(p \vee q)$, i.e. $\models_{s_x}(\sim p \wedge \sim q)$

By CL, $\models_{s_x}(\sim p \wedge \sim q)$ entails $\models_{s_x} \sim p$ and $\models_{s_x} \sim q$; but the converse needn't hold.

Hence $\mathbf{a}_2(p \vee_G q) = 1$ or 0 iff $\mathbf{a}_1(\sim p) = \mathbf{a}_1(\sim q) = 1$, i.e. $\mathbf{a}_2(p) = \mathbf{a}_2(q) = 1$; $\mathbf{a}_2(p \vee_G q) = 0$, otherwise.

$\mathbf{a}_3(p \vee_G q) = 1$ iff $\not\models_{s_x}(p \vee q)$ and $\not\models_{s_x} \sim(p \vee q)$, i.e.

$\not\models_{s_x} p$ and $\not\models_{s_x} q$ and $\not\models_{s_x}(\sim p \wedge \sim q)$, i.e. $\not\models_{s_x} p$ and $\not\models_{s_x} q$ and ($\not\models_{s_x} \sim p$ or $\not\models_{s_x} \sim q$)

$\mathbf{a}_3(p \vee_G q) = 1$ iff ($\not\models_{s_x} p$ and $\not\models_{s_x} q$ and $\not\models_{s_x} \sim p$) or ($\not\models_{s_x} p$ and $\not\models_{s_x} q$ and $\not\models_{s_x} \sim q$)

For every sentence α: if $\not\models_{S_x} \alpha$, then $\mathbf{a}_1(\alpha) \neq 1$, i.e. $\mathbf{a}_2(\alpha) = 1$ or $\mathbf{a}_3(\alpha) = 1$

Hence $\mathbf{a}_3(p \vee_G q) = 1$ iff $\mathbf{a}_3(p) = \mathbf{a}_2(q) = 1$, or $\mathbf{a}_3(p) = \mathbf{a}_3(q) = 1$, or $\mathbf{a}_2(p) = \mathbf{a}_3(p) = 1$;
$\mathbf{a}_3(p \vee_G q) = 0$, otherwise.
Therefore: $\mathbf{A}(p \vee_G q) =$

$$\langle \mathbf{a}_1(p) \cup \mathbf{a}_1(q); \mathbf{a}_2(p) \cap \mathbf{a}_2(q); (\mathbf{a}_3(p) \cap \mathbf{a}_2(q)) - (\mathbf{a}_3(p) \cap \mathbf{a}_3(q)) - (\mathbf{a}_2(p) \cap \mathbf{a}_3(q))\rangle.$$

Disjunction 2: $\vee_M$

$\mathbf{a}_3(p \vee_M q) = 1$ iff $\models_{S_x} (p \vee q)$ and $\models_{S_x} \sim(p \vee q)$, i.e. $\models_{S_x} (p \vee q)$ and $\models_{S_x} \sim p$ and $\models_{S_x} \sim q$

By CL, we have: $\models_{S_x} p$ entails $\models_{S_x} (p \vee q)$, and $\models_{S_x} q$ entails $\models_{S_x} (p \vee q)$

$\mathbf{a}_3(p \vee_M q) = 1$ iff ($\models_{S_x} \sim p$ and $\models_{S_x} \sim q$ and $\models_{S_x} p$) or ($\models_{S_x} \sim p$ and $\models_{S_x} \sim q$ and $\models_{S_x} q$)

Hence $\mathbf{a}_3(p \vee_M q) = 1$ iff $\mathbf{a}_3(p) = \mathbf{a}_2(q) = 1$, or $\mathbf{a}_3(p) = \mathbf{a}_3(q) = 1$, or $\mathbf{a}_2(p) = \mathbf{a}_3(q) = 1$;
$\mathbf{a}_3(p \vee_M q) = 0$, otherwise.
Therefore: $\mathbf{A}(p \vee_M q) =$

$$\langle \mathbf{a}_1(p) \cup \mathbf{a}_1(q), \mathbf{a}_2(p) \cap \mathbf{a}_2(q), (\mathbf{a}_3(p) \cap \mathbf{a}_2(q)) - (\mathbf{a}_3(p) \cap \mathbf{a}_3(q)) - (\mathbf{a}_2(p) \cap \mathbf{a}_3(q))\rangle$$

Thus, $\vee_G$ and $\vee_M$ are identical. ■

$f_\vee$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
(2)	(1)	(2)	(3)-(1)	(4)	(5)	(6)-(4)	(7)-(5)
(3)	(1)	(3)-(1)	(3)-(1)	(5)	(5)	(7)-(5)	(7)-(5)
(4)	(1)	(4)	(5)	(4)	(5)	(4)	(5)
(5)	(1)	(5)	(5)	(5)	(5)	(5)	(5)
(6)	(1)	(6)-(4)	(7)-(5)	(4)	(4)	(6)-(4)	(7)-(5)
(7)	(1)	(7)-(5)	(7)-(5)	(5)	(5)	(7)-(5)	(7)-(5)

In bold red: **A**(p∨q) = ⟨0,1,0⟩ or ⟨0,0,0⟩, hence **A**(p∨q) = ⟨0,1,0⟩

CONDITIONAL

Conditional 1: $\supset_G$

$\mathbf{a}_1(p \supset_G q) = 1$ iff $\models_{s_x}(p \supset q)$, i.e. $\models_{s_x}(\sim p \vee q)$

Hence $\mathbf{a}_1(p \supset_G q) = 1$ iff $\mathbf{a}_1(\sim p) = 1$, i.e. $\mathbf{a}_2(p) = 1$, or $\mathbf{a}_1(q) = 1$; $\mathbf{a}_1(p \supset_G q) = 0$, otherwise.

$\mathbf{a}_2(p \supset_G q) = 1$ iff $\models_{s_x}\sim(p \supset q)$, i.e. $\models_{s_x}(p \wedge \sim q)$

$\mathbf{a}_2(p \supset_G q) =$ 1 or 0 iff $\mathbf{a}_1(p) = \mathbf{a}_1(\sim q)$, i.e. $\mathbf{a}_2(q) = 1$; $\mathbf{a}_2(p \supset_G q) = 0$, otherwise.

$\mathbf{a}_3(p \supset_G q) = 1$ iff $\not\models_{s_x}(p \supset q)$ and $\not\models_{s_x}\sim(p \rightarrow q)$, i.e.

$\not\models_{s_x}(\sim p \vee q)$ and $\not\models_{s_x}(p \wedge \sim q)$, i.e. $\not\models_{s_x}\sim p$ and $\not\models_{s_x} q$ and ($\not\models_{s_x} p$ or $\not\models_{s_x}\sim q$)

$\mathbf{a}_3(p \supset_G q) = 1$ iff ($\not\models_{s_x}\sim p$ and $\not\models_{s_x} q$ and $\not\models_{s_x} p$) or ($\not\models_{s_x}\sim p$ and $\not\models_{s_x} q$ $\not\models_{s_x}\sim q$)

Hence $\mathbf{a}_3(p \supset_G q) = 1$ iff $\mathbf{a}_3(p) = \mathbf{a}_2(q) = 1$, or $\mathbf{a}_3(p) = \mathbf{a}_3(q) = 1$, or $\mathbf{a}_1(p) = \mathbf{a}_3(q) = 1$; $\mathbf{a}_3(p \supset_G q) = 0$, otherwise.

Therefore: $\mathbf{A}(p \supset_G q) =$

$$\langle \mathbf{a}_2(p) \cup \mathbf{a}_1(q), \mathbf{a}_1(p) \cap \mathbf{a}_2(q), (\mathbf{a}_3(p) \cap \mathbf{a}_2(q) - (\mathbf{a}_3(p) \cap \mathbf{a}_3(q) - (\mathbf{a}_1(p) - \mathbf{a}_3(q)\rangle$$

Conditional 2: $\supset_M$

$\mathbf{a}_3(p \supset_M q) = 1$ iff $\models_{s_x}(p \supset q)$ and $\models_{s_x}{\sim}(p \supset q)$, i.e. $\models_{s_x}({\sim}p \vee q)$ and $\models_{s_x}(p \wedge {\sim}q)$, i.e. $\models_{s_x}({\sim}p \vee q)$ and ($\models_{s_x}p$ and $\models_{s_x}{\sim}q$)

By CL, we have: $\models_{s_x}{\sim}p$ entails $\models_{s_x}({\sim}p \vee q)$, and $\models_{s_x}q$ entails $\models_{s_x}({\sim}p \vee q)$.

$\mathbf{a}_3(p \supset_M q) = 1$ iff ($\models_{s_x}p$ and $\models_{s_x}{\sim}q$ and $\models_{s_x}{\sim}p$) or ($\models_{s_x}p$ and $\models_{s_x}{\sim}q$ and $\models_{s_x}q$)

Hence $\mathbf{a}_3(p \supset_M q) = 1$ iff $\mathbf{a}_3(p) = \mathbf{a}_2(q) = 1$, or $\mathbf{a}_3(p) = \mathbf{a}_3(q) = 1$, or $\mathbf{a}_1(p) = \mathbf{a}_3(q) = 1$;
$\mathbf{a}_3(p \rightarrow_M q) = 0$, otherwise.

Therefore: $\mathbf{A}(p \supset_M q) =$

$$\langle \mathbf{a}_2(p) \cup \mathbf{a}_1(q), \mathbf{a}_1(p) \cap \mathbf{a}_2(q), (\mathbf{a}_3(p) \cap \mathbf{a}_2(q)) - (\mathbf{a}_3(p) \cap \mathbf{a}_3(q)) - (\mathbf{a}_1(p) \cap \mathbf{a}_3(q))\rangle$$

Thus, $\supset_G$ and $\supset_M$ are identical. ■

$f_{\supset}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	(1)	(2)	(3)	(4)	(5)	(6)-(4)	(7)-(5)
(2)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
(3)	(1)	(3)	(3)	(5)	(5)	(7)-(5)	(7)-(5)
(4)	(1)	(4)	(5)	(4)	(5)	(4)	(5)
(5)	(1)	(6)-(4)	(7)-(5)	(4)	(5)	(6)-(4)	(7)-(5)
(6)	(1)	(5)	(5)	(5)	(5)	(5)	(5)
(7)	(1)	(7)-(5)	(7)-(5)	(5)	(5)	(7)-(5)	(7)-(5)

In bold red: **A**(p⊃q) = ⟨0,1,0⟩ or ⟨0,0,0⟩, hence **A**(p⊃q) = ⟨0,1,0⟩

2.5 What is a logical inference in J_7?

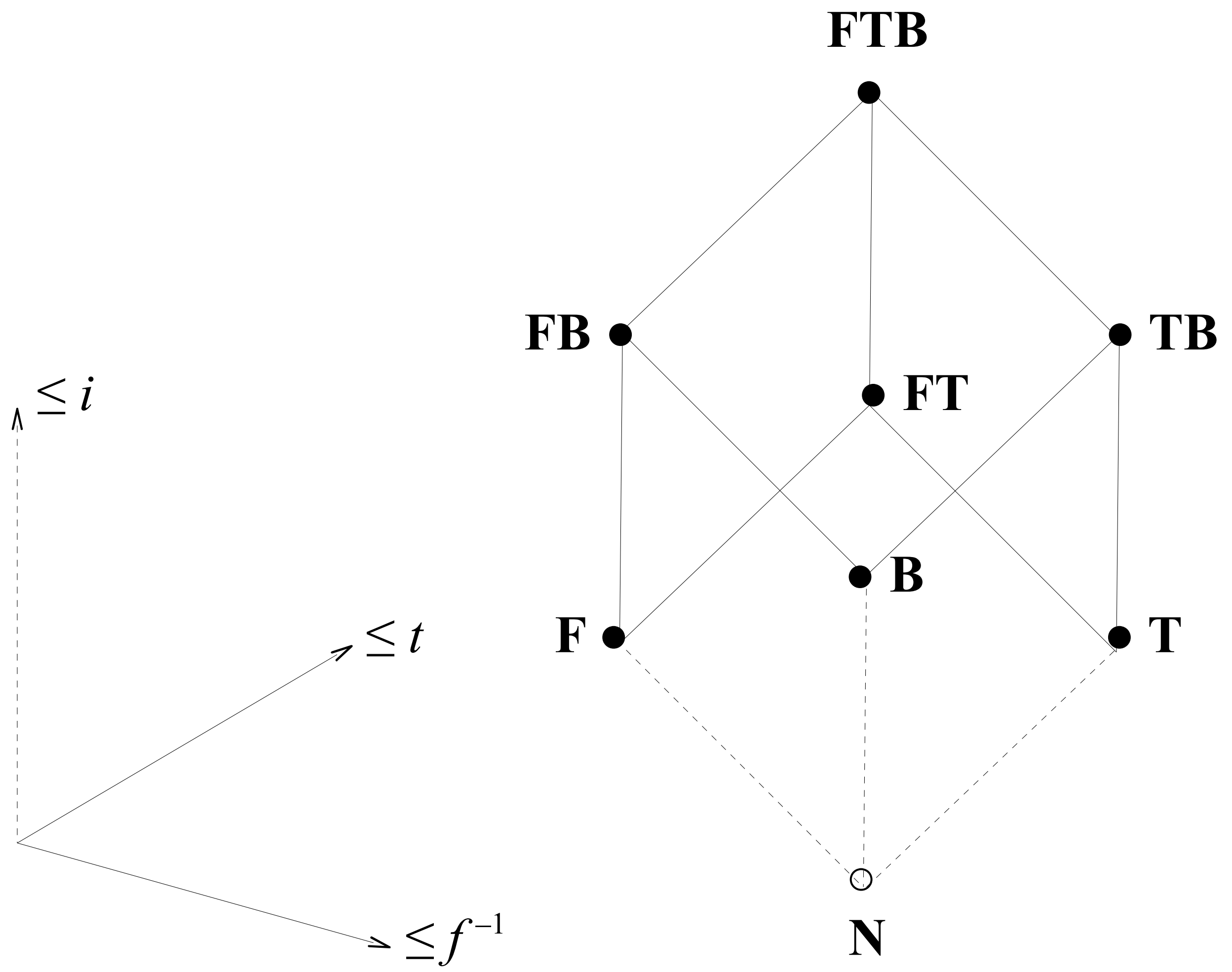
Two general trends for a characterization of entailment (logical consequence)

- A entails B iff $v(A) \leq v(B)$ — $\leq$: ordering relation
- A entails B iff $v(B) \in D$ whenever $v(A) \in D$ — D: designated values

What counts as a good argument? This is certainly a topic that exercised Jaina and other Indian logicians. Generally speaking they seem to have endorsed an account of validity in terms of the preservation of, as we would now put it in the context of modern many-valued logics, designated values. That, at any rate, is the natural path to go down, given the preceding machinery. What, then, should we take to be the designated values, that is, the values that licence assertion?

(Priest (2008): 266)

Bi-and-a-half lattice SEVEN$_{2.5}$



- A Jaina sentence α is held to be true whenever it is asserted. That is:
  $\alpha \in D$ iff $\mathbf{A}(\alpha) = \langle 1,–,– \rangle$, i.e. $D = \{(1),(3),(5),(7)\}$ in $\mathbf{J}_{7G}$
  $D = \{(1),(3),(4),(5),(6),(7)\}$ in $\mathbf{J}_{7M}$

- No formula is valid in $\mathbf{JL}_{G}$: no $\models \alpha$

PNC: $\sim(\alpha \wedge \sim\alpha)$ is valid in $\mathbf{JL}_{M}$, but not **Explosion**: $(\alpha \wedge \sim\alpha) \models \psi$

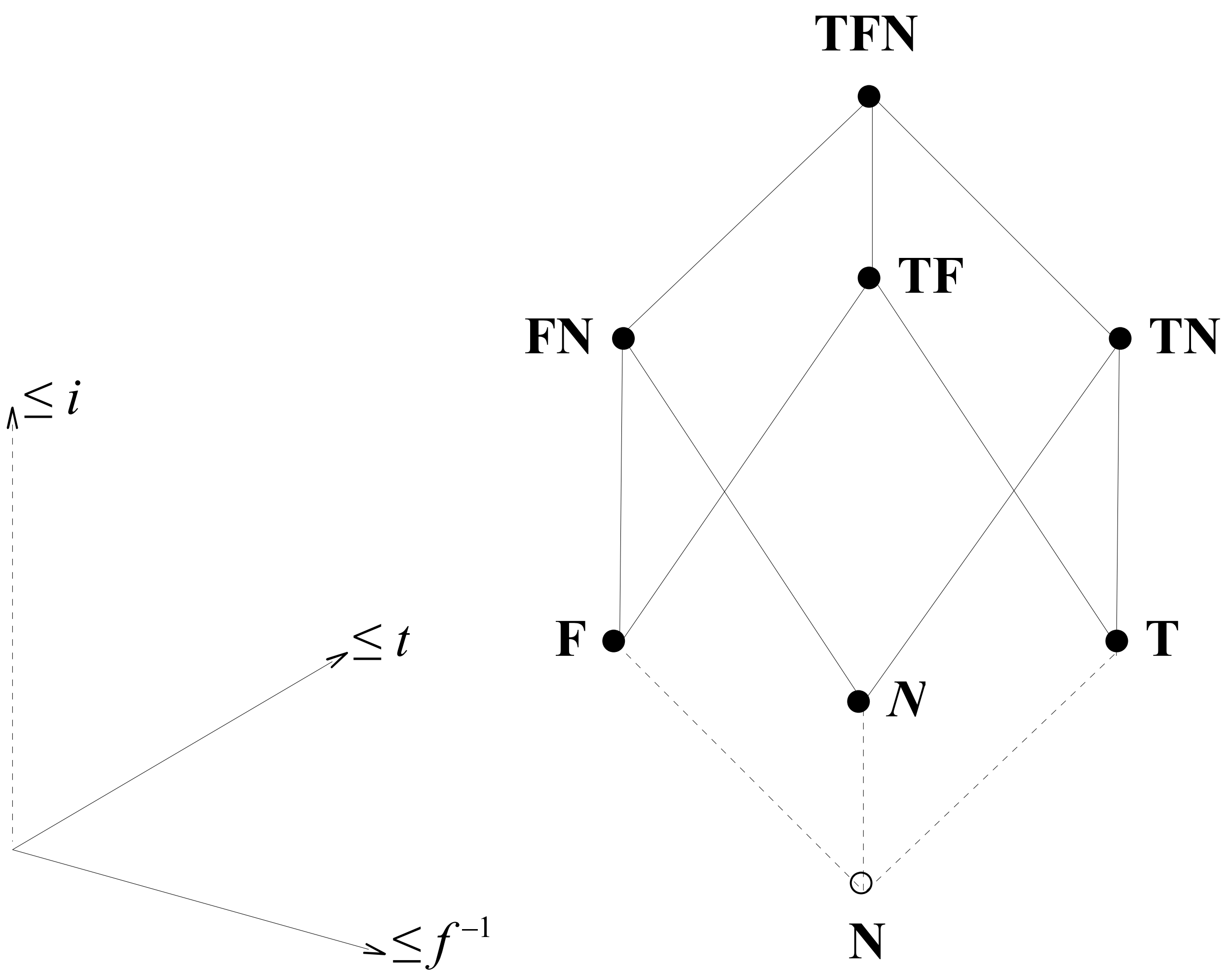
Schang (2009a):

Proposition 1. $\mathbf{J}_{7}$ is quasi-equivalent with Type 2 Semantics in Priest (2008)

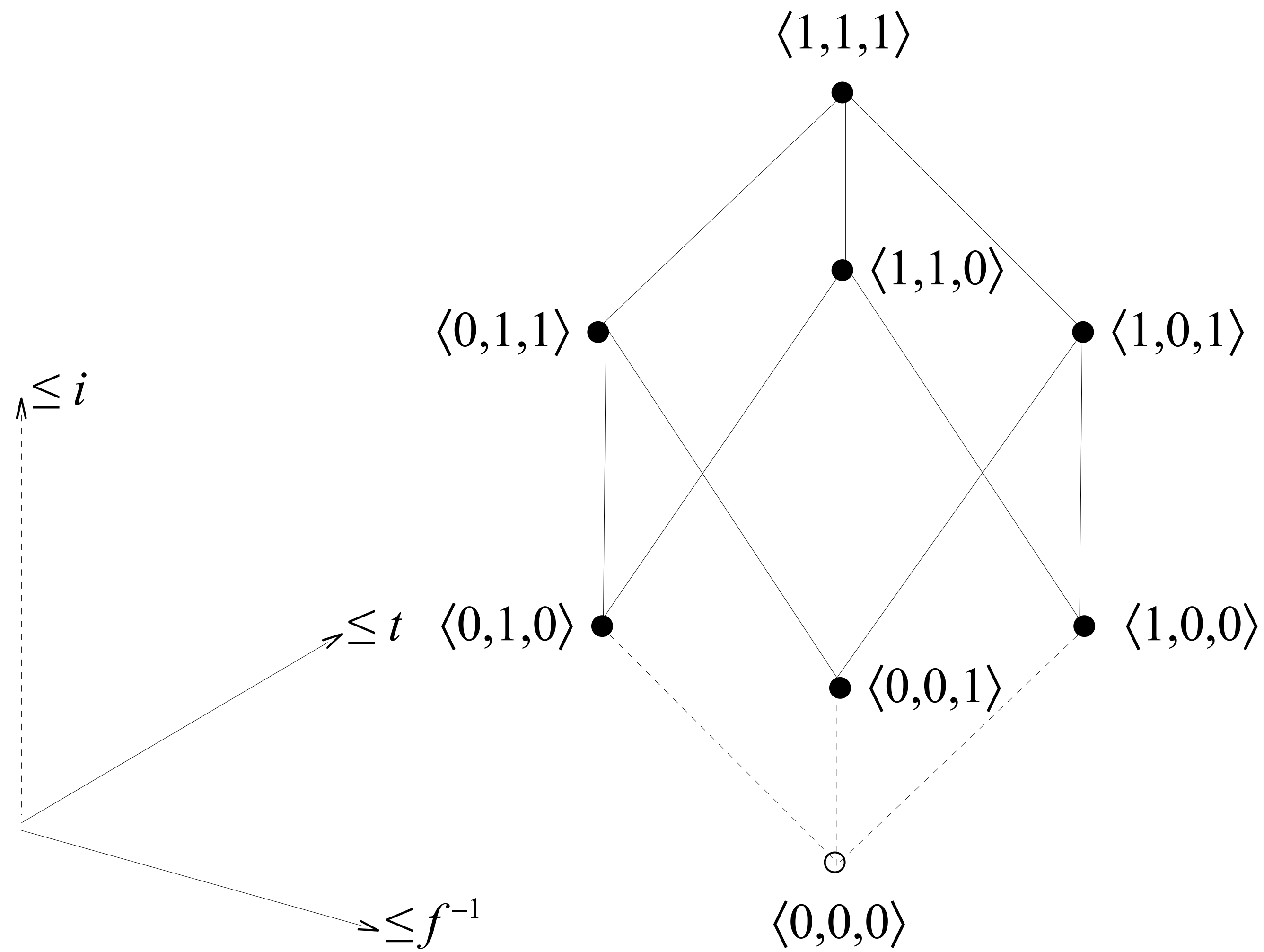
Proposition 2. $\mathbf{J}_{7G}$ is quasi-equivalent with $\mathbf{K}_{3}$ (Kleene's (strong) 3-valued logic); $\mathbf{J}_{7M}$ is quasi-equivalent with **LP** (Priest's 3-valued Logic of Paradox)

Proposition 3. $\mathbf{J}_{7}$ is equivalent with an extension $\mathbf{J}_{15}$ (including (a) and (b) for #)
$\mathbf{Q}(p) = \langle \mathbf{q}_{1}(p),\mathbf{q}_{2}(p),\mathbf{q}_{3}(p),\mathbf{q}_{4}(p) \rangle$, where $\mathbf{q}_{3}(p)$: "$v(p) = N$?" and $\mathbf{q}_{4}(p)$: "$v(p) = B$?"

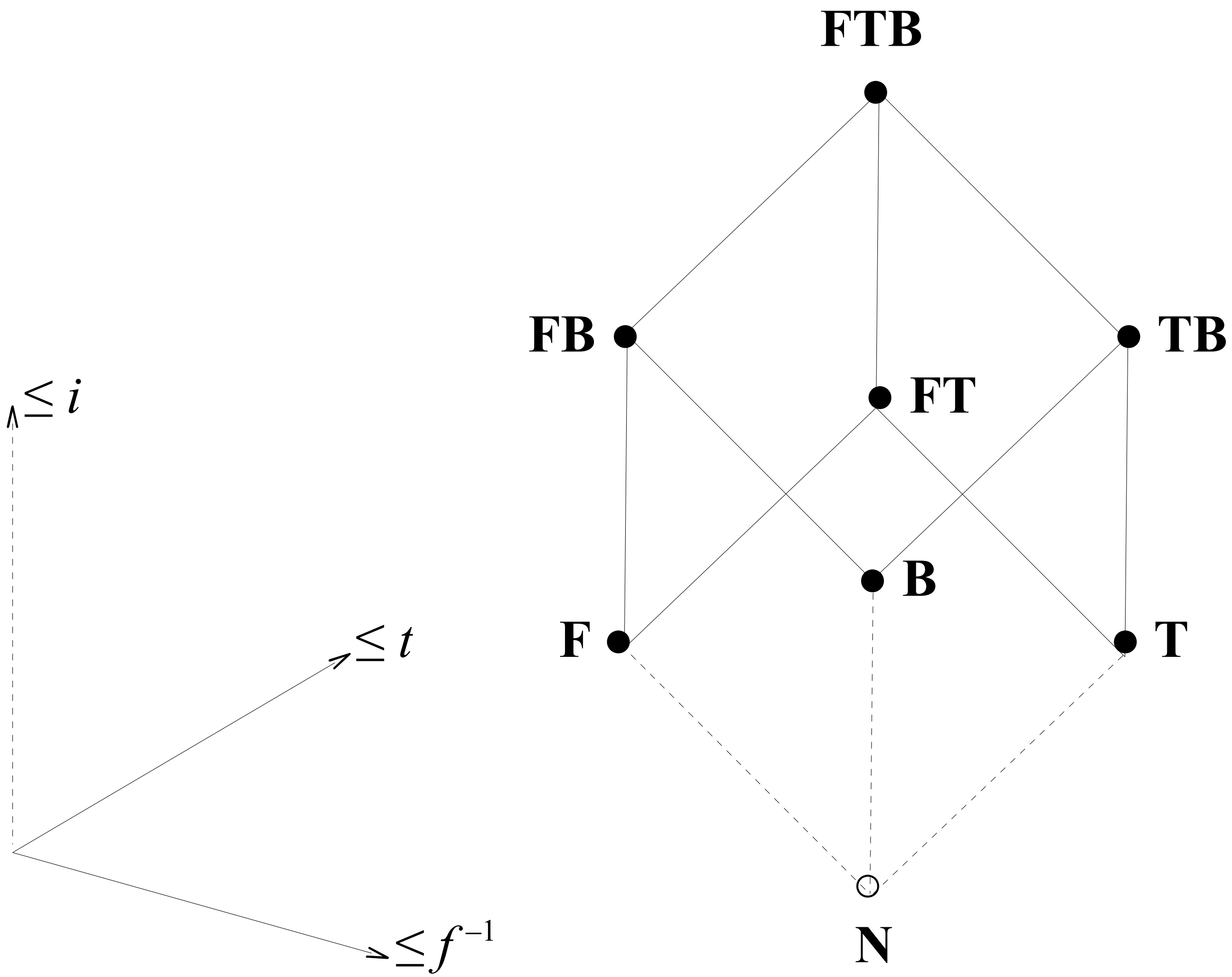
One bi-and-a-half lattice for $\mathbf{J_{7G}}$



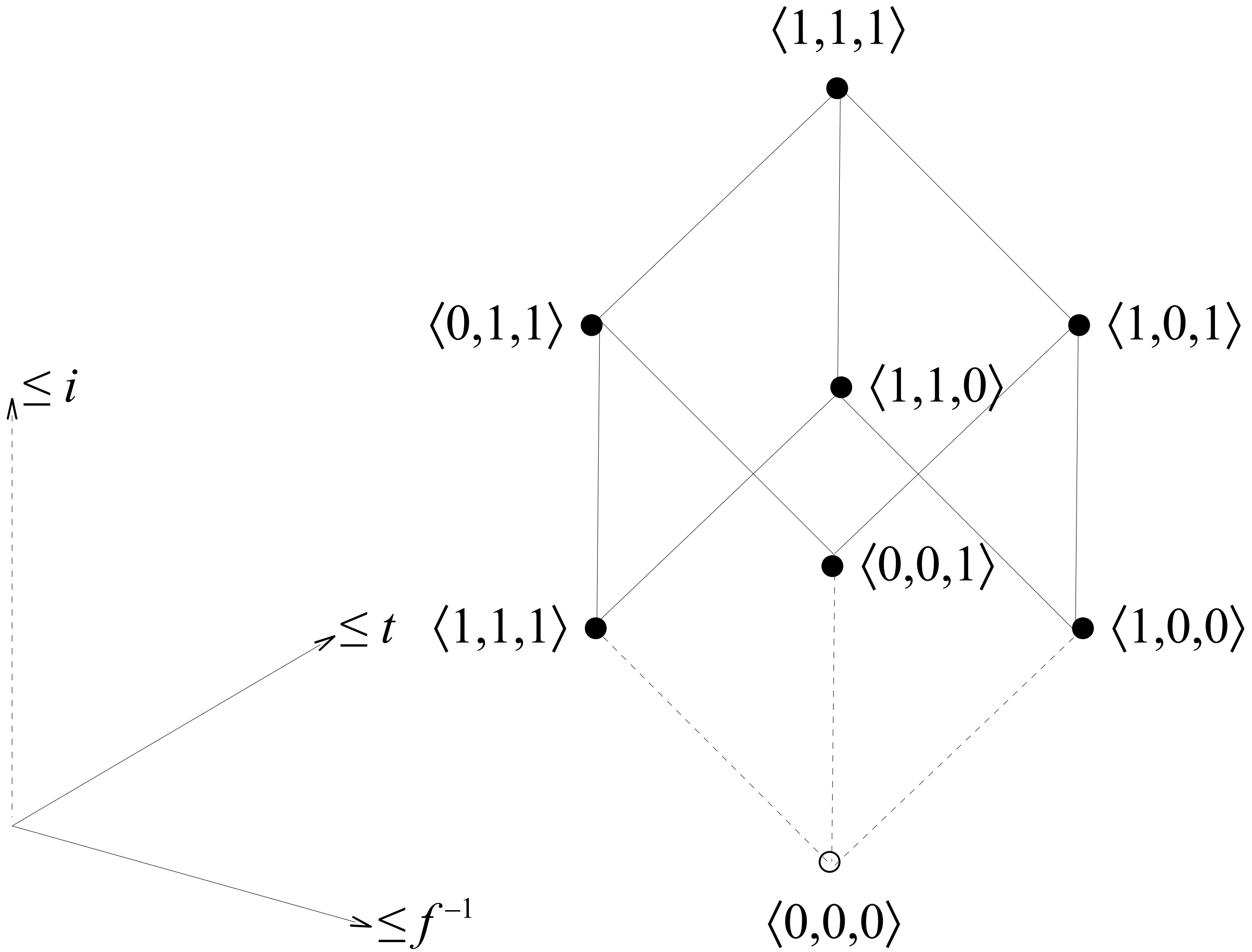
One bi-and-a-half lattice for $\mathbf{J_{7G}}$



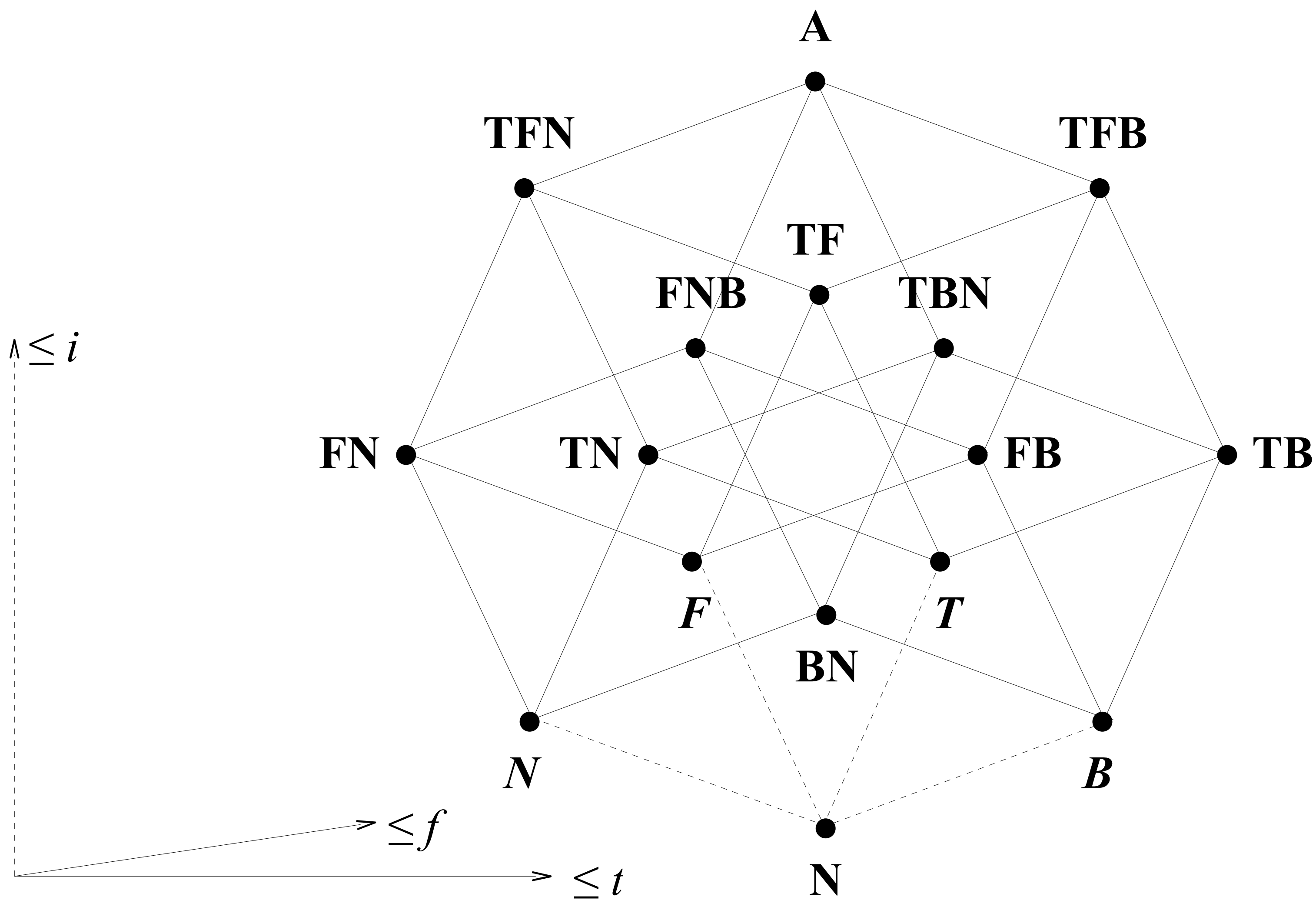
One bi-and-a-half lattice for $\mathbf{J_{7M}}$



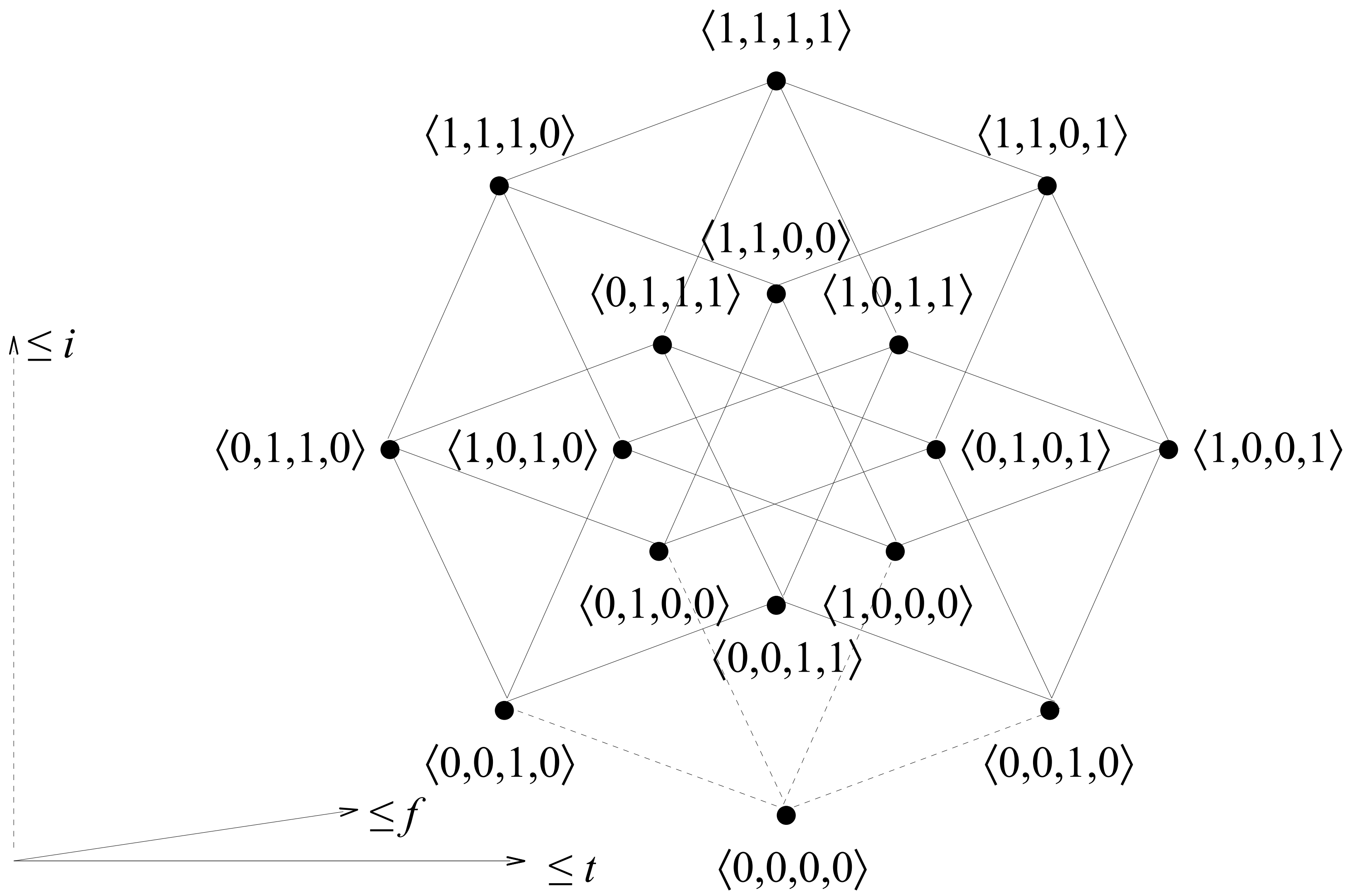
One bi-and-a-half lattice for $\mathbf{J}_{7M}$



One bi-and-a-half lattice for $\mathbf{J_{15}}$



One bi-and-a-half lattice for $\mathbf{J}_{15}$



2.6 Is $\mathbf{J_7}$ a modal logic?

- "Modal logic": logic with modalities (modes of truth) and iterations

Possibility: *syād* **A**(p) ≠ (8)

Its dual of necessity: adhgajanyāyah **A**(p) = (1) or (2)

A counterpart of Jaśkowski (1969): Discussive Logic $\mathbf{D_2}$?

Being true = being true in some respect p in $\mathbf{J_7}$ = p in $\mathbf{D_2}$ = ◊p in **ML**

Can iterations make sense in $\mathbf{J_7}$? ◊◊p, □◊p, ◊□p, □◊◊□p, etc.

$\mathbf{D_2}$ is equated with **S5**, therefore $\mathbf{J_7}$ cannot be equated with $\mathbf{D_2}$

A difference in translation: ~~p in $\mathbf{D_2}$ = (◊~)(◊~)p = ◊□p in **ML** $\alpha = (\Diamond\sim)\alpha$

~~p in $\mathbf{J_7}$ = ◊(~~p) = ◊p in **ML** $\alpha = \Diamond(\alpha)$

3 Conclusion: Truths and Beliefs

- Many-faceted reality **vs** Bivalence

  What is the truth-bearer in the Jaina system?
  In Western (Aristotelian) logic: a (true or false) proposition
  In $\mathbf{J}_7$: a (many-valued) statement

  What is the difference between truth and truth-claim?
  Truth: not a property of Fregean propositions
  the synthesis of every truth-claims

- Propositional vs. Sentential Logics

  What is the truth-bearer in the Jaina system?
  In Fregean logics: a proposition (*Gedanke*, Quine's "standing sentence")
  In $\mathbf{J}_7$: a sentence (context-dependent) … a non-Fregean logic

- Inconsistency and Paraconsistency

  $\mathbf{J_7}$ doesn't challenge the Aristotelian **PNC** (true/false in different respects)
  A strong paraconsistent logic: a logic where $\mathbf{A}_i(p) = \{1,0\}$
  Does Priest's dialetheism require strong inconsistency for the Liar Paradox?

- Two rival "epistemic logics" (with competing truth-assignments)

Roughly, the difference between Buddhism and Jainism in this respect lies in the fact that the former avoids by rejecting the extremes altogether, while the latter does it by accepting both with qualifications and also by reconciling them.
(Matilal (1998): 129)

Application 2: *Catuṣkoṭi* (a logic for skepticism)

- According to the Madhyamika's two-truth theory, truth is either conventional (partial: *saṃvṛti-satya*) or absolute (*paramārtha-satya*).
  The Jains defended a **partial** theory of truth (anekanta: non-one-sided), whereas the Madhyamikas endorsed an **absolute** theory of truth.

- According to their theory of emptiness (*sūnyatāvāda*), whatever is not self-originated cannot be predicated truly of anything.

- Nāgārjuna's resulting skepticism is summarized in the first verse of his *Mūlamadhyamaka-kārikā*: four sentences (or lemmas) are equally denied by means of stances (*dṛṣṭis*, or *koṭi*) in the Principle of Four-Cornered Negation (**4CN**) or Tetralemma (*catuṣkoṭi*).

- **4CN**: four sentences are equally denied by Nāgārjuna's:

(a)	"Does a thing or being come out itself?"	"No."
(b)	"Does a thing or being come out the other?"	"No."
(c)	"Does it come out of both itself and the other?"	"No."
(d)	"Does it come out of neither?"	"No."

- How can (a)-(d) be consistently denied together?

- 1st reading: classical negation ~ (*paryudāsa pratiṣedha*)

(a')	Not (S is P):	$\vdash \sim(p)$
(b')	Not (S is not P):	$\vdash \sim(\sim p)$
(c')	Not (S is P and S is not P):	$\vdash \sim(p \wedge \sim p)$
(d')	Not (neither S is P nor S is not P):	$\vdash \sim(\sim(p \vee \sim p))$

By (b'): $\sim(\sim p) \Rightarrow p$

By (a')-(b'): $\sim p, \sim(\sim p) \Rightarrow p \wedge \sim p$

By (a')-(b') and (c'): $p \wedge \sim p, \sim(p \wedge \sim p) \Rightarrow (p \wedge \sim p) \wedge \sim(p \wedge \sim p)$

4CN is not a paraconsistent system: the Madhyamakas were said to respect the Principle of Contradiction as a basic metaprinciple (*paribhāsā*)

Therefore: for every sentence α, $\nvdash (\alpha \wedge \sim\alpha)$

- 2nd reading: intuitionistic negation $\neg$

(a")	Not (S is P):	$\vdash \sim(p)$
(b")	Not (S is not-P):	$\vdash \sim(\neg p)$
(c")	Not (S is P and S is not-P):	$\vdash \sim(p \wedge \neg p)$
(d")	Not (neither S is P nor S is not-P):	$\vdash \sim(\sim(p \vee \neg p))$

By (d"): $\sim(\sim(p \vee \neg p)) \Rightarrow (p \vee \neg p)$

4CN is not an intuitionistic system, given that $\nvdash (p \vee \neg p)$ in IL

- 3rd reading: illocutionary negation (*prasajya pratisedha*)

(a''')	Not (S is P):	$\not\vdash$ p
(b''')	Not (S is not-P):	$\not\vdash$ ~p
(c''')	Not (S is P and S is not-P):	$\not\vdash$ (p ∧ ~p)
(d''')	Not (neither S is P nor S is not-P):	$\not\vdash$ ~(p ∨ ~p)

Denial is a no-answer to a preceding question ($\vdash$ ~p, or $\not\vdash$ p)

to be distinguished from negative assertion ($\vdash$ ~p)

- Denial is not a truth-functional operator, but an attitude toward a sentence

- A logical of such attitudes is required to make (a)-(d) consistent within **QAS**

Definition 4. A logic of acceptance and rejection is a model $\mathbf{AR}_4 = \langle \mathfrak{M}, \mathbf{A} \rangle$ upon a sentential language $\mathscr{L}$ and its set of logical connectives $f_c = \{\sim, \wedge, \vee, \supset\}$. It includes a logical matrix $\mathfrak{M} = \langle \mathbf{Q}; \mathbf{4}; D \rangle$, with :

- a function $\mathbf{Q}(\alpha) = \langle \mathbf{q}_1(\alpha), \mathbf{q}_2(\alpha) \rangle$
  $\mathbf{q}_1(\alpha)$: "Is p held to be true?", $\mathbf{q}_2(\alpha)$: "Is p held to be false?"
- a set of logical values $\mathbf{4} = \{\langle 1,0 \rangle, \langle 1,1 \rangle, \langle 0,0 \rangle, \langle 0,1 \rangle\}$
  $\langle 1,0 \rangle$: strong affirmation, $\langle 1,1 \rangle$: weak affirmation
  $\langle 0,0 \rangle$: weak denial, $\langle 0,1 \rangle$: strong denial
- a subset of designated values $D \subseteq \mathbf{4}$, where $D = \{\langle 1,0 \rangle, \langle 1,1 \rangle\}$
- a total ordering relation in V: $\langle 1,0 \rangle < \langle 1,1 \rangle < \langle 0,0 \rangle < \langle 0,1 \rangle$

Definition 5. For every sentence α such that $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha) \rangle$:

$\mathbf{A}(\sim\alpha) = \langle \mathbf{a}_2(\alpha), \mathbf{a}_1(\alpha) \rangle$

$\mathbf{A}(\alpha \wedge \psi) = min(\mathbf{A}(\alpha), \mathbf{A}(\psi))$

$\mathbf{A}(\alpha \vee \psi) = max(\mathbf{A}(\alpha), \mathbf{A}(\psi))$

$\mathbf{A}(\alpha \supset \psi) = max(\mathbf{A}(\sim\alpha), \mathbf{A}(\psi))$

Differences with **FDE**: $(B \cap N) = N$

$(B \cup N) = B$

- A consequence system for **AR**$_4$ includes the following axiom schemes and rules of inference (= **FDE** – R4*)

A1. $\alpha \wedge \psi \Rightarrow \alpha$

A2. $\alpha \wedge \psi \Rightarrow \psi$

A3. $\alpha \Rightarrow \alpha \vee \psi$

A4. $\psi \Rightarrow \alpha \vee \psi$

A5. $\alpha \wedge (\psi \vee \gamma) \Rightarrow (\alpha \wedge \psi) \vee \gamma$

A6. $\alpha \Rightarrow {\sim}{\sim}\alpha$

A7. ${\sim}{\sim}\alpha \Rightarrow \alpha$

R1. $\alpha \Rightarrow \psi, \psi \Rightarrow \gamma \,/\, \alpha \Rightarrow \gamma$

R2. $\alpha \Rightarrow \psi, \alpha \Rightarrow \gamma \,/\, \alpha \Rightarrow \psi \wedge \gamma$

R3. $\alpha \Rightarrow \gamma, \psi \Rightarrow \gamma \,/\, \alpha \vee \psi \Rightarrow \gamma$

R4*. $\alpha \Rightarrow \psi \,/\, {\sim}\psi \Rightarrow {\sim}\alpha$

- **4CN** in **AR4**: $\mathbf{A}(p) = \langle 0,0 \rangle$

(a''')	Not (S is P):	$\mathbf{a}_1(p) = 0$
(b''')	Not (S is not P):	$\mathbf{a}_1(\sim p) = 0$
(c''')	Not (S is P and S is not P):	$\mathbf{a}_1(p \wedge \sim p) = 0$
(d''')	Not (neither S is P nor S is not P):	$\mathbf{a}_1(\sim(p \vee \sim p) = 0$

- $\mathbf{AR_4}$ embeds the three mutually opposite attitudes of doctrinalism, pluralism, and skepticism as subsets of V^4.
  Doctrinalism: $V \backslash \{\langle 1,1 \rangle, \langle 0,0 \rangle\} = \{\langle 1,0 \rangle, \langle 0,1 \rangle\}$
  Pluralism: $V \backslash \{\langle 0,0 \rangle\} = \{\langle 1,0 \rangle, \langle 1,1 \rangle, \langle 0,1 \rangle\}$
  Skepticism: $V \backslash \{\langle 1,1 \rangle\} = \{\langle 1,0 \rangle, \langle 0,0 \rangle, \langle 0,1 \rangle\}$

The Jain attitude overcomes Aristotle's **elenctic** strategy (without violating **PNC**!)

DIALOGUE 1: ARISTOTLE VS. VĀDIVEDA SŪRI

1.	**Q**: “Do you accept p?”	$\mathbf{q}_1(p) = 1$
2.	**A**: “Yes, I accept p.”	$\mathbf{a}_1(p) = 1$
3.	**Q**: “Therefore you reject ~p?”	$\mathbf{a}_1(\sim p) = 0$?
4.	**A**: “No, I do not reject ~p.”	$\mathbf{a}_1(\sim p) \neq 0$
5.	**Q**: “Does it mean that you also accept ~p?”	$\mathbf{a}_1(\sim p) = 1$?
6.	**A**: “Yes, I also accept ~p.”	$\mathbf{a}_1(\sim p) = 1$
7.	**Q**: “Therefore you accept p and ~p?”	$\mathbf{a}_1(p \wedge \sim p) = 1$?
8.	**A**: “Yes, I accept both.”	$\mathbf{a}_1(\sim(p \wedge \sim p)) = 1$
9.	**Q**: “Does it mean that you also accept ~(p ∧ ~p)?”	$\mathbf{a}_1(\sim(p \wedge \sim p)) = 1$?
10.	**A**: “Yes, I accept ~(p ∧ ~p).”	$\mathbf{a}_1(\sim(p \wedge \sim p)) = 1$
11.	**Q**: “Therefore you reject ~((p ∧ ~p) ∧ ~(p ∧ ~p))?”	$\mathbf{a}_1((p \wedge \sim p) \wedge \sim(p \wedge \sim p)) = 0$?
12.	**A**: “No, I don't reject ~((p ∧ ~p) ∧ ~(p ∧ ~p)).”	$\mathbf{a}_1((p \wedge \sim p) \wedge \sim(p \wedge \sim p)) \neq 0$
13.	**Q**: “Therefore you also accept ~((p ∧ ~p) ∧ ~(p ∧ ~p))?”	$\mathbf{a}_1((p \wedge \sim p) \wedge \sim(p \wedge \sim p)) = 1$?
14.	**A**: “Yes, I accept ~((p ∧ ~p) ∧ ~(p ∧ ~p)).”	$\mathbf{a}_1((p \wedge \sim p) \wedge \sim(p \wedge \sim p)) = 1$

...

Proposition 4. For every sentence α (including p, ~p, p $\wedge$ ~p, ~(p $\wedge$ ~p), and so on), the answer of the Jaina in $\mathbf{AR_4}$ is $\mathbf{A}(\alpha) = \langle 1,1 \rangle$.
Proof: if $\mathbf{a}_1(p \wedge {\sim}p) = 1$ then $\mathbf{a}_1(p) = \mathbf{a}_1({\sim}p) = \mathbf{a}_2(p) = 1$. And if $\mathbf{a}_1({\sim}(p \wedge {\sim}p)) = 1$ then $\mathbf{a}_2(p \wedge {\sim}p) = 1$, i.e. $\mathbf{a}_2(p) = 1$ or $\mathbf{a}_1({\sim}p) = 1$. Hence for every α, $\mathbf{a}_1(\alpha) = \mathbf{a}_2(\alpha) = 1$. Hence $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha), \mathbf{a}_2(\alpha) \rangle = \langle 1,1 \rangle$. ■

DIALOGUE 2: ARISTOTLE VS. NĀGĀRJUNA

1.	**Q**: “Do you reject p?”	$\mathbf{q}_1(p) = 0$
2.	**A**: “Yes, I reject p.”	$\mathbf{a}_1(p) = 0$
3.	**Q**: “Therefore you accept ~p?”	$\mathbf{a}_1(\sim p) = 1$?
4.	**A**: “No, I do not accept ~p.”	$\mathbf{a}_1(\sim p) \neq 1$
5.	**Q**: “Does it mean that you also reject ~p?”	$\mathbf{a}_1(\sim p) = 0$?
6.	**A**: “Yes, I also reject ~p.”	$\mathbf{a}_1(\sim p) = 0$
7.	**Q**: “Therefore you reject p and ~p?”	$\mathbf{a}_1(p \vee \sim p) = 0$?
8.	**A**: “Yes, I reject both.”	$\mathbf{a}_1(\sim(p \vee \sim p)) = 0$
9.	**Q**: “Does it mean that you also reject ~(p ∨ ~p)?”	$\mathbf{a}_1(\sim(p \wedge \sim p)) = 0$?
10.	**A**: “Yes, I reject ~(p ∨ ~p).”	$\mathbf{a}_1(\sim(p \wedge \sim p)) = 0$
11.	**Q**: “Therefore you accept ~((p ∧ ~p) ∨ ~(p ∧ ~p))?”	$\mathbf{a}_1((p \wedge \sim p) \vee \sim(p \wedge \sim p)) = 1$?
12.	**A**: “No, I don't reject ~((p ∧ ~p) ∨ ~(p ∧ ~p)).”	$\mathbf{a}_1((p \wedge \sim p) \wedge \sim(p \wedge \sim p)) \neq 1$
13.	**Q**: “Therefore you also reject ~((p ∧ ~p) ∨ ~(p ∧ ~p))?”	$\mathbf{a}_1((p \wedge \sim p) \wedge \sim(p \wedge \sim p)) = 0$?
14.	**A**: “Yes, I reject ~((p ∧ ~p) ∨ ~(p ∧ ~p)).”	$\mathbf{a}_1((p \wedge \sim p) \wedge \sim(p \wedge \sim p)) = 0$

…

Proposition 5. For every sentence α (including p, $\sim p$, $p \vee \sim p$, $\sim(p \vee \sim p)$, and so on), the answer of the Madhyamika in $\mathbf{AR_4}$ is $\mathbf{A}(\alpha) = \langle 0,0 \rangle$.
Proof: if $\mathbf{a}_1(p \vee \sim p) = 0$ then $\mathbf{a}_1(p) = \mathbf{a}_1(\sim p) = \mathbf{a}_2(p) = 0$. And if $\mathbf{a}_1(\sim(p \vee \sim p)) = 0$ then $\mathbf{a}_2(p \vee \sim p) = 0$, i.e. $\mathbf{a}_2(p) = 0$ or $\mathbf{a}_1(\sim p) = 0$. Hence for every α, $\mathbf{a}_1(\alpha) = \mathbf{a}_2(\alpha) = 0$. Hence $\mathbf{A}(\alpha) = \langle \mathbf{a}_1(\alpha),\mathbf{a}_2(\alpha) \rangle = \langle 0,0 \rangle$. ■

Summary

- Two paranormal logics
  a logic for pluralism: paraconsistency
  a logic for skepticism: paracompleteness

- Two levels of paranormality in **QAS**: strong and weak
  weak paranormality: $\mathbf{a}_i(p) = \mathbf{a}_j(p) = 1$, or $\mathbf{a}_i(p) = \mathbf{a}_j(p) = 1$
  strong paranormality: $\mathbf{a}_i(p) = \{1,0\}$

- Priest's "impossible values", Shramko & Wansing's generalized truth-values are weakly paranormal: no set of values {X,not-X} occurs there

- A logical value is a structured object: an ordered set of answers

- Not a single property of propositions, but a set of data about a sentence