Universal Logic III
Estoril 2010
Tutorial on Truth Values
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Session 2, Part 3

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3.
An algebraic logic of oppositions

Abstract

In this third part, the question-answer machinery is applied to develop a general theory of oppositions. A logical value characterizes each opposite term of an opposition, and a calculus of oppositions is made possible by recursive functions upon these values. The nature of logical negation and some of their non-classical variants is investigated.

What is a logical opposition?

DEF. 1-1: opposition.
An opposition is a 2-ary **relation** OP(a,b) standing between 2 opposite terms *a* and *b*
a and *b* stand for concepts or propositions, to be reduced to propositions

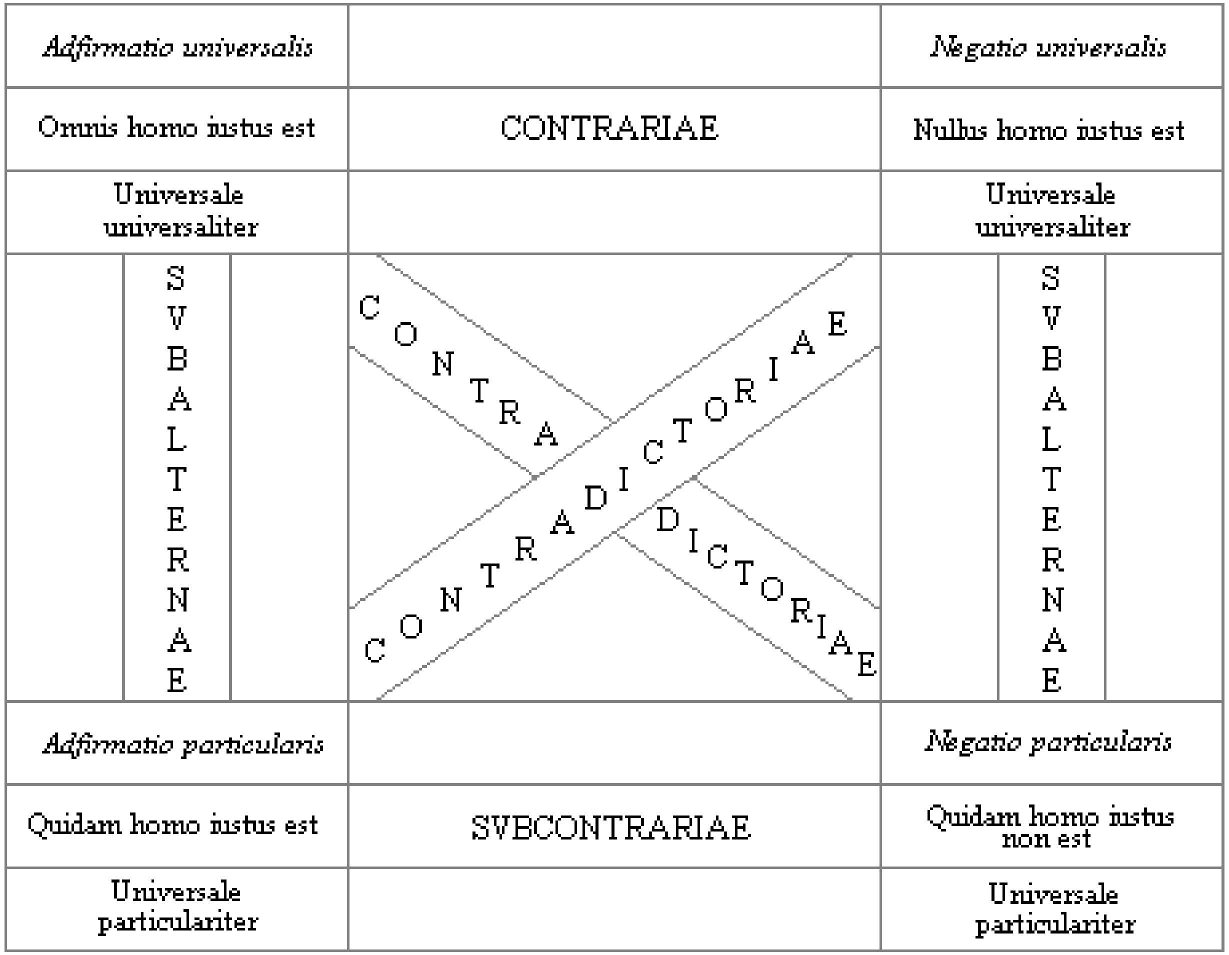
In a bivalent domain of interpretation, the set of truth-values $V = \mathbf{2} = \{T,F\}$
Proposition: a sentence that is true ($v(a) = T$) or false ($v(a) = F$)
2 includes $(\mathbf{2})^2 = 4$ ordered pairs of truth-values in the set $\mathbf{4} = \{\langle T,T\rangle,\langle T,F\rangle,\langle F,T\rangle,\langle F,F\rangle\}$

DEF. 1-2: oppositions.
Each case of opposition is a set of **compossible** truth-values
Let $\mathbf{Q}_1$ and $\mathbf{Q}_2$ be 2 questions about OP, $\mathbf{Q}_1$: "$v(a) = v(b) = F$?" and $\mathbf{Q}_2$: "$v(a) = v(b) = T$?"
Let $\mathbf{A}_i = 1$ a yes-answer to $\mathbf{Q}_i$, $\mathbf{A}_i = 0$ a no-answer to $\mathbf{Q}_i$
There are $(2)^2 = 4$ ordered pairs of answers
$OP = \{\langle 1,1\rangle,\langle 1,0\rangle,\langle 0,1\rangle,\langle 0,0\rangle\}$

How many logical oppositions are there?

The "Aristotelian" oppositions: 4 oppositions
OP = {CD,CT,SCT,SB}

DEF. 1-3: Each opposition is a set of ordered answers $\mathbf{A} = \langle \mathbf{A}_1, \mathbf{A}_2 \rangle$
Contrariety: CT = $\langle 1,0 \rangle$ = $\{\langle T,F \rangle, \langle F,T \rangle, \langle F,F \rangle\}$
Contradiction: CD = $\langle 0,0 \rangle$ = $\{\langle T,F \rangle, \langle F,T \rangle\}$
Subcontrariety: SCT = $\langle 0,1 \rangle$ = $\{\langle T,T \rangle, \langle T,F \rangle, \langle F,T \rangle\}$
Subalternation: SB = $\langle 1,1 \rangle$ = $\{\langle T,T \rangle$,~~$\langle T,F \rangle$~~,$\langle F,T \rangle, \langle F,F \rangle\}$



Boethius' own diagram of the square of opposition (Meiser, adapted from Seuren)

The “Aristotelian” oppositions: 4 oppositions, whatever *n* may be
OP = {CD,CT,SCT,SB}

DEF. 1-3: Each opposition is a set of ordered answers $\mathbf{A} = \langle \mathbf{A}_1, \mathbf{A}_2 \rangle$
Contrariety: CT = ⟨1,0⟩ = {⟨T,F⟩,⟨F,T⟩,⟨F,F⟩}
Contradiction: CD = ⟨0,0⟩ = {⟨T,F⟩,⟨F,T⟩}
Subcontrariety: SCT = ⟨0,1⟩ = {⟨T,F⟩,⟨F,T⟩,⟨F,T⟩}
Subalternation: SB = ⟨1,1⟩ = {⟨T,T⟩,~~⟨T,F⟩~~,⟨F,T⟩,⟨F,F⟩}

2 problems:

Problem #1. Aristotle acknowledged **2** oppositions, only
Problem #2: **DEF. 1-3** does not take into account the non-compossibility of ⟨T,F⟩ in SB
SB(a,b) is an **asymmetric** relation between *a* and *b*

Verbally ***four*** *kinds of opposition are possible, viz. universal affirmative to universal negative, universal affirmative to particular negative, particular affirmative to universal negative, and particular affirmative to particular negative: but really there are only* ***three****: for the particular affirmative is only verbally opposed to the particular negative. Of the genuine opposites I call those which are universal* ***contraries****, the universal affirmative and the universal negative, e.g. 'every science is good', 'no science is good'; the others I call* ***contradictories****.*

(Aristotle, *Prior Analytics*, 63b 21-30)

Aristotle's oppositions: relations of incompatibility

DEF. 1-4: OP is a relation of opposition OP(a,b), such that: $v(a) = T \Rightarrow v(b) = F$

$v(b) = T \Rightarrow v(a) = F$

Since there are ***three*** *oppositions to affirmative statements, it follows that opposite statements may be assumed as premisses in* ***six*** *ways; we may have either universal affirmative and negative, or universal affirmative and particular negative, or particular affirmative and universal negative, and the relations between the terms may be reversed.*

(Aristotle, *Prior Analytics*, 64a37-38)

I think that neither subalternation nor superalternation can be considered as relations of opposition. For example P *is subaltern of* $P \vee Q$*, and it does not really make sense to consider them as opposed.*

(Béziau (2003): 225)

Solution #1 to Problem #2: turning **DEF. 1-3** into **DEF. 1-5**

Sion (1996)

DEF. 1-5: Each opposition is a set of ordered answers $\mathbf{A} = \langle \mathbf{A}_1,\mathbf{A}_2,\mathbf{A}_3,\mathbf{A}_4 \rangle$ to 4 questions, namely: $\mathbf{Q}_1$: "v(a) = v(b) = T?", $\mathbf{Q}_2$: "v(a) = T, v(b) = F?", $\mathbf{Q}_3$: "v(a) = F, v(b) = T?", $\mathbf{Q}_4$: "v(a) = v(b) = F?".

According to Sion (1996), there are 6 oppositions: OP ∪ {IM,UNC}

	CT:	CT(a,b)	=	⟨0,1,1,1⟩
	CD:	CD(a,b)	=	⟨0,1,1,0⟩
	SCT:	SCT(a,b)	=	⟨1,1,1,0⟩
	SB:	SB(a,b)	=	⟨1,0,1,1⟩
+	IM: Implicance	IM(a,b)	=	⟨1,0,0,1⟩
	UN: Unconnectedness	UN(a,b)	=	⟨1,1,1,1⟩

Problem #3: **DEF. 1-5** results in $(4)^2 = 16$ ordered pairs of answers
what of the $(2)^4 - 6 = 10$ remaining pairs?

Solution #2 to Problem #1 and Problem #2: to define opposition by means of a **function**

Let us recall that Aristotle does not introduce explicitly the notion of "subcontraries", but refers to them only indirectly as "contradictories of contraries".

(Béziau (2003): 224)

Solution #2 to Problem #1 and Problem #2: to define opposition by means of a **function**

Logical form: $R(x,f(y))$, R: relation; f,g,h: predicate functions; x,y: individual functions

Example 1: "my mother's son is my brother"
f: son; *g*: mother, R: brother $\forall x \forall y\ (f(g(x)) \leftrightarrow R(x,y))$
Example 2: "Mother's sons are brothers"
R_1: son, R_2: brother $\forall x \forall y \forall z\ ((R_1(x,z)) \wedge R_1(y,z)) \leftrightarrow R_2(x,y)))$
Example 3: "Subcontraries are contradictories of contraries"
SCT: subcontrariety, *cd*: contradictory, CT: contrariety $\forall x \forall y\ (SCT(x,y) \leftrightarrow CT(cd(x),cd(y)))$

OP = {CT,CD,SCT,SB}: set of the relations of **opposition**
op = {*ct*,*cd*,*sct*,*sb*}: set of the functions of **opposites**

DEF. 1-6: OP is a relation of opposition such that, for any *a* and *b*, *b* is the opposite of *a*

CD(a,b): "*a* and *b* stand into a contradictory relation"
b = *cd*(a): "*b* is the contradictory of *a*"

What is a logical opposite?

DEF. 2-1: A logical opposite is a function *op* from *op*(a) to b such that, for any *a*,*b*, OP(*v*(a),op(*v*(a))) = OP(*v*(a),*v*(b))

Problem #4: how to determine the value of *b*, given the value of *a*?
only *cd* is a **truth-functional** function in the modern, Fregean logic

For any *a*,*b*: *v*(a) = T if and only if *v*(*cd*(a)) = *v*(b) = F

DEF. 2-2: classical negation ~ is a contradictory-forming operator, such that *v*(*cd*(a)) = *v*(~a) and CD(*v*(a),*v*(b)) = CD(*v*(a),*v*(~a))

What about *v*(*ct*(a)), *v*(*sct*(a)), and *v*(*sb*(a)) ?
Solution to Problem #4: to assign another interpretation for the opposite terms

Piaget (1949) constructed a theory of intrapropositional 2-ary connectives ○

DEF. 2-3: Every 2-ary proposition *a* = (p○q) is characterized by its Disjunctive Normal Form (DNF)
Each DNF is a set of ordered answers $\mathbf{A} = \langle \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4 \rangle$ to 4 questions: $\mathbf{Q}_1$: "v(p) = v(q) = T", $\mathbf{Q}_2$: "v(p) = T, v(q) = F?", $\mathbf{Q}_3$: "v(p) = F, v(q) = T?", $\mathbf{Q}_4$: "v(p) = v(q) = F?".
It results in $(4)^2 = 16$ ordered answers that characterize a 2-ary connective ○ in *a* = p○q

DEF. 2-4: A logical opposite *op* is a valuation function from *a* to *b* such that, for every interpretation of *a*,*b* into **A**, **A**(*op*(a)) = **A**(b)

	$\mathbf{A}(p \circ q)$	$a = (p \circ q)$
1.	$\langle 1,1,1,1 \rangle$	$\top$
2.	$\langle 1,1,1,0 \rangle$	$p \vee q$
3.	$\langle 1,1,0,1 \rangle$	$p \leftarrow q$
4.	$\langle 1,0,1,1 \rangle$	$p \rightarrow q$
5.	$\langle 0,1,1,1 \rangle$	$p \uparrow q$, $\sim(p \wedge q)$
6.	$\langle 1,1,0,0 \rangle$	p
7.	$\langle 1,0,0,1 \rangle$	$(p \leftrightarrow q)$
8.	$\langle 0,0,1,1 \rangle$	$\sim p$
9.	$\langle 0,1,1,0 \rangle$	$\sim(p \leftrightarrow q)$
10.	$\langle 1,0,1,0 \rangle$	q
11.	$\langle 0,1,0,1 \rangle$	$\sim q$
12.	$\langle 1,0,0,0 \rangle$	$p \wedge q$
13.	$\langle 0,1,0,0 \rangle$	$\sim(p \rightarrow q)$
14.	$\langle 0,0,1,0 \rangle$	$\sim(p \leftarrow q)$
15.	$\langle 0,0,0,1 \rangle$	$p \downarrow q$, $\sim(p \vee q)$
16.	$\langle 0,0,0,0 \rangle$	$\perp$

How many opposites are there?

A theory of **reversibility**: Piaget INRC Group

DEF. 2-5: there is a set *op* = {I,N,R,C} of **4** opposite functions such that, for any *a*,*b*, OP(**A**(a)),*op*(**A**(a)) = OP(**A**(a),**A**(b))
Let ($\mathbf{A}_i$)' be the **denial** of $\mathbf{A}_i$, such that ($\mathbf{A}_i$)' = 0 iff ($\mathbf{A}_i$) = 1 and (($\mathbf{A}_i$)')' = ($\mathbf{A}_i$)

The set of 4 operations (Klein four-group)

I	=	identity	I(**A**(a))	=	$\langle \mathbf{A}_1(a),\mathbf{A}_2(a),\mathbf{A}_3(a),\mathbf{A}_4(a)\rangle$
N	=	inversion	N(**A**(a))	=	$\langle \mathbf{A}_1(a)',\mathbf{A}_2(a)',(\mathbf{A}_3(a)',(\mathbf{A}_4(a)'\rangle$
R	=	reciprocity	R(**A**(a))	=	$\langle \mathbf{A}_4(a),\mathbf{A}_3(a),\mathbf{A}_2(a),\mathbf{A}_1(a)\rangle$
C	=	correlation	C(**A**(a))	=	$\langle \mathbf{A}_4(a)',\mathbf{A}_3(a)',\mathbf{A}_2(a)',\mathbf{A}_1(a)'\rangle$

Example: $a = (p\wedge q)$. Thus:

$$I(\mathbf{A}((p\wedge q)) = \langle 1000\rangle$$
$$N(\mathbf{A}(p\wedge q)) = \langle(1)'(0)'(0)'(0)'\rangle = \langle 0111\rangle = \mathbf{A}(\sim(p\wedge q)),\mathbf{A}((p\uparrow q))$$
$$R(\mathbf{A}(p\wedge q)) = \langle 0001\rangle = \mathbf{A}(\sim(p\vee q)),\mathbf{A}((p\downarrow q))$$
$$C(\mathbf{A}(p\wedge q)) = \langle(0)'(0)'(0)'(1)'\rangle = \langle 1110\rangle = \mathbf{A}(p\vee q)$$

DEF. 2-6: N is a contradiction-forming operator *cd* such that OP(a,*cd*(a)) = CD(a,b)
R is a (sub)contrariety-forming operator (*s*)*ct* such that OP(a,*ct*(a)) = CT(a,b) or SCT(a,b)
C is a subalternation-forming operator *sb* such that OP(a,*sb*(a) = SB(a,b)

The 4 operations in *op* = {I,N,R,C} are **commutative** functions

I = NRC
N = RC
R = NC
C = NR

Piaget

	I	N	R	C
I	I	N	R	C
N	N	I	C	R
R	R	C	I	N
C	C	R	N	I

Let us recall that Aristotle does not introduce explicitly the notion of "subcontraries", but refers to them only indirectly as "contradictories of contraries".

(Béziau (2003): 224)

Proof. SCT(a,b) = CT(*cd*(a),*cd*(b))
Let *a* = (p∨q). Then SCT(p∨q,b) = OP(p∨q, *sct*(p∨q)) = OP(p∨q,p↑q)
cd(p∨q) = (p↓q), and *cd*(p↑q) = (p∧q)
OP(p↓q,p∧q) = OP(p↓q,*ct*(p↓q)) = CT(p↓q,p∧q)
Hence SCT(p∨q,p↑q) = CT(p↓q,p∧q) ■

What is logical negation?

Again, **classical** negation is the contradictory-forming operator *cd*:

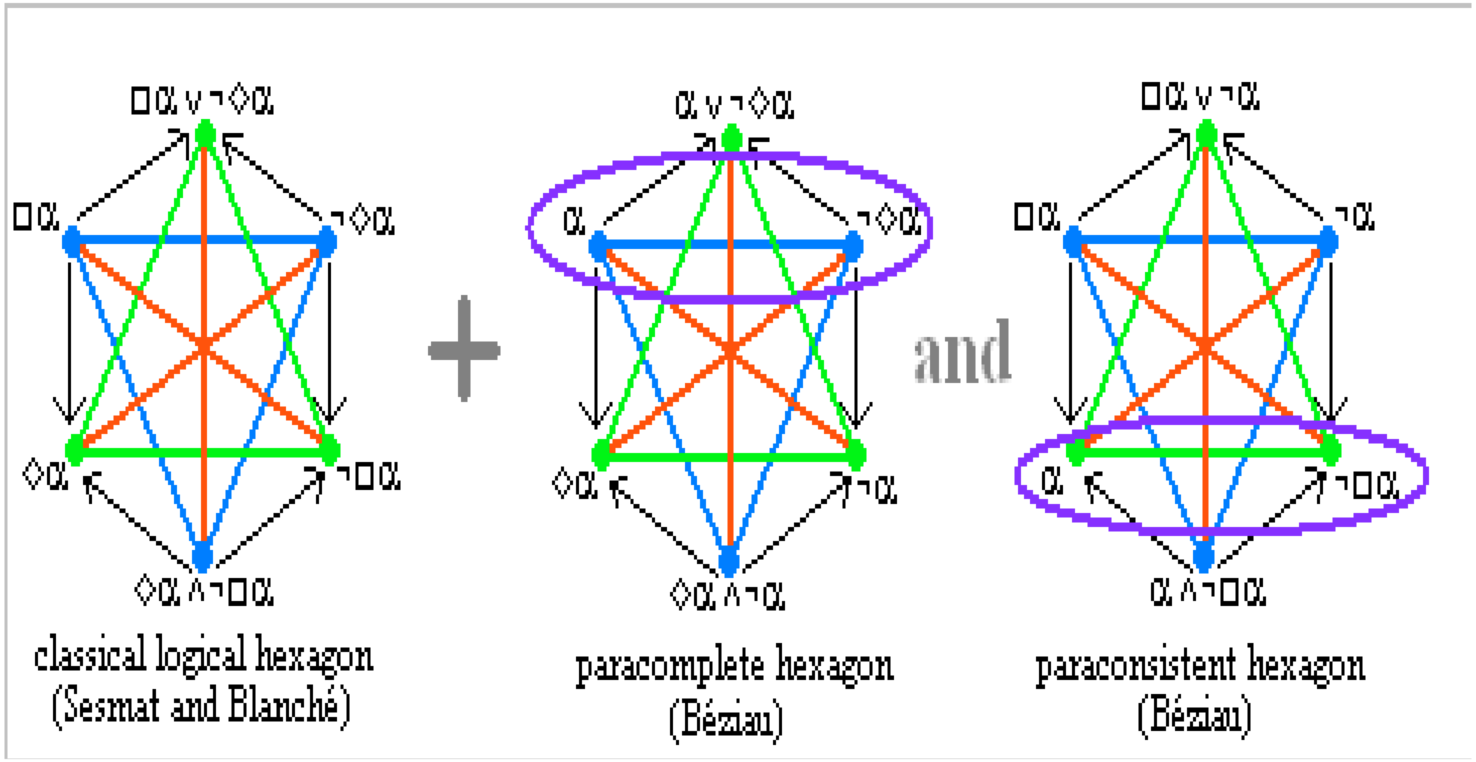
DEF. 2-2: classical negation ~ is a contradictory-forming operator, such that $v(cd(\text{a})) = v({\sim}(\text{a}))$ and $\text{CD}(v(\text{a}),v(\text{b})) = \text{CD}(v(\text{a}),v({\sim}(\text{a}))$

What about the other opposite-forming operators?

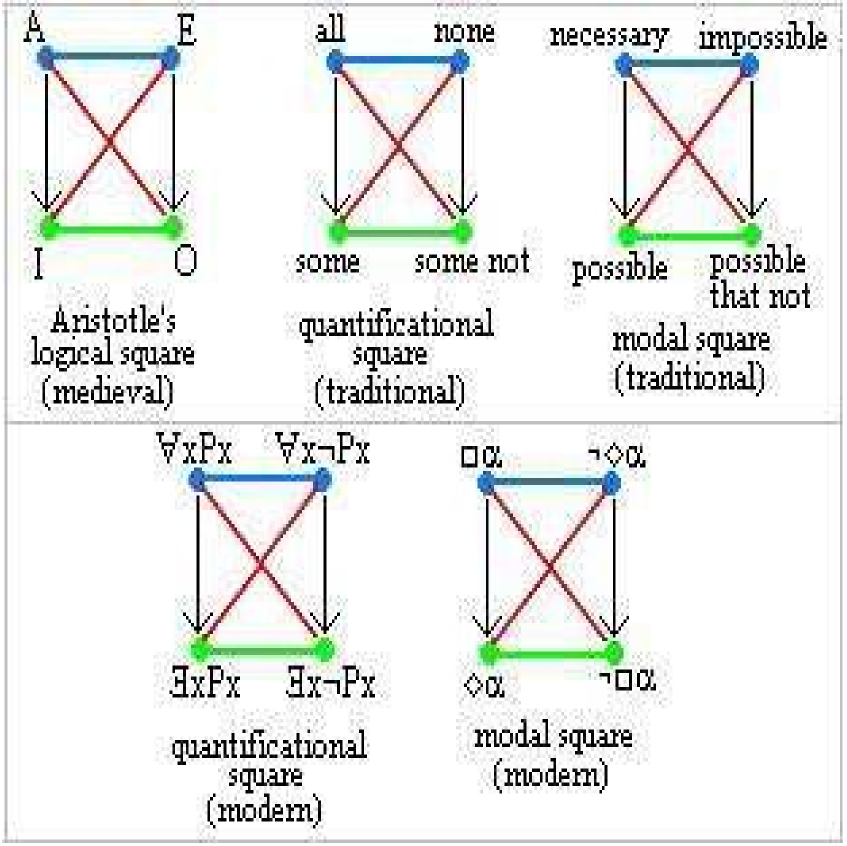
Béziau (2003): a translation of negations from a **modal** standpoint

The E-corner, impossible*, is a paracomplete negation (*intuitionistic negation *if the underlying modal logic is S4) and the O-corner,* not necessary*, is a* paraconsistent negation.
I argue that the three notions of opposition of the square of oppositions (contradiction, contrariety, subcontrariety) correspond to three notions of negation (classical, paracomplete, paraconsistent).

(Béziau (2003): 218)


□α ∨ ¬◇α
□α
¬◇α
◇α
¬□α
◇α ∧ ¬□α
classical logical hexagon
(Sesmat and Blanché)
α ∨ ¬◇α
α
¬◇α
◇α
¬α
◇α ∧ ¬α
paracomplete hexagon
(Béziau)
and
□α ∨ ¬α
□α
¬α
α
¬□α
α ∧ ¬□α
paraconsistent hexagon
(Béziau)

Problem #5: how to characterize modalities within the theory of opposition?
Modalities are structurally similar to **quantifiers**


A
E
I
O
Aristotle's logical square (medieval)
all
none
some
some not
quantificational square (traditional)
necessary
impossible
possible
possible that not
modal square (traditional)
∀xPx
∀x¬Px
∃xPx
∃x¬Px
quantificational square (modern)
□α
¬◇α
◇α
¬□α
modal square (modern)

Problem #5: how to characterize modalities within the theory of opposition?
Modalities are structurally similar to **quantifiers**

Problem #6: how to algebraize modalities, in the line of **Q-A**?
modalities cannot be characterized algebraically, according to Dugundji (1940)

There is no finite characteristic matrix for any of Lewis and Langford's systems.
(Dugundji (1940): 150)

Solution to Problem #6: to algebraize modalities in a fragment of modal logics: S5

Modalities are defined in terms of **generalized quantifiers** in Smessaert (2009)

DEF. 3-1: modalities as generalized quantifiers.
Each modality a = X(p) is a set of ordered answers **A**(p) = $\langle \mathbf{a}_1(p), \mathbf{a}_2(p), \mathbf{a}_3(p), \mathbf{a}_4(p) \rangle$ to 4 questions, namely: $\mathbf{Q}_1$: “Is *p* always F?”, $\mathbf{Q}_2$: “Is *p* actually (but not always) F?”, $\mathbf{Q}_3$: “Is *a* actually (but not always) T?”, $\mathbf{Q}_4$: “Is *a* always T?”.

It results in a set of $4^2 = 16$ modal sentences, where each logical value $\langle \mathbf{a}_1(p), \mathbf{a}_2(p), \mathbf{a}_3(p), \mathbf{a}_4(p) \rangle$ is defined by the operations of meet and join

	$\mathbf{A}(X(p))$	$a = X(p)$
1.	$\langle 1,1,1,1 \rangle$	$\top$
2.	$\langle 1,1,1,0 \rangle$	$\sim\Box p$
3.	$\langle 1,1,0,1 \rangle$	$\sim p \vee \Box p$
4.	$\langle 1,0,1,1 \rangle$	$\sim\Box p \vee p$
5.	$\langle 0,1,1,1 \rangle$	$\sim\Box\sim p$
6.	$\langle 1,1,0,0 \rangle$	$\sim p$
7.	$\langle 1,0,0,1 \rangle$	$\Box\sim p \vee \Box p$
8.	$\langle 0,0,1,1 \rangle$	p
9.	$\langle 0,1,1,0 \rangle$	$\sim\Box\sim p \wedge \sim\Box p$
10.	$\langle 1,0,1,0 \rangle$	$\Box\sim p \vee (p \wedge \sim\Box p)$
11.	$\langle 0,1,0,1 \rangle$	$(\sim p \wedge \sim\Box\sim p) \vee \Box p$
12.	$\langle 1,0,0,0 \rangle$	$\Box\sim p$
13.	$\langle 0,1,0,0 \rangle$	$\sim p \wedge \sim\Box\sim p$
14.	$\langle 0,0,1,0 \rangle$	$p \wedge \sim\Box p$
15.	$\langle 0,0,0,1 \rangle$	$\Box p$
16.	$\langle 0,0,0,0 \rangle$	$\bot$

DEF. 3-2: Each opposition OP(a,b) can be characterized by the values of its relata *a* and *b* within a Boolean algebra $\mathbb{A} = (\cap,\cup,\subset,',1,0)$. Let $\cap$ and $\cup$ the operations of meet and join such that, for every x_i and y_i ($1 < i < n$): $(\langle x_1,\ldots,x_n\rangle \cap \langle y_1,\ldots,y_n\rangle) = (\langle x_1 \cap y_1,\ldots,x_n \cap y_n\rangle)$, and $(\langle x_1,\ldots,x_n\rangle \cup \langle y_1,\ldots,y_n\rangle) = (\langle x_1 \cup y_1,\ldots,x_n \cup y_n\rangle)$. Then:

OP(a,b) = CT(a,b) iff $(\mathbf{a}(a) \cap \mathbf{a}(b)) = \langle 0000\rangle$ and $(\mathbf{a}(a) \cup \mathbf{a}(b)) \neq \langle 1111\rangle$

OP(a,b) = CD(a,b) iff $(\mathbf{a}(a) \cap \mathbf{a}(b)) = \langle 0000\rangle$ and $(\mathbf{a}(a) \cup \mathbf{a}(b)) = \langle 1111\rangle$

OP(a,b) = SCT(a,b) iff $(\mathbf{a}(a) \cap \mathbf{a}(b)) \neq \langle 0000\rangle$ and $(\mathbf{a}(a) \cup \mathbf{a}(b)) = \langle 1111\rangle$

OP(a,b) = SB(a,b) iff $(\mathbf{a}(a) \cap \mathbf{a}(b)) \neq \langle 0000\rangle$ and $(\mathbf{a}(a) \cup \mathbf{a}(b)) \neq \langle 1111\rangle$

<u>Example</u>: Let $a = p \leftrightarrow q$ and $b = p$; then $\mathbf{a}(a) = \langle 1001\rangle$ and $\mathbf{a}(b) = \langle 1100\rangle$; $(\langle 1001\rangle \cap \langle 1100\rangle) = \langle 1000\rangle$ and $(\langle 1001\rangle \cup \langle 1100\rangle) = \langle 1101\rangle$; hence $(\langle 1110\rangle \cap \langle 1001\rangle) \neq \langle 0000\rangle$ and $(\langle 1110\rangle \cup \langle 1001\rangle) \neq \langle 1111\rangle$. Therefore OP(p↔q,p) = SB(p↔q,p)

Note: subalternation includes, but is not equivalent with, entailment (logical <u>consequence</u>)

OP(a,b) = SB_1(a,b) iff $(\mathbf{a}(a) \subset \mathbf{a}(b)) = \langle 1111\rangle$, i.e. $(N(\mathbf{a}(a)) \cup \mathbf{a}(b)) = \langle 1111\rangle$

Solution to Problem #6: to algebraize modalities in a fragment of modal logics: S5

Modalities are defined in terms of **generalized quantifiers** in Smessaert (2009)

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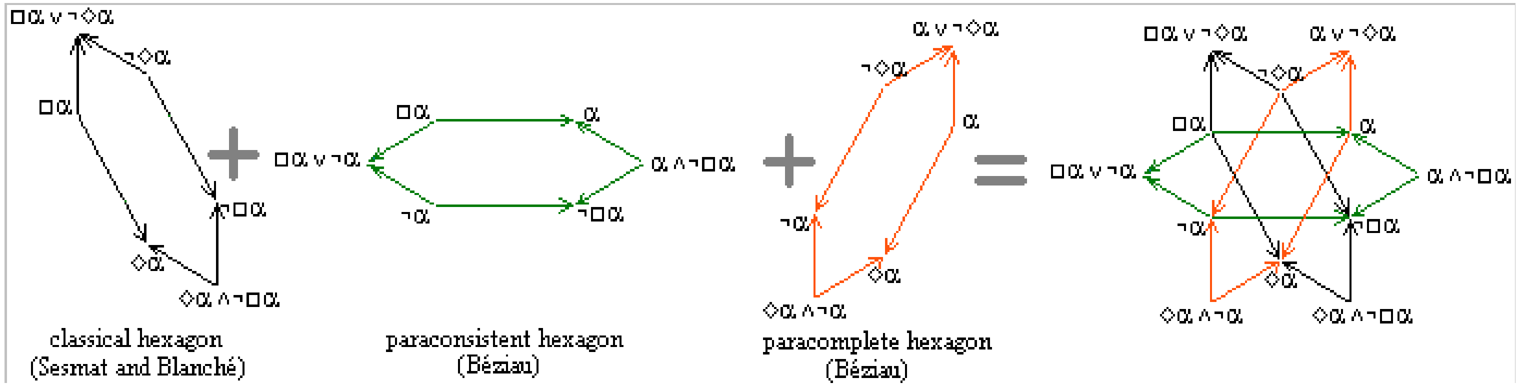
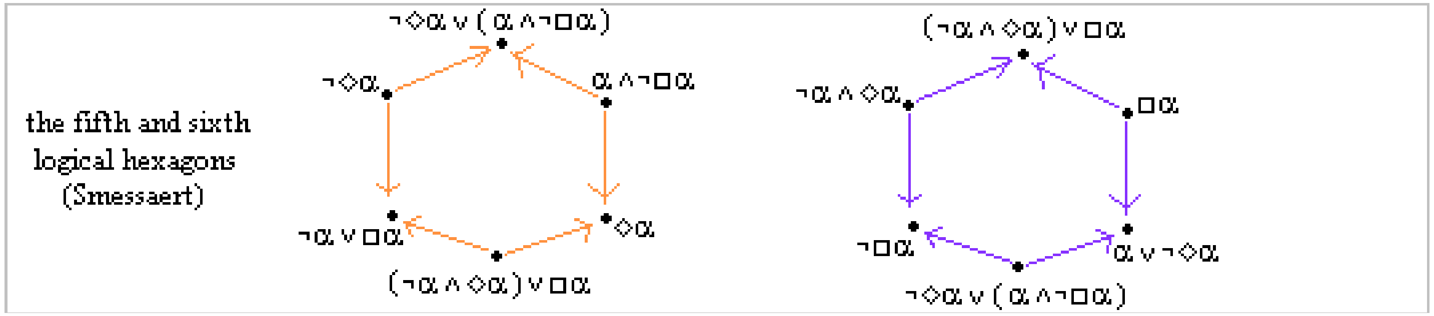
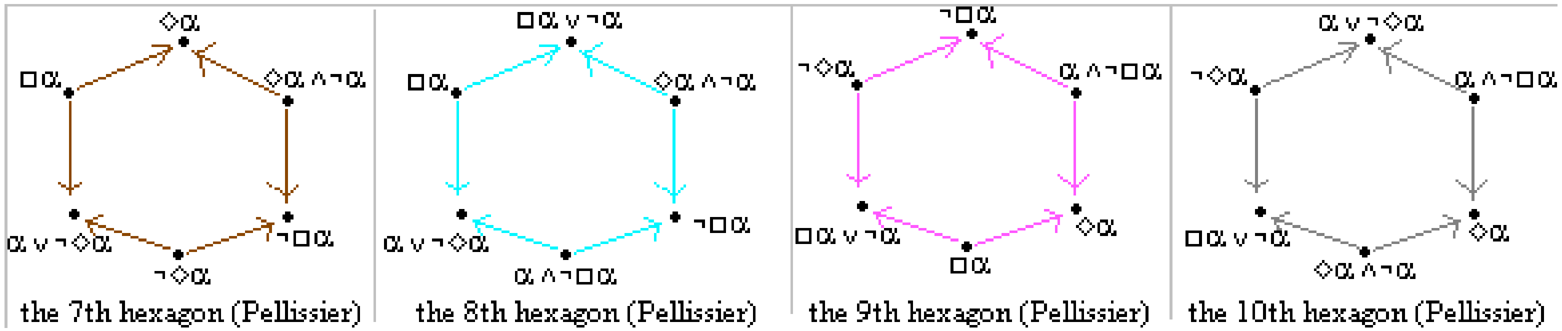
It results in a set of $4^2 = 16$ modal sentences, where each logical value $\langle \mathbf{a}_1(p),\mathbf{a}_2(p),\mathbf{a}_3(p),\mathbf{a}_4(p)\rangle$ is defined by the operations of meet and join

According to the characterization of OP in **DEF. 3-2**, there is more than just 1 logical hexagon of modalities


□α ∨ ¬◇α
¬◇α
□α
¬□α
◇α
◇α ∧ ¬□α
classical hexagon
(Sesmat and Blanché)
□α ∨ ¬α
□α
α
α ∧ ¬□α
¬α
¬□α
paraconsistent hexagon
(Béziau)
α ∨ ¬◇α
¬◇α
α
¬α
◇α
◇α ∧ ¬α
paracomplete hexagon
(Béziau)
□α ∨ ¬◇α
α ∨ ¬◇α
¬◇α
□α
α
□α ∨ ¬α
α ∧ ¬□α
¬α
¬□α
◇α
◇α ∧ ¬α
◇α ∧ ¬□α


the fifth and sixth
logical hexagons
(Smessaert)
¬◇α ∨ (α ∧ ¬□α)
¬◇α
α ∧ ¬□α
¬α ∨ □α
◇α
(¬α ∧ ◇α) ∨ □α
(¬α ∧ ◇α) ∨ □α
¬α ∧ ◇α
□α
¬□α
α ∨ ¬◇α
¬◇α ∨ (α ∧ ¬□α)


◇α
□α
◇α ∧ ¬α
α ∨ ¬◇α
¬□α
¬◇α
the 7th hexagon (Pellissier)
□α ∨ ¬α
□α
◇α ∧ ¬α
α ∨ ¬◇α
¬□α
α ∧ ¬□α
the 8th hexagon (Pellissier)
¬□α
¬◇α
α ∧ ¬□α
□α ∨ ¬α
◇α
□α
the 9th hexagon (Pellissier)
α ∨ ¬◇α
¬◇α
α ∧ ¬□α
□α ∨ ¬α
◇α
◇α ∧ ¬α
the 10th hexagon (Pellissier)

DEF. 3-3: intuitionistic negation ¬ is a contrary-forming operator, such that **A**(*ct*(a)) = **A**(¬(a)), CT(**A**(a),**A**(b)) = CT(**A**(a),**A**(¬(a)), and

Compare with Gödel's translation:
¬p := □~p

DEF. 3-4: paraconsistent negation – is a subcontrary-forming operator, such that **A**(*sct*(a)) = **A**(–(a)), SCT(**A**(a),**A**(b)) = SCT(**A**(a),**A**(–(a)), and

Compare with Béziau's translation:
–p := ~□p

Corollary about intuitionistic and paraconsistent negations:
they are **dual** to each other: X(p) :=: ~X~(p)
defined by the same opposite-forming operator: R

Problem #7 with **DEF. 3-3**: by double negation, RR = I; therefore $\mathbf{A}(\neg\neg(a)) = \mathbf{A}(a)$
But the Law of Double Negation doesn't hold in intuitionistic logic: $\neg\neg a \nrightarrow a$

Solution to Problem #7: R = Nelson's **strong** negation, not Heyting's intuitionistic negation

Heyting's intuitionistic negation cannot be characterized by any of $op = \{cd, ct, sct, sb\}$
Béziau's modal translations differ from our algebraic translation: $R \neq \Box\sim$
Compare with the difference between Jain logic $\mathbf{J_7}$ and Jaśkowski's Discussive Logic $\mathbf{D_2}$

Schang (2009a)

Jaina logic $\mathbf{J_7}$: $- p = \Diamond(\sim p)$ vs. Jaśkowski's $\mathbf{D_2}$: $- p = (\Diamond\sim)p$

$- - p = \Diamond(\sim\sim p)$ vs. $- - p = (\Diamond\sim\Diamond\sim)p$

DEF. 3-5: a subaltern-forming operator is a **mixed double** negation, such that $\mathbf{A}(sb(a)) = \mathbf{A}(ct(cd(a)) = \mathbf{A}(\neg\sim(a))$

Problem #8: does a double negation result in a proper negation?
Solution to Problem #8: a distinction between negation and **falsification**

What is falsification?

DEF. 3-6: every opposite-forming operator *op* is a negation, such that *op*(a) is the negation of *a*.

« Falsification »: to turn a sentence *a* into something false, given that every such sentence is a combination of truth- and falsity-assignments: $\mathbf{A}(a) = \langle \mathbf{a}_1(a), \mathbf{a}_2(a), \mathbf{a}_3(a), \mathbf{a}_4(a) \rangle$

DEF. 3-7: any opposite-forming operator *op* is a falsifying operator if and only if, for any opposite terms *a* and *op*(a) = b, $\mathbf{A}(a) \cap \mathbf{A}(b) = \langle 0,0,0,0 \rangle$

Not every negation in *op* is a falsifying negation, accordingly: only *cd* and *ct* are so

Summary

- A logical opposition OP is a **relation** OP(a,b)
  A logical opposite *op* is a **function** *op*(a) = b, in OP(a,*op*(a))

- Opposite functions gives rise to a variety of **negations**

cd: classical, *ct*: paracomplete, *sct*: paraconsistent, *sb*: mixed double

Negations are not the recursive functions in *op* (given $\neg \neq$ R), but **intensional** functions

- Falsification is a subset of negation, i.e. {*cd*,*ct*}

A term *a* and its negation *op*(a) can be both true or false, according to *op* and **A**(a)

- No wonder if Aristotle saw SCT (let alone SB) as a "verbal" opposition

sct(a) = *sb*(*cd*(a)) = *cd*(*sb*(a))

sb(a) = *cd*(*ct*(a)) = *ct*(*cd*(a)) "The contradictor**y** of the contrar**y** (of *a*)"

AND NOT

SCT(a,b) = CT(*cd*(a),*cd*(b)) "The contradictori**es** of the contrari**es** (*a* and *b*)" !!!

SCT and SB relate a term and its double mixed negation = its weak affirmation