The Structure of Natural Deductions

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Abstract

The theory of natural deduction of pure intuitionistic logic for the \((\to, \land, \forall)\)-fragment is very elegant. It includes a strong normalization theorem and a Church-Rosser property. This elegant treatment extends to pure second-order intuitionistic logic (Girard’s polymorphic system \(F\)). The technical details are more complicated in this case because of the impredicativity of the second-order quantifier, but strong normalization (and Church-Rosser) still holds.

The other connectives (absurdity, disjunction, existential quantification), whose features are more typical of intuitionism, do not have such an elegant treatment. Girard sees their elimination rules as defective. In second-order logic, these connectives can be circumvented because they are definable in terms of the others. It is not widely known that these connectives can also be circumvented in the treatment of first-order logic if we embed it into a version of predicative second-order logic.

Our tutorial will explain these issues.

\begin{center}
Sunday, April 18, 14h-15h30m, Sala Atlântico
\end{center}

Natural deduction for the intuitionistic propositional calculus

- Reductions and normal proofs
- Weak normalization of the \((\to, \land)\)-fragment
- The operational side: typed \(\lambda\)-calculus
- Strong normalization, Church-Rosser and all that
- Typed vs untyped \(\lambda\)-calculus
Monday, April 19, 14h-15h30m, Sala Atlântico

The Curry-Howard correspondence
The beauty of the ($\rightarrow, \land, \forall$)-fragment
Negative translation
Herbrand’s theorem
Girard’s system $F$

Tuesday, April 20, 14h-15h30m, Sala Atlântico

Prawitz’s embedding
A predicative restriction of $F$
On the new embedding (I)
The strong normalization of the $\lambda^{2,P}\eta$-calculus
On the new embedding (II)
The right way to see the “bad conectors”?

References


