The Structure of Natural Deductions

Fernando Ferreira
Universidade de Lisboa

UNILOG Tutorial
Hotel Eden
Monte Estoril, Portugal
April 2010

Abstract

The theory of natural deduction of pure intuitionistic logic for the $(\to, \wedge, \forall)$-fragment is very elegant. It includes a strong normalization theorem and a Church-Rosser property. This elegant treatment extends to pure second-order intuitionistic logic (Girard's polymorphic system F). The technical details are more complicated in this case because of the impredicativity of the second-order quantifier, but strong normalization (and Church-Rosser) still holds.

The other connectives (absurdity, disjunction, existential quantification), whose features are more typical of intuitionism, do not have such an elegant treatment. Girard sees their elimination rules as defective. In second-order logic, these conectives can be circumvented because they are definable in terms of the others. It is not widely known that these conectives can also be circumvented in the treatment of first-order logic if we embed it into a version of predicative second-order logic.

Our tutorial will explain these issues.

Sunday, April 18, 14h-15h30m, Sala Atlântico

Natural deduction for the intuitionistic propositional calculus
Reductions and normal proofs
Weak normalization of the $(\to, \wedge)$-fragment
The operational side: typed λ-calculus
Strong normalization, Church-Rosser and all that
Typed vs untyped λ-calculus

$\boxed{\textit{Monday, April 19, 14h-15h30m, Sala Atlântico}}$

The Curry-Howard correspondence
The beauty of the $(\to, \wedge, \forall)$-fragment
Negative translation
Herbrand's theorem
Girard's system F

$\boxed{\textit{Tuesday, April 20, 14h-15h30m, Sala Atlântico}}$

Prawitz's embedding
A predicative restriction of F
On the new embedding (I)
The strong normalization of the $\lambda^{2,P}\eta$-calculus
On the new embedding (II)
The right way to see the "bad conectives"?

References

F. Ferreira. "Comments on predicative logic," Journal of Philosophical Logic 35, pp. 1-8 (2006).

J.-Y. Girard et al. Proofs and Types, Cambridge University Press, 1989.

G. Mints. A Short Introduction to Intuitionistic Logic, Kluwer Academic, 2000.

D. Prawitz. Natural Deduction, Dover Publications, 2006.

T. Sandqvist, "A note on definability of logical operators in second-order logic," unpublished manuscript, 2008.

W. Tait. "Intensional interpretation of functionals of finite type I," The Journal of Symbolic Logic 32, pp. 198-212 (1967).

A. S. Troelstra & H. Schwichtenberg. Basic Proof Theory, Cambridge University Press, 2000.