Handbook of the
6th World Congress and School on Universal Logic

June 16–26, 2018
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2 – What is Universal Logic?

In the same way that universal algebra is a general theory of algebraic structures, universal logic is a general theory of logical structures. During the 20th century, numerous logics have been created: intuitionistic logic, deontic logic, many-valued logic, relevant logic, linear logic, non monotonic logic, etc. Universal logic is not a new logic, it is a way of unifying this multiplicity of logics by developing general tools and concepts that can be applied to all logics.

One aim of universal logic is to determine the domain of validity of such and such metatheorem (e.g. the completeness theorem) and to give general formulations of metatheorems. This is very useful for applications and helps to make the distinction between what is really essential to a particular logic and what is not, and thus gives a better understanding of this particular logic. Universal logic can also be seen as a toolkit for producing a specific logic required for a given situation, e.g. a paraconsistent deontic temporal logic.

Universal logic helps to clarify basic concepts explaining what is an extension and what is a deviation of a given logic, what does it mean for a logic to be equivalent or translatable into another one. It allows to give precise definitions of notions often discussed by philosophers: truth-functionality, extensionality, logical form, identity, existence, negation, etc.

The idea of universal logic is not to build a monolithic system of logic but to develop comparative study of ways of reasoning and their systematizations, promoting better understanding and knowledge of the logical realm and its connections with other fields.

3 – Aim of the event

This is the 6th edition of a world event dedicated to universal logic called "UNILOG", standing for "World Congress and School on Universal Logic". Here is the list of previous UNILOGs:

— 1st UNILOG, Montreux, Switzerland, 2005

— 2nd UNILOG, Xi'an, China, 2007

— 3rd UNILOG, Lisbon, Portugal, 2010

— 4th UNILOG, Rio de Janeiro, Brazil, 2013

— 5th UNILOG, İstanbul, Turkey, 2015

This event is a combination of a school and a congress. The school offers many tutorials on a wide range of subjects. The congress will follow with invited talks by some of the best alive logicians and a selection of contributed talks. As in previous editions there will also be a contest and a secret speaker.

This event is intended to be a major event in logic, providing a platform for future research guidelines. Such an event is of interest for all people dealing with logic in one way or another: pure logicians, mathematicians, computer scientists, AI researchers, linguists, psychologists, philosophers, etc.

The 6th edition of UNILOG will take place at the Campus Albert Londres, located close to the Célestins spring, near the banks of the river Allier, in the thermal city of Vichy, in a region of France called Bourbonnais.

4 – Call for papers

To submit a contribution send a one page abstract to unilog2018@yandex.com by December 1st, 2017.

All talks dealing with general aspects of logic are welcome, in particular those falling into the categories below.

See also the workshops where you can submit your abstract if it is appropriate and the logic prizes. Participants of the school are also strongly encouraged to submit a contribution.

General Tools and Techniques

- consequence operator
- diagrams
- multiple-conclusion logic
- labelled deductive systems
- Kripke structures
- logical matrices
- tableaux and trees
- universal algebra and categories
- abstract model theory
- combination of logics
- lambda calculus
- games

Scope of Validity
Domain of Applications of Fundamental Theorems

- completeness
- compactness
- cut-elimination
- deduction
- interpolation
- definability
- incompleteness
- decidability
- Lindenbaum lemma
- algebrization
- Dugundji's theorem

Study of Classes of Logics

- modal logics
- substructural logics
- linear logics
- relevant logics
- fuzzy logics
- non-monotonic logics
- paraconsistent logics
- intensional logics
- temporal logics
- many-valued logics
- high order logics
- free logics

Philosophy and History

- axioms and rules
- truth and fallacies
- identity
- lingua universalis vs. calculus ratiocinator
- pluralism
- origin of logic
- reasoning and computing
- discovery and creativity
- nature of metalogic
- deduction and induction
- definition
- paradoxes

Part II

6th World School on Universal Logic

5 – Aim of the School

A great variety of tutorials

For the 6th edition of this school there will be many tutorials on all aspects of logic:

- history of logic (Aristotle, Stoic logic, Medieval logic, Leśniewski, Couturat, etc.)
- relations/applications of logic to other fields (Logic and the Brain, Logic and Religion, Conceptual Engineering, etc.)
- mathematical logic and foundations (Topos theory, Lindenbaum methods, Arithmetics. etc.)
- computational logic (Data linkage, semantic technologies, programming, etc.)

Contact: vichy@uni-log.org.

A School to Promote Logical Research

Each tutorial will be presented in three sessions of one hour. The tutorials will be given by a wide range of logical scholars from all over the world.

The idea is to promote interaction between advanced students and researchers through the combination of a school and a congress. Participants of the School are strongly encouraged to submit a paper for the congress that will happen in June 21–26, just after the school.

The school will open with a round table "Why study logic?" and will end with a round table on "Why, when, where and how to publish?".

Logic Around the World

For PhD students, postdoctoral students and young researchers interested in logic, artificial intelligence, mathematics, philosophy, linguistics and related fields, this will be a unique opportunity to get a solid background for their future researches.

6 – ¿Why Study Logic?

It is the Opening Session of the 6th World School on Universal Logic, on June 16, 2018.

This topic will be discussed by a variety of people in a round table animated by Jean-Yves Beziau, UFRJ* and CNPq† (Brazil) / Visiting Researcher of École Normale Supérieure (Paris, France), organizer of the School of Universal Logic since 2005:

- Franca D'Agostini, Polytechnical University of Turin, Italy
- Mykola Nikitchenko, Taras Shevchenko National University of Kyiv, Ukraine
- Julio Michael Stern, University of São Paulo, Brazil
- Ioannis Vandoulakis, Hellenic Open University, Greece

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7 – Speakers of the 6th World School on Universal Logic

Each tutorial will be presented in 3 sessions of 1 hour. The tutorials will be given by a wide range of logical scholars from around the world:

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8 – Tutorials

The Logic of Lying

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The recent literature about lying, deceiving, misleading and other forms of deceit in philosophy of language is quite rich. The problem is also at the centre of the public debate nowadays. (See the fortune of the concept of "post-truth politics", or the widespread worry concerning the circulation of fake-news on the Internet.)

The tutorial aims at making the logic of deception clear, by stressing the connection between the practice of deceptive processes and typically logical issues related to the theme, such as the semantic behaviour of truth, the inferential force of falsity and negation, and liar-like paradoxes.

I. The many ways of deception

The first lecture provides a brief introduction to the different forms of deceit as currently studied and defined in the philosophy of language, in semantic and pragmatic perspective. We will focus on the definitions of 'lying', 'misleading', 'manipulating', 'spinning' and their respective doxastic force.

II. The role of truth in the practice of conveying falsity

The second lecture will deal with the notions of falsity and partial truth in logic and in everyday interactions. We will look at the basic logical perspectives concerning the failure of truth: classical (truth excludes falsity), paracomplete ('untrue' does not mean 'false'), paraconsistent (there might be true and false assertions) and gradualistic (there are degrees of truth and degrees of falsity — in fuzzy or probabilistic sense). A systematic confrontation between logic and our usual practices of assertion will be presented.

III. Is the Liar lying?

In the third lecture, I propose a very brief introduction to semantic Liar-like paradoxes. The presentation will focus on some paradoxes (such as Pinocchio Paradox or the Blushing Liar) that specifically enlighten the nexus between Liar-like paradoxes and the effective pragmatic of lying. The question is whether a person who says 'I am lying' (or similar assertions) can be said 'a liar', in the strict sense of the term. Another more interesting question is: can Machiavelli's Prince lie, given that we know he will lie (because Machiavelli told us)?

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Topos theory and Caramello's bridge technique

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This tutorial will offer an introduction to topos theory and geometric logic, and to the theory of topos-theoretic bridges developed by O. Caramello [1,2].

Grothendieck toposes can be seen as common generalization of the concepts of a universe of sets and of a topological space. There is an abundance of examples from topology, algebraic geometry, differential geometry and logic.

A Grothendieck topos is a kind of category, in which one can interpret the language of *geometric logic*, a certain infinitary first order language, in a way that generalizes the usual set-theoretic interpretation. Geometric logic is an intuitionistic infinitary first order logic based on that language, which is sound and complete with respect to the topos interpretation. As usual, for the completeness part one has to show that if a theory $\mathbb{T}$ does not imply a formula φ, then there is a model of $\mathbb{T}$ in some topos where φ is not satisfied. In topos theory, the completeness theorem takes a particularly nice form: there exists a topos $B[\mathbb{T}]$, and a model of $\mathbb{T}$ in it which satisfies *only* those sentences implied by $\mathbb{T}$, and thus takes care of *all* sentences φ as above simultaneously. The topos $B[\mathbb{T}]$ is called the classifying topos of the theory $\mathbb{T}$, and the said model of $\mathbb{T}$ is called the universal model. Every model of $\mathbb{T}$ in some topos arises as an image of the universal model.

Like a group can be presented by generators and relations between them, a Grothendieck topos can be presented by a *site*, i.e. a small category together with a specification of when a family of morphisms with common codomain is a covering of that codomain. The inspiring example is the category of open subsets of a topological space together with the usual notion of covering from topology. Just as different presentations can give rise to isomorphic groups, different sites can give rise to equivalent toposes.

Given a geometric theory, one can explicitly construct a site presenting its classifying topos, the syntactic site of $\mathbb{T}$. It can happen that two different theories, giving rise to two different sites, have equivalent classifying toposes. Caramello's bridge technique studies and exploits such situations: one can try to translate properties of the classifying topos into properties of the theories, and back, and thus obtain relations between the two different theories.

In this tutorial we will introduce Grothendieck toposes, the interpretation of geometric logic in them, classifying toposes and Caramello's bridge technique, all with examples. The prerequisite for the course is knowledge of the basic notions of category theory: categories, functors, natural transformations, (co)limits, adjunctions and the Yoneda lemma.

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Logic in the Brain

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Recent advances in fuzzy and paraconsistent logic confirm the complexity of the human brain. However, are we only logical beings? In addition, what role do emotions play in rational processes? And how does stress effect moral decision making? In the current tutorial, we will address these questions, taking into consideration recent studies in cognitive, affective and contemplative neuroscience and psychology of logic, focusing on decision-making, morality and free will and their underlying neuronal mechanisms. Everybody who is interested in these questions is welcome to join, and there are no specific prerequisites. The tutorial will be divided into three sessions, as a metaphor for the journey between the current state of man and the state he may achieve.

I. The bio-logic nature of 'paraconsistency' of man

Although there are contradictions inside our brain, it contains them, also through the mind's interpreter [3,8]. In fact, humans are a three brain being [2,5]. We have all experienced that emotions can interfere with reason and decision-making, and that different thoughts can simultaneously co-exist. An additional challenge is that we are capable of having many feelings at once. Logical and rational thinking requires that we pay attention, but that is hard to do if we feel threatened. Thus, we may have trouble paying attention to an abstract problem when our amygdala is sending danger signals to our logical brain. Logic and its pleasures can also suddenly seem inconsequential when we see an attractive person. The issue here is competition between different brain areas. Different sensory signals physically compete for attention in the brain, and those that are the strongest win out [9]. Seven features must be kept in mind when discovering the niceties of medieval logic, many of them closely connected: the exegetical dimension of medieval — logic a feature shared with medieval thought as a whole; the wide range of fields included in what was called "logic" by then (epistemology, philosophy of language, semantics, philosophy of science, etc.) and the strong connection to sister disciplines (rhetoric, grammar, metaphysics); the non-formality of medieval logic, even in its "formal" aspects; the philosophical and scientific orientation of logic as both an instrument for knowledge and a part of philosophy; the non-distinction between logic and philosophy of logic; the disputational approach to logic as a theory and a practice (the latter is also true of medieval university in general); last but not least, the major social and pedagogical role played by logic, before the rise of mathematics as a new standard in educational systems and sciences. This last aspect probably explains the existence of a fairly stable logical culture in the Middle Ages and pre-modern period.

II. The Sphere Model of Consciousness

The Sphere Model of Consciousness [6] suggests three axes of human experience, pointing towards the center of the sphere as the locus of human psychological development. Based on the Sphere Model, the consciousness state space has been formulated, suggesting a unifying neuroscientific model for consciousness and self [4]. In this session, we will discuss the characteristics of being in the Logos in different traditions and their possible neuronal correlates. In addition, examples of reaching similar states of being will be compared and discussed.

III. Uniting the fragmented mind: it is logical to train

Recent neuroscientific studies have confirmed that our brain is fragmented, and that increased neuronal synchronization can aid in enhancing internal integrity. Increased neuronal synchronization is related to increased cognitive flexibility, reflectivity and attention. Several brain-based integrity scales have been developed to measure state of consciousness, and were found to be connected to moral judgments. These results will be shortly discussed in connection to models of Deontic logic. Importantly, additional research consistently demonstrate that neuronal synchronization, cognition and consciousness can be elicited by training such as mindfulness, meditation and the Quadrato Motor Training [7,1]. These results and others suggest that training can greatly help in moral problem solving and creativity.

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The Adventures of the Turnstile $(\vdash)$

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"$\vdash$" is one of the most famous symbols of modern logic. It has been introduced by Frege and for this reason is called "Frege's stroke". But it is also called by other names, in particular "turnstile", a name which has more to do with its form than its meaning. Its meaning has considerably evolved and variations of its original design have sprung, in particular its most famous double: "$\models$". In this workshop we will combine an analysis of the history of this symbol and its variations with critical reflections about their meanings and uses. This will be a way to reflect on the evolution and central features of modern logic.

I. Origin of the symbol "$\vdash$" and its early history

In this first session we will recall the original meaning of Frege's stroke, when and in which circumstances it was introduced and its reception and use or non-use by Hilbert, Whitehead-Russell, Wittgenstein and Leśniewski. We will in particular focus on the distinction between truth and logical truth. We will furthermore discuss the symbolic dimension of "$\vdash$" within a general discussion on symbolism, mathematics and modern logic.

II. Syntax vs Semantics, Proof Theory vs Model Theory, "$\vdash$" vs "$\models$"

In the second session we will discuss the crystallisation of the opposition in modern logic between syntax and semantic, proof-theory and model theory, typically symbolized by "$\vdash$" vs. "$\models$". An opposition which makes sense but is also overcome by the completeness theorem. We will also discuss the incompleteness theorem from the perspective of these two symbols. We will in particular emphasize the ambiguity of the use of "$\vdash$" in sequent calculus instead of the original symbol used by Gentzen "$\rightarrow$", explaining how this confuses one of the most important results of proof theory, the cut-elimination theorem. We will also emphasize the ambiguity of the double use of the double "$\models$" in model theory: as a symbol for a semantical consequence relation and as a symbol used for a relation between models and formulas.

III. "$\vdash$" as an Abstract Consequence Relation

In the third session we will focus of the use of "$\vdash$" as a symbol for an abstract consequence relation, beyond the dichotomy proof-theory/model-theory. It denotes a fundamental relation for logical structures, slight variation of Tarski's consequence operator. We will focus in particular on the completeness theorem from this abstract perspective. We will also discuss some related notions such as logical equivalence expressed by "$\dashv\vdash$" and the notion of self-extensionality.

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History of Medieval Logic

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In the same manner as medieval philosophy, medieval logic includes a large range of cultures and languages in Byzantine, Syriac, Arabic, Hebrew and Latin traditions. It extends from the sixth century to the fifteenth century and beyond, as far as logic alone is concerned. Though challenged by Renaissance logics in the sixteenth century, especially in Reformed countries, and by new logics of discovery designed for the scientific revolution, it survived the collapse of the Aristotelian sciences up till the nineteenth century, under labels such as "scholastic logic", "Aristotelian logic", or "traditional logic". Elaborating from the late ancient legacy accessible to them, that is few sketchy textbooks, some Neoplatonic commentaries to Aristotle's Organon and a "peripatetized" version of Stoic logic, i.e. "hypothetical syllogistic", medieval logicians have introduced many novelties nowhere found before and often still discussed today: a sophisticated conception of modalities, a general theory of consequences, the notion of a (contextual) reference, distinct from signification, a distinction between truth-bearers and truth-factors, a focus on the semantics of proper names and indexicals, a disputational, pragmatic, approach to logic, the distinction between the "form" and the "matter" of the arguments within a rich and varied conception of formality, etc. Even if schematic letters have been used, as they were already in Aristotle's tracts, medieval theories are based upon a regimentation of already regimented natural languages, such as scholastic Latin.

Despite the wealth of discussions and logical innovations found in Arabic logic, the tutorials are essentially dedicated to Latin logicians. They use only English translations and terminology. They will explain and contextualize every reference to authors and texts. Everybody interested in the history of logic is welcome. A drastic selection of topics has been made in a rich history which extends over ten centuries. After a general presentation of medieval Latin logic in context (Session I), I will present only two aspects: theories of consequences (Session II), and theories of reference and truth (Session III).

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I. General Presentation of Medieval Logic in Context

Medieval Latin logic can be roughly divided in five periods. They often correspond to a "Renaissance", that is to a "re-discovery" of ancient texts not yet "available" (translated, circulated, taught, etc.). The High Middle Ages see the domination of a Roman logic (and grammar); the twelfth century witnesses a full Renaissance of logical inquiries based on Aristotle and Boethius (c. 6th AD) and focussed on "topical inferences"; the thirteenth century can be labelled a "Golden Age" of Aristotelian logic, with a strong focus on the recently rediscovered Prior and the Posterior Analytics; the fourteenth century is extremely innovative and introduces the notion of a general theory of inference; the fifteenth and the sixteenth centuries are a transitional period when the original text of Aristotle's Organon is rediscovered, and scholastic logic reaches a (too?) high level of sophistication and formalization. It is challenged as "barbarous" and fruitless by Renaissance authors, and tentatively replaced by Renaissance logics.

Seven features must be kept in mind when discovering the niceties of medieval logic, many of them closely connected: the exegetical dimension of medieval logic — a feature shared with medieval thought as a whole; the wide range of fields included in what was called "logic" by then (epistemology, philosophy of language, semantics, philosophy of science, etc.) and the strong connection to sister disciplines (rhetoric, grammar, metaphysics); the non-formality of medieval logic, even in its "formal" aspects; the philosophical and scientific orientation of logic as both an instrument for knowledge and a part of philosophy; the non-distinction between logic and philosophy of logic; the disputational approach to logic as a theory and a practice (the latter is also true of medieval university in general); last but not least, the major social and pedagogical role played by logic, before the rise of mathematics as a new standard in educational systems and sciences. This last aspect probably explains the existence of a fairly stable logical culture in the Middle Ages and pre-modern period.

II. Theories of Consequences

This tutorial studies some aspects of the transformation of the discussions about inferences (or "consequences"), deductions, syllogisms, arguments and proofs, from the twelfth to the fifteenth century. All the logicians of the Middle Ages shared an inclusive approach to logic where the study of formal reasoning is only a (small) portion of logic, even within this part of the logical teaching dedicated to the theory of inferences. Each period developed original approaches, which were based not only on a distinctive notion of what should be the basis of a successful inference, with a focus on the problem of relevance, but also on a specific conception of the relationship between inferences, deductions, syllogisms and proofs. In the twelfth century, the notion of "topical inference" means that all inferences, even formal ones, are based upon the topics and general rules derived from them, as described by Boethius (c. 6th AD), a conception that survived long in the thirteenth century, despite Abelard's (c. 12th AD) fierce defence of the idea of a purely formal inference, i.e. based only on its form regardless of any content. In the thirteenth century, a "hylomorphic" conception of the syllogism as the subject matter of the Prior Analytics means that syllogistic studies as much the matter as the form of the syllogism. In the fourteenth century, great logicians such as Walter Burleigh, William of Ockham, and John Buridan developed general theories of consequences and were very much divided about what can count as a definition of formal consequences. Not before the fourteenth century (with the notable exception of William of Ockham) was the syllogism considered a formal inference only, rather than an argument or a proof based upon a formal inference studied regardless of its (particular) contents, a conception recovered at the end of the fifteenth century with the Renaissance rediscovery of Aristotle's Organon .

III. Semantic: Reference and Truth

From the twelfth century on, two important topics were discussed in medieval logic: the notion of reference, often contextually understood, and a vigorous debate about the truth-bearers, the propositions and their signification as distinct from that of the terms, as well as the truth-factors, facts and states of affairs. This last aspect underwent original reformulations in the thirteenth century, when the idea that (necessary) universal propositions had existential import was condemned, and in the fourteenth century, especially with Walter Burleigh, who promoted an "extreme realism" and the idea of "propositions in reality". The medieval theory of reference of terms, called "supposition", has known two canonical formulation in two distinctive periods, in the thirteenth century "terminist logic" and the famous Tractatus by Peter of Spain, with a strong realistic flavour, and in the fourteenth century in the new, nominalist, terminist logic, where a universal term do not have a referent distinct from the referent of each singular terms to which it corresponds, and where universal propositions do have existential import, though not necessarily for presently existing individuals. The various ways in which the propositions "every man is an animal" and "every man is white" are analysed will be taken as an example of the various approaches to reference, signification and truth in the period.

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Useful Links

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- Peter Abelard (IEP†)
- Buridan (SEP)
- Ockham (SEP)
- Ockham (IEP)
- Consequence — Medieval (SEP)
- Insolubles (SEP)
- Medieval Terms (SEP)
- Medieval Logic and Philosophy
- Medieval Logic — Special Issue of Logica Universalis

*Stanford Encyclopedia of Philosophy
†Internet Encyclopedia of Philosophy

Lindenbaum Method

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During his brief life, the Polish mathematician and logician Adolf Lindenbaum (∗1904–1941†) contributed to mathematical logic, among other things, by several significant achievements. Some results of Lindenbaum's, which bear his name, were published without proofs by other people from the Lvov-Warsaw School and the proofs later were provided by some others, though the authorship of Lindenbaum has never been challenged. Many may have heard about Lindenbaum's lemma, asserting the existence of Lindenbaum's extension, and Lindenbaum–Tarski algebra; less known is Lindenbaum's logical matrix. This tutorial is devoted to the two last concepts rather than the first one. However, the latter can be understood in a purely algebraic fashion, if one employs the notion of Lindenbaum-Tarski algebra. In general, the notions of Lindenbaum matrix and Lindenbaum–Tarski algebra have paved a way to further algebraization of logic, which had been begun by George Boole in the 19th century, as well as to a new branch of logic, model theory. For this reason, the present tutorial is also a gentle introduction to algebraic logic.

A uniting idea of the aforementioned concepts is a special view on the formal judgments of a formal language. It is this view we call the Lindenbaum method. Although Lindenbaum expressed merely a starting viewpoint in the tradition of Polish logic of the time, this viewpoint became a standard ever since and its development goes on until this day, continuing to shape the field of algebraic logic. Our main objective is to demonstrate how this view gave rise to formulating the aforementioned concepts and how it opens door to unexplored paths.

I. Lindenbaum's logical matrix

The idea to interpret symbolic judgments in mathematical structures goes back to George Boole. It was Lindenbaum who took for an interpretation of judgments the judgments themselves. But prior to this, he started treating the entire class of judgments as an abstract algebra, nowadays known as a formula algebra. Some experts call this Lindenbaum's move a milestone in the history of algebraic logic and universal algebra.

The turning point distinguishing the Boole-De Morgan-Schröder tradition in algebraic logic from modern tradition is the algebraization of formal deduction. The first step in this direction is the introduction of the notion of deductive system and that of consequence relation. There are a few standard ways to define a deductive system; in this part, we focus on two of them — rules of inference and logical matrix. On the one hand, the Lindenbaum logical matrix is just a special case, on the other, it characterizes all formal theorems (or theses) of any deductive system (Lindenbaum theorem).

II. Characterization of deductive systems

Powerful enough to characterize any class of theses, the notion of Lindenbaum matrix does not suffice to determine any deductive system. This part will address the question of characterization of deductive systems. Two Wójcicki's theorems will be discussed. The first deals with the notion of a bundle of logical matrix; in terms of the latter any deductive system can be determined. The second theorem finds conditions under which a deductive system can be characterized by a single logical matrix.

The theorems of Lindenbaum's and Wójcicki's were merely first steps towards algebraization of deduction based on sentential formal language. Next came analysis of matrices and algebras which "separate" premises from not derived from them sentences in deductive systems, thereby introducing the conception of separating means. This in turn has led to the notion of Lindenbaum-Tarski algebra. The latter often is obtained by a transformation of a Lindenbaum matrix with the use of a special congruence. In unital deductive systems the Lindenbaum-Tarski algebra of such a system is adequate for the set of its theses.

III. Effectiveness issues

The idea of effectiveness (in a broad sense of the word) and its importance had gradually established itself by the middle of the 20th century, when the notion of cardinal number and that of effective method, that is computability, were fully realized. The problems like the following were raised and solved: whether a deductive system formulated in a countable language can always have a finite logical matrix adequate for its theorems (J.C.C. McKinsey and A. Tarski); whether it is effectively decidable that any two finite logical matrices or any two finite bundles have the same set of theorems (J. Kalicki for matrices, A. Citkin for bundles); whether any deductive system formulated in a countable formal language can be determined by a single denumerable matrix (A. Wrónski). Some of these problems and related to them, as well as the finite model property of a system, will be discussed in this part.

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Useful Links

- **Encyclopedia of Mathematics**
  — Lindenbaum method (propositional language)
  — Abstract algebraic logic
  — Decision problem
- **Internet Encyclopedia of Philosophy**
  — Lindenbaum
  — Logical consequence
- **Stanford Encyclopedia of Philosophy**
  — Algebraic propositional logic
  — Lvov-Warsaw school
  — Logical consequence
  — Truth values

Weak arithmetics and applications

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Proving the existence of some meta-mathematical object (e.g. a method for solving polynomial equations of degree 3 or 4) does not need a mathematical definition of that object: a general agreement about the correctness of each answer is achievable. Proving the inexistence of some meta-mathematical object (e.g., a method for solving polynomial equations of degree 5 or more) needs a mathematical definition. Among the inexistence problems inducing the main concepts of logic, let us start with the following two.

Problem 1 (Hilbert's tenth problem). *Prove that there is no universal method correctly asserting wether any given diophantine equation has (at least) a solution or has no.*

Problem 2. *Prove that there is some arithmetical statement that cannot be proved or disproved.*

These problems have no sense without a precise definition of an algorithm and of a proof. Together with the concept of *algorithm* formalizing the notion of *method*, it is possible to define a concept of *complexity*, also to define a *formal proof* and a *strength scale* for theories. Weak arithmetics study these statements needing a few axioms or weak rules of reasoning for proving them. Surprisingly, numerous links with the complexity of algorithms appear.

This tutorial is intended to provide an introduction to the topic, its problems and its methods. It will avoid both technical difficulties and ambiguity. It will be divided in the following three sessions.

I. Decidable fragments of arithmetic

Peano arithmetic admits various equivalent families of axioms formalizing the properties of the successor function (and of addition and multiplication) together with a family of *Induction Axioms* formalizing the usual mathematical induction. It is not possible to algorithmically determine, for

*Laboratoire d'Informatique, de Modélisation et d'Optimisation des Systèmes any sentence in its language, whether that sentence is provable from its axioms: the theory is *undecidable*. Presburger arithmetic is the well known first-order theory of the natural numbers with addition and equality. The axioms include the schema of induction. It is much weaker than Peano arithmetic and has been proved to be decidable. However the algorithms for decision require more than exponential run time. Stronger fragments than Presburger arithmetic have been proved to be decidable, e.g. the existential theory of addition and divisibility. Decidable fragments of Peano arithmetic are more and more involved in automatic reasoning.

II. Definability

Let us consider a set on words on a finite alphabet with k letters denoting as digits $\{0, 1, 2, ..., k-1\}$. There is a natural correspondence between such words and natural numbers using base k representation. Let us now consider the set of words recognizable by a finite automaton. It turns out that the correspondent set of integers is definable by a formula of the language $(+, \mathrm{V}_k)$, where $z = \mathrm{V}_k(x)$ is the relation "z is the greatest power of k dividing x". The converse is true. This correspondence provides insight in the area of complexity: a relation which is definable both in $(+, \mathrm{V}_k)$ and $(+, \mathrm{V}_{k'})$ for which there is no n and m such that $k^n = {k'}^m$ is definable in Presburger arithmetic.

Other correspondence are sources of problem or of solutions! The unary relation "x is not prime nor 0 nor 1" is definable using the formula $\exists u < x \, \exists v < x \, (x = u \times v)$. More generally, the Δ_0-definability is essentially definability with a formula in the language of arithmetic where the quantified variables are bounded by terms. Most of the natural notions have been proved to be Δ_0-definable, and classical diagonalization methods provide ad hoc non Δ_0-definable ones. A major open problem is to find a "natural" arithmetical relation which is NOT Δ_0-definable. The relation $z = \mathrm{Card}\{i \leq y \mid i \text{ is a prime number}\}$ is not known to be Δ_0-definable and is a candidate. An answer could provide the strict inclusion LOGSPACE $\subsetneq$ LINSPACE.

III. Provability

Let E be a subset of $\mathbb{N}^n$, for which there exists an algorithm that will ultimately halt when a member of the set is provided as input, but may continue indefinitely when the input is a non-member. There is a Δ_0-formula φ such that E is defined by $(\exists y \in \mathbb{N}^m)\,(\varphi(x, y) = 0)$. This very last formula is called a Σ_1–formula. The fundamental step of the answer to problem 1 is the following theorem of Y. Matiyasevich, J. Robinson, M. Davis and H. Putnam:

Theorem 1. For all Σ_1–formula ψ, there are two polynomials P and Q with natural coefficients such that for all a in $\mathbb{N}^n$, $\psi(a)$ is true if and only if there exists b in $\mathbb{N}^m$ such that $P(a,b) = Q(a,b)$.

Let us consider the following question: "*What axioms are really useful in the proof of this theorem?*". We say that a set of axioms T proves the MRDP-theorem for the following mathematical statement: For all Σ_1–formula ψ, there are two polynomials P and Q with natural coefficients such that

$$T \vdash \forall x\,(\psi(x) \leftrightarrow (\exists y \in \mathbb{N}^n)\,(P(x,y) = Q(x,y))).$$

Let $\mathrm{I}\Delta_0$ be the fragment of Peano arithmetic where the induction axioms schema is reduced to Δ_0-formulas.

Theorem 2. If $\mathrm{I}\Delta_0$ proves MRDP, then NP = co-NP.

A much stronger theory is obtained if we add an axiom, denoted by EXP, which guarantees the totality of the exponential function:

$$\forall x\,\forall y\,\exists z\,(z = x^y).$$

Most of the usual arithmetic is provable in the theory $\mathrm{I}\Delta_0$ + EXP:

Theorem 3. $\mathrm{I}\Delta_0$ + EXP proves MRDP.

But EXP is not a theorem in $\mathrm{I}\Delta_0$. A weaker axiom than EXP is the following axiom Ω_1: $\forall x\,\forall y\,\exists z\,(z = x^{\lfloor \log_2(y) \rfloor})$

Theorem 4. If $\mathrm{I}\Delta_0 + \Omega_1$ proves the MRDP-theorem, then NP = co-NP.

We are very far from proving the premise of this theorem, but more and more parts of the natural arithmetic turn out to be formally proved to be in $\mathrm{I}\Delta_0 + \Omega_1$. For example:

Theorem 5. $\mathrm{I}\Delta_0 + \Omega_1$ proves the infinity of the prime numbers.

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Useful link

The web site of the regular conference *Journées sur les Arithmétiques Faibles* (in French): http://lacl.u-pec.fr/jaf.

Dialectics. An Introduction

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The philosophical historiography concerning dialectics is immense and complex. As Hintikka [14, p. 109] writes "dialectic has the tendency to multiply itself beyond necessity". In this context, my method is to focus on the definitions of the term in texts from Plato to contemporary philosophy, and on one idea that permeates the whole history of dialectics, and is defended by authors as different as Aristotle, Hegel, Adorno, Rescher. It is the view that dialectics is a kind of philosophical logic, more specifically the *logic of philosophy*.

The first part of the tutorial is devoted to ancient dialectics, in particular to the conception of dialectics in Plato's *Parmenides* and *Aristotle's Topics*. The second examines the meaning of dialectics in Hegel's philosophy and in two stages of its reception (in Croce and Adorno). The third is on dialectics and contemporary philosophical logic and focuses, more specifically, on the nexus dialectics-dialetheism.

I. Dialectics in Plato and Aristotle

On the first day we will first analyse the difference between Zeno's method of *reductio ad absurdum* and Plato's dialectical method [see 11,15, 5,18]. Second we will consider the relation between Plato's dialectics in the *Parmenides* and Aristotle's *Topics*. In Aristotle's *Topics* Plato's dialectic is systematized and methodically articulated as logic of our thinking about *éndoxa* (the *éndoxa* are theses concerning controversial questions of universal interest such as: is justice the advantage of the stronger?). I will stress the fundamental continuity between Plato and Aristotle, and the genuinely dialectical nature of Aristotle's philosophy. In so doing, I share the interpretation of those philosophers (in particular Berti [5]) who see the continuity between Plato and Aristotle in the idea of dialectics as the logic of philosophy.

II. Hegel's dialectics

On the second day we will first discuss the passage of the *Logic* in the *Encyclopaedia* (at the end of the "Preliminary Considerations") in which

Hegel presents the three moments/sides of every conceptual thought (Hegel calls conceptual thought also true thought and *das Logische*). The passage is fundamental for two reasons: it contains Hegel's own definition of the formal structure of every dialectical and speculative inference, and presents the idea that this dialectical structure corresponds to the behaviour and method of every true thought. Then we will see what two 20th century thinkers — Benedetto Croce and Theodor Wiesengrund Adorno — write about the meaning of Hegel's dialectic. Their concern is on two aspects: the meaning of Hegel's dialectic as logic of philosophy, and the role of negation in dialectical inferences.

III. Dialectics and contemporary philosophical logic

The third lesson is focused on the relation between dialectics and paraconsistent logics, more specifically dialetheism, the theory according to which there are true contradictions. Apostel [3] recalls that paraconsistent logics, which were impressively growing in the 70ies, and were developed by da Costa school in Brazil, by Jaśkowski in Poland and by Routley in Australia, present the necessary condition and the formal basis of dialectics. However, he also claims that they cannot be said to be dialectical logics in the Hegelian sense, and, more importantly, that they need dialectical logic. They allow us to see how to logically deal with contradictions without explosion, but they do not let us see why and how we can affirm a contradiction. Hence Apostel [3, p. 459] formulates the following task for a dialectical foundation of paraconsistentism: "in dialectical logic we have to show which contradictions are admissible and which ones are not". We will ask in what sense the Hegelian theory of dialectical contradictions can fulfil the task envisaged by Apostel.

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Stoic Logic: the dialectic of the Gods

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Stoic logic was the main alternative to Aristotelian logic in Antiquity. Developed less than a century after Aristotle's death by Chrysippus, the third head of the Stoic school, it was considered the most impressive logical system of the ancient world, up to the point that the Ancient Greeks said: "if the Gods had a dialectic, it would be Chrysippus' dialectic". Chrysippus wrote 108 books of logic in 311 volumes, almost half of his writings, and by far the most considerable corpus ever written by a logician, including his major work, the *Logical Investigations*, in 39 volumes. Unfortunately, as the rest of the works of the first generations of the Stoic school, it is almost completely lost: only ten pages of the *Logical Investigations* remain, badly preserved in a papyrus in a poor state of conservation. For the rest, we have to rely on short quotations, handbook accounts and hostile criticisms. As a consequence, despite its considerable influence in Ancient and (indirectly) in Medieval times, Stoic logic remained largely ignored or misunderstood for centuries, until its progressive rediscovery by scholars between the end of the nineteenth century and the second half of the twentieth century. In the 1930s, the pioneering work of Łukasiewicz defined Stoic logic, in contrast to Aristotle's logic, as the ancient form of propositional logic. Łukasiewicz's work impulsed two decades later a new trend of work on Stoic logic, which expanded with the development of the study of Hellenistic philosophies in the 1970s. Łukasiewicz's interpretation has been improved and refined, by developing aspects of Stoic logic not touched by Łukasiewicz, such as Stoic semantics, the theory of the 'sayable' and the proposition, the modalities, and the analysis of complex syllogisms. It is probably too simple to present Stoic logic as a propositional logic, even if its basic rules are propositional inference rules. And it remained unnoticed by Łukasiewicz that Stoic logic anticipated three important features of modern symbolic logic: (1) the Fregean theory of signification *(Bedeutung)*, since Frege was probably introduced in Jena to Stoic semantics by his colleague Rudolf Hirzel who was Frege's tenant for many years; (2) the logical asymmetry between function and argument expressed by the predicate/case distinction; (3) the analysis of universal propositions as conditionals (which allowed the Stoics to introduce a procedure rival to Aristotle's quantifiers). However, what one must not forget when studying Stoic logic is that the Stoics conceived it as dialectic, and attached the greatest importance to the dialogical context of its procedure and to its relationship to the other parts of philosophy, ethics and physics. As a consequence, Stoic logic or dialectic was a science, a part of philosophy (as opposed to the 'instrument' or organon that defined Aristotle's syllogistic) and even a virtue. All these features make a quite distinctive form of logic.

The aim of this tutorial is to present Stoic logic. Stoic logic is probably the most important step in the history of logic between Aristotle and Frege. Not only does it have an historical importance but it is also still worth reading and studying for its fascinating insights, even if the fragmentary state of the evidence does not allow to know all the refined details of their theory. The tutorial will present the main sources and their alternative interpretations to give an idea as accurate as possible of the nature of Stoic logic. Everybody interested in logic and ancient philosophy is welcome to join. There is no specific prerequisites. The tutorial will be divided in the following three sessions.

I. Stoic semantics

This part will be devoted to an overview of Stoic semantics: the Stoics distinguished between the vocal sound, for instance 'Dion', the real object of the world bearing the name, for instance the man called Dion, and an intermediate incorporeal entity, which they called the 'sayable' *(lekton)* and which they described as the signification of the vocal sound. An alternative presentation distinguish between what is signified by a common or proper name ('man', 'horse', 'Dion', 'Socrates'), namely a quality (and not a substance as in Aristotle) and what is signified by a verb ('walks', 'talks'), namely, according to the Stoics, a predicate or 'what happens' to someone. We will examine the logical and philosophical implications of these alternative presentations of Stoic semantic theory and the status of the 'sayable'.

II. 'Sayables' and propositions

This part will be dedicated to the exposition of the Stoic theory of the different sayables, namely the propositions and the non-propositional items such as predicates, questions, orders, prayers. We will also examine the Stoic typology of propositions, the distinction of simple propositions and the non-simples (i.e. molecular) propositions, in particular the truth-conditions for the conditional (including the equivalent of a debate on strict implication).

III. Syllogistic (inference rules) and analysis

In this last part, we will discuss the two important aspects of Stoic syllogistic: (1) the inference rules known as the Stoic 'five indemonstrables', including the *Modus ponens* and the *Modus tollens*; (2) the rules of analysis of complex syllogisms. We will also explain how the Stoic understood the notion of a 'demonstration' as a stronger requirement for an argument than logical validity due to the epistemological nature of the propositions involved and how they translated universal and non-universal propositions by using simple propositions and conditionals. As a conclusion, the tutorial will indicate the place of Stoic logic within Stoic philosophy and within the history of logic.

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The inconsistency theory of truth and nominalistic mathematics

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This tutorial will bring together two subjects that are not normally discussed together, namely the inconsistency theory of truth and nominalistic philosophies of mathematics.

Session 1

According to the inconsistency theory of truth, our conflicting intuitions when it comes to determining the truth value(s) of the Liar Sentence and its siblings are due to the fact that the linguistic rules for the truth predicate are inconsistent. This was first argued by Chihara [2] and later by Eklund [5] and Scharp [10]. I will explain this solution to the semantic paradoxes and provide what I believe to be the best defence of it. Doing so involves bringing in Lewis' [8] theory of language conventions and Nagel's [9] idea of a *view from nowhere.*

Session 2

One conclusion from session 1 will be that we, as a language community, have a high degree of freedom to decide by convention on what logic to use, roughly in the sense of Carnap [1]. This puts the many formal theories of truth that have been proposed in a new light: they can be evaluated on the basis of how useful they would be as potential conventions, rather than on the basis of whether they are correct. From this perspective, we will take a closer look at Kripke's theory of truth [7] and van Fraasen's concept of supervaluation [11]. Then we will tinker a little with the possible convention they in effect describe until we get something that is useful for mathematics.

Session 3

The idea of a nominalistic mathematics is to give a philosophical account of what mathematics is that does not inflate our ontology with ad-hoc abstract objects. Chihara [3,4] proposed that we can do so by constructing mathematics on possible open-sentences. Field [6] approached the same goal by using the system of all spatial regions of the universe as his foundation. I will argue that they both fail. Chihara assumes that there are uncountably many open-sentences, which is to stretch the concept of language beyond the nominalistic and into the abstract. And the spatial regions of the universe must either be understood as abstract collections (of concrete entities) or as collections determined by language, which means that Field's foundation is either not nominalistic or not sufficient for his purpose. However, using the lesson from session 2, we can do better: by relying on a non-classical logic convention, we can make the ontology to which we are limited as nominalists suffice for a scientifically adequate mathematics.

Prerequisites

I will assume a basic acquaintance with the Liar Paradox and philosophy of mathematics. Session 3 in addition presupposes knowledge of Cantor's theorem about the cardinality of the set of the real numbers and of mathematical analysis up to and including the Fundamental Theorem of Analysis.

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Aristotle's Principle of Non-Contradiction

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In *Metaphysics Gamma*, Aristotle understands the principle of non-contradiction (PNC) as the most certain principle of all such that it is impossible to be mistaken about it. Yet, Aristotle is also concerned with the fact that some people may reject this principle. In that respect, he constructs arguments aiming to defend PNC as a true opinion. There is then a difficult contrast to explain: on the one hand, PNC is a necessary principle of the highest importance; on the other, it is merely justified as a true opinion against those who challenge it. So the essential question is as follows: if PNC is postulated as the most certain principle of all, why does Aristotle feel the need to speak of it as a mere opinion?

Many have been puzzled by this contrast. Łukasiewicz [8] concludes about Aristotle: "he may himself have felt the weakness of his arguments; and that may have led him to present his Law as an ultimate *axiom* — an unassailable *dogma*" [8, p. 62, original emphases]. Others have used Aristotle's weak and problematic arguments as a way to illustrate the failure of PNC [9].

These reactions show that Aristotle's defence of PNC is, at worst, not understood or, at best, not taken seriously. The aim of this tutorial will be to answer this concern by accounting for Aristotle's method. We shall explain why PNC is defendable only as a true opinion, even though it is said to be the most certain principle of all, and we shall conclude that Aristotle's weak defence of PNC is perfectly compatible with the postulate of PNC as a strong axiom.

Everybody is welcome to join, and there are no specific prerequisites. The tutorial will be divided into three sessions.

I. Aristotle's PNC and Łukasiewicz's formulations

A first session will focus on Aristotle's definition of PNC, as it is exclusively based on predicates and requires two conditions, namely simultaneity and similarity. PNC is also to be distinguished from two derived principles, namely the excluded middle and bivalence. Finally, contradiction is more than mere contrariness, in so far as two contraries are contradictory, if and only if one is true and the other false. We shall then compare Aristotle's PNC with Łukasiewicz's [8] interpretations of it through an ontological, a logical, and a psychological formulation. Influenced by Frege's logical formalism, Łukasiewicz then accuses Aristotle of "logicism in psychology".

II. An Aristotelian contextualization of PNC

In a second session, we shall analyze the context in which Aristotle's PNC takes place. *Metaphysics Gamma* introduces a hierarchy of sciences: philosophy is the universal science, which includes the particular sciences of physics and mathematics. Aristotle assesses PNC with respect to philosophy, in so far as PNC is a necessary principle only for those who know about the general nature of things, and which goes beyond any specific mathematical or physical nature. It is within this epistemic context that Aristotle's PNC has to be understood, meaning that non-philosophers express an opinion about it, without being knowledgeable about its necessity. As such, the definition of Aristotle's PNC is inseparable from the way PNC is either intrinsically cognized or merely believed.

III. Aristotle on the rejection of PNC

A third session will study why Aristotle explicitly admits the possibility of rejecting PNC. Indeed, he has to convince all non-philosophers that PNC should not be regarded as a false opinion. According to him, there are two ways of challenging PNC. One is for physicists to assume that things are endlessly changing, making their meanings indefinite and thereby irrelevant to PNC. Aristotle's reply is that physical motion cannot be used against the postulate that things have definite meanings. The other objection is that any proof of PNC already uses PNC in the premises of the proof. Aristotle acknowledges this *petitio principii*, and then concludes to the absence of a direct proof. Nevertheless, he suggests an indirect refutation to this objection, aiming to show that it is impossible not to use PNC in language; thus, even the rejection of PNC will have to rely on the use of PNC.

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Useful Links

- Aristotle on Non-Contradiction (Stanford Encyclopedia of Philosophy)
- Aristotle's Metaphysics (Wikipedia)
- Library of Ancient Texts Online (Google)

Definite Descriptions in the Proof-Theoretic Setting

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Definite descriptions are ubiquitous in natural languages. Phrases like "the capital of France" or "the youngest sister of Jack" usually do not lead to any problems in communication. However, we can easily run into troubles when we try to provide a satisfactory logical analysis of their behavior. In fact, proper definite descriptions having a unique designatum, are rather not problematic, in contrast to those which fail to designate, called improper (or unfulfilled) definite descriptions. The famous Russellian "the present King of France", is of this kind but even innocent-looking "the son of Jack" may be problematic in case Jack has no son, or more than one.

History of logical and philosophical investigations devoted to the explanation of definite descriptions is fascinating and illuminating. Famous logicians like Frege, Russell, Hilbert, Bernays, Carnap, Quine, Rosser and Hintikka — to mention only a few scholars from the earliest stage of investigation — were strongly engaged in this enterprise. We can find numerous brilliant analyses and even complete formal theories of this apparently simple linguistic phenomenon. Yet, despite the efforts, it can be hardly agreed that a fully satisfactory and commonly accepted theory was provided.

On the other hand, a proof-theoretic apparatus was not yet applied in this field and we would like to explore this possibility. In particular, we will show that the application of techniques taken from modern structural proof theory may shed a new light on the good and bad sides of different approaches to definite descriptions. No prerequisites are assumed. The tutorial will be structured in the following way:

I. Survey of the most important and interesting theories of definite descriptions

In the context of classical logic we will focus on the well known reductionist approach of Russell and the chosen object theory of Frege and its formalization provided by Kalish and Montague. Then we describe some of the theories developed in the framework of free logic by Lambert, Scott, van Fraasen and others. We finish the presentation with three different theories developed on the ground of modal logic by Thomason and Garson, Goldblatt, Fitting and Mendelsohn.

II. Presentation of some elements of proof theory required for further study

We introduce a suitable version of generic sequent calculus, discuss some of its properties, the problem of cut elimination and extension by extra rules. Finally we provide a sequent calculus equivalent to Kalish and Montague version of Fregean theory and prove cut elimination theorem for it.

III. Sequent calculus for modal system based on free logic which is equivalent to Thomason and Garson's theory

We prove cut elimination theorem for this system and discuss some possible extensions of it taken from free logic hierarchy. We provide also a system for Goldblatt's theory and explain why cut rule is not eliminable for it. Finally we consider an open problem of providing a sequent calculus for Fitting and Mendelsohn's theory.

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Conceptual Engineering: A Systematic Unified Framework

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We use concepts all the time to make sense of reality. The quality of our cognition thereby crucially depends on that of our conceptual schemes and repertoires, so that: *the better our concepts are, the better our cognitive activities will be.* Conceptual engineering is the fast-moving research field [3,5,6] that means to provide a method to assess, criticize, and improve any of our concepts working as such cognitive devices [4,5,10,13] [see also 18,20], that is: to identify conceptual deficiencies, elaborate ameliorative strategies, and prescribe normative guidelines as to whether and how to use a concept (vs. to describe how it works as a matter of fact) [1,4,5,19,20]. The aim of the SUFCE tutorial is to provide a systematic overview of conceptual engineering, to be divided into three sessions:

- **S1: Research Program.** The first session of the tutorial will introduce the overall research program of conceptual engineering: its starting point, its main goal and objectives, along with its most pressing challenges [6]. A typology of its main variants will be presented, [e.g. 3,5,17] and the standard objections against them will be critically analyzed, [e.g. 12] [cf. 18,20].

- **S2: Theoretical Foundations.** The second session of the tutorial will then consist in laying down the foundations of conceptual engineering by developing the theories of cognition (viz. 'cognitive engineering') [10,11,15] and concepts [14,16,21] that are needed to effectively implement conceptual engineering as a widely applicable method for the cognitive optimization of our conceptual devices.

- **S3: Methodological Framework.** Finally, the third session of the tutorial will deliver a method of conceptual engineering constructed as a fully recast Carnapian method of explication [1] [cf. 7,8], upgraded with other complementary template procedural methods for re-engineering concepts (namely, that of 'conceptual modeling' [13], 'levels of abstraction' [9], and 'reflective equilibrium' [2]).

Basic knowledge in philosophy language, mind and cognition, as well as interest in meta-philosophical issues are expected. Further material will be available in due course. At least one-quarter of each session will be devoted to discussion (Q&A).

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Leśniewski's Evolutional Logic

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Different in many respects from standard versions of symbolic logic, Stanisław Leśniewski's systems of logic (called Protothetics, Ontology and Mereology) present a lot of original and unusual features that continue to be stimulating for modern logicians and thinkers, since they have been elaborated in Warsaw between the two World Wars. Among these aspects, one of the certainly most interesting is the way Leśniewski conceived definition as a process that has to be counted among the usual inference tools, like Modus Ponens or Universal Instanciation. This peculiarity makes Leśniewski's symbolic language quite unusual. Instead of being determined once for all with a set of symbols and a list of rules for the specification of well formed formulae, Leśniewski's language has to remain open and able to integrate the many novelties and evolutions that can be step by step introduced by definitions. With these specific symbolic languages, Leśniewski was able to show that very tiny systems of axioms (including for example only equivalence, the universal quantifier and a modern sort of copula) can give rise to very powerful systems of logic.

Leśniewski's systems are often considered to be very interesting but technically difficult. With this tutorial my aim is to show that the main stimulating aspects of this non standard logic are actually perfectly accessible, without specific prerequisite, just an intellectual interest in general logical matters. Everybody who has this interest is welcome to join. The tutorial will be divided in the following three one-hour sessions.

I. An open and evolutional symbolic language

In this session, we are going to understand how to build a complete propositional logic, resting only on the single connective "if and only if". The main ideas of this construction are in Leśniewski himself, in Alfred Tarski (his unique PhD student), but also in Bertrand Russell's early logical writings. Leśniewski's achievement in this matter was strongly based on the new kind of formal language he elaborated: an evolutional language, in which every part of a formula takes its symbolic status and determined meaning from the *context* in which it occurs. Like in natural languages, the meaning of a word or a symbol depends on the combination of words in which it is used and the meaning of an expression (sentence or formula) depends on the expressions (in particular definitions) that have been previously asserted. As we will see, Leśniewski discovered very nice notational solutions in order to warrant both contextuality and logical accuracy.

II. A new Organon

This second session is devoted to the powerful logic of terms Leśniewski conceived introducing as a single new logical constant a modern version of the traditional *copula* (in the tradition, the word *"est"* in the Latin sentence *"homo est animal"* was called a *copula*). As we will see, this system of logic includes as a part the standard first order calculus, but it allows, among a lot of other possibilities, to develop a rich system of oppositions. As an example, we are going to examine how the definitions of different negations allows to rebuilt in modern terms the famous system of oppositions studied by Aristotle in the *Organon*.

III. Classes and paradoxes

As other logicians of his time Leśniewski developed his logical systems with the aim to give a foundation to mathematics. In this perspective, one of the most important issues was the status of classes or sets and the way to prevent from Russell's paradox. Leśniewski was not at all satisfied by Russell's solution. As a strong nominalist, he was also completely opposed to any theory supposing the existence of abstract objects, like set theory. In this session, we are going to explore the brilliant analysis he gave of Russell's paradox. This analysis led him to conceive his famous theory devoted to the part-whole relation: Mereology. This will be a good example to see how Leśniewski's logic allows formalizing an applied theory.

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Logic of Desires

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An important and general distinction in philosophy of mind is between epistemic attitudes and motivational attitudes. This distinction is in terms of the direction of fit of mental attitudes to the world. While epistemic attitudes aim at being true and their being true is their fitting the world, motivational attitudes aim at realization and their realization is the world fitting them. The philosopher John Searle calls "mind-to-world" the first kind of direction of fit and "world- to-mind" the second one. There are different kinds of epistemic and motivational attitudes with different functions and properties. Examples of epistemic attitudes are beliefs, knowledge and opinions, while examples of motivational attitudes are desires, preferences, moral values and intentions. The course is aimed at discussing logics for modeling static and dynamic aspects of motivational attitudes whose most representative example is the logic of desires.

The first session of the tutorial will devoted to discuss the logic of desires in opposition to the logics of knowledge and belief (epistemic logic and doxastic logic).

The second session of the tutorial will be devoted to the problems of preference generation and intention formation: (i) how preferences of agents are determined both by her desires and by her moral values, and (ii) how beliefs and preferences determine choices and are responsible for the formation of new intentions about present actions (present-directed intentions) and future actions (future- directed intentions).

The third session will be devoted to the dynamic aspects of desires including desire expansion and desire revision as well as the connection between desire and belief change, on the one hand, and preference change on the other hand.

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On the complexity of the model checking problem

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The apparently benign task of checking whether a finite structure models a given sentence from first order logic (FO) and how efficient it might be to run this task on a computer reveals a vivid realm at the interface between computer science and mathematics mixing numerous and diverse fields.

For example, the model checking of a primitive positive sentence (a first-order sentence using only $\exists$ and $\wedge$) is better known as the Conjunctive Query Containment in Database theory; it can be recast as the existence of a homomorphism between two structures which is a well studied extension of Graph Colouring in Combinatorics; it is nothing else than a Constraint Satisfaction Problem popular in the Artificial Intelligence community. Perhaps more surprisingly its complexity is governed by algebraic properties of the model from Universal Algebra studied in Clone theory.

The dichotomy conjecture first proposed by Feder and Vardi in the early nineties stipulates that according to the model this problem is either tractable (solvable in Polynomial time) or intractable (NP-complete). Around January 2017 three independent proofs have been proposed for this conjecture.

We will give a personal view of this field by focusing on fragments of first order logic where again algebra plays a prominent role in understanding and studying the complexity of the model checking problem. These syntactic fragments will be defined by selecting allowed symbols among the following: $\forall, \exists, \wedge, \vee, \neg, = and \neq$.

There are no specific prerequisites. The tutorial will be divided in three sessions detailed hereafter.

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I. Constraint Satisfaction Problem and the dichotomy conjecture

We will briefly recall the context from Complexity theory (P vs NP, Ladners's theorem) before introducing formally the dichotomy conjecture. So, we focus in this session on the complexity of the model checking of a given primitive positive sentence (fragment $\exists\ \wedge$) when parameterised by the model, a problem known as the constraint satisfaction problem (CSP). We will in fact concentrate on the special case when the model has only two elements, which amounts to a variant of propositional Satisfiability (SAT) and sketch the proof of its dichotomy — this is a result know as Schaefer's dichotomy. Methodologically, the proof relies on a non trivial case analysis that amounts to finding the border between tractable and intractable cases on an underlying algebraic object known as Post's lattice. We will explain in some details why this is the case. In particular, it will be quite illuminating to see how preservation of the model under certain well behaved Boolean functions will make complete certain well known incomplete algorithms.

If times allow, we will conclude this session with glimpses of the proofs of more general partial results supporting the dichotomy conjecture.

II. What about other fragments of FO?

Bounded model checking from verification is often reduced to the satisfiability problem of quantified Boolean sentences (QBF) that is propositional sentences with variables that are either existential or universal. Many Sat solvers go beyond instances in conjunctive normal form (CNF) and allow some disjunction. This motivates us to investigate fragments of FO allowing the universal quantifier or the disjunction as a connective. Another more prosaic motivation is that studying a fragment of FO that is very expressive will limit the number of cases to study and one might obtain a complexity classification that is still rather elusive in the case of more restricted fragments of FO.

We will briefly recall the complexity context when one throws universal quantifiers to the mix (Alternating Turing machines, Pspace). We will show that some fragments of FO such as primitive positive first order logic with disequalities (fragment $\exists\ \wedge\ \neq$) can be classified as corollaries of Schaefer's theorem.

With the exception of these and the fragment corresponding to CSP and its universal extension the QCSP, all other fragments can be classified and exhibit a strange behaviour : tractability is not explained by complicated algorithms but rather by very simple logical properties of the model, namely that a type of quantifier can be relativised to a specific constant of the model.

We will discuss in particular the case of equality free positive first-order logic (fragment $\exists\ \forall\ \wedge\ \vee$) a fragment for which one obtains a tetrachotomy governed by the surjective hyper endormorphisms of the model.

III. Quantified CSP, some progress for the last remaining open case

If one assumes that one of the recent proof proposed for the dichotomy conjecture is correct, there is a single fragment for which the complexity is not classified, namely positive Horn (fragment $\exists\ \forall\ \wedge$). The model checking problem known as the QCSP is to the CSP what QBF (or QSAT) is to SAT. Some partial results seem to suggest that for a given model the QCSP is either as hard as the general problem (Pspace-complete) or of the same complexity as a hard CSP (NP-complete) or tractable (polynomial time solvable). That is QCSP would follow a trichotomy between Pspace complete, NP-complete and P.

The drop in complexity from Pspace to NP seems to be explained also by a slightly more advanced form of relativisation of the universal quantifiers, best explained in terms of restricted games and interpolation of complete Skolem functions from families of partial ones. One natural example known as the collapsibility property enjoyed by some models amounts to the case when it suffices to check the cases where all universal variables of the sentence but a bounded number take a constant value known in advance.

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C.S. Peirce's Logic of Relations: Graph-theoretical and Surface-theoretical Models

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Charles Sanders Peirce is, as Alfred Tarski has rightly reminded us, the father of the logic of relations. Although Augustus de Morgan pioneered investigation into the logic of dyadic relations as an outgrowth of his mathematical study of syllogistic, it was Peirce who first developed a general logic of relations, that is, a logic for relations of any adicity (valency) whatsoever. The *corazon de corazon* of this logic is Peirce's so-called "Reduction Thesis", consisting of two controversial clauses. The first of these is a necessity clause stating that, besides monadic relations (one-place predicates) and dyadic relations, a relationally complete logic must also have genuine triadic relations, that is, three-place relations which cannot be analyzed into combinations of relations of lesser adicity. The second clause is a sufficiency clause, specifically, the claim that genuine triadic relations, together with monadic and dyadic relations, suffice for a relationally-complete logic. The means for composing all other (n>3)-adic relations are two logical operations, namely, the unary operation of auto-relative multiplication and the binary operation of relative multiplication. Peirce's Reduction Thesis has been all but universally rejected, often even by scholars sympathetic to Peirce and his work in logic. This tutorial explicates his contentious thesis and subsequently presents two topological models for his logic of relations. One is a variant of topological graph theory, called *Peircean Relational Graph Theory*, and the other uses surface theory, called *Peircean Relational Surface Theory*. These two models provide justification for his remarkable contribution to a universal logic of relations including proofs of his Reduction Thesis, one in each model.

> "We homely thinkers believe that, considering the immense amount of disputation there has always been concerning the doctrine of logic, and especially concerning those which would otherwise be applicable to settle disputes concerning the accuracy of reasonings in metaphysics, the safest way is to appeal for our logical principles to the science of mathematics, where error can only long go unexploded on condition of not being suspected."
> — C.S Peirce, *The Regenerated Logic*

Synopses of Tutorial Sessions

I. Peirce's Logic of Relations and Peircean Relational Graph Theory

C.S. Peirce's view that mathematics is the science of necessary reasoning about hypothetical possibilities by means diagrams will be introduced. Further, his contention that logic requires topology will be briefly examined. Peirce's diagrammatic logic of relations will be explicated including his "Reduction Thesis," specifically, the thesis, that a relationally complete logic requires, but only requires monadic, dyadic, and triadic relations. The fundamentals of Peircean Relational Graph Theory (PRGT), a radical variant of standard graph theory will be delineated. It will be shown that PRGT is able to represent straightforwardly both relations of one, two, and three adicities and the logical operations of auto-relative and relative multiplication.

II. Garnering the First Fruits of PRGT and Those of a Later Gleaning

The representational scope and power of PRGT will be presented via relevant combinatorial formulas as well as diagrams of relational networks. Several key theorems will be demonstrated culminating in a proof of Peirce's Composability-of-Relations Theorem (The Reduction Thesis justified). A taxonomy of general varieties of relational networks willed be tabulated. As a preamble and a propaedeutic to the third session, surface diagrams which are two-dimensional counterparts to the one-dimensional diagrams of PRGT will be introduced. The gluing of surfaces with boundaries will be presented as the means to represent auto-relative and relative multiplication.

III. Peircean Relational Surface Theory

Employing some insights of such pioneers in topology as A.F. Möbius and Max Dehn, three surface models for Peirce's logic of relations will be explored, specifically:

1) a cap/sleeve/pair of pants model,
2) a model of spheres with one, two, and three discs excised,
3) a disc/annulus/bi-annulus model. While a disc, an annulus, and a bi-annulus are homotopically distinct from each other, the above three models are homotopically equivalent. This will be diagrammatically displayed and algebraically demonstrated.

A problem with using these surface models to represent Peirce's logic of relations will be discussed and then solved, involving the use of deformation retractions of the disc, the annulus, and the bi-annulus as necessary aspects of an adequate model Peirce's logic of relations in two dimensions.

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Useful Links

- Charles Peirce Society
- Peirce.org
- Centro de Sistemática Peirceana
- Grupo de Estudios Peirceano
- Institute for Studies in Pragmaticism
- Helsinki Peirce Research Centre
- Centro de Estudos de Pragmatismo, São Paulo, Brazil
- Charles Sanders Peirce: Logic, Internet Encyclopedia of Philosophy

Wittgenstein's Logic

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The tutorial will be focused on Wittgenstein's logic in the Tractatus. It is divided into three sections:

I. Quantification

In the first section, we will explore Wittgenstein's account of quantification. See in particular sections 5.3, 5.501 and 5.52 of the Tractatus.

II. Decidability

In the second section, we will explore Wittgenstein's philosophy of logic. In particular, we will focus on Wittgenstein's claim that "proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases" (6.1262). See also section 6.1203.

III. The color exclusion problem

Finally, in the last section, we will discuss the color exclusion problem and Wittgenstein's later attempt to overcome the shortcomings of his logical atomism. See in particular section 6.3751:

> "For example, the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour. Let us think how this contradiction appears in physics: more or less as follows — a particle cannot have two velocities at the same time; that is to say, it cannot be in two places at the same time; that is to say, particles that are in different places at the same time cannot be identical. (It is clear that the logical product of two elementary

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propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colours at the same time is a contradiction.)"

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Reasoning on data: the ontology-mediated query answering problem

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Knowledge representation and reasoning (KR) is the field of artificial intelligence that studies formalisms, mostly based on logics, to represent and do reasoning with various kinds of human knowledge. Modern information systems often comprise a knowledge base expressed in a KR language. At the core of a knowledge base, there is a so-called *ontology*, which defines the conceptual vocabulary of the knowledge base and describes general knowledge about a domain of interest. Formally, an ontology is a logical theory in a fragment of first-order logic, which may be more or less expressive. The simplest ontologies define hierarchies of concepts and relations, while richer ontologies are often expressed in description logics, a prominent family of KR languages devoted to representing and reasoning with ontologies, or rule-based languages. Another classical component of a knowledge base is the fact base, which contains assertions about specific individuals.

In the last decade, the increasing amounts of available data, which may be large, complex, heterogeneous and/or incomplete, have deeply impacted the field. How to better access data by incorporating knowledge, typically expressed in ontologies, has become a crucial issue, at the crossroad of KR and data management. On the KR side, the challenge was to tackle a new reasoning task, namely querying data (whereas classical KR problems such as consistency checking or classification can be recast as very specific query answering problems), which required to find new languages and algorithmic techniques offering various tradeoffs between expressivity and tractability of reasoning. On the data management side, the challenge was rather to extend query answering techniques to take into account knowledge. The issue of querying data while taking into account inferences enabled by an ontology has received several names, it will be called *ontology-mediated query answering* in this talk. It can also be seen as querying a knowledge base, composed of an ontology and a (possibly virtual) fact base linked to data sources.

The aim of this tutorial is to give an overview of ongoing research in KR on ontology-mediated query answering. The main KR formalisms investigated in this context will be presented, and compared with respect to expressivity, decidability and computational complexity, with a special focus on a recent family of formalisms, namely existential rules.

The tutorial will be divided in three parts:

I. I will first present the context and the main notions related to ontology-mediated query answering: the logical view of queries and data; ontologies in computer science; knowledge bases; relevant knowledge representation and reasoning formalisms; fundamental problems on knowledge bases.

II. Then I will present in more detail the main formalisms studied in the context of ontology-mediated query answering: *Horn description logics* and *existential rules*. Description logics are decidable fragments of first-order logic, and their Horn subset is roughly obtained by disallowing any form of disjunction. Existential rules are also known as the Datalog± family, or tuple-generating dependencies in database theory, and they generalize both Horn description logics and Datalog, the querying language for deductive databases. The basic algorithmic approaches to ontology-mediated query answering will be reviewed.

III. The last part will be devoted to decidability issues in the existential rule framework. Logical entailment with general existential rules is not decidable, however many subclasses for which it is decidable have been defined. I will present the landscape of decidable classes of rules and explain the ideas behind decidability properties.

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Logic and Computer Programming

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The aim of this tutorial is to acquaint attendees with the primary models of program semantics; to present logics based on formal program models; to study relationship between such logics; to discuss applicability of program logics in program analysis and verification.

Computer Programming, as well as Software Engineering in general, is a grateful area of logic application. Logics can be used at every stage of software development cycle, in particular, during requirement analysis, specification, design, verification, and testing.

To be successful, such logics should adequately represent essential features of the development stages. Among various logics, oriented on software development, the central place belongs to logics describing main properties of computer programs. Such logics should be based on formal program models.

The tutorial consists of three sessions:

I. Review of program-oriented logics. Formal models of programs.
II. Program-oriented first-order logics of predicates and functions with non-fixed arity. Their relationships with classical first-order logic. Soundness and completeness of logics.
III. Program logics of Floyd-Hoare style of partial predicates and functions over hierarchical data structures. New consequence relations, their properties. Applicability of program logics.

The main questions to be discussed during the first session are a short review of program-oriented logics and main methods of description of formal semantics of programs:

- denotational semantics in style of Scott-Strachey;
- operational semantics in style of Gordon D. Plotkin;
- axiomatic semantics in style of Floyd-Hoare.

Then we describe various classes of mappings used to represent program semantics such as n-ary mappings, mappings with non-fixed arity (quasiary mappings), and mappings over hierarchical data. We demonstrate that these classes have different compositional properties that affect program construction and investigation.

During the second session, we construct various first-order logics based on the described classes of mappings. We demonstrate that each logic has specific features which are not characteristic for classical logic based on n-ary mappings. Soundness and completeness of such logics are discussed.

The last session is devoted to construction of various types of Floyd-Hoare program logics.

We investigate classical Floyd-Hoare logic, logics with partial predicates and functions, logics over hierarchical data. Such analysis demonstrates that even in a case of simple programs we have to introduce new rather complicated consequence relations and new rules of calculi.

In conclusion, we formulate the main challenging problems of program logics construction and investigation and discuss approaches to their solution.

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Analogical Reasoning

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Analogical reasoning has been known as a noticeable form of plausible and creative reasoning since Antiquity. Still it has remained apart from logic, since its conclusions do not offer the guarantees of syllogistic and more generally deductive reasoning. Closely related to analogical reasoning is the notion of analogical proportions. They are statements of the form "a is to b as c is to d". For about two decades now, their formalization and use have raised the interest of a number of researchers. Ten years ago, a propositional logic modeling of these proportions has been proposed. This logical view makes clear that analogy is as much a matter of dissimilarity as a matter of similarity.

Moreover, an analogical proportion is a special type of logical proportions, a family of quaternary operators built as a conjunction of two equivalences linking similarity or dissimilarity indicators pertaining to pairs (a,b) and (c,d). Homogeneous logical proportions (which include analogical proportion) and heterogenous logical proportions are of particular interest. These remarkable proportions play a key role in the solving of various intelligence quizzes. Moreover analogical proportion-based inference has been experimentally shown to be quite good at classification tasks. Recent theoretical results suggest why.

The tutorial provides an introduction and a detailed discussion of the above points and related issues. It is organized as follows:

I. The first lecture singles out analogical proportion among logical proportions. Logical proportions, a family of particular quaternary Boolean operators built from similarity or dissimilarity indicators between pairs, are first introduced. Then, different sub-families are identified according to their definitional structure, or some characteristic properties. Analogical proportion appears as one of the four symmetrical logical proportions that are code independent (which means that their truth value does not change when 0 and 1 are exchanged). Analogical

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proportion is uniquely characterized among these four proportions by satisfying reflexivity ("a is to b as a is to b") and the central permutation property (if "a is to b as c is to d" then "a is to c as b is to d"). Other noticeable properties of analogical proportion and relations with other proportions are presented, as well as a discussion in terms of structures of opposition.

II. The second lecture is devoted to analogical proportion-based inference. Indeed analogical proportions are at the basis of an inference mechanism (which can be related to the basic analogical reasoning pattern) that enables us to complete or create a fourth item (described by means of Boolean attributes) from three other items. The good results of this inference in solving quizzes and in classification problems are then reported. The fact that this inference can never be wrong in case the classification function is an affine Boolean function is emphasized. We also discuss the differences with case-based reasoning and case-based decision.

III. The third lecture is devoted to extensions of analogical proportion beyond the Boolean case on the one hand and to the use of other logical proportions on the other hand. Multiple-valued logic extensions enable us to handle items described with numerical attributes, while the extension of analogical proportion to non distributive lattices make possible to define and identify such a proportion between concepts in a formal context, in the sense of formal concept analysis. Besides, the four non symmetrical code independent logical proportions are also worth of interest since they express that there is an intruder in a 4-tuple that is not in some definite position in the tuple. Lastly we explain how these proportions can be used as well in classification.

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MMT — Meta-Meta-Theory and/or Tool: A Framework for Defining and Implementing Logics

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MMT is a framework for designing formal languages and building knowledge management applications for them. It systematically avoids a commitment to a representational paradigm, a particular concrete or abstract syntax, or a particular semantics and thus naturally subsumes type theories, logics, set theories, ontology languages, etc. Despite this high degree of generality, MMT includes generic solutions to deep problems including IDE, web browser, module system, and type checking. Therefore, designing logics and applications inside MMT can yield very strong systems at extremely low cost.

I. Overview and demo

Optionally bring your notebooks to install MMT

II. Language Design in MMT

III. Application Development in MMT

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Useful Link

- MMT homepage

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Logic-based reasoning for information integration and data linkage

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Biography: Marie-Christine Rousset is a Professor of Computer Science at the University of Grenoble Alpes and senior member of Institut Universitaire de France. Her areas of research are Knowledge Representation, Information Integration, Pattern Mining and Semantic Web. She has published around 100 refereed international journal articles and conference papers, and participated in several cooperative industry-university projects. She received a best paper award from AAAI* in 1996, and has been nominated ECCAI† fellow in 2005. She has served in many program committees of international conferences and workshops and in editorial boards of several journals.

Information integration and data linkage raise many difficult challenges, because data are becoming ubiquitous, multi-form, multi-source and musti-scale. Data semantics is probably one of the keys for attacking those challenges in a principled way. A lot of effort has been done in the Semantic Web community for describing the semantics of information through ontologies.

In this tutorial, I will show that description logics provide a good model for specifying ontologies over Web data (described in RDF), but that restrictions are necessary in order to obtain scalable algorithms for checking data consistency and answering conjunctive queries. I will explain that the DL-Lite family has good properties for combining ontological reasoning and data management at large scale.

Finally, I will describe a unifying rule-based logical framework for reasoning on RDF ontologies and databases. The underlying rule language allows to capture in a uniform manner OWL constraints that are useful in practice, such as property transitivity or symmetry, but also domain-specific rules with practical relevance for users in many domains of interest.

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I will illustrate the expressivity of this framework for modeling Linked Data applications and its genericity for developing inference algorithms. In particular, I will show how it allows to model the problem of data linkage in Linked Data as a reasoning problem on possibly decentralized data. I will also explain how it makes possible to efficiently extract expressive modules from Semantic Web ontologies and databases with formal guarantees, whilst effectively controlling their succinctness. Experiments conducted on real-world datasets have demonstrated the feasibility of this approach and its usefulness in practice for data integration and information extraction.

Everybody interested in description logics, databases and information integration is welcome to join. There is no specific prerequisites. The tutorial will be divided in the following three sessions:

I. This part will be devoted to introduce the problems of information integration and data linkage from heterogeneous data sources, in particular in the setting of the Web of data (also called Linked Data), and the ontology-based approach to address these problems.

II. This part will be devoted to description logics, their use for specifying ontologies and the associated inference algorithms for reasoning on data in presence of ontologies.

III. In this last part, we will present a unifying rule-based logical framework for reasoning on RDF ontologies and databases, based on Datalog and its extensions.

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Useful Link

- Website of the book *Web Data Management*

Louis Couturat (1868–1914): Early symbolic logic and the dream of a characteristica universalis

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It is known that Bertrand Russell turned to logic after having become acquainted with the work of the Italian mathematician Giuseppe Peano. He was personally introduced to the latter in 1900, at the First International Congress of Philosophy in Paris, by a French colleague, responsible for the Logic Section of the Congress: Louis Couturat.

Who was this French philosopher? History of logic almost completely ignores him, because he didn't contribute to the field. Nevertheless he played an important role in the development of the discipline. He was among the first who grasped the appeal of the new "algorithmic logic", renewing Leibniz' dream of a *characteristica universalis*, and started very early to integrate modern logic into the philosophy curriculum at the French university. He wrote several introductory works on logic for the French public. And he created a vast network of correspondents, extended from Argentina to Russia, including among others Russell, Peano, Peirce, MacColl, Frege and Schroeder. He devoted himself to mutually connect these scholars and to make circulating their ideas through the scholarly world at a maximum speed.

In this tutorial, we will try to grasp the work of Couturat in its entire scope, ranging from his work on Leibniz to the philosophy of mathematics, epistemology, and logic. Beyond his published work we will also consult his correspondence and his unpublished manuscripts (e.g. on the history of mathematical logic).

I. Louis Couturat

In the first session, I will provide an overview over the life and the work of Louis Couturat. In particular, I will elaborate his philosophical programme which eventually led him to study contemporary advances in symbolic logic and to make considerable efforts for introducing symbolic logic in France. I will also present and analyze his various activities as a reviewer, editor, conference organizer, international "mail box", partisan of international auxiliary languages, and so on.

II. Philosophy of logic

In the second section, I will outline Couturat's philosophy of logic, focussing on two major topics: firstly, the relation between logic and mathematics and in particular the question of logicism, discussed by Couturat in form of the alternative "algebra of logic or logic of algebra"; secondly, I will show how Couturat's criticisms of various systems of symbolic systems fits into a larger semiotic approach, covering also mathematics, the algebras of the natural sciences (e.g. chemical formulae) and even natural languages.

III. History of logic

The third and last session will focus Couturat's construction of a "History of mathematical logic" in his unpublished series of lectures at Collège de France in 1904/05. We will especially analyze the relation between the contemporary discussions in logic and the kind of questions Couturat tried to answer in his historical account.

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Logic and Religion

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From a historical point of view, logic has been a constant companion of philosophical reflections about religion. Arguments for and against the existence of God have been proposed and subjected to logical analysis in different periods of the history of philosophy. In discussions on the concept of God too logic has played a considerable role. With the rise of modern logic, in the beginning of twentieth century, and the analytic philosophy of religion, in the fifties, the connection between logic and religion has become much more established. A result of this development was the series of events World Congress of Logic and Religion, whose first and second editions took place, respectively, in João Pessoa, Brazil, in 2015, and in Warsaw, Poland, in 2017; the 3rd World Congress on Logic and Religion will take place in Varanasi, India, in 2019. The purpose of this tutorial is to introduce the field of Logic and Religion from the perspective of philosophical inquiry; nonetheless, something will be said about the role played by logic in world religious traditions.

I. General perspectives on Logic and Religion

In the first part of the tutorial I will speak about the role played by logic in religion, both from the philosophical and religious perspective. I will point out how logical notions appear in different religious traditions and how a good deal of logical reasoning is needed to make sense of good part of what they say. I will also speak about two of the most traditional philosophical undertakings related to God and religion: the construction and appraisal of arguments for and against the existence of God and the logical analysis of the concept of God.

II. Arguments for and against the Existence of God

In the second part of the tutorial I will deepen the issue of the construction and appraisal of arguments for and against the existence of God. After giving a short historical background, I will explain the varieties of arguments found in both religious and philosophical traditions. After that I will concentrate on two instances of such arguments. On the side of theist arguments, I will examine Anselm's ontological argument found in the second chapter of his *Proslogion* and some recent attempts to logically formalize it. On the side of the atheist arguments, I will examine the role played by logic in Hume's exposition of the problem of evil and the response given by Alvin Planting known as the free-will defense.

III. The Concept of God

In the last part of the tutorial I will move to the analysis of the concept of God. I will first speak about the project inaugurated by Anselm nowadays called *Perfect Being Theology* (which consists in, from some definition of God as a maximally perfect being, logically derive God's properties or perfections such as uniqueness, omniscience, omnipotence, moral perfection, omnipresence, eternality, impassibility and simplicity). After that I will look on how this project and the logico-philosophical inquiry about divine properties has been conducted in recent philosophy of religion.

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Useful Links

- Anselm: Ontological Argument for God's Existence, Internet Encyclopedia of Philosophy
- World Congresses on Logic and Religion

Tractarian Logic and Semantic Technologies

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From voice recognition and natural language processing, to semantic interoperability and automated reasoning, semantic technologies are the latest and quite possibly last frontier in information science. From banking to defense, the modern world runs on semantic technologies. Semantic technologies find the best route, identify friends, make economic predictions, and translate languages. Yet, they do not stand on their own. Rather they are grounded in the more abstract world of logic which focuses on such issues as propositional form, well formedness, substitution criteria, quantification, logical grammars and certainty versus probability.

Early computer science pioneers were well versed in the logic models inspired by Boole, Frege and Russell (and later by Carnap, Church, Tarski and Quine to name but a few) — what became classical first order logic 'FOL'. As a result, semantic technologies such as Relational Databases, Natural Language Processing and OWL† (the predominant model for semantic/knowledge representations) were all grounded in FOL.

However, the intellectual lineage that became FOL was not without its opponents, almost from the beginning. The Cambridge of pre-war England was also home to Ludwig Wittgenstein whose Tractatus provided, in significant respects, an alternative approach to logic from that espoused by Russell.

The divide between Russell and Wittgenstein still lives today. And it is of supreme relevance to both theoretical and applied logicians because it points to unanswered foundational issues in logic AND practical consequences stemming from foundational problems. Moreover, semantic tech-

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nologies have been evolving, based on empirical feedback, in directions that make them look less classical; rather more non-classical — specifically, Tractarian.

So what was Wittgenstein's Tractatus really about? And what makes it relevant today? Though written in a dense and aphoristic style, the Tractatus dealt squarely with many of the foundational issues that must be addressed by any semantic technology including:

1. Boundaries between lexical and semantic processing
2. Boundaries between abstract typing systems and semantic types
3. The structure of knowledge
4. The interplay of formal and probabilistic reasoning
5. Meaning versus reference
6. Saying versus exemplifying/showing

The tutorial is thus divided into three sections:

I. In the first section, we describe how Wittgenstein's logic in the Tractatus (and his lectures from the early 1930s) differs from what became absorbed into consensus first order logic FOL. Towards that end we will revisit the Tractatus in the light of Wittgenstein's lecture notes from 1930 where he first rearticulated central points in the Tractatus having had ten years to think about them. We will look in depth at several passages in the Tractatus in this light including 2.0131, 3.314, 3.333, 3.342, 4.0312, 4.1272 and 5.

II. In the second section, we make the link to show where Wittgenstein's ideas about logic are relevant for the design of semantic technologies. We will focus on knowledge representation and natural language processing. For example, we will show that for Wittgenstein, all semantic technologies must be grounded in abstract typing systems. And logical operators link experiential (sense) propositions to molecular/composite representations.

III. In the third section, we describe the limitations of current semantic technologies especially in the areas of natural language and multi-sensory (i.e., multi-modal) representation and how those limitations can be traced to limitations in the consensus understanding of first order logic FOL. Finally, we describe some current semantic engineering efforts in the fields of multi-domain semantic fusion and natural language understanding that are explicitly based on Tractarian logic.

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On Logical Modeling of the Information Fusion

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Information fusion is one of the most successful theories developed since about 20 years. However its meaning is still the subject of intense debate [2]. Despite its interpretational problems, recently researchers started successfully applying the apparatus of information synthesis to the economy, finances, sensory fusion, databases integration etc. W.A. Sander in "Information Fusion" [6] describes the domain as follows:

> Information Fusion or Data Fusion is the process of acquisition, filtering, correlation and integration of relevant information from various sources, like sensors, databases, knowledge bases and humans, into one representational format that is appropriate for deriving decisions regarding the interpretation of the information, system goals (like recognition, tracking or situation assessment), sensor management, or system control.

The aim of the tutorial is to give an overview of a few chosen models and information synthesis formalisms. We introduce three models of the fusion operator on theories/specifications. See e.g. [2] for other fusion models. No previous knowledge of information fusion is assumed, but we will do assume basic knowledge of propositional and first-order logic.

Everybody interested in logical modeling is welcome to join. The tutorial will be divided in the following three sessions.

I. Fusion by Products

We start with a quick historical overview of the fusion problem and we present the first fusion formalization under the generalized products of relational structures. Fraissé-Hintika-Galvin Autonomous Systems are the main tool for the decision synthesis of models of first-order theories under products of models [4,7,8].

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II. Los Ultrasynthesis

This lecture will be devoted to the exposition of the theory synthesis extracted from the analysis of the celebrated Los Ultraproduct Theorem [7,8]. A special case of such an Ultrasynthesis Operator for theories of initial segments of a standard model of arithmetics [1], formulated by M. Mostowski, will be the principal subject of our investigations.

III. Sensory Minimization

We conclude by the Dasarathy's [5] Sensory Fusion Minimization question on the minimal number of sensors necessary for the recognition of any object. Here the formalism of the sensory fusion is based on the multi-head finite automata recognition. Under the sensing multi-head automata model we prove the so called '3-sensory Theorem' [3], saying that three sensors only are sufficient.

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Mathematics and Logic in Ancient Greece

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The question of the relation of mathematics and logic in ancient Greece has puzzled many historians, who viewed no connection between Euclidean geometrical demonstration and logical reasoning as conducted within Aristotle's syllogistics and Stoic propositional logic.

The aim of this tutorial is to identify logical principles and modes of reasoning as applied in mathematics and in philosophical thinking. Logical thinking manifests itself in mathematical and philosophical reasoning over such fundamental questions, as the problem of the finite and the infinite and thereby of the finitary and infinitary methods of handling the infinite and the modes of reasoning about it, the problem of classes of finite objects and the status of their existence, and other relevant problems.

I. The finitary arithmetic of Euclid's Elements

(1) The *"domain" of Euclid's "Elements", Book VI.* The Euclidean number — *arithmos* — has the following formal structure: $A \risingdotseq \{aE\}$, where E designates the unit and a is the number of times *(multitude)* that E is repeated to obtain the number A, denoted by a segment.

Euclid constructs his arithmetic for the numbers-*arithmoi*, that is for the numbers designated as segments, while the arithmetic of multitudes is taken for granted. Thus, arithmetic is constructed as formal theory of numbers-*arithmoi*, while the concept of multitude or iteration number has a specific meta-theoretical character.

(2) *Equality.* The concepts "equal", "less", "greater", to which today are ascribed a purely quantitative meaning, in Euclid seems to be also associated with the geometric notion of relative position, but also applied to multitudes when Euclid compares two sets of numbers-*arithmoi*.

(3) *Generality.* Euclid sometimes uses quantificational words applied to numbers-*arithmoi*, although such expressions are very rare. The most common way by which Euclid expresses generality is to speak about *arithmos*

without article. Thus, most enunciations in Euclid's arithmetical Books state some property about numbers, where *arithmos* is used without article. However, when he proceeds to the *ekthesis* of a proposition, general statements about numbers are interpreted as statements about an arbitrary given (indicated) number. In virtue of the instantiation described above the process of proof takes places actually with an *arbitrary* given number. This "rule of specification" is considered inversible, although Euclid applies explicitly the inverse rule very rarely in the arithmetical Books. The degree of generality attained in this way is no higher than generality expressible by free variables ranging over numbers.

(4) *Fundamental concepts.* The basic undefined concept of Euclidean arithmetic is that of *to measure (katametrein)*, which underlies most of the kinds of numbers defined by Euclid. The concept "a number B measures a number A" can be interpreted as follows: B measures $A \rightleftharpoons (B < A)$ & $(A = nB)$, that is A is obtained by n repetitions of B.

(5) *Implicit assumptions concerning reasoning over infinite processes.* In the proofs of Proposition 1 and 2, exposing the process of *anthyphairesis*, Euclid uses the following implicit assumptions:

i. The least number principle: a set of multiples nB, such that $nB \geq A$ has a least element n_0, such that $n_0B \geq A$, yet $(n_0 - 1)B < A$.

ii. The *infinite descent principle*: the process of *anthyphairesis* will terminate in a finite number of steps, that is the chain $A > B > B_1 > B_2 > \ldots > B_k > \ldots$ is finite.

iii. If X measures A and B, then X measures $A \pm B$, that is if $A = mX$, $B = nX$, then $A \pm B = (m \pm n)X$.

The first assumption is equivalent to the principle of mathematical induction if the following axiom is added: every number (except the unit) has a predecessor. The second assumption is equivalent to the principle of mathematical induction and is used in Proposition 31. However, the use of these principles has always *finitary* character in Euclid.

(6) *Introduction of entities of higher complexity.* In Propositions 20–22, Euclid uses the "class" of all pairs that "have the same ratio". Each such class is uniquely associated with one pair of numbers, namely the least pair of numbers that have the same ratio. Euclid gives an effective procedure for finding such a least pair.

(7) *The finitary principle and the use of effective procedures.* Euclidean arithmetic is constructed from below, beginning from the unit. Further, a number of arithmetical concepts are introduced in the Definitions of Book VII. From these, the concepts of part, multiple, parts, proportionality, and prime numbers are not defined effectively. However, they become effective in virtue of Propositions 1, 2, and 3 that provide an effective procedure for any numbers to find their common measure. In this way, the proofs of the Propositions 4–19 should be considered as effective either. The introduction of more complex objects is realised through the comparison of these objects and the establishment of an equality-type relation between them. Euclid always provides an effective procedure for finding the least pair of the objects found in equality-type relation.

Therefore all propositions that involve existence of numbers appear, in Euclid's arithmetic, associated with some effective procedure for finding the required number. This kind of arithmetic is constructed without assumptions of axiomatic character. It lacks the concept of absolute number or any elaborated concept of equality.

(8) *Reductio ad absurdum.* Nowhere Euclid makes use of the assumption that all numbers form a fixed universe of discourse that is given beforehand. Hence, he never postulates or proves existence of numbers having a certain property, but always 'constructs' the required numbers by means of effective procedures. Existence of numbers is never deduced by strong indirect arguments. The use of *reductio ad absurdum* relies on a specific propositional form of the law of excluded middle and applies to decidable arithmetical predicates. Moreover, Euclid seems to avoid the law of excluded middle in the arithmetical proofs. All propositions of the form $P(A) \vee \neg P(A)$ are proved by consideration of each part of the disjunction separately.

(9) *Underlying logic.* The approach adopted by Euclid does not need any special predicate logic. Euclid's arithmetic can be characterised as a finitary fragment of classical arithmetic; hence, it does not necessarily presuppose the full force of first-order predicate logic.

II. Arithmetic reasoning in the Neo-Pythagorean tradition

Pythagorean number theory in the form survived in the texts of later authors has the following distinctive features:

(1) Arithmetical reasoning is conducted over a 3-dimensional "domain" that extends indefinitely in the direction of increase.

(2) The *monas*, denoted by an alpha, is taken to be a designated object (yet, not a number), over which a (potentially infinite) iterative procedure of attaching an alpha is admitted. Numbers are defined as finite suites (finite instances of the natural series). Various kinds of numbers can be defined as suites constructed according to certain rules, following a finitary form of inductive definition.

(3) Arithmetic is then developed by genetic constructions of various finite (plane or spatial) schematic patterns. Therefore, Pythagorean arithmetic represents a visual theory of counting over a distinctive combinatorial "domain".

(4) Arithmetical reasoning is conducted in the form of mental experiments over concrete objects of combinatorial character. Any assertion about numbers utters a law, which can be confirmed in each case by pure combinatorial means.

(5) Arithmetic concerns affirmative sentences stating something 'positive' that can be confirmed by means of the construction of the corresponding configuration *(deixis)*. No kind of 'negative' sentence is found. It is a 'positive' finitary fragment of classical arithmetic.

III. Self-reference in Plato and Aristotle: the Third Man Paradox

In Plato's *Parmenides* (132a–133b), the widely known *Third Man Paradox* is stated, which has special interest for the history of logical reasoning, because of the *self-reference* involved. Many papers call attention to the violation of a metalogical principle — the type rules — because of the Third Man Paradox. This view is encouraged by the linguistic difficulties which Plato has faced in his attempt to formulate an ontology of abstract entities, i.e. that in Greek language abstract and concrete terms are formally indistinguishable: *to leukon* (literally 'the white') may signify both 'the white thing' and 'whiteness'. The root of this misconception stems from the fact that in English literature the Platonic terms *eidos* and *idea* are usually rendered by the same word: Form or Idea. However, we find a clear distinction in Plato's texts, that corresponds to a fundamental logical distinction between *class-as-many*, distributed to its elements by predication, and *class-as-one*, standing as an individual in an extended sense and capable of being an element of a further class-as-many (Russel), or between *distributive* and *collective class* (Leśniewski), etc.

The Third Man paradox is obtained, speaking in modern terms, as follows: out of *all* the things of an initial domain of particulars to which the property '... is large' *(idea)* applies is formed an *eidos* ('the large'). Further, *this eidos* is added to the initial domain of particulars and the scope of the universal quantifier 'all' is extended over it, taken for individual. The construction results in an impredicative generation of a (potentially) infinite sequence of new *eide* (infinite regress).

Plato puts the following solution into Parmenides' mouth. The *eidos* is defined as a *paradigm*, which expresses the form of instances of the *eidos*, considered as a singular thing 'found' in nature. Further, participation in an *eidos* is identified with instantiation of the *eidos*. Further, the *eidos* is compared with a fixed instance of it and the following question is posed: can we conclude that an *eidos* is similar to an instance of it on the basis that the latter is an instantiation of the *eidos*?

Plato defines similarity in such a way that leads to a negative answer to the above question: entities are *similar* to each other if and only if they participate in one and the same *eidos*. In this way, what is today called *domain* of the class (the domain of 'participants' of the *eidos* is taken into consideration. This domain consists of *homogeneous* things ("similar" to each other). Therefore, neither a thing is "similar" (homogeneous) to an *eidos*, nor an *eidos* to another thing that participates in it; otherwise, if an *eidos* is "made similar" to a thing, we obtain the Third Man Paradox. In this way, Parmenides makes a clear demarcation between two kinds of homogeneous entities: the level of particulars and the level of *eidon*, and the confusion between them is *ad hoc* removed.

The Peripatetic commentaries of the Third Man Paradox focus primarily on the statement of the argument and the premises on which it is grounded, rather than on its solution by means of the predicate of similarity. The first scholar of antiquity who explicitly ascribes a solution to Plato is Proclus.

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Useful Link

- Francis Jeffry Pelletier & Edward N. Zalta, "How to Say Goodbye to the Third Man": https://mally.stanford.edu/plato.pdf.

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Natural language argument, the fallacies and p-logic

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This tutorial reviews the main contemporary approaches to natural language argument, explains the role these approaches assign to the fallacies, and contrasts this with applications of probability theory (aka "p-logic") to select fallacies.

As John Woods [19, p. 15] put it: "Formal logic is a theory of logical forms; and informal logic is all the rest". Informal logicians [e.g. 11,1] as well as proponents of the Pragma-dialectical school of argumentation [4,3] tend to view "all the rest" as shouldering the real work in the analysis and evaluation of natural language argumentation.

Indeed, many reject formal methods. In place of the proof techniques of the truth-functional calculus, for instance, typical resources rather include argument diagrams, schemes, and the fallacies. Similarly, rather than endorsing soundness (premise truth and deductive inferential validity) as a standard of good argument, informal logicians speak of cogency (premise acceptability, relevance, and inferential sufficiency).

In the 1960s, this anti-formalist stance arose in reaction to the only widely available formal apparatus being first-order deductive logic. The breath of formal resources available today, however, makes a continued disenchantment with them at least questionable. In fact, their neglect deprives of useful resources in appraising defeasible reasoning and argument in ways that let formal and informal realign resources.

The tutorial starts by reviewing the informal resources. We particularly study the role of the fallacies [10] in Walton's [16,17] dialogical approach, and reconstruct the rules for critical discussion in the Pragma-dialectical model, whose consensualism particularly epistemologists have criticized [15,13]. This critique demarcates an import difference, and entails a distinct view on what the fallacies are (not) [20,21].

Against this background, we offer a brief technical introduction to probability theory (p-logic), then apply it to give an analysis of argument cogency. This not only clarifies a core concept of informal logic. P-logic also provides an important corrective to its usual applications. In application to select (alleged) fallacies, indeed, formal and informal normative approaches to natural language argumentation can align.

Building on groundwork by Oaksford and Hahn [14] and Korb [12], among others, this contributes to a burgeoning area of research that successfully applies probabilistic reasoning to natural language argumentation. It also supplements recent work by Hahn and Hornikx [7], for instance, who use p-logic to formalize argument schemes such as those proposed by Walton, Reed, and Macagno [18].

Please note: The three tutorial sessions build on each other. Rather than pick one or two sessions, participants would do well to attend all three. We provide learning materials in class as online resources; there is no prior reading assignment. A background in formal logic or probability theory is neither required nor harmful to profit from the tutorial. The main learning outcome is the improved ability to orient oneself within the field of argumentation studies, and correctly apply p-logic to such crucial notions as argument cogency, fallacy, or argument strength, among others.

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Introduction to Unified Logic

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Unified logic, also called *mutually-inversistic logic*, is constructed by the author. It unifies Aristotelian logic, classical logic, relevance logic, modal logic, dialectical logic, ancient Chinese logic, Boolean algebra and lattice, natural deduction, fuzzy logic, rough set, non-monotonic logic and paraconsistent logic. It is also a unification of extensional logic and intensional logic, a unification of inductive logic and deductive logic and a unification of two-valued logic and many-valued logic.

Session 1

— Material implication vs. mutually inverse implication
— Composition operators vs. connection operators
— Formations of terms and propositions
— Truth tables for composition operators
— Inductive compositions vs. decompositions
— Truth tables of connection operators
— Mutually inverse diagrams for connection operators
— The principle of meaningfulness and meaninglessness duality for distinguished propositions

Session 2

— First-level single quasi-predicate calculus
— Second-level single quasi-predicate calculus

Session 3

— Unified logics unify more than a dozen logics.

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9 – Poster Session for Students

During the school part of UNILOG'2018, in June 16–20, we are organizing a poster session for students and young researchers (Post-docs). A good opportunity to interact. It is a way to:

- present what you are doing and/or what you want to do
- to receive feedback and counseling from advanced researchers
- to know what other people are doing

If your poster is selected for presentation at the Universal Logic School you should register at the school, but we will waive for you the fee for the congress. Moreover the three best posters will be selected for presentation during the congress, in June 21–26.

If you are interested, send your poster before March 15 to vichy@unilog.org.

The size of the poster should be: 100 cm x 140 cm / 40 inches x 55 inches.

The Logic of Public Debates

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Faithful Semantical Embedding of Dyadic Deontic Logic E in HOL

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Some logical and algebraic aspects of C_∞-rings*

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The Possibility Implies the Necessity: Gödel's Proof for the Existence of God

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A paraconsistent approach to da Costa's deontic logic: beyond contradictions and triviality

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An Abstract Approach to Algebraizable Logics with Quantifiers

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Efficient Protocols for Privacy and Integrity in the Cloud

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Multirings, Quadratic Forms and Functors: Relationship between axiomatizations on quadratic forms

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10 – ¿Why, what, when, where and how to publish?

At the end of the school part of UNILOG'2018, June 20 at 18h-19h, there will be a round table about publication, a central activity of research it is worth to reflect on. The participants are:

- Jean-Yves Beziau, Federal University of Rio de Janeiro, Brazil, founder and Editor-in-Chief of the journals *Logica Universalis* and *South American Journal of Logic*, the book series *Studies in Universal Logic* and *Logic PhDs*, Editor of the Logic Area of the *Internet Encyclopedia of Philosophy*

- Pierre Cartier, IHES*, Bures-sur-Yvette, France, Bourbaki Member and Editor (1955-1983)

- Didier Dubois, IRIT†, France, Editor of *Fuzzy Sets and Systems*

- Clemens Heine, Executive Editor of Mathematics and Applied Sciences at Birkhäuser/Springer, Basel, Switzerland

- Rohit Parikh, City University of New York, USA, Former Editor of *International Journal of Foundations of Computer Science* (1990–1995) and *Journal of Philosophical Logic* (2000–2003)

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Part III

6th World Congress on Universal Logic

11 – Opening Ceremony of the 6th World Congress on Universal Logic

It will take place on June 21, 2018, 11–12h, at Vichy University Campus.

The following authorities and professors have already confirmed they will come:

- Charlotte Benoit, Elected of the Regional Council and Deputy Mayor of the City of Vichy, France
- Jean-Yves Beziau, Professor of Logic, University of Brazil, Rio de Janeiro, co-chair of UNILOG'2018
- Olivier Cavagna, Vice-Director of Vichy Community, France
- Cécile Charasse, Associate Professor of Management Sciences, Head of the Allier Institute of Technology, Université Clermont-Auvergne, France
- Vedat Kamer, Professor of Logics, University of İstanbul, Turkey, co-chair of UNILOG'2015
- Christophe Rey, Associate Professor of Computer Science, Université Clermont-Auvergne, France, co-chair of UNILOG'2018
- Farouk Toumani, Professor of Computer Science, Head of the LIMOS laboratory[*], CNRS[†] & Université Clermont-Auvergne, France

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12 – Secret Speaker

The secret speaker is a speaker whose identity is revealed only at the time of her/its/his speech. The presence of the secret speaker gives a dramatic touch to the UNILOG event since the first edition in Montreux in 2005.

Previous secret speakers at UNILOG include Saul Kripke, Jaakko Hintikka, Grigori Mints, Benedikt Löwe and exclude Brigitte Bardot, Kurt Gödel, Aristotle Schwarzenegger, Saharon Shelah...

The talk of the secret speaker will be at a secret time in a secret place. Keep your eyes open!

Guess who she/it/he is and win a free banquet dinner!

Send your guess before June 15 midnight to unilog2018@yandex.com.

The happy winner will be the first to send the right answer. All participants of UNILOG are welcome to play, except the secret speaker.

Hint: "What I tell you three times is true." (*The Hunting of the Snark*, Lewis Carroll)

13 – Talks of Keynote Speakers

Argument-based logics

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Argumentation is an alternative approach for handling inconsistency, which justifies conclusions by arguments. Starting from a knowledge base encoded in a particular logical language, an argumentation logic builds arguments and attack relations between them using a consequence operator associated with the language, then it evaluates the arguments using a semantics. Finally, it draws conclusions that are supported by "strong" arguments.

In this talk, I present two families of such logics: the family using extension semantics defined in [1] and the one using ranking semantics introduced in [2]. I discuss the outcomes of both families and compare them. I also compare the argumentation approach with other well-known paraconsistent logics.

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*Keynote speaker at the session "Argumentation" (page 460).

Material exclusion, contradictions and other oppositions

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It is notoriously difficult to argue against the dialetheist: one cannot easily lead her to revise her beliefs by pointing to a contradiction, given that dialetheists do accept some contradictions as being true. As a result, it seems that there is very little for the dialetheist to fear. Recently, Francesco Berto (for instance, in [1]) has argued that there is a sense of contradiction that even a dialetheist should concede is unacceptable: a sense involving material exclusion. Roughly, if one sentence represents a state of affairs A that materially excludes a state B, then, A and B cannot both be the case. This would be non-question begging, given that it does not involve semantical notions such as truth and falsity, the core notions that are in question for the dialetheist. However, we shall argue that in most cases the notion of material incompatibility gives us only a weaker kind of opposition, the one known from the square of opposition as *contrariety*. As a result, that is not the kind of contradiction that the dialetheist has in mind. However, the dialetheist is not on better grounds. In claiming that some contradictions are true, the negation employed represents a weaker kind of opposition, also known from the square of opposition: subcontrariety. In fact, both approaches fail to grant the target notion of contradiction, the one present in the square. That concept of contradiction, we shall argue, allows for no exception. We shall provide for evidence that what has been conflated in these and related discussions is the notion of contradiction present in the square of oppositions and a version of the law of non-contradiction (LNC), $\neg(\alpha \wedge \neg\alpha)$, which is valid for negations representing contrariety and violated by negations representing subcontrariety. Validating the LNC is not enough to grant a contradiction in the target sense; violating LNC is enough to grant that we are no longer having a contradiction in the target sense.

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*Keynote speaker at the workshop "Reflections on Paraconsistency" (page 295).

Analogies in Civil Law

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Whereas in Common Law legal reasoning is based on analogical reasoning, in Civil Law analogies are only exceptions. We will discuss the existing logical approaches for analogies in Civil Law and try to develop a new one.

Analogies are based on similarities. The talk will deal with two issues concerning similarity. Firstly, it will be discussed whether the legal prerequisites or the interests behind the rule are the adequate point of reference for the similarity. Secondly, we will deal with the question of the adequate degree of similarity. We will define a necessary (but not sufficient) minimum standard for the overweighing of interests. Based on the minimum standard we will develop a more adequate model.

Exploring the internal language of toposes

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Since the work of the early pioneers in the 1970s, it's known that any topos supports an *internal language*, which allows to speak and reason about its objects and morphisms in a naive element-based language: From the internal perspective, objects of the topos look like sets, morphisms look like maps between sets, epimorphisms look like surjective maps, group objects look like plain groups and so on; and any theorem which has an intuitionistic proof also holds in the internal universe of a topos.

With recent discoveries of new applications of the internal language in algebra, geometry, homotopy theory, mathematical physics and measure theory, the study of the internal language of toposes is currently experiencing a resurgence. Our goal is give an introduction to this topic and illustrate the usefulness of the internal language with two specific examples.

Firstly, the internal language of the "little Zariski topos" allows us to assume without loss of generality that any reduced ring is Noetherian and in

*Keynote speaker at the workshop "Logic, Law and Legal Reasoning" (page 381).
†Keynote speaker at the workshop "Categories and Logic" (page 368).

fact a field, as long as we restrict to intuitionistic reasoning. This technique yields for instance a simple one-paragraph proof of Grothendieck's generic freeness lemma, because it is trivial for fields. We thereby improve on the substantially longer and somewhat convoluted previously known proofs.

Secondly, the internal language of the "big Zariski topos" can be used to develop a synthetic account of algebraic geometry, in which schemes appear as plain sets and morphisms of schemes appear as maps between these sets. Fundamental to this account is the notion of "synthetic quasicoherence", which doesn't have a counterpart in synthetic differential geometry and which endows the internal universe with a distinctive algebraic flavor.

Somewhat surprisingly, the work on synthetic algebraic geometry is related to an age-old question in the study of classifying toposes. The talk closes with an invitation to the many open problems of the field.

Peircean logic as semiotic and biosemiotics as transdisciplinary framework

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Peircean pragmaticism is close to Poppers critical rationalism in its fallibilism and evolutionary thinking. Peirce's synechistic continuity thinking includes a biosemiotics that has been develop over the last 30 years [2] represents a form of postmodern semiotic realism attempting to encompass qualitative and quantitative methods. Herby it represents a unity of science that the logical positivist could not produce and offers an alternative to constructivist postmodernism's many incommensurable small stories. So what is the ontology that makes such a common framework for quantitative and qualitative sciences possible? Peirce produces a transdisciplinary process philosophy through his triadic pragmaticist semiotic realism [1]. For Barbieri — and many other well-established researchers in the natural sciences — to be scientific is to be able to give mechanistic model explanations and eventually extend them with dualist theories of codes and information. In [3] I have argued that this foundation is not enough. It does not even embrace a systems and cybernetic foundation making self-organization possible. Peirce is inspired by German idealism, Especially Shelling and exchanges Hegel spirit and dialectics with his triadic semiotic logic. It is based

*Keynote speaker at the workshop "The Logic of Social Practices" (page 214).

on his three phaneroscopic (phenomenological) categories and views logic as semiotic and as a normative science for right thinking. He integrates this with empirical quantitative science, since he was educated as a chemist and did empirical work in physics [4]. This integration of a phenomenological and hermeneutical aspect at the foundation of his semiotic view of logic and empirical science is possible because of a changed view on reality and science [5]. The talk explains this construction.

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A categorical presentation of probabilistic logic

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Since the invention of categories by Eilenberg and MacLane (a logician by training), in 1948, most of the mathematical theories have been reformulated using the new paradigm. It is a common opinion that measure theory and probability theory don't fit in this paradigm. Going back to Boole and Tarski, I plan to sketch a development of measure theory (and probability) putting categories in the heart of the matter.

*Keynote speaker at the workshop "Categories and Logic" (page 368).

Quantum Theory for Kids

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In [1] we present an entirely diagrammatic presentation of quantum theory with applications in quantum foundations and quantum information. This was the result of many years of work by many, and started of as a category-theoretic axiomatisation motivated by computer science as well as axiomatic physics. However, I have always felt that the diagrammatic presentation is of great use in its own right, be it to bridge disciplines, make quantum theory more easy to grasp, or, for educational purposes, in [2] we made the bolt claim that using diagrams high-school kids could even outperform their teachers, or university students. Now, we will put this claim to the test. To do so, we have written two tutorials [3,4], covering exactly the same material, but one only using diagrams, while the other contains the standard Hilbert space presentation. There are corresponding sets of examples too. We will present the pictorial tutorial, as well as provide the logical underpinning of this material.

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*Keynote speaker at the workshop "Logic for Children" (page 361).

A unified view of some formalisms handling incomplete and inconsistent information

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Sets of formulas in classical logic are often called knowledge or belief bases, as containing explicit information held by an agent. This framework does not allow for reasoning about ignorance. The issue of reasoning about incomplete information or ignorance has been addressed independently in three communities:

- in uncertainty management, scholars have for a long time used additive set-functions to represent belief often using numerical measurement methods like in subjective probability theory, and more recently using non-additive monotonic set functions like possibility and necessity measures, Shafer's belief and plausibility functions, Walley's upper and lower previsions.
- in logic there has been two main trends. Very early in the XXth century, some logicians have tried to handle the notion of ignorance by means of an additional truth-value, like Kleene and Łukasiewicz for instance. More recently, the full power of modal logic has been exploited to develop epistemic or doxastic logics, especially using extensions of system KD45.

This paper proposes a formal framework in the form of a two-tiered propositional logic, which can capture the three approaches in the setting of possibility theory. We recall a simplified version of epistemic logic that can be extended to graded beliefs and can capture three-valued logics of incomplete information. The graded version of this minimal epistemic logic is an expressive generalization of possibilistic logic. Then we propose a general framework where any set function representing uncertainty can be accommodated. It can account for multiple conflicting sources of information, and in particular, Belnap logic can be encoded in this formalism.

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Importance of distinction of levels in a logical discourse: an investigation from the perspective of a theory of graded consequence

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In order to follow the objective of the title, let us list some quotations by Alonzo Church. These quotations are well enough to give a good account of the ideas we shall be venturing in. Our attempt in this presentation would be to bring to the fore the usual practice of the logical systems, where some of the following requirements are lacking. The theory of graded consequence (GCT) [2], in contrast, would be presented as a formal set-up where the following prescriptions are preserved.

> *In order to set up a formalized language we must of course make use of a language already known to us... Whenever we employ a*

*Keynote speaker at the workshop "Logics and Metalogics" (page 337).

language to in order to talk about some language...we shall call the latter language the object language, and we shall call the former the meta-language.
— [3], p. 47

In defining a logistic system..., we employ as meta-language the restricted portion of English...
— [3], p. 50

After setting up the logistic system as described, we still do not have a formalized language until an interpretation is provided. This will require a more extensive meta-language than the restricted portion of English...However, it will proceed not by translations of the well-formed formulas into English phrases but rather by semantical rules...
— [3], p. 54

The semantical rule must in the first instance be stated in a presupposed and therefore unformalized meta-language...Subsequently, for their more exact study, we may formalize the meta-language (using a presupposed meta-meta-language and following the method already described for formalizing the objet language)...As a condition of rigor, we require that the proof of a theorem (of the object language) shall make no reference to or use of any interpretation...
— [3], p. 55

The study of the purely formal part of a formalized language in abstraction from the interpretation, i.e., of the logistic system, is called ...logical syntax. The meta-language used in order to study the logistic system in this way is called the syntax language.
— [3], p. 58

...the reader must always understand that syntactical discussions are carried out in a syntax language whose formalization is ultimately contemplated, and distinctions based upon such formalization may be relevant to the discussion...In such informal development of syntax, we shall think of the syntax language as being a different language from the object language.
— [3], p. 59

Following...Quine, we may distinguish between use and mention of a word or symbol...As a precaution against univocation, we shall

> *hereafter avoid the practice... of borrowing formulas of the object language for use in the syntax language (or other meta-language) with the same meaning that they have in the object language.*
> — [3], pp. 61–63

These issues are also addressed in some other works [1,4,5,6,7]. Our aim is to briefly touch on others' perspectives, keeping the focus on the treatment offered by GCT.

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Kripke and Łukasiewicz: A Synthesis

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In classical logic the naïve theory of truth and satisfaction is inconsistent. Kripke provided a well-known partial solution to the paradoxes in a non-classical logic. But it has a big limitation: it doesn't work for logics with serious conditionals, or restricted universal quantification.

Another partial non-classical solution is given by Łukasiewicz continuum-valued logic. It allows naïve truth for sentences containing a rather natural

*Keynote speaker at the session "Philosophy" (page 488).

conditional. But it has a different limitation: it doesn't work for sentences containing even unrestricted quantifiers. (Kripke's partial solution handled those.)

So neither result handles restricted quantifiers. It would be nice to synthesis the two: to have an account which handled both unrestricted quantifiers and a Łukasiewicz-like conditional. (And to do so in "essentially" the way that Łukasiewicz and Kripke did.) It will thereby also handle restricted universal quantification, which is interdefinable with the conditional given unrestricted quantification.

I'll show how to do so in this talk. The synthesized approach improves on my previous work on conditionals and restricted quantifiers, in essentially preserving the attractive features of the Łukasiewicz resolution of the quantifier-free semantic paradoxes, including the easy calculation of solutions.

Logic construction and computability on algebraic abstract structures

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The construction of Computability on abstract structures was founded in the theory of semantic programming in [1–6]. We will discuss some problems in this approach connected with computability and definability. The main idea of this constructions was created on the base of restricted quantifiers. In [1–4], a construction of a programming language of logical type was proposed for creating the programming systems that provide control of complex systems in which control under different conditions depends on the type of the input data represented by formalisms of logical type on the basis of logical structures. For constructing an enrichment of the language with restricted quantifiers, we extend the construction of conditional terms. We show that the so-obtained extension of the language of formulas with restricted quantifiers over structures with hereditary finite lists is a conservative enrichment. For constructing some computability theory over abstract structures, in [6,7], Yu.L. Ershov considered a superstructure of hereditarily finite sets. From the problems in Computer Science the superstructure of

*Keynote speaker at the workshop "Model Theory" (page 226).

hereditarily finite lists was constructed in [3], and the computability theory was developed in terms of Σ-definability in this superstructure. From the standpoint of constructing a programming language, such an approach seems more natural for accompanying logical programs since for a specific implementation of a language of logical type on sets, we must externally define the sequence of an efficient exhaustion of their elements. In choosing a list of elements, the order is already contained in the model, and we have a definition in the model of operations that explicitly defines the work with the list items. However, from the viewpoint of the construction of programs, taking into account the complexity of their implementation, it is preferable to consider their constructions based on the Δ_0-construction while retaining sufficiently broad logical means of definitions, and on the other hand, ensuring more imperative constructions in the required estimates of performance complexity.

In this talk, we consider the questions of definability on the basis of the Δ_0-formulas whose verification of truth has bounded complexity with respect to the basic terms and relations in the basic model, as well as the implementation of the list operations in the superstructure. From the standpoint of specific applications of this logical programming system, the two types of problems we solve can be distinguished: (1) the local problems of constructing specific computations with data from the domain under investigation and searching for fast ways of computing these characteristics from making operative decisions in real time; (2) the strategic multipurpose problems that use large data for solving them and require search and definition already in a language allowing unrestricted existence quantifiers. To solve problems of the first type, we propose to extend the class of terms of our language by conditional terms which can be determined using only Δ_0-formulas and by recursive terms which can be determined using only Δ-formulas.

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'La question est précisément de l'âge' [Rousseau, Emile]: Natural logic and the pre-history of modern psychology

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The history of logic is inextricably linked to the history of the human sciences. Approaches derived from sociology and anthropology can help us to stand outside logic as an objective system, not by relativising or deconstructing it but by way of historical reconstruction. What about psychology, though? Even assuming it to be a 'human' rather than a 'hard' science, we can hardly speak about 'approaches' to logic derived from psychology. Rather than an approach, we must speak about a relationship, and an incestuous one. The idea of 'natural logic', as a capability embedded in the human mind, was a precursor to modern psychological concepts of intelligence and cognitive ability, along precise historical pathways many of which have not yet been traced in detail.

In my book A History of Intelligence and 'Intellectual Disability': The Shaping of Psychology in Early Modern Europe, I argued that the idea of a 'subjective' logic is rooted in the beginnings of Christianity and Empire. Only a hindsighted misreading of Aristotle can turn him into the source of

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this essentially modern picture of the human being as a natural logician. Such a misreading supports psychology's hard-science claims by implying the universality of that picture across historical eras, thus promoting too the modern ethical acceptance of cognitive ability (at the expense of all else) as the essence of what it is to be human.

My paper reprises some of the argument in that book, and ends with a critique of today's absolute presupposition that psychological 'development' is a natural kind. Developmentalism in its broadest form sees the human being as an essentially cognitive interiority, structured by linear time and tending towards the goal of perfection ('normality'). From its roots among the early Christian fathers, this idea has blossomed in abundance in today's psychological disciplines.

From the early modern period there is the notion of l'ordre in Pascal, Malebranche and Rousseau and its echoes in Piaget. This has had a major impact on modern reconceptualisations of childhood and on the invention of the category known as 'developmental disability' in adults. In short, I ask: how did the sense of order in natural logic stop being spatial and become temporal?

The Indispensability of Logic

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The Putnam-Quine indispensability argument is a well-known attempt at establishing the existence of mathematical objects. Very roughly, the line of argument is that since mathematical claims play an indispensable role in our best scientific theories, the mathematical claims receive indirect confirmation. This in turn gives us a reason to believe that objects quantified over in mathematical claims exist. In this paper I formulate a number of corresponding indispensability arguments for logical laws. The thought is that if a logical law plays an indispensable role in our best scientific theories, then it receives indirect confirmation. I compare and assess a variety of indispensability arguments, and I argue that none of the arguments tell conclusively in favour of the laws of classical logic.

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Category theory and its foundations: the role of diagrams and other "intuitive" material

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When analyzing, in *Tool and object* [1], the historical development of category theory and the early debate on its foundations, I was led to discuss some general philosophical aspects of the formation of new mathematical concepts (in learners and in a community as a whole) and of mathematical research programmes; motivating examples were discussed under the headings of "intended models" and "technical common sense". It turned out to be crucial to focus on the respective background of the people involved in these processes, in particular, the attitude of "people without expertise in a certain area" was shown to play a role.

This observation lends itself to discussion within the perspective of the workshop (which speaks about such groups of people as "children in a wider sense of the term"); therefore, the talk will review this issue to some extent. A special focus will be laid on the role of diagrams in the debates on category theory. On the one hand, I intend to compare the role of diagrams played in proofs of category theory with the role of diagrams played in proofs of classical Euclidean geometry (as analyzed by Manders [2], among others). In both cases, one should focus on the ways in which a diagram is used to *prove* a proposition, on the one hand, or to *display* a proposition, on the other. And there is a tension playing an eminent role, in my opinion, in the foundational debate, namely the tension between diagrams as displaying propositions about finite sets of objects of a category on the one hand and the consideration of a category as an infinite diagram (or graph) on the other.

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CERES: automated deduction in proof theory

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CERES (cut-elimination by resolution) (see [1]) is a method of cut-elimination which strongly differs from cut-elimination a la Gentzen. Instead of reducing a proof φ stepwise (and thereby simplifying the cuts) CERES computes a formula $\mathrm{CL}(\varphi)$ represented as so-called *characteristic clause set*. $\mathrm{CL}(\varphi)$ encodes the structure of the derivations of cuts in φ and is always unsatisfiable. In classical logic any resolution refutation ρ of $\mathrm{CL}(\varphi)$ can be taken as a skeleton of a *CERES normal form* φ^* of φ (in φ^* all cuts are atomic). CERES was mainly designed as a computational tool for proof analysis and for performing cut-elimination in long and complex proofs; an implementation of the method was successfully applied to Fürstenberg's proof of the infinitude of primes [2].

There is, however, also an interesting theoretical aspect of the CERES method: reductive cut-elimination based on the rules of Gentzen can be shown to be "redundant" with respect to CERES in the following sense: if φ reduces to φ' then $\mathrm{CL}(\varphi)$ *subsumes* $\mathrm{CL}(\varphi')$ (subsumption is a principle of redundancy-elimination in automated deduction). This redundancy property can be used to prove that reductive methods (of a specific type) can never outperform CERES. Moreover, subsumption also plays a major role in proving the completeness of intuitionistic CERES (CERES-i) [3]. CERES-i is based on the concept of proof resolution, a generalization of clausal resolution to resolution of cut-free proofs. The completeness of CERES-i can then be proven via a subsumption property for cut-free proofs and a subsumption property for proof projections under reductive cut-elimination. The results demonstrate that principles invented in the area of automated deduction can be fruitfully applied to proof theory.

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Alternating truth in argumentative dispute resolution

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In my talk I propose a three-component arguments evaluation procedure as an essential part of an algorithm for the argumentative dispute resolution. The core idea of the resolution algorithm is to provide a coherent reply to the question whether a certain dispute contains a nonempty set of defensible arguments. The algorithm will be based on the Dung-style abstract argumentation approach [1] and on its further developments towards creating formalisms with structured arguments, as outlined by H. Prakken and G. Vreeswijk [2], including their practical application to modelling argumentation [3]. Implementing the structured arguments into the Dung-style argumentation framework opens a perspective for creating expressively rich formalisms, which are able to capture the following three important dispute properties: combining rigor and plausible arguments based on defeasible and indefeasible rules, reinstating attacked arguments by counterattacks and estimating arguments defensibility in the skeptical or credulous way. However, any of these properties may generate an aggregation problem putting the formalism at a risk of collapse, as the rationality postulates by L. Amgoud and M. Caminada demonstrate [4].

The three-component evaluation of arguments solves the aggregation problem by means of first discriminating among the three levels of argumentative dispute and then combining the evaluation outcomes on each level in a special order. It discriminates truth-based and logic-centered evaluation of the structured arguments' validity from the inside, evaluation of arguments' subsets expressing the disputants' positions which is coherency-based with respect to the set inclusion and Dung-style abstract approach-based evaluation of the dispute arguments' set on top of the first two levels. The level-wise aggregation of those evaluations leads to a non-standard ordering of truth-values inside arguments. The idea of the non-standard ordering of

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the truth-values is borrowed from the many-valued logic of Dmitry Bochvar [5] and Victor Finn [6]. The non-standard ordering of truth-values expresses the idea of alternating truth-values in justifying, rebutting and reinstating arguments from diverse standpoints, as it often happens in disputes and glimmers the post-truth. On the one hand, such ordering enables us to identify the propositions in arguments, which the disputants evaluate alternatively, although it does so at the cost of losing some standard properties of propositional connectives. To show why it may be considered a reasonable price I reconstruct Karamazov's and Raskolnikov's cases from F. Dostoyevsky's 'The Karamazov's brothers' and 'Crime and Punishment' as examples.

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Diagrammatic Reasoning in Peirce and Frege

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The ancient paradigm of mathematical reasoning is diagrammatic, the sort of reasoning one finds, for example, in Euclid's *Elements*. In the seventeenth century this practice gave way to the constructive algebraic problem solving characteristic of, for instance, Euler. And in the nineteenth century, mathematical practice was again transformed to become, as it remains today, a practice of deductive reasoning from the contents of concepts as set out in definitions. Both Peirce and Frege, knowing nothing of each other's work, took this mathematical development to show (*pace* Kant) that, as Peirce thinks of it, even deductive reasoning involves constructions, in particular, the construction of diagrams, among which Peirce includes algebraic formulae. As Frege puts what is essentially the same point, even deductive reasoning can be ampliative, a real extension of our knowledge. And both devised two-dimensional logical languages aimed at showing how this works, how through the construction and manipulation of diagrams (in a broad sense) one can make discoveries in mathematics. There are, nonetheless, very significant differences in the notations each devised for this purpose. My aim is to understand salient differences in the logical languages of Peirce and Frege in light of the similarities in their overall outlooks.

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A New Perspective for Relevance Logic

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Over the years, relevance logics have been generated in many ways. Among them may be mentioned syntactic generation by Hilbertian axiom systems, natural deduction rules, and Gentzen consecution calculi; and semantic production via Meyer-Routley relational structures and semi-lattice semantics. Each approach has brought valuable insights and techniques, but it seems fair to say that none has been really satisfying. That, perhaps, is the main reason for the gradual decline of interest in the subject since the turn of the century. In this talk, we will discuss a fresh perspective that has recently been developed. It suitably adapts the procedure of semantic decomposition trees, well-known for their usefulness in classical and modal contexts, to give a 'syntactically monitored semantics' for relevance logic. The semantics is perfectly classical, the syntactic monitoring is not.

Jan Łukasiewicz: his many-valued logic

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The roots of many-valued logics can be traced back to 4^{th} century BC. In Chapter IX of *De Interpretatione* Aristotle considers the timely honoured sentence "There will be a sea-battle tomorrow". Since the battle-sentence refers to not actually determined events, it is a future contingent. Accordingly, the Philosopher from Stagira suggests the existence of the "third" logical status of propositions.

In 1920 Łukasiewicz and Post successfully formulated many-valued systems. Their constructions were possible in the result of an adaptation of the truth-table method used to the classical logic by Frege and Peirce (in 1879 and 1885, respectively). Incidentally, the priority lies on the side of the Polish scholar, who presented his three-valued logic already in his official university lecture in 1918.

*Keynote speaker at the session "Non-Classical Logics" (page 439).
†Keynote speaker at the workshop "The Lvov-Warsaw School: Past, Present and Future" (page 270).

Our first aim is to present the rationale, a philosophical background and some technical issues of Jan Łukasiewicz ingenious logical construction in which logical values were multiplied. I also outline further research, development of original ideas, impact of Łukasiewicz settings and their applications.

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Contradiction, triviality, inconsistency toleration and other misunderstandings in the empirical sciences

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A prevalent view during the last decades in the logic and the philosophy of science submits the thesis that — contrary to what the traditional view might suggest [1,2] — inconsistent theories do not always have to be rejected, as history of science has shown that inconsistencies are often present and tolerated in scientific practice [3,4,5,6]. But, while the coherence of this view has nowadays been widely defended, there is still no consensus on how this toleration takes place, and more precisely, on why are inconsistencies tolerable to begin with. My aim here is to address two important questions concerning this view, namely: how do we usually characterize 'inconsistency toleration' in empirical sciences? and how should we characterize it?

The first question has already been answered by the defenders of two different approaches to inconsistency toleration, namely, (a) the 'handling inconsistency' projects [7,8,9,5] and (b) and the 'avoiding triviality' projects [10,11,12,13,14]. Projects of the first type often assume that inconsistencies

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are falsities that, while most of the time are problematic for the scientific endeavor, almost never, are an actual risk for scientific reasoning. In contrast, the defenders of the 'avoiding triviality' approaches assume that when faced with a case of inconsistent science, one needs to explain how Explosion is avoided in that very case, as Explosion has always been thought of as a danger when talking about contradictions in science. With that in mind, the 'avoiding triviality' approach has characterized inconsistency toleration as the avoidance of triviality when facing a contradiction.

To finally provide an answer to the second question, I will claim that the second approach is unsuccessful for the following reasons. First, while cases of inconsistent and trivial formal theories are well documented in the literature, the same does not happen with inconsistent and trivial empirical theories. This has prevented philosophers and logicians of science from grasping how triviality looks like in the context of empirical sciences. In contrast, in the latest decades, much understanding has been gained on how to handle contradictions in science; different strategies have been proposed, explained and extended, and, as a matter of fact, many of the handling inconsistencies maneuvers have been explicative of the cases of inconsistency toleration in empirical sciences.

Furthermore, I will conclude that in order to provide a better understanding of inconsistency toleration in empirical sciences, we should modify the way in which contradictions are often understood (as a risk of explosion), the way in which triviality is often characterized and, of course the way in which inconsistency toleration is often defined. All these modifications should be done in such a way that these concepts are sensibly explanatory for the actual scientific activity when facing contradictions in empirical sciences.

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Foundational Issues: Still Meaningful

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Keywords: formal systems, first-order logic (FOL), consistency, completeness, decidability, relational, machine learning.

Dominant computing practice relies upon assumptions, perspectives and conclusions that arose primarily from the philosophical investigations of the 19th and 20th centuries. We struggle with the relationship between syntax and semantics; deduction and interpretation; and data and knowledge, with no consensus on these matters. When we put powerful application platforms in the hands of the uninitiated, we risk not merely inappropriate uses but the introduction of subtle errors, both deductive and interpretive. Lacking a common formal foundation and common semantic framework, we may blindly integrate systems that are fundamentally inconsistent. Our use of formal systems to represent some aspect of the physical world is more than an abstract game: there are consequences if "the game" is not an appropriate interpretation (i.e., a semantic model) of the formalism. Most often, the first indications of negative consequences are the frustrations and dissatisfactions of end users. For reasons we don't really comprehend, inconsistencies in the underlying formal foundations of computer applications usually result in a "soft fail". For example, the analytics that consume derived data depend on possibly complex chains of logical inference, logic of which most users are simply unaware and so cannot be expected to reason about consistently.

We have been fortunate that catastrophic failures are not more common. Yet the catastrophic failures that occur in terms of scalability, security, statistical models, and even predictive analytics are arguably related to unresolved issues within or among the underlying logical systems. More worrisome than catastrophic failures having deep roots in foundational issues, our world embraces rapid data accumulation and integration. Integration principles for ensuring semantic consistency remain an open issue. Coupled with minimal attention to semantics, both logical validity and semantic consistency should be suspect. Despite reassuring ourselves by measuring the statistical correlation of predictions with historical results, how should we

*Keynote speaker at the workshop "Logic for Dynamic Real-World Information" (page 321).

respond to a computational result interpreted as requiring or suggesting a decision in the real world? Can any confidence be placed in the next query results, analytic conclusions, machine learning, or our interpretations?

As we develop new methods of reasoning, algorithms, and representations of knowledge, we must reassess the foundations on which we have relied. We must go beyond superficial assignments of "meaning" and question the completeness, consistency, and decidability of the formal deductive system we are implicitly relying upon, identifying the axioms we have unwittingly embraced and their consequences. (For example, we usually embrace ZF and ZFC set theories, but fail to recognize that they make no provision for individual objects — every "thing" is a set, possibly empty. At the same time, FOL does not permit quantification over empty sets and assumes that sets have individuals.) We must also be very clear about the universe of discourse, and what it means to assert that data or information are "missing".

In this survey talk, I argue the importance of foundational issues, briefly mentioning examples in database theory and practice, financial predictive analytics, machine learning, and computational semantics (linguistic logic).

Three Probabilistic Generalizations of Deducibility

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It is not so much a commonplace as an unstudied presumption that the natural way to generalize the relation of deducibility is by means of a logical (epistemic, judgemental) probability measure: the probability $\mathfrak{p}(\mathsf{c}|\mathsf{a})$ takes the value 1 when the inference from the premise or assumption a to the conclusion c is classically valid, and generally a lower non-negative value when it is invalid. This presumption needs to be contested, since there are several other functions, defined in terms of a probability measure $\mathfrak{p}$, that provide tenable necessary conditions for classical deducibility. Indeed, there exist essentially eight distinguishable pairs of truth functions $\langle \mathsf{Z}, \mathsf{X} \rangle$ of a and c such that Z is deducible from X if and only if c is deducible from a, and in consequence there exist eight distinct functions $\mathfrak{f}$ such that $\mathfrak{f}(\mathsf{c}|\mathsf{a}) = \mathfrak{p}(\mathsf{Z}|\mathsf{X}) = 1$ when c is deducible from a (and conversely too if $\mathfrak{p}$ is regular).

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Of particular interest, in addition to the pair $\langle \mathsf{c}, \mathsf{a} \rangle$, are the pair $\langle \mathsf{a}', \mathsf{c}' \rangle$ (where the prime represents negation) and the pair $\langle \mathsf{c}, \mathsf{a} \vartriangle \mathsf{c} \rangle$ (where the triangle represents exclusive disjunction or symmetric difference). Since c is deducible from a if and only if a' is deducible from c', and also if and only if c is deducible from $\mathsf{a} \vartriangle \mathsf{c}$, the three functions $\mathfrak{c}(\mathsf{c} \,|\, \mathsf{a}) = \mathfrak{p}(\mathsf{c} \,|\, \mathsf{a})$ (usually called *credence*), $\mathfrak{q}(\mathsf{c}|\mathsf{a}) = \mathfrak{p}(\mathsf{a}'|\mathsf{c}')$ (called *deductive dependence* in [2]), and $\mathfrak{n}(\mathsf{c}|\mathsf{a}) = \mathfrak{p}(\mathsf{c}|\mathsf{a} \vartriangle \mathsf{c})$ (which may be called *nearness*) all take the value 1 when c is deducible from a. They generalize deducibility in diverse ways: provided that c is not deducible from a, $\mathfrak{c}(\mathsf{c} \,|\, \mathsf{a}) = 0$ when a and c are contraries, $\mathfrak{q}(\mathsf{c}|\mathsf{a}) = 0$ when a and c are subcontraries, and $\mathfrak{n}(\mathsf{c}|\mathsf{a}) = 0$ when a is deducible from c (and conversely too if $\mathfrak{p}$ is regular).

$\mathfrak{c}(\mathsf{c}|\mathsf{a})$ is almost universally understood to measure the *degree of belief* of the hypothesis c given the evidence a (or the appropriate *betting quotient*). $\mathfrak{q}(\mathsf{c}|\mathsf{a})$ measures the extent to which the content of c is included within the content of a, the *deductive dependence of* c *on* a. It is often a good substitute for $\mathfrak{c}(\mathsf{c} \,|\, \mathsf{a})$, for example as a measure of *degree of confirmation.* In [1] it is shown how the replacement of $\mathfrak{c}$ (or $\mathfrak{p}$) by $\mathfrak{q}$ resolves some outstanding problems besetting the interpretation of indicative conditionals.

The present paper will consider whether there is any illuminating interpretation of the function $\mathfrak{n}$. It will also seek to demystify the surprising fact that the three functions $\mathfrak{c}, \mathfrak{q}, \mathfrak{n}$, though demonstrably distinct functions, can be transformed into one another by means of simple linguistic translations.

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Lewis Carroll's seven charts (and many others)

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It is well known that Lewis Carroll designed a diagrammatic method to solve syllogisms and more complex problems. These diagrams have received growing attention in recent years among scholars who acknowledged their merits and limitations [1]. It is less known that Carroll has also left a series of seven diagrams, known as the seven charts, of a rather different kind. These figures depict different propositions, represented in various notations, interconnected by lines or double-lines. These charts have been printed by Carroll himself around 1887, presumably, to collect the opinion of his logical friends. However, one has to wait 1977 for their (first) publication in William W. Bartley's reconstruction of Carroll's lost logic fragments (new edition in 1986 [2]). They have also been reproduced in 2010 by Francine Abeles in her edition of Carroll's logic pamphlets [3]. Both Bartley and Abeles reproduced additional charts that have not been printed by Carroll but were found in his logic notebook and among his manuscripts. Further (unpublished) charts are known to exist. Interpretations of these charts have been provided by Bartley himself and Mark Richards (both are reported in [2]) and more recently by Alessio Moretti [4]. The aim of this talk is to make sense of these charts and inquire how they stand within a long tradition of Aristotelian diagrams in logic. For the purpose, we provide an overview of Carroll's charts. Then, we assess the interpretations that were made of them. Finally, we investigate what they might teach us on Carroll's logical project and on the place of Aristotelian diagrams in it.

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Tones and Chords: Fuzzy and Intuitionistic Approaches to Musical Elementhood

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Rudolph Carnap in his famous book *The Logical Structure of the World* instances musical chords in order to illustrate his concept of *quasi analysis*. And he sets the bar high, when he characterizes chords as “uniform totalities, which are not composed of constituents”. On the basis of similarity circles Carnap suggests the possibility of regaining tones as quasi-constituents of chords. So it is interesting to ask: what are the contributions of present day working music theorists to the identification of chord-constituents (or quasi-constituents)?

The prevalent study of musical chords as subsets $X \subseteq \mathbb{Z}_{12}$ of the chromatic 12-tone system provides a quite restricted and abstract level of description. Nevertheless it plays a quite productive role for the generation of new ideas and for the reconsideration of old ones. It provides insights into aspects of musical actuality and constitutes a manageable playground for theoretical explorations. Therefore it serves as a good starting point for the lecture.

Linchpin and point of departure in two directions of study is the conversion of a chord $X \subseteq \mathbb{Z}_{12}$ into a characteristic function $\chi_X \colon \mathbb{Z}_{12} \to \{0,1\}$:

1. The interpretation of $\chi_X \colon \mathbb{Z}_{12} \to \{0,1\} \subset \mathbb{C}$ as a function into the complex numbers offers a fruitful transfer into Fourier space. Furthermore it can be easily extended to the study of fuzzy chords or *pitch class profiles* $\chi_X \colon \mathbb{Z}_{12} \to [0,1] \subset \mathbb{C}$, which play a central role in cognitive studies and statistical music theory.

2. The study of the subsets $X \subset \mathbb{Z}_{12}$ through their affine stabilizisers, i.e affine endomorphisms $f \colon \mathbb{Z}_{12} \to \mathbb{Z}_{12}$, satisfying $f(X) \subseteq X$ leads to an intuitionistic interpretation of elementhood, where a refined characteristic function $\chi_X \colon \mathbb{Z}_{12} \to \Omega$ takes values in the subobject classifier (truth value object) Ω of a topos $Set^{\mathcal{M}}$ of monoid actions. The special case of the 8-element triadic monoid $\mathcal{M}$ and a 6-element truth value object Ω is music-theoretically illuminating and moreover it provides a nice *vade mecum* into topos theory and Lawvere-Tierney topologies.

*Keynote speaker at the workshop “Logic and Music” (page 204).

There are remarkable parallels between prominent objects in the two approaches: in particular between the *Fourier prototypes* (chords, for which one of the Fourier coefficients of χ_X takes a maximal magnitude among all non-empty subsets $X \subsetneq \mathbb{Z}_{12}$) and the Lawvere-Tierney-extensions of the triad (i.e. chords, which are classified by the concatenation $j \circ \chi_X$ of the characteristic function of a triad and a Lawvere-Tierney topology $j\colon \Omega \to \Omega$). If time allows I will report on some explorations in the direction of a synthesis of both approaches. The interpretation of the subobject classifier Ω in terms of suitable maps $\kappa\colon \Omega \to \mathbb{C}$ allows to study the characteristic functions $\chi_X\colon \mathbb{Z}_{12} \to \Omega$ in terms of the discrete Fourier transforms $\widehat{\kappa \circ \chi_X}\colon \mathbb{Z}_{12} \to \mathbb{C}$.

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Formalizing Umwelts

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The *Umwelt* is a notion suggested by Jakob von Uexküll, a German Baltic biologist in the early XXth century [2]. The umwelt is his (Kantian) notion of how an animal or any living being like a child, "sees" the world, and how it can act in it. This notion carries a potential for developing a language to talk about the "subjective world" so that certain commonsense notions can be talked about more precisely. While the notion is old, it ties up with contemporary research in animal psychology. Also, von Uexküll anticipated many computer science ideas, particularly in the field of robotics, some 25 years before these were invented. Thus Uexküll's ideas overlap with the discussion of the *Wumpus world*, a common example in [1], p. 27.

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We will offer a formalism for the umwelt of an agent consisting of a set W of worlds, a partition P of the world corresponding to the agent's perception, a set E of "effectors" which allow the agent to alter its environment, i.e., to move from one world to another, and finally a utility function U which corresponds to what the agent currently wants. The effectors are (possibly non-deterministic) maps from W to itself and U is a map from $W \times W$ to the real numbers. $U(u, v)$ represents an agent's utility of v when the agent is at u. The agent's goal is to choose its actions, using its current information, so as to maximize its overall utility.

Can we now think of the *logic* of an animal? Obviously not in the sense in which we use the word logic. But if logic is thought of as a *motivated internal process* from one internal state to another then such a thing does become feasible. Ditto for the notion of communication where communication between two agents must be in terms of their common partition.

Two agents can combine their umwelts so that the resulting umwelt is a sort of least upper bound to the two individual umwelts. The partition is a common refinement of the two individual partitions and the set of effectors is a union of the two individual sets. The utility function for the pair is no longer linearly ordered, but a notion of *Pareto optimality* will still apply. If the utilities are even partially aligned then the two agents can cooperate. This model can then be used to explain how two species of animals can be in symbiosis.

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Capturing Consequence

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The ability to capture implicational structure is a significant virtue in a logic. First-order formalisations are for instance often preferred to propositional ones because they are thought to underwrite the validity of more natural-language arguments than the latter.

My talk will compare and contrast the ability of some well-known logics — propositional and first-order in particular — to capture the implicational structure of natural language. I show that there is a precise and important sense in which first-order logic does not improve on propositional logic as far as respecting natural-language validity is concerned. One moral concerns the correct way to state first-order logic's superiority vis-à-vis propositional logic. The second moral concerns semantic theory, and the third the use of logic as a tool for discovery. A fourth and final moral is that second-order logic's transcendence of first-order logic is greater than first-order logic's transcendence of propositional logic.

David Hilbert's Early Logical Notation

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In David Hilbert's early axiomatic programme as presented in his "Grundlagen der Geometrie" (1899) logic and, with this, logical notation became relevant around the turn to the 20th century. Hilbert style axiomatic systems are based on sets of axioms which are independent from intuition or any extra-mathematical reality. They are justified by meta-axiomatical investigations of independence, completeness and consistency.

The tool used for proving consistency were relative consistency proofs. The consistency of the axioms of Euclidean geometry was proved, e.g., under the presupposition of the consistency of arithmetic. This required the

*Keynote speaker at the session "Universal" (page 407).
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axiomatization of arithmetic and the proof of its consistency. Hilbert demanded a logicistic solution, i.e. proving the consistency of arithmetic under the presupposition of the consistency of logic. Logic became part of the program. It had to be reformulated in an axiomatic form. A relative consistency proof was impossible for logic, so Hilbert unspecifically demanded a "direct" proof.

Subsequently the constitution of a suitable logical system became a task within Göttingen research on the foundations of mathematics. Hilbert and his colleagues were looking for a logical system more feasible for mathematical means than the systems on the market, in particular Gottlob Frege's *Begriffsschrift* and Ernst Schröder's Algebra of Logic.

Notational features were instrumental for the practicability of logic for mathematical means. The early ideas can be drawn from Hilbert's 1905 lecture course "Logical Principles of Mathematical Reasoning". They may have influenced some idiosyncratic features of the logical notation later used in the textbook by David Hilbert and Wilhelm Ackermann *Grundzüge der Theoretischen Logik* of 1928.

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Diagrammatic quantum reasoning

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The ZX-Calculus is a powerful graphical language for quantum reasoning and quantum computing introduced by Bob Coecke and Ross Duncan. The language is universal: any pure qubit quantum evolution can be represented, and it comes with a strong equational theory which provides an axiomatisation of some fundamental quantum properties like the complementary quantum observables within a general framework of dagger symmetric monoidal categories.

ZX-calculus has multiple applications in foundations of Physics but also in quantum computing (e.g. quantum error correcting codes, measurement-based quantum computation), and can be used through the interactive theorem prover Quantomatic. The main obstacle to wider use of the ZX-calculus was the absence of a completeness result for a universal fragment of quantum mechanics, in order to guarantee that any true property is provable using the ZX-calculus.

In this talk, we present the first complete and approximatively universal diagrammatic language for quantum mechanics. We make the ZX-Calculus complete for the so-called Clifford+T quantum mechanics by adding two new axioms to the language.

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To Peirce Hintikka's Thoughts

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We compare Peirce's and Hintikka's logical philosophies, especially their "action-first" ("knowledge-last") epistemologies. We identity a number of close similarities on the following fronts:

A. Epistemology: Both developed a Socratic theory of the method of discovery, which resulted in fallible epistemology and included abductive moves in the interrogative model of inquiry.

B. Meaning: Both were proponents of subjunctive (pragmaticist) formulation of meaning, rejecting sense-data and taking justification of reasoning grounded on observational facts.

C. Philosophy of Science: Both Peirce and Hintikka emphasized the importance of the theory of the economy of research: Peirce in terms of methodeutic scientific values and the cost-benefit analysis, Hintikka in terms of strategic aspects of inquiry and the question-answer structures in scientific reasoning. In both, the decisions to select/omit data and hypothesis are considered under a realist methodology.

D. Philosophy of Logic: Both Peirce's and Hintikka's thoughts are characterized by algebraic and relational thinking, meta-theoretical ideas, centrality of epistemic modalities, and taking syntax, semantics and pragmatics as a unity. Their respective philosophies of logic were guided by viewing logic as a model-building activity, not inferentialism, and taking possibilities as real.

Moreover, the origins of epistemic logic, KK-thesis and cross-identification date back to Peirce's writings on graphical logic, which Hintikka studied when he was doing research for his Knowledge and Belief.

I conclude that Hintikka's version of nominalism that he took to result from IF logic might be the only version of nominalism acceptable to Peirce, given the "extreme scholastic realism" of the latter and the fact that his theory of quantification was aimed at capturing what is going on in actual mathematical practices.

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Grounding as meta-linguistic relation: grounding rules for implication

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The concept of *grounding* has a long and venerable history that starts with Aristotle and continue through philosophers such as Ockham or Bolzano. Quite recently we assist to an impressively flourishing and increasing interest for the notion of grounding, which is studied and analyzed from many different angles. Amongst them, scholars have been trying to capture the structural and formal properties of the concept in question by proposing several logics of grounding [e.g. see 1,2,3,4]. In these logics grounding is formalized either as an operator or as a predicate. The main aim of this talk is to present a different approach to the logic of grounding, where grounding is formalized as a meta-linguistic relation, just like the notion of derivability or that of logical consequence. Let me call such an approach LG. The central characteristics of LG can be resumed in the following list:

— LG allows a rigorous account of ground-theoretic equivalence.
— In LG grounding rules are unique; in particular it is possible to formulate an unique grounding rule for negation.
— In LG it is also possible to formulate grounding rules for implication which are quite different from everything that has been proposed so far and that seem to better reflect our intuitions on the issue.
— Finally LG allows to prove important results such as the soundness and completeness theorems, but also the deduction theorem.

The main aim of this talk is to present a different approach to the logic of grounding, where grounding is formalized as a meta-linguistic relation, just like the notion of derivability or that of logical consequence [see 4,5].

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A Compendium for Positive Logic

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Formulae, in a fixed language $\mathcal{L}$:
General formulae are obtained from atomic formulae by iteration of $\wedge$, $\vee$, $\exists$ and $\neg$ (no $\forall$, to be replaced by $\neg\exists\neg$).
Positive formula: no negation, use only $\wedge$, $\vee$ and $\exists$. Its prenex form is $(\exists\underline{x})\varphi(\underline{x})$, where φ is boolean positive.

Truth = Satisfaction in a $\mathcal{L}$-structure (Tarski, defined by induction):
Atomic formula: basic fact.
$\varphi(a) \wedge \psi(b)$ is true $\Leftrightarrow$ $\varphi(a)$ and $\psi(b)$ are true.
$\varphi(a) \vee \psi(b)$ is true $\Leftrightarrow$ $\varphi(a)$ or $\psi(b)$ are true.
$(\exists y)\varphi(\underline{a}, y)$ is true $\Leftrightarrow$ for some b in M, $\varphi(\underline{a}, b)$ is true.
$\neg\varphi(\underline{a})$ is true $\Leftrightarrow$ $\varphi(\underline{a})$ is not true.

Forcing (Cohen, and then Robinson, defined by induction):
Atomic formula: basic fact.
$\varphi(a) \wedge \psi(b)$ is forced $\Leftrightarrow$ $\varphi(a)$ and $\psi(b)$ are forced.
$\varphi(a) \vee \psi(b)$ is forced $\Leftrightarrow$ $\varphi(a)$ or $\psi(b)$ are forced.
$(\exists y)\varphi(\underline{a}, y)$ is forced $\Leftrightarrow$ for some b in M, $\varphi(\underline{a}, b)$ is forced.
$\neg\varphi(\underline{a})$ is forced $\Leftrightarrow$ in no continuation of M $\varphi(\underline{a})$ is forced.

All depends of the meaning that we give to the word "continuation"!

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Homomorphism:
Definition. A map f from M to N, such that if the atomic formula $\varphi(\underline{a})$ is true in M, then $\varphi(f.\underline{a})$ is true in N.
Observation. If $\varphi(\underline{a})$ is positive and true in M , then $\varphi(f.\underline{a})$ is true in N.
N is a continuation of $M \Leftrightarrow$ there is an homomorphism from M to N.
Therefore: $\neg\varphi(\underline{a})$ is forced in $M \Leftrightarrow$ for no homomorphism f from M into another $\mathcal{L}$-structure N, $\varphi(f.\underline{a})$ is forced in N.

Coherence of forcing:
Preservation Lemma. If $\varphi(\underline{a})$ is forced in M , it is forced in any continuation of M.
Definition. M is generic if truth in M coincides with forcing in M.
Lemma. M is generic iff one of the following holds, for every $\varphi(\underline{a})$, $\underline{a} \in M$:
(i) If $\varphi(\underline{a})$ is true in M, then it is forced in M.
(ii) $\varphi(\underline{a})$ is forced or $\neg\varphi(\underline{a})$ is forced.
(iii) If $\varphi(\underline{a})$ is forced in some continuation of M, then it is forced in M.

Existence Lemma. Every model of T can be continued into a generic one.
Observation. A model M of T is generic iff every homomorphism from M into a generic model of T is an elementary embedding. Because of the JCP, all the generic models satisfy the same sentences.
Conclusion (weak forcing). M forces $\neg\neg\varphi(\underline{a})$ iff $\varphi(f.\underline{a})$ is true in every generic continuation of M.

Positively closed models:
Definition. M is positively closed if, for any homomorphism f into a model N of T and positive $\varphi(\underline{a})$, $\varphi(\underline{a})$ is true in M iff $\varphi(f.\underline{a})$ is true in N.
If M is pc, and if a tuple $\underline{a}$ in M does not satisfy a positive formula φ, then it satisfies another positive formula ψ which is contradictory to it, that is T entails $\neg(\exists\underline{x})\varphi(\underline{x}) \wedge \psi(\underline{x})$.
Every generic model is positively closed.
Definition. A pc model M is positively ω-saturated if, for any tuple $\underline{a}$ of elements of M, any consistent set $\varphi_i(\underline{a}, x)$ of positive formulae has a realisation in M.
Positively ω-saturated pc models are generic. All the pc ω-saturated models have the same h-inductive theory.

Types:
Definition. A type in n variables $\underline{x} = (x_1, ...x_n)$ is a maximal set of positive formulae $\varphi_i(\underline{x})$ which is consistent with T.

We topologize the set $S_n(T)$ of types by declaring that the positive formulae define closed sets. We obtain a precompact set which does not necessarily satisfy Hausdorff separation condition.
The sets of types are Hausdorff if and only if the h-inductive theory of its generic models has the Amalgamation Property for homomorphisms.

Model theory:
Tarski: we consider the class of models of a (complete) theory, and elementary embeddings between them.
Robinson: we consider the class of existentially closed models of an inductive theory (with the Joint Embedding Property), and embeddings between them.
Ben-Yaacov: we consider the class of positively closed models of an h-inductive theory (with the Joint Continuation Property), and homomorphisms between them.
Morleyisation interprets Tarski in Robinson and Robinson in Ben-Yaacov.
The universal domains, that is, the ω-saturated models, are more general in Ben-Yaacov setting than in Robinson setting, and more general in Robinson setting than in Tarski setting.

Ben-Yaacov observed that Positive Logic gives the most general Model Theory if we want to preserve Compacity, and that the classical manipulations that were done in Tarski's setting can be extended to the positive (and Robinson's!) frame provided that Hausdorff separation is assumed.

Decolonizing "Natural Logic"

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"Natural logic" was proposed by Henry Lewis Morgan (1818–1881) [5] as the engine of cultural evolution, concluding that the "course and manner" of cultural development "was predetermined, as well as restricted within narrow limits of divergence, by the natural logic of the human mind". Inherited from Kant [2] as the logic of common sense, late 19th century philosophers and anthropologists accepted "natural logic" as the common ground of rationality that ensured that human differences were ultimately tied together

*Keynote speaker at the workshop "Sociology and Anthropology of Logic: Past and Present" (page 342).

by a shared system of order. Lucien Lévy-Bruhl [4] recognized natural logic, but argued as well for a pre-logical stage of human development. Others including Franz Boas, Benjamin Whorf, and Claude Lévi-Strauss [1,7,3] took the Kantian distinction between natural logic and scientific logic as marking the difference between "savage" and "civilized" cultures. In this case, natural logic is the foundation upon which the latter is built, and which serves as the basis for the survival of so-called primitive peoples through their assimilation to its "civilized" system of order. Even as "natural logic" shaped the 20th century response to such peoples, American logician Josiah Royce (1855–1916) [6] proposed an alternative "primary logic" or system of order that, like natural logic, was foundational to human agency, but also rejected natural logic's inherent reductionist and assimilating power. In this discussion, I will examine the emergence of natural logic in anthropology and its trajectory in support of the dominant colonizing system of order. I will conclude by arguing that the primary logic of Royce provides a critical tool that can support the work of decolonization in the present world.

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It Was So Revolting I Couldn't Take my Eyes Off It

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Dialetheism is the view that some contradictions are true. One might naturally ask for examples. This paper offers a new one. There is a well known psychological phenomenon (noted, for example, by Plato in *The Republic*) in which something is so repulsive that one is compelled to look at it. One is attracted and repelled. Prima facie, that is a contradiction, and, given the context, a true one. I argue that is exactly what it is. A brief discussion of dialetheism frames the topic.

General principles for the design of logical notations

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The context of this talk are two mathematical traditions in the 19th century, namely those of *symbolical algebra* and the *algebra of logic*, which were both very sensitive to the development and use of good notations.

On the one hand, Peacock, Herschel and Babbage explicitly advocated the replacement of the fluxional notation and geometric methods in England by the notations employed on the continent. On the other hand, many of the disagreements between Boole, Jevons, MacColl and others frequently concerned the notational variants that they used in their respective systems of logic. This general interest in notations is also reflected in many comments and remarks about this subject in the writings of the above mentioned authors. In my talk, I will focus in particular on the reflections of Charles Babbage [1,2,3] and Hugh MacColl [4,5], because they also formulated and discussed a number of general principles that characterize a good notation. These principles, which are not always compatible with each other, will be used to assess and discuss various notations for logic.

*Keynote speaker at the workshop "Reflections on Paraconsistency" (page 295).
†Keynote speaker at the workshop "Practices of Writing and Reading in Logic" (page 179).

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Place and Value of Logic at Louis Couturat

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Louis Couturat is known for having made known and defended the works of logic which were contemporary to him, especially those of Russell and the Italian school. It is a well-deserved reputation, the work of conceptual analysis and which, different, of putting in relation the mathematicians of his time are exceptional. But if you take a closer look at it along its work, you realize that the place and value of logic are not simple problems for Couturat. The logic is no longer the one taught in Greek studies, and yet it has no place in mathematics, unlike the algebra of logic to which Couturat always returns. On the other hand, he discovers the logic of Russell as a novelty and invention, to which he gives, in agreement with Lalande and Itelson, the old name of Logistics. But later, in his unpublished *Manuel de Logistique* (1905), he will present it as an ancient science to which symbols have been added, yet we know he does not like the use of symbols as Peano instituted. Couturat is therefore in a complex relationship with the logic of his time, and we will try, in this presentation, to unfold the causes.

*Keynote speaker at the session “History” (page 521).

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Ill-Defined Attitudes

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A classical logician, such as A.J. Ayer, regards the law of identity, $(x)(x = x)$, as trivially true [1]. He has trouble understanding an agnostic who cautiously withholds assent to the sweeping generalization. Ayer is more puzzled by Hegelians and Marxists who propose counterexamples. But his most puzzling adversary says the law is not even false. Peter Geach ([2], p. 241) dismisses $(x)(x = x)$ as ill-defined on the grounds that there is no absolute identity.

*Keynote speaker at the workshop "Reflections on Paraconsistency" (page 295).

When the topic changes 'God exists', Ayer ([1], pp. 115–116) and Geach [2] reverse roles. Now, it is Geach who must make sense of Ayer's attribution of nonsense. He will receive little help from standard models of belief.

Nevertheless, Geach poses the deeper enigma because he is a non-cognitivist about a simple logical tautology. In "The Deviant Logician's Dilemma", W.V. Quine ([3], pp. 80–83) says that any attempt to deny a logical law just changes the subject. But Geach only denies that '$(x)(x = x)$' is well-defined.

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The Validity of Validity

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VALIDITY, in many guises and shapes, is an omnipresent notion within modern logic.

However, is the current practice with respect to validity a valid one?

I shall argue that it is not; in particular the customary conflation between inferential validity and the logical holding of consequence will be discussed and the role (need?) for a completeness theorem will be discussed. Some consequences for epistemology and the philosophy of mathematics are also noted.

Five notions that are all known as "validity" will be unscrambled, to wit:

1) Validity of a proof (demonstration);
2) Validity of an inference;
3) (Logical) Validity of a wff (proposition);
4) (Logical) Validity of a consequence among wff's (propositions);
5) (Prawitz-)Validity of derivations in the sense of Proof-Theoretical Semantics.

*Keynote speaker at the workshop "Naming Logics II" (page 332).

Here the common conflation of 1) and 2), as well as the almost universal reduction of 2) to 4), will be given special attention at the hand of writings of Frege and Tarski.

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The Ace of the Second Generation of the Lvov-Warsaw School. Bolesław Sobociński and some of his unknown philosophical views

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Bolesław Sobociński (1906–1980) was a pupil of Jan Łukasiewicz and Stanisław Leśniewski. He was one of the main representatives of the second generation of the Lvov-Warsaw School, a member of the Warsaw School of Logic. He is known as the most influential popularizer of Leśniewski's prothotetics, ontology and mereology, as well as the author of many achievements in set theory, theory of algebras, and symbolic logic (including many-valued logics, modal logics). The international scientific community knows Sobociński as the founder and for many years editor of the prestigious scientific periodical *Notre Dame Journal of Formal Logic*. In this lecture we present an overview of his scientific biography. We also aim to highlight the

*Keynote speaker at the workshop “The Lvov-Warsaw School: Past, Present and Future” (page 270).

lesser known side of Sobociński — that is, his philosophical formation following in the footsteps of his two great teachers. Sobociński combined the philosophical acumen and 'mathematical' preciseness of Łukasiewicz with Leśniewski's deep philosophical interests expressed in his general and systematic framework. His early philosophical interests are known because of his collaboration with the Cracow Circle — the Catholic 'branch' of the Lvov-Warsaw School. Sobociński did not publish any papers concerning the topics researched by the school, having a merely 'advisory' role on the subject of logic. However, as evidenced by his unpublished correspondence with Father Bocheński, Sobociński had his own, original philosophical views on the topics addressed by the Circle. In his letters he discussed the issue of the existence of universals and developed his original metaconceptualistic point of view; he considered the possibility to formalise the concept of the Universe on mereological grounds, and outlined the applications of mereological concepts to theological issues. This lecture will survey his views in to the original manuscripts and suggest an modern formulation of their main tenets.

Abstract Agent Argumentation (Triple-A)

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Triple-A is an abstract argumentation model, distinguishing the global argumentation of judges from the local argumentation of accused, prosecutors, witnesses, lawyers, and experts. In Triple-A, agents have partial knowledge of the arguments and attacks of other agents, and they decide autonomously whether to accept or reject their own arguments, and whether

*Keynote speaker at the session "Argumentation" (page 460).

to bring their arguments forward in court. The arguments accepted by the judge are based on a game-theoretic equilibrium among the argumentation of the other agents. The Triple-A theory can be used to distinguish various direct and indirect ways in which the arguments of an agent can be used against his or her other arguments.

The logic of causation

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The notion of cause and effect underpin our understanding of reality. Indeed the whole business of experimental science is attempting to predict whether a certain (actual or hypothetical) intervention at a given point in space and time will affect what happens at another, typically later point. To abstractly capture which such points can affect each other, we can employ *causal structures*. Broadly, a causal structure is one of a family of graph-like structures, where the presence or absence of connections capture which events could, in principal, have causal affects on each other. A natural example is the causal structure coming from relativistic spacetime, where connections appear whenever one point in spacetime each reachable from another without exceeding the speed of light.

In this talk, I will describe a new framework for expressing and reasoning about causal structures, based on categorical logic. It operates by first fixing a universe of 'raw materials' from which a category of causal and higher-order causal processes is constructed. The internal logic of this category is multiplicative linear logic, and I will show how the interplay between this logic and the fundamentally graphical structure of process composition yields a means of reasoning about causality, which applies to traditional statistical causal inference, as well as reasoning about more exotic situations, such as indefinite and quantum causal structures.

*Keynote speaker at the workshop "Logic and Physics" (page 398).

Polish Contributions to Universal Logic

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More precisely speaking, the paper concerns those ideas developed in Warsaw School of Logic which could be considered as contributions to universal logic. The term 'universal logic' was not used by Warsaw logicians. In fact, it seems that they would have reservations with respect to the view that universal logic is something analogous to universal algebra. Warsaw School considered logic as autonomous field, entirely independent of mathematics and philosophy. Thus, any algebraization of logic was considered as being at odds with the priority of logic as *ars artium scientia scientiarum ad omniam aliarum scientiarum methodorum principiam viam habent* (Petrus Hispanus).

Roughly speaking, the analysis of logic as such it, according to Warsaw logicians, the business of metalogic, considered as a part of logic itself. Perhaps Leśniewski's attempts to embed metalogic into his logical systems, more precisely into protothetic, can be considered as the purest attempt to unify logic and metalogic. Later views, instantiated mostly by Tarski's approach, located metalogic in metamathematics. Consequently, metalogic consisted in mathematical analysis of logical systems. According to Tarski, all methods accepted as mathematically standard can be and should be used in metalogic, independently whether they are constructive or not, finitary or not, etc. This ideology allowed to analyze logic regardless of various philosophical orientations in the foundations of mathematics.

More special investigations, which can be eventually subsumed under the label 'universal logic' include (the list is incomplete):
1. The relation between classical logic and non-classical systems;
2. The methodology of propositional calculus;
3. The theory of consequence operation;
4. The calculus of systems;
5. The Lindenbaum algebra;
6. The relations between logic and topology;
7. Various studies on the nature of logical concepts as the most universal.

*Keynote Speaker at the workshop "The Lvov-Warsaw School: Past, Present and Future" (page 270).

On the Formal Evolution of Islamic Juridical Dialectic

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The practice of dialectical disputation among Muslim jurists may be nearly as old as the Islamic juristic enterprise itself. But just as Islamicate intellectual projects (law, philosophy, theology, etc.) are marked by a rich pluralism of opinion (khilāf/ikhtilāf), so too is the development of Islamicate dialectical theory (jadal/munāẓara) pluralistic (and nonlinear), even within the juridical domain itself. Still, there are deeper trends which may be noted, perhaps the most important being the general infusion of post-Avicennan syllogistic into legal theoretical and dialectical argument, commonly understood to have begun in the 11th century CE. This trend to greater logical formalism culminated in methods developed in a little-known school of juristic dialectic in Transoxiana in the 12th and 13th centuries, gaining momentum and moving westward in step with the Mongol expansion. This more rigorously syllogistic juridical dialectic in turn gave birth to a new, universal dialectical method: the ādāb al-baḥth wa'l-munāẓara, or "protocol for dialectical inquiry and disputation", equally applicable in theology, philosophy, and law; and this streamlined system quickly grew into a core discipline in the Islamic Sciences, generating a massive commentary tradition. In this talk, I will present a small number of vignettes — snapshots from various moments in variant streams of Islamic juridical dialectic — marking certain key features at each stage. Special focus will be maintained on the formative dynamic of dialectical disputation in shaping both legal theory and dialectical theory itself.

*Keynote speaker at the workshop "Logic, Law and Legal Reasoning" (page 381).

Paraconsistency: Theory and Practice

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The first part of this talk will be concerned with *theory* of paraconsistent logics. Namely, a new upcoming book titled 'Effective Propositional Paraconsistent Logics' (joint work with Arnon Avron and Ofer Arieli) will be introduced and discussed. The purpose of this book is to provide a comprehensive methodological presentation of the rich mathematical theory that exists by now concerning what is the heart of paraconsistent reasoning: paraconsistent propositional logics. Among those logics it mainly concentrates on those which are effective (in the sense that they are decidable, have a concrete semantics, and can be equipped with implementable analytic proof systems). We will start by defining basic notions related to paraconsistency, considering some important approaches to paraconsistency, such as multi-valued logics (both truth functional and non-deterministic); logics of formal inconsistency; paraconsistent logics which are based on modal logics. Each logic in the book is studied from both a semantical and a proof theoretical points of view.

The second part of this talk will focus on practical aspects of paraconsistency in the context of requirement engineering, one of the fundamental stages of software development. The problem of inconsistency in requirements specifications has been in the spotlight of the software engineering community for many years. While in the previous decades, it was perceived as a problem that needs to be eliminated on sight, recently, it has been more widely recognized that maintaining consistency at all times is not only infeasible but even counterproductive. Over the last decades, a more tolerant approach toward inconsistency has emerged [1], along with tools supporting inconsistency management (e.g., [2,3]). However, their adoption in practice has remained quite modest. I will describe an empirical study (joint work with Irit Hadar and Daniel Berry [4,5,6]), which investigated practitioners' perceptions and attitudes towards inconsistency management, aiming to better understand the practical barriers of the adoption of inconsistency management tools in practice.

*Keynote speaker at the session "Non-Classical Logics" (page 439).

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[*] Conference on Advanced Information Systems Engineering
[†] Institute of Electrical and Electronics Engineers
[‡] Cognitive Aspects of Information Systems Engineering
[§] Institute of Electrical and Electronics Engineers
[¶] Requirements Engineering
[‖] Evaluation of Novel Software Approaches to Software Engineering

14 – Workshops

Logical Geometry and its Applications

This workshop is organized by

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Aristotelian diagrams are compact visual representations of the elements of some logical, lexical or conceptual field, and various logical relations holding between them (e.g. contradiction and contrariety). These diagrams have a rich history in philosophical logic, which can ultimately be traced back to the works of Aristotle and Apuleius. Without a doubt, the oldest and most widely known example is the so-called 'square of opposition' for syllogistics, but throughout history, authors have also developed larger, more complex diagrams, such as hexagons, octagons, and even three-dimensional diagrams. In contemporary research, Aristotelian diagrams have been used in nearly all subbranches of logic, such as modal logic, various families of non-classical logics, probabilistic and fuzzy logic, consequential logic, and logics of rational agency. Furthermore, because of the ubiquity of the relations that they visualize, Aristotelian diagrams are also frequently used outside of logic, in disciplines such as philosophy, linguistics, computer science, law, cognitive science, and natural language processing.

In recent years, Aristotelian diagrams have also begun to be studied as objects of independent logical and diagrammatic interest, giving rise to the burgeoning field of logical geometry. Rather than focusing on the specific details of any given application, logical geometry aims to develop a systematic theory of Aristotelian diagrams in general. On the logical side, it studies topics such as information level, logic-sensitivity and the interplay between Aristotelian, duality and Boolean structure; on the visual/geometrical side, it is concerned with informational vs. computational equivalence in Aristotelian diagrams, and with analyzing these diagrams as purely geometrical entities (in terms of symmetry, Euclidean distance, polyhedral duality, etc.).

The keynote speaker at this workshop is Amirouche Moktefi (page 145).

Call for papers

The Workshop on Logical Geometry and its Applications (WoLGA) at UNILOG'2018 aims to deepen our theoretical understanding of the logical and diagrammatic behavior of Aristotelian diagrams, as well as to broaden our perspective on their (historical and contemporary) applications. Relevant topics include (but are not restricted to):

- Aristotelian diagrams for non-classical logics
- Aristotelian diagrams and metalogical considerations
- the interplay between Aristotelian and duality relations
- the interplay between the Aristotelian relations and Boolean structure
- probabilistic interpretations of the Aristotelian relations
- relations between (families of) Aristotelian diagrams
- Aristotelian diagrams from the perspective of diagram design
- various kinds of symmetry in Aristotelian diagrams
- logical and geometrical distance in Aristotelian diagrams
- case studies on Aristotelian diagrams used in medieval logic (Western and Arabic)
- case studies on Aristotelian diagrams used by Modern logicians (e.g. Keynes, Carroll)
- case studies on contemporary uses of Aristotelian diagrams in logic
- case studies on contemporary uses of Aristotelian diagrams in other disciplines (e.g. computer science, linguistics)

A one-page abstract should be sent via email before October 5th, 2017 (extended deadline!) to lorenz.demey@kuleuven.be and hans.smessaert@kuleuven.be.

Notifications will be sent out by November 15th, 2017.

Logical Oppositions in Avicenna's Hypothetical Logic

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In his hypothetical logic, Avicenna introduces new kinds of hypothetical propositions by using quantifications ranging over situations (or times) and distinguishing between universal and particular, affirmative and negative, conditionals or disjunctives. For instance, the conditionals are expressed thus:

- A-hypothetical conditional: "Whenever A is B, then C is D";
- I-hypothetical conditional: "Maybe when A is B, then C is D";
- E-hypothetical conditional: "Never if A is B then C is D";
- O-hypothetical conditional: "Not whenever A is B, then C is D".

In these propositions the elements are predicative but simple. However, in section 7 of *al-Qiyās* [1], pp. 361–384, he goes further by considering hypothetical propositions, where the elements are themselves quantified propositions of the form A, E, I and O. These propositions have structures like the following ones: "Whenever every A is B, then every C is D" ("whenever A_1 then A_2" for short) or "Never when every A is B, then Some C is D" ("Never if A_1 then I_2" for short), and so on. In [1], chapter 1 of section 7, pp. 361–372, he provides sixteen different A-hypothetical conditional propositions by combining their A, E, I or O elements in all possible ways. In the same way, he provides 16 E-hypothetical conditionals, 16 I-hypothetical conditionals and 16 O-hypothetical conditionals by combining their quantified elements in all possible ways, and, in [1], chapter 2, pp. 373–384, he makes the same thing with the disjunctive hypothetical propositions. He also says that the logical relations of contradiction, contrariety, subcontrariety and subalternation hold between all these propositions.

In this contribution, I will consider only the hypothetical conditional propositions listed in chapter 1 of section 7, and will analyse the logical relations between all of them. Now, in Avicenna's frame, all A-conditional and I-conditional propositions, whether categorical or hypothetical have an import (i.e. they require the truth of their antecedents to be true), while all E-conditional and O-conditional propositions, whether categorical or hypothetical do not have an import. As a result, the 16 A-hypothetical conditionals are different from the 16 E-hypothetical ones, while the 16 I-hypothetical conditionals are different from the sixteen O-hypothetical ones. So the total number of distinct propositions is 64. This gives rise at first sight to 8 octagons, each containing two A-hypotheticals and two I-hypotheticals and their contradictories. These octagons are not necessarily of Buridan's kind, for we find some octagons of Johnson-Hacker's kind plus another new kind of octagons, different from them both. The octagons can also be grouped two by two, which gives rise to several figures containing 16 vertices and allows for more relations between the propositions. This shows the richness of the theory.

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On the Interaction of Tense and Aspect — Merging Kites

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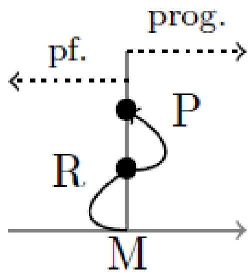
As illustrated in [1, p. 333], a past-oriented binary choice $[\pm R(etroject)]$ followed by a future-oriented binary choice $[\pm P(roject)]$ suffices to distinguish the four basic finite tense types of English: present $[-R, -P]$, future $[-R, +P]$, past $[+R, -P]$, conditional $[+R, +P]$.

name	example	Tense1	Tense2	Aspect1	Aspect2
		past($\pm R$)	fut($\pm P$)	perf	prog
simple pres	works	–	–	–	–
simple fut	will work	–	+	–	–
simple past	worked	+	–	–	–
simple cond	would work	+	+	–	–

In our talk, we shall first review the evidence that these two choices form an asymmetric ordered pair $\langle \pm R, \pm P \rangle$, with the *past* oriented binary choice $\pm R$ linearly before and vertically higher in the syntactic structure than the *future* oriented binary choice $\pm P$. This state of affairs parallels the asymmetrical *earlier to later* iconicity that characterizes path expressions such as spatial *from Brussels to Paris* or temporal *from 2 to 5*, where the temporally prior source expression precedes — and its phrase structurally arguably includes — the temporally later goal expression.

That the $\langle \pm R, \pm P \rangle$-asymmetry might well be relatable to the source-goal asymmetry of path expressions is reinforced by the equally and similarly fixed relation between the two aspectual binary features [±*perfect*] and [±*progressive*]. The perfective aspect restricts the situation expressed by the root verb *work* to a finite past-oriented time-segment starting *before* and leading up to the point P. The progressive (or continuous) aspect, for its part, restricts the situation expressed by the root verb to a future-oriented time-segment that includes P, but is longer and stretches to some finitely distant point *after* P. This is what creates the still-going-on effect in *John is working*.



And here too, what is involved is an ordered pair: $\langle \pm perf, \pm prog \rangle$. Though the two aspectual choices involve finite time-segments rather than jumps to tense reference points, they are characterised by the same *source-goal* or *before-after*-asymmetry as $\langle \pm R, \pm P \rangle$, witness the only possible order of the perfective and progressive auxiliaries in the tense forms below.

name	example	past(R)	fut(P)	perf	prog
pres perf prog	has been working	−	−	+	+
fut perf prog	will have been working	−	+	+	+
past perf prog	had been working	+	−	+	+
cond perf prog	would have been working	+	+	+	+

While the tense-pair $\langle \pm R, \pm P \rangle$ has been analysed in terms of a connected pair of Jacoby-Sesmat-Blanché-hexagons, the aspectual pair will be shown to involve a similar connected double-kite grafted onto each of the four basic tense vertices in the kite-representation provided in [1, p. 135, fig. 12], more specifically the A- and Y-corners of each of the two connected kites.

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Squares, Cubes and Circles. Sketches of Oppositional Geometry between Geulincx and De Morgan

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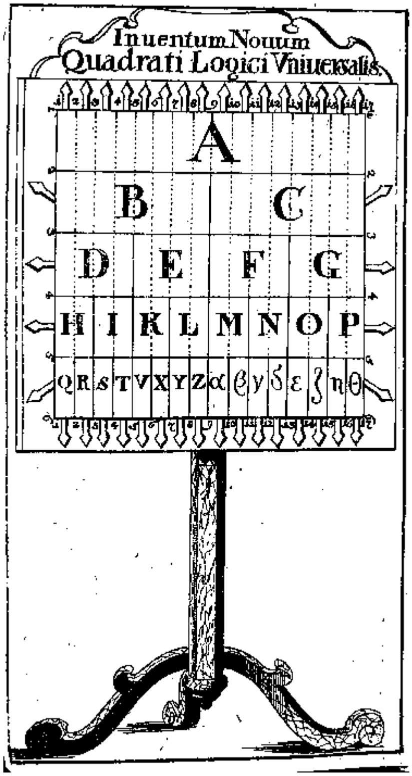
Normally the history of oppositional logic and especially "oppositional geometry" is focused on squares of opposition in the middle ages and the 20th century: Aristotle has used terms of "oppositional geometry" such as "contrariety" and "contradiction" [1] which were elaborated by (Ps-)Lucius Apuleius Madaurensis in form of a "quadrata formula" [2]. This square of opposition became popular by scholastic philosophers. The oldest document of this geometrical form was found in a church in Gotland [3]. Finally, in the 20th century, the square of opposition was transformed in a "logical hexagon" and a "logical tetrahexahedron" ([1] and [4, ch. 7]) by Augustin Sesmat (1951) and Robert Blanché (1953).

In addition to that history, my talk brings up a discussion on the geometry of logical opposition between the 17th and the 19th century. A few examples can be mentioned to show the diversity of topics: at first, I will argue that Arnold Geulincx has invented a logical cube in which a square of opposition is integrated [cf. 5, ch. 7]. But another more sophisticated form of the logical cube was provided in Johann Christian Lange's *Inventum novum quadrati logici universalis*, published in 1714. This cube (see the frontispiece) combines Eulerian diagrams (by using cubes in a vertical order) with oppositional geometry (by using arrows). In the last part of my talk, I will show the connection between the early modern forms of oppositional geometry with modern logic, especially the analysis of Augustus De Morgan [6].



Cube of Johann Christian Lange

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End of the square?

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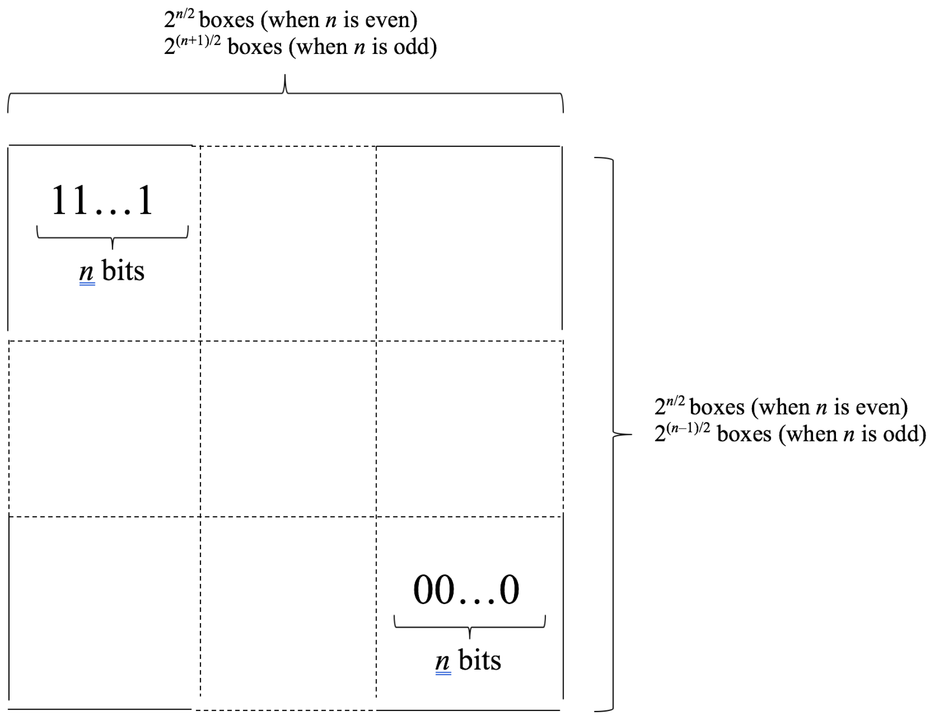
It has been recently argued that the well-known square of opposition is a useless gathering that can be reduced to a one-dimensional figure, viz. an ordered line segment of positive and negative integers [1]. However, one-dimensionality leads to some difficulties once the structure of opposed terms goes beyond categorical statements, including logical hexagons.

An alternative structure is proposed in the present talk, relying upon a semantics of bitstrings and leading to a systematic gathering for any length n of the bitstrings [3]: the structure is a rectangle whenever n is odd; it is a square whenever n is even, although the latter are not structured like the Aristotelian square [2].



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Category Theory and Logical Geometry — Is a commutative diagram an Aristotelian diagram?

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Category theory is concerned with structural equivalence between different objects in the same and between different formal frameworks (categories). Its most important tool are commutative diagrams (structure-preserving arrow-diagrams), which serve (as the drawings in classical geometry) as the foundation for a special kind of diagraphical reasoning. In its form of topos theory it is powerful enough to provide an analysis and a reconstruction of classical-mathematical and intuitionist logic. My presentation revolves around the question "Are the diagrams of category theory a kind of Aristotelian diagram?".

One of the main selling points of category theory is that it provides a macro view on formal structures and their inter-relations that would not be possible by any "direct" comparison of these structures. The same is true of Aristotelian diagrams. Both provide an overview by representing structures and their relations by a picture. In both cases, this picture can, according to a special kind of grammar, get a step-by-step interpretation. But this pictorial mode of representing is itself not sequential and the information it contains cannot be reduced to one sequential reading. Furthermore, the geometrical features of the diagrams can "show" new ways how to read (or sometimes to rearrange) it.

In my presentation, I will give a short introduction into the way diagrams are utilized in category theory. I will — by using a simple example — show how the contemplation of such a diagram can trigger the discovery of new inter-structural features and point in a way how to prove it. I will compare this to simple examples of classical Aristotelian diagrams and the way they are utilized. In this comparison the focus will be on the function of the geometrical property of symmetry.

$$\begin{tikzcd}
 & f^*E_V \arrow[rr] \arrow[dl] \arrow[dd] & & E_V \arrow[dl] \arrow[dd] \\
f^*E \arrow[rr, crossing over] \arrow[dd] & & E & \\
 & U \arrow[rr] \arrow[dl] & & V \arrow[dl] \\
M \arrow[rr] & & N \arrow[from=uu, crossing over] &
\end{tikzcd}$$

$$\begin{tikzcd}
T \arrow[drr, bend left, "x"] \arrow[dr, dotted, "{(x,y)}"] \arrow[ddr, bend right, "y"'] & & \\
 & X \times_Z Y \arrow[r, "p"] \arrow[d, "q"] & X \arrow[d, "f"] \\
 & Y \arrow[r, "g"] & Z
\end{tikzcd}$$

Practices of Writing and Reading in Logic

This workshop is organized by

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A great deal of the working logician's job is to write and read. This holds in at least two senses:

First, in order to tackle a task in logic, it is necessary to apply the rules for transformation or deduction as stated relative to a given logical system. In order to apply these rules correctly, you may produce certain inscriptions and watch a sequence of transformations of an initially given formula, i.e., you may write down the consecutive steps and eventually read off the result.

But secondly, there is a broader sense in which writing and reading are relevant to logic. Communicating logical problems (and solutions) inevitably requires activities of writing for an audience, and most commonly at least some bits of prose. Then, participating in the 'logical community' will typically require to disseminate your outcomes. But participating in the 'logical community' also requires to work through others' contributions. Hence activities of reading are necessary to assess received input — which may then again be commented on, corrected or disproved. Moreover, the range of available input may depend on individual or collective activities of selecting and systematizing items of logical work which is deemed as relevant. Hence what there is for you to read may to great extent depend not only on what has been written, but also on what has been read by others.

The presently announced workshop aims at an account of logic as construed from logicians' practices of writing and reading in both respects. Further interests are activities of commenting or reviewing, and of publishing and collecting. In order to take an interdisciplinary stance, the workshop will allow for a variety of approaches.

The keynote speakers at this workshop are Dirk Schlimm (page 158) and Volker Peckhaus (page 149).

Call for papers

Topics for contributions may include, but are not restricted to:

- Questions of notation in logicians' formalizations
- Questions of literary style in logicians' prose
- Tools for collaborative research in logic
- Bibliometrical research on logic publications
- The role(s) of logic journals
- The role(s) of reviewing sections in logic journals
- Translations of logical literature
- Logicians' publishing activities
- Logicians' reviewing activities
- Logicians' perusal of public or research libraries
- Logicians' private libraries and collections
- Correspondence among logicians
- Correspondence among logicians and publishers

Contributed talks should not exceed a duration of 30 minutes including discussion. To submit a contribution, please send a one-page abstract by November 15, 2017 to annasoph@mail.uni-paderborn.de.

Logic as Subject and Method of a Logician's Work

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A good book on logic or mathematics usually has a transparent and detailed overall structure, takes care to state its definitions and theorems in a well-marked fashion and routinely draws its inferences without commenting on each of them. — Notably in formal logic there does not seem to be one strand that managed to put these three features into one *object-language* system. In addition, most of the common systems have none of the three features. The features vaguely alluded to in that sentence are these:

(A) representation of super-argumentative structures
(B) consistent signalling of discursive functions of what is being said
(C) confinement to the borders of *one* language

The main part of the talk will consist in outlining a formal system which has all three features while not diverging too much from ‹mainstream logic› before describing how such a system may guide logicians in developing their thoughts in writing. Said system is most intuitively presented as a modification of standard first-order logic in three respects. First, the atomic category of performators is added, object-language indicators of the discursive functions. Second, the resulting language gets a natural deduction calculus allowing one to shed all commentary devices (like rule commentary). Third, different kinds of sentence sequences are acknowledged leading to a *corpus logic*, meaning a logic that allows one to arrange formal texts of thetic, argumentative, inquisitive or other character in a linear fashion. These three measures together fulfill the features (A–C).

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Writing and Drawing in Logic — the Case of Aristotelian Diagrams

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When communicating their research, logicians not only write words and formulas, but they also draw various kinds of diagrams. The use of diagrams has a very long history in logic, including diagrams such as the Porphyrian tree and the pons asinorum in medieval logic, and Euler diagrams, Venn diagrams and Peirce graphs in more recent times [2]. In this contribution, however, I will focus on yet another broad category of diagrams used in logic, viz. *Aristotelian diagrams.* These diagrams visually represent the elements of some logical, lexical or conceptual field, and the logical relations holding between them (in particular, the relations of contradiction, contrariety, subcontrariety and subalternation). Without a doubt, the oldest and most well-known example is the 'square of opposition' for the categorical statements from syllogistics; however, throughout history, several larger, more complex Aristotelian diagrams have also been devised, such as hexagons, octagons, cubes and rhombic dodecahedra.

The central question of this contribution is: what exactly is the role of Aristotelian diagrams in the practice of logicians? Given their widespread use, it seems obvious that these diagrams indeed do have an important methodological role to play, but it is unclear what that role consists in precisely. Previous work has tried to address this issue from a highly *theoretical* perspective. For example, Smessaert and Demey [3] develop a sophisticated mathematical account of the information contents of Aristotelian diagrams; based on this account, they then argue that the widespread use of Aristotelian diagrams is due to their informational optimality. In this contribution, these theoretical approaches will be complemented with a more *practice-based* perspective [1]. In particular, I will present a detailed examination of the writings of logicians regarding Aristotelian diagrams. In other words: which reasons do logicians themselves offer for their use of Aristotelian diagrams? I will distinguish four broad views on the use of Aristotelian diagrams.

First of all, the received view holds that Aristotelian diagrams primarily serve as *mnemonic* devices, used mainly when introducing novice students to the abstract discipline of logic. However, this view has become untenable, because today, most Aristotelian diagrams are no longer found in logic textbooks, but rather in research-level papers/monographs from a wide variety of reasoning-related disciplines (logic itself, but also linguistics, psychology, computer science, etc.).

A second view focuses on the cognitive advantages that Aristotelian diagrams have in virtue of their *multimodal* nature (symbolic/textual + visual). This second view is related to the first one, but it is still fundamentally different: whereas the first view focuses exclusively on the use of Aristotelian diagrams in pedagogical contexts, the second one accommodates both teaching and research-level contexts. However, this account has difficulties to explain the use of larger, more visually complex diagrams, such as octagons and, especially, three-dimensional diagrams.

Thirdly, certain authors motivate their use of Aristotelian diagrams by emphasizing their rich and respectable *tradition* within the broader history of logic. In this way, the tradition of using Aristotelian diagrams gets endowed with a certain degree of (implicit) normativity. This view is, at best, incomplete, because it cannot offer an explanation as to why the tradition of using Aristotelian diagrams came about in the first place.

The fourth, and in my opinion most plausible view, holds that Aristotelian diagrams have a powerful *heuristic* potential. They function as a new layer of abstraction that enables researchers to draw high-level analogies between seemingly unrelated frameworks, and to introduce new concepts (by transferring them across frameworks). On this view, Aristotelian diagrams primarily function as a unifying *language* for a broad interdisciplinary research community working on logical reasoning.

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Teaching Begriffsschrift: Frege's Notation and the Problems of Pedagogy

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Logical concepts become legible in the medium of written notation. Logic's celebrated metamorphosis into a mathematical discipline in the late nineteenth and early twentieth centuries depended upon systems of writing that transformed no less dramatically than the ideas they expressed. Perhaps the most striking notational novelty of that period was Gottlob Frege's two-dimensional Begriffsschrift, or "concept-script", introduced in [1]. Frege's contemporaries criticized his notation, and its reputation as a strange, difficult system persisted. But Frege himself always defended his Begriffsschrift, and recently scholars have argued that it is far more perspicuous than Frege's peers appreciated [2,3,4].

Alongside these welcome discussions of the Begriffsschrift's philosophical value in the present, I focus here on Frege's notation in its historical connection with his teaching. A logician who introduces a new notation does so with the hope that a community of researchers will actually use it; in this sense notation is a social project. Frege had little success on the social side of notation-building, but he understood its importance. His sustained efforts to teach students in Jena to use Begriffsschrift spanned decades. As Frege drew and explicated his sprawling symbolism on the blackboard, however, he did not invite students into any sort of dialogue. I argue that by conducting his lectures without interaction, Frege performed in practice his theoretical position that logic was already there to be observed, not subject to human intervention or manipulation. Just as Frege promoted his notation as an observational technology for arraying logic on the page, so was attending his lectures a passive, observational experience. By exploring Frege's concept of notation as he enacted it in the classroom, I aim to reveal the enmeshed social and theoretical layers of the ultimately practical question of how people write logic down.

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Practices of Writing and Reading in Logic: the 14th Century case of Thomas Manlevelt

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Not much is known about the fourteenth-century logician Thomas Manlevelt, but his work is remarkable enough. His fame rests chiefly on a series of short treatises on the then newly-developed terminist logic, comprising *De suppositionibus*, *De confusionibus* and *De consequentiis*. Widely popular in the fourteen hundreds, they were in use as textbooks and commented upon at universities all over the continent.

In this paper I will rather concentrate on his extensive commentaries on the so-called Old Logic: the *Isagoge* by Porphyry and the *Categories* by Aristotle. Manlevelt's commentary on the *Isagoge*, the *Questiones libri Porphirii*, is edited in full, with introduction and indices, by the presenter of this paper, as volume 113 in the series *Studien und Texte zur Geistesgeschichte des Mittelalters (STGM)*, Brill, 2014. An edition of Thomas Manlevelt's commentary on the *Categories* is currently in preparation.

Following in the footsteps of William of Ockham, Manlevelt stresses the individual nature of all things existing in the outside world. In his commentaries on the *Isagoge* and the *Categories*, Manlevelt radically challenges our conceptual framework, by extending Ockhamist tenures and insights to any logical, and if need be metaphysical or theological subject matter. We are confronted with a radical variety of nominalism, outdoing Ockham in a number of ways. With Manlevelt, early Ockhamism is being pushed to its extremes. He applies Ockham's razor in an unscrupulous manner to do away with all entities not deemed necessary for preservation. In the end, Manlevelt even maintains that substance does not exist.

In relation to Thomas Manlevelt's 14th century commentary (and my 21st century edition of it) most if not all of the topics mentioned in your call for papers will be cursorily or thoroughly dealt with — from either the medieval or the medievalist perspective.

Thus, on the topic of notation in logicians' formalizations, I will take a look at Manlevelt's rudimentary use of the letters of the alphabet to stand not only for people and things, or their accidental properties, but also for propositions — which may be looked upon as a token of liked-mindedness among a certain school of logicians, as well as a tools for collaborative research in logic. On the topic of literary style in this particular logicians' prose, I will have a stern verdict to make over Manlevelt's arid prose style, combined with a highly sophisticated modeling of the argumentation: the *questio* in optima forma. Of course, hard data on any kind of correspondence between medieval logicians are hard to find, but a keen reader of any of these logician's texts may well try and establish which other logicians' works may have been at his elbow while composing his own tracts or commentaries. And at whose elbow his own works came to be lying in their turn. Review activities in our present day sense of the term are not be expected from Manlevelt, if only because of the lack of logic journals in those days. But implicitly and sometimes quite explicitly he does praise some of his colleagues and predecessors, and harshly blames others.

From a more medievalist and to some extent bibliometrical perspective I will spend some words on Manlevelt's afterlife (following a seven centuries' slumbering) in his present days editions, mentionings in logic journals, et cetera. The only thing to be said about translations of logical literature in the vein of Manlevelt and logicians like him, is that to a large extent translations are unnecessary. Everyone interested in this branch of logic reads the works in Latin. Translations will only become an issue when the philosophical community at large gets wind of this extraordinary thinker. The day we are all eagerly looking forward to.

Note on Paul Hertz and the Origins of the Sequent-Notation

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In his PhD Dissertation [1], Gerhard Gentzen (1909–1945) made use of the sequent notation in order to develop a structural framework for logic systems that combined features of axiomatic systems and Natural Deduction. This notation enabled him to prove the famous *Hauptsatz*, regarded as a landmark in the history of mathematical logic and essential in the development of (ordinal) proof-theory. Gentzen adopted this sequent-notation from the "Systems of propositions" (German: *Satzsysteme*) of Paul Hertz (1881–1940), previously studied by him in [2]. Hertz developed in the second decade of 20th Century these systems as an original approach to the formalization of logic, on the basis of a *sole logical concept*, represented by the arrow '$\rightarrow$'. In Hertz's sense, a "proposition" is an expression of the forms:

(1) $a \rightarrow b$,

(2) $a_1, a_2, \ldots, a_n \rightarrow b$,

where $a, a_1, a_2, \ldots, a_n$ are called the antecedents of the propositions and b their succedent. In [3], examples of relations between propositions in a system are represented geometrically. In this context, the notion of "ideal element" is introduced, whose function consists in the reduction of relations between the propositions. Hertz considered the rules for these propositions as the "essence of logic" [4]. The very idea of sequent calculi and of structural rules was conceived in Hertz's systems. Their goal consisted in achieving reduction methods for axiomatic systems from which some sort of "minimal" and independent system could be obtained, that is, a system where proofs should be as elemental as possible. Hertz's proposal was quite

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idiosyncratic at this time (dominated by the notation of *Principia Mathematica*) and can be compared with previous ideas by others logicians.

The aim of this paper is to evaluate Hertz's ideas from the point of view of their *notational innovation* and to analyze Gentzen's interpretation of this innovation in order to determine Hertz's *specific* influence in Gentzen's work. In the paper its place in the context of the development of symbolic logic will be also discussed.

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Truth-tables and Tautologies in Early Logical Empiricism: Hans Hahn as a Pioneer of Logical Pluralism

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Outlines of the history or philosophy of logical empiricism mention Hans Hahn in passing, if at all. He is usually characterized as a typical proponent of the Vienna Circle, embracing both empiricism and modern logic, defending logicism, and rejecting synthetic judgements a priori. However, by taking the specific audience of his few philosophical writings into consideration, this paper argues that a second look reveals momentous idiosyncrasies in Hahn's positions.

Goldfarb, Ricketts, Uebel and others [1,2,3,4,5,6] have meticulously reconstructed how early logical empiricists appropriated Russell's logicism and Wittgenstein's notion of tautology in different ways. Particularly, in contrast to Schlick, Carnap used truth-tables in order to characterize tautologies. This notation facilitated his development towards the principle of tolerance and logical pluralism. Compared with this, Hahn's role as a pioneer of logical pluralism has not been sufficiently acknowledged yet, partly due to the informal style of his few philosophical writings, which were directed towards a broad public. It is only transcripts of Hahn's courses at the University of Vienna which document his use of truth-tables. Along with hitherto barely discussed remarks in the meetings of the Vienna Circle, Hahn's more technical lectures to students and experts corroborate the claim that he indeed systematically adopted logical pluralism prior to Carnap and Menger.

As a consequence of the more nuanced reading proposed in this paper, Hahn's logicist and nominalist philosophy of mathematics is not subject to the otherwise fatal critique brought forward by Gödel [7].

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On the Notation of Fred Sommers' Traditional Formal Logic

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This talk will be about the notational choices made by American philosopher Fred Sommers in the design of his system of formal logic, called Traditional Formal Logic (TFL). Starting in the 1960s, Sommers objected to the mainstream practice in formal logic, rooted in the work of Frege: in particular, Sommers criticized the quantifier-variable rendering of general statements, because this practice produces formal sentences which are often syntactically distant from their natural language equivalents. Sommers instead proposed that a logic like Aristotle's, that takes sentences of the form "[Some/All] X [are/are not] Y" to be primitive, could be formalized to achieve the same inferential power as the mainstream Fregean logic, while maintaining the ease for users and learners that comes with syntactic closeness to natural language. Thus, in developing TFL, Sommers paid close attention to how his logic would be used, an important aspect of which is its notational design: Sommers revised TFL's notation many times over three decades, as he invented new notational devices and his design principles changed.

At the beginning, Sommers' desiderata for his notation were ease of use and extendibility to sentences of arbitrarily high complexity, but over time these evolved to include psychological realism, i.e., the belief that a logical notation can and should mirror the actual cognitive processes of reasoning. These desiderata sometimes came into conflict, and in later versions of TFL concerns with psychological realism seem to take precedence over ease of use. Ease of use, moreover, is difficult to characterize, in part because it depends on the examples on which the logic is employed — in particular, the frequent choice of Lewis Carroll's logic puzzles as in-text examples by Sommers and his followers shows the ease of using TFL for those puzzles, but that easiness may not extend to more "everyday" situations of inference. Sommers' notational choices can also be fruitfully compared to those of historical designers of formal logical systems. For instance, like Boole, Sommers chose to exploit analogies of logic to arithmetic and algebra in his notational design, despite important and potentially confusing disanalogies. Also, contra Frege, Sommers uses mathematical symbols in his logical notation, reflecting his vision of TFL as a tool for philosophers, to aid in natural-language reasoning, rather than as a tool for mathematicians, to organize proofs.

This talk will present TFL's notation at several stages of development, and assess to what extent the notation realizes Sommers' design principles at that stage, making comparisons to past developments of logical systems to illuminate the assessments.

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Truth Tables without Truth Values. On 4.27 and 4.42 of Wittgenstein's Tractatus

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In 4.27 Wittgenstein [2] presents a formula, which calculates the possible combinations "with regard to the existence of n atomic facts":

$$K_n = \sum_{\nu=0}^{n} \binom{n}{\nu} \tag{WF}$$

This formula is much more complicated than 2^n, which, as Max Black pointed out [1], is equivalent to (WF). However, I will argue that Wittgenstein actually has good reasons to present (WF) instead of 2^n: In (WF) Wittgenstein doesn't need to assume truth values, which he refutes to assume as independent objects, but only atomic facts.

While 2^n calculates the amount of possibilities to assign 2 truth values to n atomic facts, (WF) calculates the sum of the number of possibilities to chose ν from n atomic fact for all ν from 0 to n. Thus, truth values are nor considered in (WK), but only the existence and non-existence of atomic facts.

When Wittgenstein presents a truth table for the conditional in 4.442 he only marks the possible combinations of atomic facts with "W" and leaves a blanket for the impossible combination. Thus, Wittgenstein uses "W" not in order to assign a truth value, but to mark the combinations of atomic facts. I will present a notation, which consequently puts Wittgenstein's idea forward by replacing this kind of truth tables by pure tables of atomic facts or, to put it in other words, truth tables without truth values.

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Proof Theory

This workshop is organized by

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Proof theory is one of the core disciplines of mathematical and philosophical logic and needs no further explanation or advertising.

The keynote speakers at this workshop are Francesca Poggiolesi (page 153) and Alexander Leitsch (page 134).

Call for papers

We invite contributions on all aspects of proof theory, philosophical or technical. Topics include:

- general proof theory
- categorial proof theory
- type theory (including foundations)
- computational aspects of proofs
- consistency
- proof systems for non-classical logics
- proof editing
- ordinal analysis
- structural and substructural proof theory
- proof-theoretic treatment of paradoxes
- historical aspects of proof theory
- proof-theoretic semantics

Abstracts (one page) should be sent by October 5, 2017 via e-mail to cfp-proof-theory@informatik.uni-tuebingen.de.

Tomographs for Substructural Display Logic

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The central feature of Belnap's Display Logic [1] is the possibility of displaying every formula occurring in any given sequent as the only formula in either the antecedent or succedent. This is accomplished by means of structural connectives that retain the positional information of the contextual formulae as they are moved aside. Goré accommodates substructural, intuitionistic and dual intuitionistic logic families by building upon a basic display calculus for Bi-Lambek logic. His version uses two nullary, two unary, and three binary structural connectives. Since the structural connectives are not independent of one another, display equivalences are required to mediate between the binary structural connectives.

I propose an alternative approach in which two graph-like ternary structural connectives express one set of three structural connectives each. Each of these new connectives represents all three sequents making up one of the two display equivalences. The notion of sequent disappears and is replaced by that of a structure tomograph consisting of systems of ternary connectives in which nodes mark the linking of the connectives and of formulae to those connectives. The turnstile of a sequent is represented by the highlighting of a single node linking connectives.

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*Interest Group in Pure and Applied Logics

The Existence of Pure Proofs

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Topic of our talk is the notion of pure proofs from a proof theoretical point of view. In a first step, we explain how to deal with this informal philosophical notion in a formal way. We identify formal counterparts to the relevant philosophical concepts and notions and provide a formal definition of pure proofs, this means a definition of pure derivations (in the calculus of Natural Deduction).

The main goal of our talk is to show that every derivation can be transformed into a pure derivation, namely into a derivation satisfying the following condition: every non-logical symbol (the counterparts of mathematical notions) occurring in the derivation already occur in an essential assumption or in the conclusion of this derivation.

Partial results are easily obtained via well-known results: it is a technical lemma that we may replace unnecessary constant symbols by variables. Pureness with respect to relation symbols is a consequence of the existence of the Prawitz normal form and of the subformula property. The crucial aspect is the treatment of function symbols: to prove the existence of a pure derivation, we have to replace some (only the unnecessary) occurrences of terms in a derivation by variables, and to show that the resulting derivation satisfies our demands.

In the course of our argumentation, we overcome some technical difficulties: we introduce a formal notion of occurrences of terms in a derivation. We identify congruent occurrences of terms in a derivation, namely those occurrences which have to be of the same shape due to the inference rules according to which the derivation under discussion is generated. Finally, we show under which conditions such congruent occurrences can be replaced by variables (or other suitable terms). Applying this substitution theorem to derivations in Prawitz normal form, we obtain pure derivations.

Our result also sheds light on the problem of the identity of proofs, another philosophically relevant problem of proof theory. When transforming a derivation into its pure version, we do not change its normal form, but an essential property of this derivation. This seems to be a good reason to reconsider, whether we should identify derivations having the same normal form.

Extensions of Non-Monotonic and Non-Transitive Atomic Bases

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The paper presents a proof-theoretic approach to nonmonotonic logics that are in line with inferentialism and logical expressivism [3,2,1,7]. Thus, the paper remedies the problem that previous attempts to capture logical expressivism in a formal system [3,5] have been criticized for being monotonic [6], and that a recent attempt to provide a nonmonotonic system doesn't include logical constants [9].

According to logical expressivism, it is the characteristic job of logical vocabulary to make explicit (i.e. to put into the form of something assertable) inferential relations among atomic sentences, i.e., a material consequence relation over atomic sentences. According to the kind of semantic inferentialism that goes naturally with logical expressivism, the meanings of atomic sentences are determined by this atomic consequence relation. This suggests that logical expressivists should introduce logical vocabulary by extending atomic bases, i.e. atomic languages plus consequence relation over them.

Material atomic consequence relations are plausibly nonmonotonic, and they plausibly obey Containment ($\Gamma \mathrel{|\!\sim} p$ if $p \in \Gamma$). Nonmonotonic consequence relations that obey Containment cannot obey multiplicative Cut (since it allows us to go from $\Gamma \mathrel{|\!\sim} p$ and $\Gamma, p, q \mathrel{|\!\sim} p$ to $\Gamma, q \mathrel{|\!\sim} p$). Hence, the logical expressivist needs logics that (conservatively) extend atomic bases without allowing Weakening or multiplicative Cut.

I provide a sequent calculus for such non-monotonic, non-transitive base extensions. I take the sequents in the base consequence relation as axioms in a tweaked version of G3cp. The extensions are supraclassical, and the Ketonen-style rules I am using are invertible. This allows us to give the connectives an expressivist interpretation. I show how this sequent calculus can be turned into a natural deduction system by subscripting sentences to keep track of the assumptions we used in a way familiar from natural deduction systems for relevance logics.

Along the way, I compare and contrast my expressivist base extensions to work on base extensions in the tradition of proof-theoretic semantics. Most work on extensions of atomic bases focuses on monotonic and transitive bases [10,11(pp. 313–328)]. Sometimes the assumption of transitivity is motivated by an interest in what Schroeder-Heister calls "definitional reflection" [4,12,13]. Piecha and Schroeder- Heister [8] have noted that a certain kind of monotonicity isn't plausible if we take atomic bases to be meaning determining. I go further and argue we should also reject transitivity (multiplicative Cut) and definitional reflection as constraints on meaning determining bases.

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Distributive Deductive Systems: the case of the First-Order Logic

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The objective of our research is a modern reflection on the notion of proof and on the effectiveness of its construction. In the project we take advantage of the fact that various proof systems can generate the same or closely related solutions for the same problem (a formula) with various complexity (understood both in terms of the time complexity and in terms of the size of the derivation tree). Therefore it seems like a lot can be achieved in the field of complexity of proof-search by dividing the initial problem into "subproblems" and assigning to each subproblem a "proof module" which is computationally optimal for the given subproblem.

A distributive deductive system (DDS, for short) for a given logic L consists of two layers: the *module-layer* of proof systems and the *meta-layer*. Proof systems of the first layer are understood as sets of rules enriched with procedures and/or heuristics of their use. The rules act on finite sequences of sequents. Each such proof system — called a *module* — simulates a proof method (or proof methods) to the effect of computational characteristics of the method. For example:

- Module A stores rules acting on finite sequences of left-sided sequents. The rules simulate the method of analytic tableaux in the original account and system KE with the rule of cut.
- Module D enhances the method of resolution. The rules act upon finite sequences of right-sided reversed sequents.
- Module E stores rules acting on finite sequences of right-sided sequents, the rules are of synthesizing character.

Hence the different modules of a DDS represent (and characterize) a rich collection of various proof methods. The task of the meta-layer is to *distribute parts of a derivation among different modules*. Consequently, the meta-layer will *distribute the computational costs of conducting a derivation among the modules*. More specifically, the meta-layer analyses the input data (such as a single formula) using simple functions, such as the length of a formula, the number of distinct variables in a formula, but also the pattern of connectives nested in the scope of other connectives; then, taking into account the procedures and/or heuristics available, the meta-layer chooses the form of a sequent used for the input and hence also the module (modules) from the module-layer that will be used at the start. In case of big inputs, the obtained sequents may be analysed by the tools of the meta-layer many times. For example, the initial input is analysed to a collection of sequents and the meta-layer indicates that while part of the sequents can be efficiently treated with the rules of analytic tableaux (module A), for the other part it is more convenient to decide its inconsistency with resolution system in module D.

The aim of our talk is to present the idea of distributive deductive systems and the results obtained so far for the case of the First-Order Logic.

Remarks on Sequent Calculus

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In the last section V of his thesis, after the proof of the Hauptsatz, Gentzen proved the equivalence between the main three types of formalization of the logical inference: the Hilbert-Ackermann system (H.A.), the Natural Deduction Calculus (N.D.), and the Sequent Calculus (S.C.). In this proof we can see, so to say, the birth of the same formalism of S.C., which is maybe the most important formalization of logical deduction ever provided. Also the handwritten version of the thesis, let's say *Ms.ULS*, contains a similar proof of equivalence, as we have learnt from the important researches made by Jan von Plato on the newly found Gentzen's texts. Admittedly, the last section of the thesis is normally rated "less important" than the other sections, but nonetheless it casts some important light on the emergence of the S.C., and more generally on some structural features of Gentzen's work. In the Thesis the equivalence proof proceeds through the following sequence of steps: i) a proof that every derivation within the H.A.-axiomatization can be transformed in an equivalent derivation of N.D.-calculus; ii) a proof that every N.D.-derivation can be transformed into an equivalent S.C.-derivation; iii) a proof that every S.C.-derivation can be transformed into an equivalent H.A.-derivation. The proof is conducted first for Intuitionistic logic and afterward for Classical logic. In this way, of course, the goal to prove the equivalence of all three calculi is accomplished. However, the main single component showing the origin of S.C. is *the translation of derivations built* *within N.D.-formalism into derivations built within the axiomatic logical calculus of Hilbert and Ackermann's book.* And it is interesting to note that in the pertinent part of *Ms.ULS* Gentzen provided a proof of the equivalence between N.D.-calculi and the H.A.-formalism by showing the possibility to translate every (classical) N.D.-derivation into an equivalent H.A.-derivation; in this way it is explicitly supplied a missing link which is only implicitly present, as a by-product of previous steps i)-iii), in the published version of the thesis. Gentzen proceeds as follows: given an N.D.-proof of, say A, one first lists all those assumptions which are not already discharged before the accomplishing of the inference leading to A. Let us indicate them by Γ. Then one substitutes A by $\Gamma \to A$. If A is an assumption, $A \to A$ takes its place. The steps of inference of N.D. are accordingly translated:

$$\frac{A \qquad B}{A \,\&\, B}\,\mathrm{I}\& \qquad \rightsquigarrow \qquad \frac{\Gamma \to A \qquad \Delta \to B}{\Gamma, \Delta \to A \,\&\, B}$$

Paired with the occurrence of the figure of *sequent*, here we see, probably for the first time, the disentangling of two meanings often conflated in the notion of implication: the propositional (object-language) connective, say $\supset$, and the (meta-level) notation for the formal derivability relation, say $\to$. Of course, in this step Gentzen was greatly helped by his work on Hertz-systems from the summer of 1931, which output his first published paper of 1932.

Beside trying to retrace the intricate threads leading to the proof of the equivalence, I mean to focus on the emergence of two paradigms in the conception of Cut. The paradigm of *structural reasoning*, which was preserved in the intermediate calculus LDK of *Ms.ULS* , where the Cut rule continues to play a fundamental role, and the *analytic* paradigm. In the latter paradigm analytic proofs were the new goal, and Gentzen was able to attain it thanks to the *Hauptsatz* proved for that "evolution" of LDK-calculi which is constituted by the LK-calculi. In the latter calculi, structural reasoning was sharply separated from logical meaning, and the general setting was purely inferential.

The mathematics of derivability

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Traditionally, the notion of derivability (or provability) in proof theory is defined in terms of derivations: sequences or tree-like structures consisting of formulae or sequents, satisfying certain conditions involving proof rules. The 'driving force' of derivations usually consists of conditional statements: implications in the object language ($\varphi \to \psi$), entailments in the metalanguage ($\varphi, \psi \vdash \varphi \wedge \psi$), or proof rules involving sequents (if $\Gamma \vdash \varphi$ and $\Gamma, \psi \vdash \chi$ then $\Gamma, \varphi \to \psi \vdash \chi$).

I propose an alternative definition of derivability, capitalizing on the dynamic character of conditional statements. It is based on set-valued functions $\mathcal{F}\colon \wp(\mathsf{EXP}) \to \wp(\mathsf{EXP})$, where EXP denotes a collection of expressions, with the intended meaning: *for all $E \subseteq \mathsf{EXP}$, E entails the expressions in $\mathcal{F}(E)$*. So when EXP is a collection of atomic formulae, then $\mathcal{F}$ represents the Horn sentence

$$\bigwedge_{\Gamma \subseteq \mathsf{EXP}} \bigwedge_{\varphi \in \mathcal{F}(\Gamma)} \left(\bigwedge \Gamma \to \varphi\right).$$

When EXP is a collection of formulae of some logical language, then $\mathcal{F}$ represents the collection of sequents $\Gamma \vdash \varphi$ for all $\Gamma \subseteq \mathsf{EXP}$ and all $\varphi \in \mathcal{F}(\Gamma)$. And when EXP is a collection of sequents, then $\mathcal{F}$ represents the proof rule *from $\mathcal{S}$ infer* $\Gamma \vdash \varphi$, for all collections of sequents

$$\mathcal{S} = \{\Gamma_i \vdash \varphi_i \mid i \in I\} \subseteq \mathsf{EXP}$$

and all sequents $\Gamma \vdash \varphi$ in $\mathcal{F}(\mathcal{S})$.

In [1], I experimented with this idea in the context of propositional Horn logic. This led to several results on uniform and polynomial interpolation. Along the way, a characterization of validity was established: $\mathcal{F} \models \mathcal{G}$ iff $\mathcal{G} \sqsubseteq \mathcal{F}^*$, i.e. $\mathcal{G}(P) \subseteq \mathcal{F}^*(P)$ for all sets P of atoms. In other words: (the Horn formula represented by) $\mathcal{F}$ entails (the Horn formula represented by) $\mathcal{G}$ if and only if $\mathcal{G}$ is contained in the reflexive transitive closure $\mathcal{F}^*$ of $\mathcal{F}$. Moreover, it appeared that the set-valued functions form a *weak lazy Kleene algebra* (a notion inspired by [2]), governed by axioms like:

$(\mathcal{F} \sqcup \mathcal{G}) \circ \mathcal{H} = (\mathcal{F} \circ \mathcal{H}) \sqcup (\mathcal{G} \circ \mathcal{H})$,

$I \sqcup \mathcal{F} \circ \mathcal{F}^* \sqsubseteq \mathcal{F}^*$,

if $\mathcal{F} \circ \mathcal{G} \sqsubseteq \mathcal{G}$ then $\mathcal{F}^* \circ \mathcal{G} \sqsubseteq \mathcal{G}$.

Here $\mathcal{I}$ is the identity function, and $\sqcup$ is defined by

$$(\mathcal{F} \sqcup \mathcal{G})(P) = \mathcal{F}(P) \cup \mathcal{G}(P).$$

The left distributive version

$$\mathcal{F} \circ (\mathcal{G} \sqcup \mathcal{H}) = (\mathcal{F} \circ \mathcal{G}) \sqcup (\mathcal{F} \circ \mathcal{H})$$

of the first axiom does not hold, and neither do the variants

$$\mathcal{I} \sqcup \mathcal{F}^* \circ \mathcal{F} \sqsubseteq \mathcal{F}^*$$

and

$$\text{if} \quad \mathcal{F} \circ \mathcal{G} \sqsubseteq \mathcal{F} \quad \text{then} \quad \mathcal{F} \circ \mathcal{G}^* \sqsubseteq \mathcal{F}$$

of the second and third axiom.

In the paper abstracted here, the notions and results sketched above are extended to full Horn logic, where the atomic formulae contain terms and variables and where all formulae have implicit universal quantification at the outermost level for all occurring variables. For this purpose, the theory of set-valued functions is extended with substitutions $\sigma\colon \mathsf{EXP} \to \mathsf{EXP}$. The characterization of validity now reads

$$\mathcal{F} \models \mathcal{G} \quad \text{iff} \quad \mathcal{G} \sqsubseteq \left(\bigsqcup_{\sigma \in \mathsf{SUB}} (\sigma \cdot \mathcal{F}) \right)^*,$$

where SUB denotes the set of all substitutions and where $\sigma \cdot \mathcal{F}$ is defined by

$$(\sigma \cdot \mathcal{F})(X) = \{\sigma(\varphi) \mid \exists Y \subseteq \mathsf{EXP}(X = \{\sigma(\psi) \mid \psi \in Y\} \ \&\ \varphi \in \mathcal{F}(Y))\}.$$

With the proper establishment of notions and results for the combination of set-valued functions with substitutions, we can scale up to the investigation of proof systems for algebraic theories and propositional logics, involving sequents. The next step to first-order logic requires another extension to deal with variable binders (like quantifiers). All in all, it is my goal to substantiate the claim that *set-valued functions are a core ingredient for the proper mathematical analysis of derivability.*

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From Syntactic Proofs to Combinatorial Proofs

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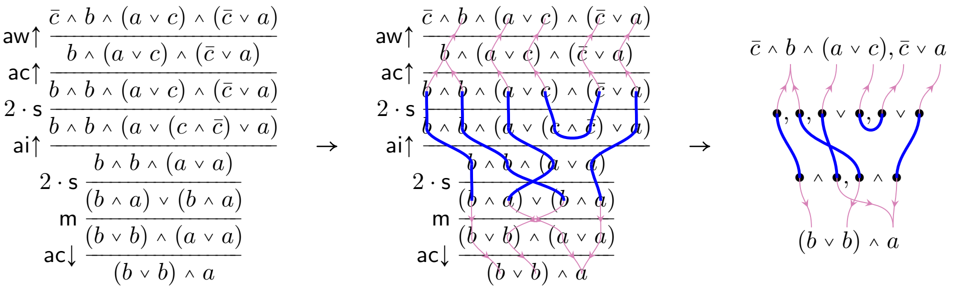
Proof theory is a central area of theoretical computer science, as it can provide the foundations not only for logic programming and functional programming, but also for the formal verification of software. Yet, despite the crucial role played by formal proofs, we have no proper notion of proof identity telling us when two proofs are "the same". This is very different from other areas of mathematics, like group theory, where two groups are "the same" if they are isomorphic, or topology, where two spaces are "the same" if they are homeomorphic.

The problem is that proofs are usually presented by syntactic means, and depending on the chosen syntactic formalism, "the same" proof can look very different. In fact, one can say that at the current state of art, *proof theory is not a theory of proofs but a theory of proof systems.* This means that the first step must be to find ways to describe proofs independent from the proof systems. In other words, we need a "syntax-free" presentation of proofs.

Combinatorial proofs form such a canonical proof presentation that (1) comes with a polynomial correctness criterion, (2) is independent of the syntax of proof formalisms (like sequent calculi, tableaux systems, resolution, Frege systems, or deep inference systems), and (3) can handle cut and substitution, and their elimination. Below is an example showing how a combinatorial proof can be extracted from a deep inference derivation:



In a nutshell, a combinatorial proof consists of a purely *linear* part (depicted above in blue/bold) and a part that corresponds to *contraction* and

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weakening (depicted above in purple/regular). Combinatorial proofs can be composed horizontally and vertically, and can be substituted into each other.

In this presentation, I will discuss the basic definition of combinatorial proofs, how they can be normalized, and how we can transform syntactic proofs into combinatorial proofs and back.

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[*]Leibniz International Proceedings in Informatics

Logic and Music

This workshop is organized by

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This workshop shall represent a privileged platform to make an important step forward to new universal approaches to logic(s) of music. The story goes back at least to the ancient Greeks: Aristotle invented his logic as a formal theory of syllogisms of categorical sentences. Pythagoreans brought mathematics and music into a very general and close contact (harmony of spheres). It depends from our understanding of the relation between logic and mathematics whether there is a direct connection between them with respect to music. In the 16th century we find an intensive use of several aspects of syllogistic forms in the context of cantus (composition). Leibniz looked on logic as "ars inviendi" on the basis of his new understanding of syllogistic forms. But he also develops his "algebra of thought". Algebraic investigations of different aspects of music can be seen as purely mathematical or as autonomously logical (model-theoretical) paradigms.

The workshop focuses on the relatively autonomous approaches to logic(s) of music and musical logic, i.e. logic in pieces of musical compositions. We invite composers, conductors, musicians and musicologists interested in the interplay between logic and music to submit a paper or just active participation. Another objective is to bring together researchers from all over the world into closer contact. The organizer of this workshop plans to edit a special issue of a journal (e.g. Logica Universalis).

The keynote speaker at this workshop is Thomas Noll (page 146).

Call for papers

Topics may include:

- syllogistic forms in musical patterns
- Leibniz's logic of music
- axiomatic and model-theoretic music theory*

*This research topic was initiated by Susanne K. Langer, "A Set of Postulates for the Logical Structure of Music", *The Monist*, vol. **39(4)**, Oxford University Press, 1929, pp. 561–570.

- music in the context of non-classical logics like intuitionistic logic, many-valued logic, modal-tense logic etc.
- logical vs. mathematical approaches to music
- rules and syntax in logic and music
- recursiveness and compositionality in logic and music
- logic of melodies
- logic of scales, intervals and chords
- chord operators
- logic of musical harmony
- logic of cadence
- logic of rhythm
- logical spaces of music
- logic of music vs. musical logic
- logic of (violation of) musical expectation
- applications of a logic of music: analysis of pieces of composition, (automatic) pattern recognition, creation of new pieces of music

Contributed talks should not exceed a duration of 30 minutes including discussion. Abstracts (500 words maximum) should be sent via e-mail, before October 5, 2017, to logic.music2018@gmx.de.

Outside-in or inside-out? A logic for human sensory system

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According to [2] there is a transfer of structure from waveforms to auditory experiences. Colour experience is completely different in this respect. The causal connections here are outside-in, but not inside-out; the experience of colour gives us information that enables us to undertake epistemic activities concerning external things, but it is not innately associated with the ability to produce or adjust the colour values of the things one sees. If colour experience is useful for influencing the world, it is so only by the intermediary of acquired generalizations. An artist can reproduce her colour experiences in paint. But she has to go to Art School to learn enough about paints to do so. When we identify the external properties correlated with the experience of colour, we go beyond untutored vision, reconstructing the external reality of colour with the help of information additional to that which is available in colour-experience alone.

If we follow [2], the component incompatibility of red and green would, however, seem to be an artifact of the human red-green visual opponent processing system. We hear sounds and smell smells, but we do not as a rule hear and smell the objects that are their causal source as qualified particulars. But, if it is just a question of causal source Matthen's thesis is trivial. Paradoxically, visual imagery a priori would be more reliable than visual perception.

Furthermore, I think that if Matthen aims to demonstrate that the logic of determinable and determinate implicit in the function of human sensory system is not conceptual, he reintroduces the same logic when he talks about musical harmony. Quoting [3], we could say that the musician not only thinks about sound, but also 'in sound'. Thinking about sound, it might be admitted, can be construed on a linguistic model — an 'inner speech' using the vocabulary of auditory qualities and relations. Indeed much of the thinking that a composer does is conceptual thinking about the relationships of sound patterns, and since the notion of conceptual thinking as analogous to language leaves open the question of how precise the analogy is, it is surely not too far-fetched to take a linguistic approach to this aspect of the composer's activity. With the aspect I referred to above as 'thinking in sound', a more intimate relationship between composition and sound, the 'linguistic model' begins to look far too narrow and specialized, a limit for Matthen's model.

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*Lecture I: Perception; Lecture II: Minds; Lecture III: Epistemic Principles.

Inferentialism and Music: the Art of Implication and Negation

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The aim of my paper is to establish link between musical experience and Brandom's "space of reasons" in its role of the linguistic proxy for Hegel's System of Spirit. My proposal goes like this: First, one takes the basic connectives of *formal logic* — implication and negation — and gives them a more general meaning of (materially good) inference and conceptual incompatibility. Since these can be found in every reasoning, the foundations of "the System" are set with formal logic adopting a more general, *dialectical function*. Accordingly, Brandom tracks the concepts of inference and incompatibility back to Hegel's notions of mediation (modeled on the middle term of a syllogism) and negation (in the specific sense of differentiation). This framework is broad enough to allow the musical experience to fit in.

In this final step, I am drawing on the work of Leonard Meyer who, in his theory of musical understanding, combines Peirce's consequential theory of meaning (according to which the meaning of some event consist in the sum of its consequences) with Dewey's conflict theory of emotions (according to which emotions are adjustments of some conflicting changes in our body). Just notice the joint presence of both, implication and negation here.

Now, according to Meyer's theory, the emotions employed in music are aroused when an expectations, a tendency to respond, is skillfully suspended or permanently blocked. The basic expectational tendencies are those traceable to the regular functions of organism such as breath and pulse. So, e.g., the strong emotional reaction typically results from the conflict between the expected regular succession of *strong and weak beats* and its inhibition by means of *syncopation*. These immediate emotions are adjusted by those concerning general matters of style, such as the existing keys in which the piece can be played or the ways one usually closes the phrase, to be negated by means such as an unexpected modulation or a deceptive cadence.

On the one hand, establishing link between musical space and space of reasons is easy now, since the expectations are obviously inferentially articulated. On the other hand, the suggested link is complicated one because in musical space the self-consciousness is at work from the very beginning: one of the musical effects is achieved by composers deliberate evoking expectations that are negated later. I will close my argument by means of an example, sketching a line in which the emotional reactions might be developed from relatively basic, transitive ones to those that are more complex and intransitive, and in which the self-conscious interplay of “implication” and “negation” is entertained in a transparent way.

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Musical Activity as the Basis for the Evolution of Joint Intentionality and Nonlinear Grammar

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The perfection of singing, drumming and dancing, performed in unison, may have driven our evolutionary ancestors to develop joint intentionality, nonlinear grammar, and additional traits which distinguish us, as humans, from the other great apes.

As Tomasello has persuasively argued, what seems to distinguish us from the great apes is our joint intentionality. Chimpanzees play different roles in hunting a monkey. But each will abandon the team if distracted by something more rewarding. One-year-old human infants, on the other hand, persist until everyone on their team receives their reward. Our ability to be an ad hoc “we” makes us human.

Humans typically manifest this solidarity physically through body language. We exhibit a “sixth sense” by which we unconsciously orient our-

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selves towards those present around us. We synchronize all of our movements. When we slow down a video of people, they seem to be dancing. Other great apes may lack this joint synchronicity.

Musical activity could have fostered such joint synchronicity. It may have started with identical twins. Rhythmic unison — singing, drumming and dancing together — may have attracted mates, and engendered a virtuous cycle of rapid evolutionary change. This would have fostered that "sixth sense" but also improved vocal chord control. For the group, musical activity could have heightened the sense of "we" before and after a shared activity, such as a hunt. Work songs fostered a sense of shared work. As the repertoire of songs grew, they could influence language.

In linguistics, Jackendoff has noted that syntax must have arisen after a protolanguage with a linear grammar which was quite robust. Such a linear grammar is used by sailors speaking pidgin; second language users who never develop fluency; people with certain brain injuries; deaf children who develop their own gestures; but also the great apes. It consists of strings of words for which there are no rules. Word order is simply determined pragmatically in context.

Joint musical activity demands a perfection of all and at all times. It thus legislates rules. Sounds or words must be annunciated exactly. Rituals develop. Words and concepts become categorized, as Levi-Strauss observed. People develop a sense of right or wrong, in-tune or out-of-tune, grammatical or ungrammatical. Rules must be followed in creating new words. Syntax arises as rules which may not be broken, and is distinct from pragmatic constraints.

Language arises from pragmatic activity. Nonlinear grammar arises with the division of labor such that we can perform a task that we do not completely understand, as when using a new word, or playing our own part in a greater musical whole. This fosters our ability to hear what others are saying as well as what we ourselves are thinking. Whereas perhaps other apes can only think one perspective at a time. Thus they can answer questions but they never ask them. Musical activity teaches us to be "I" and "you" and "they" and "we", in parallel.

Listening and Reading: Temporalities of Musical Performance and Notation

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Since McTaggart's ground-breaking article of 1908, most philosophers of time have recognised the distinction he draws between dynamic ("A-Series") and static ("B-Series") orderings of temporal sequences. Though there is much debate about the moral(s) to be drawn for the metaphysics of time, the distinction itself is firmly established and well understood (Harrington 2013). The aim of this talk is to exploit the distinction to draw out some structural differences between various modes of access to musical sources.

On the one hand, the experience of listening to music forms an ordered sequence (from start to finish), has fairly determinate extension in time (not too fast, not too slow), and requires attention from moment to moment (not just overheard). These features can be captured by an "A-Series" analysis and point to some essential characteristics of the enjoyment of music. On the other hand, the analysis of a score permits us to examine structural features (such as symmetries), to linger or skip, and to break off at will. The notation is spread out in space, not unlike the time-lines deployed by theorists of the "B-Series" to conceptualise the relations of "earlier than" and "later than".

The question arises — but can hardly be settled other than by stipulation — of the possible senses in which a reading of a score can be a source of properly musical enjoyment, and a disanalogy is suggested between the priorities here and those we find, for instance, between the spoken and the silently read in the reception of poetry: the notes on the stave represent sounds, where the words of a poem are words.

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Is there any logic of harmony?

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A logic can be understood as the fixation of a set of constitutive rules (a *codex*, a formal game). I.e., we fix the *internal* meaning of all symbols (figures, players) involved in this game. We define inference relations and prove theorems in the object language as well as in the meta language. *Harmony* as a context-free relation between chords can be understood as the internal meaning of music.

The minimum assumptions of a logic of harmony are:

1. Fixation of *logical space*: Our logical space is the *chromatic scale* of tone pitches explicated by a discrete scale of integers. Each integer can be used to represent exactly one single tone. *Tone intervals* are ordered pairs of pitches and will be represented by ordered pairs of integers of the form $\langle x_i, x_j \rangle$ with $x_i > x_j$. Each interval has a characteristic positive length l $(l > 0)$: $\mathrm{L}(\langle x_i, x_j \rangle) = l$. The chromatic scale — like the scale of integers — is to be thought as open in both directions and, therefore, infinite. Independently from our hearing capacities we have an infinite number of tones, intervals and chords.

2. Declaration of *chords* as our basic (minimal) expressions context-freely identifiable by its inner structure: A key feature of the logic of chords is that this formal theory is *not* an *atomistic* one. The basic elements are chords consisting of at least three tones, two *basic intervals* and one *reference interval*. A basic interval is the relation between directly adjacent tones in a chord. The reference interval is the relation between the sharpest and the deepest tone of any chord. If we consider chords with more than 3 tones we have at least one level of intermediate intervals. A chord is a molecular expression characterized not only by its tones but mainly by its *matrix of interval lengths*. Each chord can be uniquely identified solely by its *inner* structure. A class of (partially or totally tone-different) chords — e.g., the class of 3-tone-major-chords in root position — can be identified simply by knowing its characteristic matrix of interval lengths common to all of its elements. Chords are logically independent of each other in the sense that any sequence of chords is allowed without any restriction.

3. *Chord operators*: Chords are *harmonically dependent* in the sense that each pair of chords constitutes *internal* harmony which can be described by

using chord operators. Internal harmony is nothing else than the relation between two or more chords based solely on the inner structure of these chords. In this sense “chord” as well as “harmony” are formal concepts. Euphony is not necessary. E.g., we have of course chords and harmony in twelve-tone music (dodecaphony) and free jazz in a chromatic space. A *unary* chord operator takes a chord as its input and yields a chord as its output. There are operators permuting the lengths of basic intervals. Examples are *complete inversions* of basic interval lengths relative to tone-related or interval-related fixed points (among them a kind of *non-classical negation of chords*) and *cyclic permutation operators*. There are *tone-related operators* like barré operators (outputs with identical matrices of interval lengths. An n-ary chord operator ($n \leq 2$) takes an n-tuple of chords as input and yields a chord as its output. If it comes to more complex harmonic constructions like sequences consisting of tonic, subdominant and dominant (*cadences*) it can be useful to have at least binary operators to create them.

4. *Inference relations*: Characterization of types of tone-related inference relations as well as inference relations with respect to interval lengths: *Tonality* can be understood as rule-governed restrictions on the inner structure of chords as well as restrictions on the sequences of chords (e.g., creating cadences) accompanied by special inference properties.

If total this means that we will show that there is a partial positive answer to our initial question but the jury is still out.

Musical Performance: a Composition of Monads

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Much work has been done on computer representations of music at the physical level. Developments such as K-nets by Klumpenhouwer and Lewin provide a way for representing transformations from one pitch-class to another. Category theory should facilitate the development of a logical approach to music, which can be mapped into one of the physical approaches for implementation. Towards this aim Mazzola and Andreatta developed the idea of a category of directed graphs (objects = notes or chords, edges

= music operations such as transposition), as a topos-based approach for a description of the music, ultimately delivering the generalised PK-net with the concepts of form and support. PK-nets enable heterogeneous collections of musical objects to be naturally compared and manipulated as described by Popoff, Andreatta and Ehresmann [1].

The work to be presented builds on that developed for information systems, taking up the challenge of a testing application for the Cartesian monad approach to universal design [2]. A principal aim is to capture the performance of music as a communication between the musicians and the audience using the categorial construction of a monad. In this respect the monad, a term originally used by Leibniz, presents a musical performance as a composition over time signatures, with adjointness between each step: the monad looking backwards and then forwards and its associated comonad looking forwards and then backwards. The physical characteristics of the notes in each time-frame are complex, so it is necessary to use a strong Cartesian monad, facilitating the representation of each time-frame as a product. The monad is process, handling dynamic aspects. The category upon which the monad operates is a topos holding relatively static information such as the players, the score and the venue, together with the relationships between them. The topos is far from totally static with its arrows facilitating flexibility in all information held, including relationships; the topos is also searchable through the subobject classifier. There is no assumption of any particular musical genre. Such a categorial framework could be implemented in a functional programming language such as Haskell, under the control of a scripting language such as Forth, as employed in the Blockchain method.

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*Alternative Natural Philosophy Association

The Logic of Social Practices

This workshop is organized by

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Social Practices refer to everyday practices which are routinely performed. They integrate different types of elements such as bodily and mental activities, material artifacts, knowledge, emotions, skills, etc.

The keynote speaker at this workshop is Søren Brier (page 122).

Call for papers

We invite submissions on the following topics:

- Philosophy of Social Practices (Semiotics, Pragmatism, Analytic Pragmatism, Collective Intentionality)
- Practice Theory (Bordieau, Giddens, Foucault, Schatzki) and applications
- Role of material artifacts and material resources (Warde, Schatzki, Reckwitz and Shove, among others)
- Routine behavior as formation of habits
- Simulation models to investigate the emergence of social practices
- Routine behavior (habits) vs. socially shared behavior (rituals)

Abstracts (one page) should be sent by November 15, 2017 via e-mail to giovagnoli@pul.it.

A Computable Model of Amartya Sen's Social Choice Function in the Framework of the Category Theory Logic

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A significant part of the short history of the mathematical theory of social, political, and economical sciences, specifically in the context of welfare politics and economy, is related with the development of the notion and the theory of the so-called "social welfare function (SWF)". This theory started with the pioneering contributions of A. Bergson and of the Nobel Prize P. Samuelson leading to the "Bergson-Samuelson SWF" [1,2], but received a substantial improvement by the contribution of another Nobel Prize, K. Arrow [3]. Arrow's SWF is intended as a function ranking social states as less, more, or indifferently desirable, for every pair of them, with respect to individual welfare measures and/or preferences. One of the main uses of SWF is aimed, indeed, at representing coherent patterns (effectively, structures) of collective and social choices/preferences as to alternative social states. The essential limitation of SWF's is that they are defined in the framework of an approach to the study of social and economic systems stable at equilibrium like in statistical mechanics. They are all inspired, indeed, by Samuelson's general approach to mathematical economics in his seminal handbook [2], based on Gibbs' statistical thermodynamics of gases, to which the first two chapters of the book are significantly dedicated, because naturally consistent with the liberal individualistic vision of economy and society. Unfortunately, a fundamental unexpected and undesired consequence of Arrow's mathematical theory is the famous "Arrow's impossibility theorem", demonstrating the mathematical inconsistency for democratic systems of social choices based on the majority decisions. The main contribution of A. Sen's theory of social choice functions (SCF) [4], for which he was awarded with the Nobel Prize in Economics in 1998, was the formal demonstration that the only way for avoiding Arrow's impossibility results is introducing in the model the *interpersonal comparison of utilities* — and generally the information exchange among persons constituting homogeneous groups, on the contrary considered as irrelevant in the classical economic theory. This allows also to introduce into the mathematical modeling of SCF Theory *distributive principles* of social and economical *justice*, such as, for instance, the famous J. Rawl's *maxmin principle,* which gives priority to the interests of worst-off persons. This transforms SCF Theory into a *normative theory of social choices.* On this regard, Sen demonstrated that an effective mathematical modeling of ethical constraints in economy cannot be based on abstract and not-computable *optimal choices* defined on the complete (total) ordering of social/economical states in a society, but on concrete criteria of *maximal choices* relative to the different contexts, and then defined on *partial orderings,* not necessarily satisfying a *transitive relation* among the different social aggregates (sets) of persons so defined, and between groups and the whole society. All this means that the physical paradigm underlying Sen's mathematical theory of economy and society is no longer the gas thermodynamics stable at equilibrium of the liberalism mathematical models, but *the fluid thermodynamics* of condensed matter systems, stable in far from equilibrium conditions, characterizing a "liquid society" such as ours. The real-time information exchange among communication agents determines the fast aggregation/dissolution of interest groups in a worldwide environment — think, for instance, at the stock-exchange market and at the infinite flow of data streams it produces. Unfortunately, this condition makes unrealistic a SCF/SWF Theory based on finite [5], and then Turing-computable sets, because, on infinite sets, Sen's maximal partial orders correspond to as many ultrafilters requiring higher order functions to be calculated [6]. We propose in this contribution an original solution of this problem in the formal framework of the Category Theory, based on the categorical dual equivalence (anti-isomorphism) between co-algebras (environment) and algebras (system), originally applied to the mathematical modelling of condensed matter thermodynamic systems, stable in far from equilibrium conditions, in the context of quantum field theory of dissipative systems, human brains included [7,8,9]. The same categorical duality co-algebras/algebras is used also in theoretical computer science, for formalizing the effectiveness of dynamic computations on infinite data streams with always changing inner correlations — i.e., on infinite data sets just it is the case of Sen's SCFs. This approach is inside the paradigm of the Algebraic/Co-algebraic Universality in computations [10,11], which is wider than the classic Turing Universality, "probabilistic", "quantum" Turing Machines included.

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Rituals as "Social Habits"

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Rituals can be considered as social practices or habits in an "institutionalized" We-mode. They have the important function to create social spaces in which individuals can share emotions, experiences, values, norms and knowledge. They need human cooperation as a kind of intersubjectivity typical of human beings who, differently from apes, are able to have "Collective Intentionality" [11,7], i.e. the basic intention to cooperate and therefore to reach together a certain goal. There is a contemporary lively debate on the nature and structure of Collective Intentionality, as necessary notion to researches in the field of social ontology (the pioneers in this area are John Searle, Raimo Tuomela, Margareth Gilbert, Michael Bratman and Philip Pettit).

Our aim is to try to isolate a process that is common to habits and rituals and this process is related to a reduction of complexity (in the Aristotelian sense it entails habitus and consuetudo), that characterizes individual and social ordinary life. But, to share habits in a larger environment where ritual can become public, with specific rules of behavior, that make it recognizable from people inside and out side the community, we need a third level of behavior, i.e. the process of institutionalization of them. The set of acts which characterizes human habits can be institutionalized to form the cultural rituals that belong to human life-forms. What we must clarify here is how this institutionalization is possible and actually works. In this sense what can seem merely shared habits become "social" in a strong sense, and reinforce their function in establishing solidarity and social identity.

We can observe that human beings (but also other species) have the capacity to impose a function to an object so that the object acquires a function dependent on the peculiar scope of the agent. The continuity between individual habits and rituals (social habits) is thus showed by the fact that humans create these "agentive functions" (in Searle's terminology) in a wide variety of situations. Also non human animals have their form of creating functions for objects but there is a fundamental difference in the concept of "function" in the human case.

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Collective Phronesis? An investigation of collective judgement and professional knowledge

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In the proposed paper we will discuss whether, and if so how professional action can be understood as a collective capability, and in what way this capacity can be understood in terms of Practical wisdom, Phronesis [see 1,4]. We will investigate the idea of "collective phronesis" by use of two examples from different professional practices, such as teaching and policing. Professional action, e.g. in the role of a teacher or police officer, seems — by definition — to transcend the individual horizon of an agent. The professional identity implies that the individual (implicitly and/or explicitly) takes a representational stance regarding the profession as a collective. The question is how this can be understood.

In the contemporary debate within social epistemology on the possibility of collective agency, one often speaks of two distinct cases/forms of collective judgement [2,3]. In the first case an individual (here: professional) has to judge upon a situation that requires social evidence, i.e. the judgments of others e.g. colleagues or on routines and informal rules. The second case is about situations where a group of individuals (here: professionals) act/judge together. Looking at the everyday practice of professionals, such as police officers or teachers, one can see that professional practices often have to deal with conflicts between these two cases. This is e.g. the case when it is not clear whether the judgement of the group is based on social evidence or informal rules.

In our presentation we will in a first part present two examples from

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the everyday practice of teachers and police officers where two options presented above turn out to conflict with each other. In the second part of our presentation we will provide a critical analysis of the Aristotelian concept of "phronesis", that is traditionally discussed within the framework of individual agents and judgement [5], and discuss it in relation to the question of the possibility of collective judgment. In a third part we discuss in what way a professional judgement or action could be understood as an expression of "collective phronesis".

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Bridging Habits and Cognition: Inference and Category Learning through Neural-Dynamic Logic

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The link between, on the one hand, habits and goal-directed learning within a dual-process structure, and, on the other hand, the learning of cognitive capacities has a tradition within animal (and human) learning paradigms. Neural network models of animal learning have been used to learn associative relations between stimuli or/and processes and provide new hypotheses as to the nature of learning and relationships between learning systems. However, more biologically realistic models that account for the continuous (spatiotemporal) dynamics of the learning/decision making problems that such theorized animal learning processes are tested on may suffer from the challenge of finding an appropriate and cognitively intuitive parameterization. As a consequence, the modelled processes in learning may not be easily tuned to each other. Following recent interest in integrating principles of logic into neural computational modelling, in this article, by way of an example concerning the Associative Two-Process theory, we suggest a neural-dynamic logic approach to understanding the nature of the interaction between the two (habitual and goal-directed) learning processes. We attempt to describe how the cognitive phenomena of categorization by common outcomes, and transitive inference, can be grounded in the learning and interactions of habits with goal-directed systems.

Moral Bubbles in Action
The Logic of Cognitive Autoimmunity

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In [1], Woods described the "epistemic bubble" as an *immunized state* of human cognition [2] that compromises the awareness of the agent about her beliefs and knowledge. In my presentation I will introduce a symmetrical view on the agent immunization, focused on the agent's missing awareness of her potential or actual *violence*, also highlighting the importance of considering the actual agent as cogently *moral*.

A basic aspect of the human fallacious use of language in social settings, as far as its effects are concerned, I call "military", is the *softness* and *gentleness* granted by the constitutive capacity of fallacies to *conceal errors*. Being constitutively and easily unaware of our errors is very often intertwined with the self-conviction that we *are not at all violent and aggressive* in the argumentation we perform (and in our eventual related actions). In this last case we are dealing with what I have called a *moral bubble* [3]. I believe that the issue of the violence embedded in morality provides us with a significant clue about the existence of something akin to a *moral bubble*, that is very homomorphic with the epistemic bubble, in which an agent is "trapped". One should never forget how:

- unawareness of our error is often accompanied by lack of awareness regarding the deceptive/aggressive character of our speech (and behavior).

After all when we act morally we "want" to believe we are acting in a non violent way *a priori*, and we "want" to preserve the *moral bubble* we are in, which permits us to erase the possible violence we are dealing with. In this perspective morality is strictly intertwined with violence.

Finally, I will contend that moral bubble is also a necessary condition to the social survival of morality itself: its scope is to avoid the cognitive breakdown that would be triggered by the constant appraisal of the major or minor inconsistency of our conduct with respect to our convictions.

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John Searle as Practice Theoretician

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I will research John Searle's [1] idea of social causation as collective intentionality and (according to my understanding) the more important notion of background. I will compare Searle's notions of collective intentionality and background of the social to Maurizio Ferraris's [2] notions of text as replacement of collective intentionality. The problems that Ferraris addresses are understood here in terms of practises. In this article I will look into more contemporary debates in social ontology and in practise approach Theodore Schatzki [3] is a kind of pioneer in this approach. My viewpoint to Searle comes from two ordinary language philosophers: John Austin and Ludwig Wittgenstein. I will interpret Searle as an analytic version of the practise approach. By this I mean that Searle has analytically distinguished social practise to have three components, which are performative, background, and collective intentionality. These notions, combined with the Practice approach of Schatzki, will help me to formulate a constructive critique of Searle's theory. mostly in terms of Practise Approach of Schatzki, even though many theories are used. I will concentrate. The main focus will be in the understanding of background as the collective intentionality of being part of the practise in general.

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Polarization Dynamics in the Age of Social Media

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Information, rumors, and debates may shape the perception of reality and heavily impact public opinion. On online social networks users tend to select information that is coherent to their system of beliefs and to form polarized groups of like-minded people — i.e., echo chambers — where they reinforce and polarize their pre-existing opinions. Such a context exacerbates misinformation, which has traditionally represented a political, social, and economic risk. Indeed, since 2013 the World Economic Forum has been listing massive digital misinformation at the core of other geopolitical risks, such as terrorism or cyberattacks. In this talk we explore how we can understand social dynamics by analyzing massive data on online social media. We provide the empirical existence of echo chambers, showing that confirmation bias is the main driver behind content consumption [1]. Moreover, we address the emotional dynamics inside and between different narratives, and investigate users' response to both confirmatory and contrasting information [2,3]. Moving beyond misinformation, we show that similar patterns may be observed around both the Brexit — the British referendum to leave the European Union — and the Italian Constitutional Referendum debates, where we observe the spontaneous emergence of well-segregated and polarized groups of users around news sources [4]. Finally, we characterize the anatomy of news consumption on a global scale. By means of a tight, quantitative analysis on 376 millions users and 920 news outlets, we show the natural tendency of users to focus on a limited set of pages (*selective exposure*) eliciting a sharp and polarized community structure [5]. Our findings provide interesting insights about the determinants of polarization and the evolution of core narratives on online debating, and highlight the crucial role of data science techniques to understand and map the information space on online social media.

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The Logic of preferences and a settlement of conflicts (based on the modeling of the Nagorno-Karabakh conflict)

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Political processes can be described by means of modal logic and modal semantics (semantics of possible worlds). The suggested model for the description of conflict and its possible settlement (resolution) is based on the logic of preference, developed by von Wright [1], and complemented with some elements of temporal logic [2]. In the case of the Nagorno-Karabakh conflict we suggest to use its substantial interpretation, this makes possible to compare different state of affairs and find the compatible models of common future, where the state of peace or the state of absence of military actions are possible. This means that the preferable state of affairs is the situation which is not the best for any of the participants, but at the same time it is not the worst for any of them (there is no winner, who "takes all", but there is no explicit looser).

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However, the existing political language does not provide opportunity for such descriptions From logical and semiotic point of view it implies that the language based on binary oppositions should be abandoned and replaced by the multivalent semantics and enriched by some ambivalent complicate concepts:

$$war \mathbin{\&} peace, war \mathbin{\&} {\sim}peace, {\sim}war \mathbin{\&} {\sim}peace, {\sim}war \mathbin{\&} peace.$$

With respect to the existing conflict-settlement practice there is a strong need to change conceptual framework in such a way that will make possible to reach a true compromise, instead of alternatives which actually are acceptable only to one side. The suggested approach and the procedure of the substantial interpretation of logical pattern of preferences can be applied to the all types of conflict, if initial stands of conflicting parties can be explicated as some system of basic propositions and propositional attitudes.

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Model Theory

This workshop is organized by

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Model theory is the branch of mathematical logic dealing with the connection between a formal language and its interpretations, or models, i.e., it represents links between syntactic and semantic objects. These objects can be used to classify each others producing structural classifications of theories and their models. Solving classification questions valuable characteristics arise (dimensions, ranks, complexities, spectra etc.) for various classes of structures and their theories.

The keynote speakers at this workshop are Bruno Poizat (page 154) and Sergey Goncharov (page 129).

Call for papers

We invite contributions on all aspects of Model Theory. Topics include:

- Equational classes, universal algebra
- Basic properties of first-order languages and structures
- Quantifier elimination, model completeness
- Finite structures
- Countable structures
- Uncountable structures
- Model-theoretic constructions
- Categoricity and completeness of theories
- Interpolation, preservation, definability
- Classification theory, stability and related concepts
- Abstract elementary classes and related topics

- Models with special properties
- Properties of classes of models
- Effective and recursion-theoretic model theory
- Model-theoretic algebra
- Model theory of ordered structures; o-minimality and their variations
- Logic on admissible sets
- Second and higher-order model theory
- Nonclassical models
- Abstract model theory
- Jonsson theories
- Topologies on classes of theories and their models
- Applications of model theory

Abstracts (one page) should be sent by November 15, 2017 via e-mail to sudoplat@math.nsc.ru.

Unification in linear multi-modal logic of knowledge and non-transitive time*

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Unification problem in the field of non-classical logics often formulated as the possibility of a formula to become a theorem after the replacing of variables. In the late 90's [1] S. Ghilardi proposed an important approach to the unification using the projective formulas that allowed to get the solution of constructing finite complete sets of unifiers for a lot of logics. A number of remarkable consequences were found from the projective unification (e.g. unitary type of unification, in dealing with bases of admissible rules [2], almost structural completeness [3]).

In [4] we investigate logic, based on the idea of non-transitive time [5], which suggests that the available in the past data may not be transferred to the present. Here we consider the multi-agent case of this logic with the universal modality.

Let, in our notation, $\mathcal{LITK}$ be the logic characterized by the temporal Kripke frame $F = \langle W, R_1, \ldots, R_k, R_e, \mathrm{Next}\rangle$, where W is the disjoint union of the clots (tense moments) C^t, $t \in \mathbb{N}$; $R_1, \ldots, R_k$ are some equivalence relations within each clot; R_e is $\mathcal{S}5$-equivalence relation in clots; and Next

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is the relation such that

$$\forall a, b \in W\colon a \text{ Next } b \Leftrightarrow a \in \mathrm{C}^t \;\&\; b \in \mathrm{C}^{t+1}.$$

We extend the language of $\mathcal{LITK}$ by adding the operator of universal modality $\Box_{\mathrm{U}}$ and define the truth values of formulas containing $\Box_{\mathrm{U}}$ on the model $M = \langle F, V\rangle$:

$$\forall x \in F, \langle F, x\rangle \Vdash_V \Box_{\mathrm{U}}\varphi \Leftrightarrow \Big[\forall y \in F, \langle F, y\rangle \Vdash_V \varphi\Big].$$

The logic $\mathcal{LITK}$, which language $L^{\mathcal{LITK}}$ containing $\Box_{\mathrm{U}}$ is called the linear multi-agent logic based on non-transitive time with universal modality ($\mathcal{ULITK}$ for short).

Theorem 1. Unifiability of any formula $\varphi(p_1, \dots, p_s)$ in $\mathcal{ULITK}$ can be effectively checked using the substitution $\sigma(\varphi)$ of the following form:

$$\forall p_i \in \mathrm{Var}(\varphi)\; \sigma(p_i) \in \{\top, \bot\}.$$

Theorem 2. Any unifiable in $\mathcal{ULITK}$ formula is projective.

An algorithm for constructing most general unifier is proposed: it suffices to write out the following substitution $\sigma(p_i)$ instead of all variables p_i of a given unifiable formula φ:

$$\sigma(p_i) \coloneqq (\Box_{\mathrm{U}}\varphi \wedge p_i) \vee (\neg\Box_{\mathrm{U}}\varphi \wedge \mathrm{gu}(p_i)).$$

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Computable Modal Algebras and Contact Algebras

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It is well-known that the class of Boolean algebras is not universal from the computability-theoretic point of view: In particular, a computable Boolean algebra is computably categorical iff it is relatively computably categorical [1]. On the other hand, in general structures, the notions of computable categoricity and relative computable categoricity do not coincide.

Khoussainov and Kowalski [2] initiated the line of research that aims to answer the following question: How does expanding the language of Boolean algebras affect computability-theoretic properties of the class? One of the methods to investigate this problem involves using the approach of Hirschfeldt, Khoussainov, Shore and Slinko [3]. They introduced the notion of an *HKSS-complete* class of structures. The informal idea is the following: If a countable structure has some interesting computability-theoretic property, then for any HKSS-complete class K, one can find a structure $\mathcal{S}$ from K possessing the same property (see [3] for details). In [2], it was shown that the class of Boolean algebras with operators is HKSS-complete.

Let $\mathcal{B}$ be a Boolean algebra. A function $f\colon |\mathcal{B}| \to |\mathcal{B}|$ is a *modality* if it satisfies the following two properties: $f(0_{\mathcal{B}}) = 0_{\mathcal{B}}$, and $f(a \vee b) = f(a) \vee f(b)$ for all $a, b \in \mathcal{B}$. In [4], it was proven that the class of Boolean algebras with four distinguished modalities is HKSS-complete. A *modal algebra* is a Boolean algebra with one distinguished modality.

Theorem. The class of modal algebras is HKSS-complete.

A similar result is obtained for the class of contact algebras introduced by Dimov and Vakarelov [5] for the study of the region-based theory of space.

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Syntactic and Semantic Presentations of Scientific Theories in Abstract Model Theory

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This paper focuses on the logic relations holding between syntactic and semantic presentations of empirical theories. Two problems advanced in [2] are extensively examined here: first, the problem of defining a notion of equivalence such that the theoretical equivalence of two syntactic theories implies the model-theoretic equivalence of the two corresponding semantic theories. Secondly, the problem of establishing whether language translations of a syntactic theory are in a many-to-one or many-to-many relation with the corresponding semantic theory(ies). This paper introduces the *theory of institutions* [1] to show the logical duality holding between syntactic and semantic presentations of a given theory in a language-independent context and for any-order and multi-sorted logics.

First, syntactic and semantic presentations of scientific theories are formalized in the theory of institutions framework. The notions of theoretic and model-theoretic equivalence are then defined as isomorphisms in the categories Th and Vth of, respectively, syntactic and semantic theories. Secondly, it is proven that given an institution I, two syntactic theories over I are equivalent if and only if the corresponding semantic theories over I are equivalent. Finally, the many-to-many logic relations holding between language translations of a syntactic theory and the corresponding semantic theories are shown in terms of functors mapping syntactic theory morphisms in Th to semantic theory morphisms in Vth.

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On almost deterministic algebras of binary isolating formulas for polygonometrical theories

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We consider (almost) deterministic possibilities of algebras $\mathfrak{A}$ of binary isolating formulas [1] for polygonometrical theories [2]. Recall [1] that $\mathfrak{A}$ is said to be (*almost*) *deterministic* if for any labels u and v the set $u \cdot v$ is a singleton (respectively finite).

Proposition. If $\mathcal{P}$ is a plane and G_1 is finite then the algebra $\mathfrak{A}$, for the theory $T(\mathrm{pm})$, is $(|G_1| + 1)$-almost deterministic.

This proposition can be generalized, producing almost deterministic algebras $\mathfrak{A}$, if G_1 is finite and $\mathcal{P}$ is an almost plane, i.e., for any side parameters $g_1, g_1' \in G_1$ there are cofinitely many angle parameters $g_2 \in G_2$ such that the triple (g_1, g_2, g_1') is extensible till a tuple for parameters of a triangle. The following theorem shows that almost deterministic $\mathfrak{A}$ can be obtained by extensions of polygonometries.

Theorem. For any polygonometry $\mathrm{pm} = \mathrm{pm}(G_1, G_2, \mathcal{P})$, with a finite group G_1 and without polygons inhibiting the projectivity, there is an extension $\mathrm{pm}' = \mathrm{pm}(G_1, G_2', \mathcal{P}')$ of pm on a plane $\mathcal{P}'$ such that the algebra $\mathfrak{A}$ of binary isolating formulas for $T(\mathrm{pm}')$ is almost deterministic.

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Pregeometry on subsets of fragment of Jonsson set

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We have deal with some J-ω-stable theory [1] and its semantic model. This thesis introduced and discussed the concepts of minimal Jonsson subsets and respectively strongly minimal Jonsson subsets of this semantic model.

We want to each Jonsson subset X of the semantic model assign ordinal number (or, perhaps, -1 or ∞) and it is the rank Morley of this set, denoted by $\mathrm{MR}(X)$.

Let T is a fragment of some Jonsson set and it is a perfect Jonsson theory, C will be its semantic model. X is a definable subset of C.

Definition 1. $\mathrm{MR}(X) \geq 0$ if and only if X is nonempty; $\mathrm{MR}(X) \geq \lambda$ if and only if $\mathrm{MR}(X) \geq \alpha$ for all $\alpha < \lambda$ (λ is the limit ordinal); $\mathrm{MR}(X) \geq \alpha + 1$ if and only if in X exists an infinite family (X_i) disjoint $\exists$-definable subsets, such that $\mathrm{MR}(X_i) \geq \alpha$ for all i.

Then Morley rank of set X is $\mathrm{MR}(X) = \sup\{\alpha \mid \mathrm{MR}(X) \geq \alpha\}$.

Moreover, we assume that $\mathrm{MR}(\varnothing) = -1$ and $\mathrm{MR}(X) = \infty$ if $\mathrm{MR}(X) \geq \alpha$ for all α (in the latter case we say that X has not rank).

Definition 2. The degree of Morley $\mathrm{d_M}(X)$ of Jonsson set X is the maximum length n of its decomposition $X = X_1 \cup \ldots \cup X_n$ into disjoint existentially definable subsets of rank Morley α.

Next, we standardly define pregeometry on the set of all subsets of the semantic model and the concept of strongly minimal Jonsson sets.

On this basis, it introduces the concept of the independence in the frame of special pregeometry under subsets of some existentially closed submodel. The notion of independence leads to the concept of basis and then we have an analogue of the theorem on uncountable categoricity for fragments of Jonsson set.

In this abstract, we collect the necessary facts and notions about pregeometries, existence of bases and hence a well-defined dimension, modularity laws, etc. (like in [2]) in the frame of Jonsson sets studying.

All concepts that are not defined in this thesis can be extracted from [1].

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On definable sets in generic structures

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In this talk we will present our joint work on definable sets and generic structures [1]. First of all we will present an analysis of the diagrams which forming generative classes to describe definable sets and their links in generic structures as well as cardinality bounds for these definable sets, finite or infinite.

We will present the basic characteristics definable sets in generic structures and will compare them each other and with cardinalities of these sets.

The notion of definable set is one of the basic notions in Model Theory. Studying definable sets one can observe what properties can be described by formulas. In this presentation we will present the basic characteristics for definable sets in generic structures.

In the first section we will present several preliminary notions and necessary results of generic structures. After that we will introduce some fragments of definable sets in generic structures, characterize finite and cofinite definable sets, and describe bounds for finite definable sets and their covers, and in the final section we will show the basic characteristics and their bounds for infinite definable sets. The topics will be covered through several examples.

Theorem 1. For a definable set X the following conditions are equivalent:
(1) X is finite;
(2) $X = X_{\Phi(A)}$ for some $\Phi(A) \in \mathsf{D}_0$;
(3) there is $\Phi(A) \in \mathsf{D}_0$ such that $X_{\Phi(A)} = X_{\Psi(B)}$

$$\text{for any } \Psi(B) \text{ with } \mathcal{M} \models \Psi(B) \text{ and } \Phi(A) \leqslant \Psi(B).$$

Theorem 2. For a definable set X the following conditions are equivalent:
(1) X is cofinite;
(2) $M \smallsetminus X = (M \smallsetminus X)_{\Phi(A)}$ for some $\Phi(A) \in \mathsf{D}_0$;
(3) $M \smallsetminus X \subseteq Y_{\Phi(A)}$ for some $\Phi(A) \in \mathsf{D}_0$ and a definable set Y;
(4) there is $\Phi(A) \in \mathsf{D}_0$ such that $(M \smallsetminus X)_{\Phi(A)} = (M \smallsetminus X)_{\Psi(B)}$

$$\text{for any } \Psi(B) \text{ with } \mathcal{M} \models \Psi(B) \text{ and } \Phi(A) \leqslant \Psi(B).$$

Theorem 3. A covering set U of diagrams $\Phi(A)$ for X (with $\mathcal{M} \models \Phi(A)$) is minimal if and only if for each $\Phi(A) \in U$ there is a coordinate a_i for a tuple $\bar{a}$ in X such that a_i belongs to A and does not belong to universes B of other diagrams $\Psi(B) \in U$.

Theorem 4. For any definable set $Y \supseteq X$ in the generic structure $\mathcal{M}$,

$$\omega \leq d(\mathsf{D}_0, X) \leq d(\mathsf{D}_0, Y) \leq d(\mathsf{D}_0) \leq 2^{\max\{|\Sigma|,\omega\}}.$$

Theorem 5. For any definable set $Y \supseteq X$ in the generic structure $\mathcal{M}$,

$$|X| \leq d(\mathsf{D}_0, X) \cdot \mathrm{sc}(X) \leq d(\mathsf{D}_0, Y) \cdot \mathrm{sc}(Y) \leq |M| = d(\mathsf{D}_0) \cdot \mathrm{sc}(\mathcal{M}).$$

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On lattices in generative classes

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In this presentation will show the study of lattices in generative classes associated with generic structures [1–5]. We will show that these lattices can be non-distributive and, moreover, arbitrary enough, also heights and wights of the lattices are described. A model-theoretic criterion for the linear ordering is proved and these linear orders are described.

We will investigate the connection of generative classes with several classes of algebras as Boolean algebras, for example the first result is that in a Boolean algebras generated by the considered lattices are described.

Theorem 1. For any self-sufficient class $(\mathsf{D}_0, \leqslant)$ and a $(\mathsf{D}_0, \leqslant)$-generic structure $\mathcal{M}$, the structure $\langle L(\mathcal{M}, \mathsf{D}_0, \leqslant), \wedge, \vee \rangle$ is a lattice which can be non-distributive.

Theorem 2. For any self-sufficient class $(\mathsf{D}_0, \leqslant)$ and a $(\mathsf{D}_0, \leqslant)$-generic structure $\mathcal{M}$, the lattice $\mathcal{L} = \langle L(\mathcal{M}, \mathsf{D}_0, \leqslant), \wedge, \vee \rangle$ has the following characteristics:
(1) $1 < h(\mathcal{L}) \leq |M| + 1$ if $\mathcal{M}$ is finite, and $h(\mathcal{L}) = \omega$ if $\mathcal{M}$ is infinite;
(2) $1 \leq w(\mathcal{L}) \leq |M|$ if $\mathcal{M}$ is at most countable, and $h(\mathcal{L}) = |M|$ if $\mathcal{M}$ is uncountable.

All values in the described intervals can be realized in appropriate generic structures.

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Preserving properties at expansions of models of ordered theories by unary predicates

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The present lecture deals with the notion of *weak o-minimality*, which initially was deeply studied by D. Macpherson, D. Marker and C. Steinhorn in [1]. A subset A of a linearly ordered structure M is *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \ldots \rangle$ such that any definable (with parameters) subset of the structure M is a finite union of convex sets in M. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal structures.

Here we discuss properties that are preserved at expanding models of an $\aleph_0$-categorical weakly o-minimal theory by a convex unary predicate. By

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[2] any expansion of a model of a weakly o-minimal theory by an arbitrary family of convex unary predicates is a model of a weakly o-minimal theory. We prove that the following properties as $\aleph_0$-categoricity and convexity rank [3] are preserved under such expansions.

Theorem. Let M be a model of an $\aleph_0$-categorical weakly o-minimal theory, M' be an expansion of M by an arbitrary finite family of convex unary predicates. Then M' is a model of an $\aleph_0$-categorical weakly o-minimal theory of the same convexity rank.

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The complexity of quasivariety lattices

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We consider two complexity measures of (relative) quasivariety lattices: the Q-universality and the Nurakunov non-computability property. The concept of Q-universality was introduced by M.V. Sapir [1] in 1985. A quasivariety K is *Q-universal* if, for any quasivariety R of a finite type, the quasivariety lattice $\mathrm{Lq}(\mathsf{R})$ is a homomorphic image of a sublattice of the quasivariety lattice $\mathrm{Lq}(\mathsf{K})$. The second complexity measure was suggested by A. M. Nurakunov [2] in 2012. We say that a class K of algebraic structures of a fixed type has the Nurakunov non-computability property if the set of all (isomorphism types of) finite sublattices of the quasivariety lattice $\mathrm{Lq}(\mathsf{K})$ is not computable.

In [3], it was proved that a class K is Q-universal if and only if it contains a subclass which has the Nurakunov non-computability property. In this regard, the following questions arose [cf. 3,4]:

- Does any Q-universal class K contain a subclass which has the Nurakunov non-computability property?

- Is there a class of algebraic structures which is not Q-universal but which has the Nurakunov non-computability property?

A positive answer to the first question was given by M. V. Schwidefsky [4] for almost all the known Q-universal quasivarieties. The author gives a positive answer to the second question, cf. Theorems 1 and 2 and [5].

Theorem 1. If a class K of algebraic structures of finite type contains an AD-class then it contains continuum many proper subclasses $\mathsf{K}' \subset \mathsf{K}$ which have the Nurakunov non-computability property but which are not Q-universal.

Theorem 2. For the following classes K of algebraic structures, there are continuum many subclasses $\mathsf{K}' \subset \mathsf{K}$ which have the Nurakunov non-computability property but which are nevertheless not Q-universal:

1. *the variety of all unars*;
2. *the variety of all pointed Abelian groups*;
3. *the quasivariety of all [directed] graphs*;
4. *the variety of all differential groupoids*;
5. *the variety of all commutative rings with unit*;
6. *any finite-to-finite universal quasivariety*;
7. *the variety of MV-algebras*;
8. *the variety of Cantor algebras*;
9. *the variety of modular* $(0,1)$*-lattices*;
10. *the Sapir quasivariety which is generated by a single semigroup.*

In case (1), the classes K' can be chosen as quasivarieties.

Theorem 1 can be applied to almost all known Q-universal quasivarieties. In theorem 2, we list just some examples, where theorem 1 applies; this list is however not exhaustive.

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A definition to the concept of a model-theoretic property with applications to the expressive power of first-order logic

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The work [1] describes a normal version of the universal construction of finitely axiomatizable theories showing great expressive possibilities of separate formulas of first-order logic. In the subsequent, it became clear that for the solution of the general question on expressive power of predicate logic cannot do without an exact definition to the concept of a model-theoretic property. Within the framework of a new combinatorial approach [2,3], intended to characterize expressive power of formulas of first-order logic a version of definition to the concept of a model-theoretic property was found [3,4], that is adequate to the common practice of investigations in model theory. At the same time, it is well applicable to the solution of various problems in this direction. Notice that, although the definition to the concept of a model-theoretic property contains some informal parts, nevertheless, it provides exact mathematical statements. For instance:

(a) a standard version of the finite signature reduction procedure preserves all available model-theoretic properties;
(b) there is a computable isomorphism between the Tarski-Lindenbaum algebras of predicate calculi of finite rich (undecidable) signatures preserving all available model-theoretic properties, etc.

From the point of view of the suggested definition of a model-theoretic property, a new perspective approach arises to a solution of the common problem on expressive possibilities of first-order logic [4]. This definition also has applications in model theory. It makes it possible to bring some popular classes of complete theories (e.g. the class of all o-minimal theories) to some canonical form so that they become realistic model-theoretic properties.

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Lattices of bounded based subvarieties of discriminator varieties

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A subvariety W of some variety V of universal algebras is *bounded based* in V, if W is axiomatizable in V by some system of equations with variables $x_1, \ldots, x_n$ for some natural n. The family of all bounded axiomatizable subvarieties of the variety V is the lattice $\mathsf{L}_V^{\mathrm{rb}}$ relative the relation $\subseteq$. This lattice can be not some sublattice of the lattice L_V of all subvarieties of the variety V. Above for the discriminator varieties V the lattice $\mathsf{L}_V^{\mathrm{rb}}$ is some sublattice of the lattice L_V.

We have:

Theorem. For any discriminator variety V the lattice $\mathsf{L}_V^{\mathrm{rb}}$ is universal (in the model-theoretical sense) to the lattice L_V.

Axiomatizability of the class of subdirectly irreducible acts over a group*

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In this work we consider the questions of axiomatizability for the classes of subdirectly irreducible acts over a group. The same questions for the classes of regular, free, projective and (strongly, weakly) flat acts were considered in [1-4]. More precisely, in this works there is the description of monoids classes of regular, free, projective and (strongly, weakly) flat acts over which are axiomatizable.

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Let S be a monoid. A (left) S-act ${}_SA$ is a set A on which S acts unitarily from the left in usual way. If G is a group and H is subgroup of G then the set $G/H = \{gH \mid g \in G\}$ with the operation $g_1(g_2H) = (g_1g_2)H$ where $g_1, g_2 \in G$ is G-act. Elements x, y of a left S–act A are connected (denoted by $x \sim y$) if there exist $n \in \omega$, $a_0, \dots, a_n \in A$, $s_1, \dots, s_n \in S$ such that $x = a_0$, $y = a_n$, and $a_i = s_ia_{i-1}$ or $a_{i-1} = s_ia_i$ for any i, $1 \le i \le n$. An S-act ${}_SA$ is a connected if we have $x \sim y$ for any $x, y \in B$. It known that a connected G-act ${}_GA$ over a group G isomorphic to G-act ${}_G(G/H)$ for a some subgroup H of a group G.

Recall that S-act ${}_SA$ is subdirectly irreducible if $\cap\{\rho_i \mid i \in I\} \neq \Delta$ for every family of congruences ρ_i on A with $\rho_i \neq \Delta$ where Δ is zero congruences on A. From this definition immediately follows the proposition.

Proposition. Let G be a group and ${}_G(G/H)$ be a connected G-act. Then ${}_G(G/H)$ is subdirectly irreducible if and only if the intersection of all subgroups of the group G containing the group H is not equal to H.

Corollary. Let G be the group. Then all connected G-acts are subdirectly irreducible if and only if the set of all subgroups of the group G is linearly ordered.

A class of L-structures K for a first order language L is axiomatizable if there is a set of sentences Π in L such that an L-structure $\mathcal{A}$ lies in K if and only if every sentence in Π is true in $\mathcal{A}$.

Theorem. Let G be the group. A class K of subdirectly irreducible G-acts is axiomatizable if and only if K is a finite class.

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On e-spectra for families of theories of Abelian groups

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We consider e-spectra for families of theories of Abelian groups [1]. By A, B, Γ, E we denote the classes of all theories of Abelian groups, whose positive Szmielew invariants are exhausted by $\alpha_{p,n}$, β_p, γ_p, ε, respectively. For $\mathsf{X}, \mathsf{Y}, \mathsf{Z}, \mathsf{U} \in \{\mathsf{A}, \mathsf{B}, \Gamma, \mathsf{E}\}$ we denote by XY, XYZ, XYZU, respectively, the set of all theories of Abelian groups whose positive Szmielew invariants are exhausted by corresponding $\alpha_{p,n}$, β_p, γ_p, ε for $\mathsf{X}, \mathsf{Y}, \mathsf{Z}, \mathsf{U}$. By $\mathcal{F}$ we denote the set of all theories of Abelian groups with finite Szmielew invariants. Choose an infinite set P_0 of prime numbers and take a countable set $D \subset \mathcal{P}(P_0)$ such that $\langle D, \subseteq \rangle$ is a dense linearly ordered set isomorphic to $\langle \mathsf{Q}, \leq \rangle$ and without cuts (A, A') having $\bigcup A \neq \bigcap A'$. Denote by $\mathrm{Cl}_E(\mathsf{A})_D$ the family $\left\{ \mathrm{Th}\left(\oplus_{p \in X} \mathsf{Z}_p^{(\omega)} \right) \mid X \in D \right\}$.

Theorem.

(1) For any $\lambda \in \omega \cup \{\omega, 2^\omega\}$ there is an E-combination T of theories of finite Abelian groups (in $\mathsf{A} \cap \mathcal{F}$ and with least generating set) such that $e\text{-}\mathrm{Sp}(T) = \lambda$.

(2) There are 2^ω families in $\mathrm{Cl}_E(\mathsf{A})_D$ whose E-closures do not have least generating sets and whose E-combinations T satisfy $e\text{-}\mathrm{Sp}(T) = 2^\omega$.

(3) For any $\lambda \in \omega \cup \{\omega, 2^\omega\}$ there is an E-combination T of theories in BE (respectively, $\Gamma\mathsf{E}$, $\mathsf{A}\Gamma\mathsf{E}$, $\mathsf{B}\Gamma\mathsf{E}$) and with least generating set such that $e\text{-}\mathrm{Sp}(T) = \lambda$.

(4) There are 2^ω families $\mathrm{Cl}_E(\mathsf{BE})_D$ (respectively, $\mathrm{Cl}_E(\Gamma\mathsf{E})_D$, $\mathrm{Cl}_E(\mathsf{A}\Gamma\mathsf{E})_D$, $\mathrm{Cl}_E(\mathsf{B}\Gamma\mathsf{E})_D$) whose E-closures do not have least generating sets and whose E-combinations T satisfy $e\text{-}\mathrm{Sp}(T) = 2^\omega$.

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Transformation and Categoricity Spectrum*

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Definition 1. Structure $\mathcal{A}_0$ of signature σ_0 is called a transformation of a structure $\mathcal{A}$ of signature $\sigma = \langle P_1, \ldots, P_k \rangle$, where predicates P_i of m_i-arity, $1 < i < k$, if structure $\mathcal{A}_0$ constructed from the structure $\mathcal{A}$ by some algorithm and exist formulas

$$\varphi_0((\overline{x}, \overline{y}), \varphi_1(\overline{x}, \overline{y}_0), \overline{y_1}), \ldots, \theta_i(\overline{x}, \overline{y}_0), \overline{y_1}, \ldots, \overline{y_{m_i}},$$

$$\overline{x} = (x_1, \ldots, x_n), \overline{y} = (y_1, \ldots, y_m), \overline{y_i} = (y_1^i, \ldots, y_m^i), 1 \leq i \leq k,$$

of signature σ_0, parameters $c_1, \ldots, c_n$ from $\mathcal{A}_0$ and the following conditions hold:
(1) $\overline{B} = \{\overline{b} : \overline{b} \in |\mathcal{A}_0^m, A_0| = \varphi_0(\overline{c}, \overline{b})\}$;
(2) the formula $\varphi_1(\overline{x}, \overline{y_0}, \overline{y_1})$ defines a congruence η on the structure $\overline{B} = \langle \overline{B}, \overline{Q_i^{m_i}} \rangle$, where predicates $Q_i^{m_i}$ correspond to formulas

$$\theta_i(\overline{x}, \overline{y}_0), \overline{y_1}, \ldots, \overline{y_{m_i}}, 1 \leq i \leq k;$$

(3) the structure $\overline{B}/\eta$ is isomorphic to the structure $\mathcal{A}$.

Definition 2. For a computable structure $\mathcal{A}_0$, the categoricity spectrum is the set of all Turing degrees capable of computing isomorphisms among arbitrary computable copies of $\mathcal{A}_0$. If the spectrum has a least degree, this degree is called the degree of categoricity of $\mathcal{A}_0$.

Let $\mathcal{A}_i$ is a transformation of a structure $\mathcal{A}$ where $i = 0, \ldots, 8$.
We consider structures of the following signatures:
σ the signature of a partial order (an oriented graph);
σ_0 the signature of an irreflexive symmetric graph;
σ_1 the signature of a nilpotent group of class 2 and prime exponent;
σ_2 the signature of a lattice;
σ_3 the signature of a ring;
σ_4 the signature of an integral domain;
σ_5 the signature of a commutative semigroup;

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σ_6 the signature of a bipartite graph;
σ_7 the signature with two equivalences;
σ_8 the signature of algebraic fields.
For $i = 0, \dots, 8$, we have the following theorem:

Theorem. For any signature σ_i, $i = 0, \dots, 8$, there exist transformation $\mathcal{A}_i$ of the structure $\mathcal{A}$ of the signature σ such that the categoricity spectrums coincide.

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The nonforking notion for Jonsson sets

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We consider countable language L and complete for existential sentences perfect Jonsson theory T in language L and its semantic models $\mathfrak{C}$. Let X be the Jonsson set in T and M is existentially closed submodel of the semantic model $\mathfrak{C}$, where $\mathrm{dcl}(X) = M$. Then let $\mathrm{Th}_{\forall\exists}(M) = \mathrm{Fr}(X)$, $\mathrm{Fr}(X)$ is the Jonsson fragment of Jonsson set X.

With the help of the nonforking notion we will give the notion of independence for Jonsson sets. Let M $\exists$-saturated existentially closed model power k (k enough big cardinal) of Jonsson theory T. Let A be the class of all Jonsson subsets of M and P is the class of all $\exists$-types (not necessarily complete), let *JNF* (Jonsson nonforking) $\subseteq P \times A$ be a binary relation. There is the list of the axioms 1–7 which defined Jonsson nonforking notion *JNF* and we have result for fragment $\mathrm{Fr}(X)$ of the Jonsson set X.

Theorem. The following conditions are equivalent:
1) the relation *JNF* satisfies the axioms 1–7 relative to the fragment $\mathrm{Fr}(X)$;
2) $\mathrm{Fr}(X)^*$ is stable and, for all $p \in P$,

$$A \in A((p, A) \in JNF \Leftrightarrow p \text{ does not fork over } A)$$

(in the classical meaning of S. Shelah [1]), where $\mathrm{Fr}(X)^*$ is the center of the fragment $\mathrm{Fr}(X)$.

Independence. The nonforking extensions will be the "free" ones.

Forking as in this theorem can be used to give a notion of independence in J-ω-stable theories [2].

Definition. We say that $\bar{a}$ is independent from B over A if $\mathrm{tp}(\bar{a}/A)$ does not fork over $A \cup B$. We will denote this fact through $\bar{a} \perp_A B$.

This notion of independence for above mentioned Jonsson sets has many desirable properties: monotonicity, transitivity, finite basis, symmetry, etc.

All concepts that are not defined in this thesis can be extracted from [2].

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Similarity of definable closures of Jonsson sets

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Let L be a countable first-order language and T be some inductive theory in this language, E_T and AP_T are denoting correspondingly the following classes of this theory: class of all existentially closed models and class of all algebraically prime models.

Definition 1. An inductive theory T is called *existential-prime* (EP) if it has an algebraically prime model and $\mathrm{AP}_T \cap \mathrm{E}_T \neq \varnothing$.

Definition 2. A theory T is called *convex* (C) if, for any model $\mathfrak{A}$ and any family $\{\mathfrak{B}_i \mid i \in I\}$ of its substructures, which are models of the theory T, the intersection $\bigcap_{i \in I} \mathfrak{B}_i$ is a model theory T. It is assumed that this intersection is not empty. If this intersection is never empty, then the theory is called *strongly convex* (SC). An inductive theory is called an *existentially prime strongly convex theory* (EPSC) if it satisfies the above definitions simultaneously.

Let X be a Jonsson set in the theory T and M be an existentially closed submodel of a semantic model $\mathfrak{C}$, where $\mathrm{dcl}(X) = M$. Then let $\mathrm{Th}_{\forall\exists}(M) = \mathrm{Fr}(X)$, where $\mathrm{Fr}(X)$ is the Jonsson fragment of the Jonsson set X. Let A_1 and A_2 be Jonsson subsets of a semantic model of some Jonsson EPSC-theory, where $Fr(A_1)$ and $Fr(A_2)$ are fragments of Jonsson sets A_1 and A_2.

Then we have the following result:

Theorem. Let $\mathrm{Fr}(A_1)$ and $\mathrm{Fr}(A_2)$ be $\exists$-complete perfect Jonsson theories. Then the following conditions are equivalent:
1) $\mathrm{Fr}(A_1)$ and $\mathrm{Fr}(A_2)$ are J-syntactically similar as Jonsson theories [1];
2) $\mathrm{Fr}(A_1)^*$ and $\mathrm{Fr}(A_2)^*$ are syntactically similar to the complete theories [1], where $\mathrm{Fr}(A_1)^*$ and $\mathrm{Fr}(A_2)^*$ are respectively the centers of fragments of the considered sets A_1, A_2.

All concepts that are not defined in this thesis can be extracted from [2].

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Dimension, ranks and their applications to algebraic structures

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Dimension is one of the most important notions of geometry. It is well known that a space of an infinite dimension is more complicated than another one of a finite dimension. In Model Theory, M. Morley [2] suggested a new rank as a variant of notion of dimension, which is called *Morley rank*, and started to investigate theories whose Morley rank is not infinite. Later S. Shelah suggested a localization of Morley rank, he suggested to use not all formulas, but just some of them, say, from the set Δ. This rank is called Δ-rank, or φ-rank, if Δ consists of one formula φ. If φ-rank of the whole structure is finite for each formula φ, such structure is called *stable*.

I suggest a localization of φ-rank in the following way, which is quite natural for totally ordered structure. I define $(\varphi,(C,D))$-rank inside a cut (C,D) of a given totally ordered structure. If $(\varphi,(C,D))$-rank is finite for any formula φ and for any cut (C,D) such a structure is called *o-stable*.

This notion is fruitful for investigating such natural algebraic structures as ordered groups and fields.

It has been proved that an o-stable ordered group is Abelian, the eventual stabilizer of an unbounded definable subset is not a zero-subgroup and an o-ω-stable ordered field is real closed and its infinite definable subset has a non-zero interior.

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Strong decidability of the classification over Gl

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The classification of extensions of the minimal logic J using slices was introduced in [1]. It extends the classification of superintuitionistic logics proposed by T. Hosoi [2].

In [1] the decidability of the classification was proved, i.e. for every finite set Ax of axiom schemes it is possible to efficiently calculate the slice number of calculi obtained by adding Ax as new axioms to J.

We will consider extensions of the logic $\mathsf{Gl} = \mathsf{J} + (p \vee \neg p)$. In [3], it is established that the logic Gl is strongly recognizable over J. A logic L is *strongly recognizable over* J if there is an algorithm which decides, for every finite system Rul of axiom schemes and rules of inference, if the logic $\mathsf{J}+Rul$ coincides with L. A family S of logics is *strongly decidable over* J if there is an algorithm which decides, for every finite system Rul of axiom schemes and rules of inference, whether the logic $\mathsf{J}+Rul$ is included in S. It is proved that the family of extensions of the logic Gl is strongly decidable over J [3].

In this work we prove strong decidability of the classification over Gl:

Theorem. For every finite set Rul of axiom schemes and rules of inference, it is possible to efficiently calculate the slice number of calculi obtained by adding Rul as new axioms and rules to Gl.

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Logical Correctness

This workshop is organized by

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Typically, logical correctness is taken to concern whether or not an argument or proof follows a logical path from premises to conclusions. In recent years, however, such a view has been complicated by the proliferation of logics, approaches to logic, and uses of logic. In this workshop, we intend to discuss the philosophical and logical consequences of these changes with regard to how, or if, there is any sort of criteria by which a logical structure could be deemed correct, and whether or not those criteria are context-relevant in some specifiable manner.

In a broader sense of the word, correctness can also be understood in at least three different senses:

— meta-logical: a logical system or calculus is correct iff all provable statements in it are true (Related word: soundness.)

— logical: a statement is correct iff it refers to an implicitly or explicitly rule system. (Related word: accuracy)

— moral: an action is correct iff it obeys given norms of behavior. (Related word: political correctness)

There seems to be connections between all these three readings of correctness, to be centered around the criterion of a norm. But, while in the metalogical concept of correctness-as-soundness truth is something that is attributed or denied to sentences, with the logical concept of correctness-as-accuracy it deals with actions (also verbal actings) and allows gradations. As to the moral correctness, it refers to social norms and departs from the criterion of truth. A special emphasis is to be made on Dummett's inferentialist explication of the concept "Boche", in this respect: does such a logical explanation succeed in affording the meaning of such non-logical concepts?

The keynote speaker at this workshop is Ole Thomassen Hjortland (page 132).

Call for papers

We invite abstracts for papers dealing with any of the below topics (though not necessarily limited to them):

- Anti-exceptionalism about logic
- A priorism about logic
- Logical foundationalism
- The connection between logic and reasoning
- Logic and argumentation
- Different uses for logic (argument / computer science / scientific reasoning etc.)
- Contextual logics
- Logical pluralism
- Political correctness (semantics of slurs / norms of language and common decency)

Contributed talks should not exceed a duration of 30 minutes, including discussion. To submit a contribution, please send a one-page abstract by November 15, 2017 to schangfabien@gmail.com.

Computational Hermeneutics: Using Computers to Interpret Philosophical Arguments

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We introduce a method named *computational hermeneutics* aimed at improving the tasks of logical analysis and interpretation of arguments. This method has been developed as a result of reflecting upon previous work on the application of Automated Theorem Proving (ATP) for the formalization and assessment of arguments in metaphysics [e.g. 2,3,5] and is specially suited to the utilization of different kinds of logics (intensional, modal, higher-order, etc.) through the technique of semantic embeddings [1].

Computational hermeneutics has been inspired by Donald Davidson's theory of *radical interpretation* [4] and can be seen as an instance of the hypothetico-deductive method which exploits the computing power and us-ability of modern theorem provers: We work iteratively on an argument by temporarily choosing a logic for formalization; fixing truth-values and inferential relations among its sentences; and then working back and forth on the formalization of its axioms and theorems, by making gradual adjustments while getting real-time feedback about the suitability of our speculations. In this fashion, by engaging in a dialectic process of questions and answers — of conjectures and refutations — we work our way towards an adequate logical analysis and interpretation of an argument by circular movements between its parts and the whole (cf. hermeneutic circle).

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*'KI' is the shorthand for 'Künstliche Intelligenz' (Artificial Intelligence). KI is the German Conference on Artificial Intelligence.

Logical Instrumentalism and Linear Logic

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Logical instrumentalism is the view that norms for deductive reasoning should be evaluated based on one's aims and goals in reasoning and the domain of investigation [see 1]. This means two things. First, as long as there are two domains of investigation which are best served by different norms for deductive reasoning, this will be a logical pluralism: logical instrumentalism will license more than one "correct" logic. Second, should a domain of reasoning call for a particular logic, then logical instrumentalism must license that logic as one of the correct logics.

The bulk of what I will show in this paper is that linear logic is ideal for analyzing sentence syntax. Once this is established, using work from Michael Moortgat on categorial grammar [see 2,3], we must concede that the logical instrumentalist must accept linear logic as a legitimate logic. This has interesting implications for the meanings of the logical connectives. Since the particular linear system in question has a multitude of connectives which are not found in more orthodox logical systems, the logical instrumentalist must license a wider range of logical connectives than we might have originally thought.

One might think that given the information about linear logic and what its applicability requires the instrumentalist to license, this is a mark against instrumentalism. I conclude my paper by suggesting that licensing linear logic as a correct logic is a benefit rather than a burden.

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Evidence and self-evidence in the foundations of logic

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We relate the question of the correctness of proofs and of a possible foundation of a logical system to a general, computational, concept of a formal system as a mechanical procedure, in the sense of a Turing machine, for producing provable formulas [3]. By means of justification logic tools, the question about the evidence in a given system is united with the question of an abstract causal structure of a mechanical decision procedure. After introducing a translation procedure of the work of a Turing equivalent register machine into a suitable justification logic language, it is easy to show that, for each translated register program, the reason (evidence and cause, not necessarily by a register routine) could be proved for the program's halting/non-halting. The evidence of justification logic reasons exceeds the limits of a given formal axiomatic system (since not obeying the constrains of the incompleteness theorems [4]). Further, justification logic (including its axiomatic description of reason operators) does not satisfy Gödel's constructivity requirements [4]. Thus, as a foundational question and the question of the criteria of the correctness of reasoning, we discuss a possible "meta-justification" of the axioms about reasons by analyzing the work of a register (Turing) machine in causal terms in comparison with general self-evident structures of a human agent's reasoning. This includes a sort of abstract pragmatic considerations of the use of concepts by an abstract reasoner (attention to our own acts in using concepts [5]).

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Identifying Logical Evidence

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We take ourselves to know certain logical claims, for example that Socrates is wise and just only if he's wise. However, we currently fail to have a viable account of how we possess logical knowledge. Historical attempts to explain this knowledge, such as appeals to intuition and linguistic proficiency, have been found to be ultimately unsatisfactory, either because they are metaphysically obscure or fail to explain logical disagreements [6]. Yet, it's imperative that we have a complete understanding of logical knowledge. While we use logic to form beliefs in all areas of life, such as when testing scientific theories and engaging in rational debate, we now have many competing logics at our disposal to do so, all of which lead us to reasoning differently in certain situations. Yet, in order to make informed decisions about which logics we should use, we require suitable criteria to adjudicate between them, which can only be developed with a full understanding of what constitutes logical evidence. Without such an account of logical evidence, we lack the resources to make principled and holistic decisions about the correct logic to use. Consequently, a new, more complete, explanation of logical knowledge is needed.

In order to supply such an explanation, prominent figures such as Timothy Williamson [7], Graham Priest [5] and Ole Hjortland [2], have recently argued for a new account of logical knowledge, logical anti-exceptionalism, which emphasises that such knowledge isn't special in any sense, and that logic's method is akin to that of the natural sciences. Just as science proceeds by advancing theories attempting to best explain the relevant data, by a process known as abduction, so logic proposes theories to explain its own domain of data as lucidly and coherently as possible. Thus, we come to be justified in our logical beliefs by recognising which available logical theory best explains the relevant data.

Unfortunately, however, there is little agreement between proponents of logical anti-exceptionalism over what constitute these relevant data that logical theories must explain, and no clear indication yet of how we should settle the matter of which data are relevant. But, without a detailed account of what these data are, logical anti-exceptionalism cannot hope to provide the means to adjudicate between competing logics, a major motivation for any modern theory of logical epistemology. Thus, we need to know what type of data, exactly, logical theories must explain.

This talk argues that we can look to logical practice for help in both providing support for logical anti-exceptionalism and pinpointing the types of data logical theories must explain. While using the practice of researchers has proven a useful method to study how knowledge is acquired in the natural sciences [1] and mathematics [3], the same method has yet to be extensively used in the study of logic. Yet, just as philosophers of science have used historical scientific experiments and disputes as their data to infer how we come to know empirical claims, so we can use a practice based method in studying logical knowledge. By taking logical arguments as our data, we can infer from these arguments the methodological principles that logicians rely upon, and the data their theories attempt to explain. The rationale for using practice to inform an epistemology of logic is the presumption that generally, as with scientists, logicians provide suitable reasons for their claims even if, ultimately, they are not wholly satisfactory. Thus, we should expect logicians' arguments to provide insight into how we can come to know logical truths, and the data logical theories must accommodate.

To show the fruitfulness of this practice-based approach, the talk considers as a case study arguments from one of the most significant debates in the modern logic, the dispute between classical logic and dialetheism over the truth of inconsistent theories. Concentrating particularly on Priest's [4] initial arguments for dialetheism from the liar and Russell-set paradoxes, and classical replies to the arguments, it's proposed that both Priest and his classical opponents rely upon at least three methodological principles: Firstly, that linguistic and mathematical puzzles, such as the liar sentence and Russell set, can form part of a logical theory's explanandum; secondly, that linguistic norms form part of logical evidence, for example in admitting the need to take the meaningfulness of the liar sentences seriously; and thirdly, that mathematical concepts and findings form part of logical evidence, for example by suggesting that only classical logic can underpin mathematical results.

The talk concludes that these initial results from the practice-based approach provide both support for logical antiexceptionalism, and details on the types of evidence a logical theory should accommodate. To offer support for their logical views, rather than attempting to settle disputes on purely definitional or intuitional grounds, logicians appeal to their logic's ability to explain certain relevant phenomena, including linguistic norms and findings from mathematics. We suggest that with yet further consideration of important logical disputes, we can hope to build an even fuller picture of logical epistemology and evidence.

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Around Peirce

This workshop is organized by

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Peirce's 2014 Centennial Congress suggested many ways for "invigorating philosophy for the 21st century". One may expect Charles S. Peirce's findings to invigorate logic in particular. Not only did Peirce improve Boole's algebra, develop a logic of relatives and invent logical quantification, but he drew a whole system of diagrammatic reasoning which may not have born all its fruit yet. He also developed new prospects on informal logic and the theories of induction and abduction. While his inspiration roots in Boole, De Morgan and Schröder, his writings were very influential on Skolem, Hintikka and Polish logic among many, and will probably be on the future of logic.

The keynote speaker is this workshop is Danielle Macbeth (page 137).

Call for papers

Relevant topics include (but are not restricted to):

- Peirce's place in the history of logic
- The influence of Peirce's writings on current logical trends
- Peirce's version of pragmaticism
- Logic and semiotics
- Implications of Peirce's logic for the future of logic
- Peirce's conception of probabilities
- Alpha, Beta and Gamma graphs
- The use of Peircean diagrams as a pedagogical tool
- Peirce's logic of continuity
- The logic of abduction
- Peircean epistemology

Contributed talks should not exceed a duration of 30 minutes including discussion. A one-page abstract should be sent via email before November 15, 2017 to jeanmariechevalier@yahoo.fr and/or benoitgaultier@hotmail.fr.

A dinner with Charley

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I think we should reconsider the value and content of the Lecture V of the Cambridge Conferences [4]. Hilary Putnam, in his magnificent introduction to the volume [2], preferred to concentrate on the theme of the continuum in mathematics, defining the whole lesson in a funny way 'A dinner with Charley'. The problem of observation as access to higher cognitive processes certainly has a long history. In this work Peirce argues that the fundamental condition for the development of good reasoning skills is precisely the ability to discriminate proximal phenomena through observation. The theme, today, is at the center of great attentions. From the hidden object tracking processes studied by Baillargeon [1], to the researches on the deferred imitation conducted by Mandler [3], only to make two examples, it emerges the importance of a phenomenon often underestimated because often considered a mere proto-ability that can not be analyzed as true cognitive processes. Observational learning studies have emphasized non-modular processes, those particularly related to categorization and recall capabilities, where attention and awareness during pattern analysis would not be negotiable. In general, within the recent literature on metaphysics of intentionality, the Kantian distinction between receptivity and spontaneity, at least in relation to the topic of perceptual judgment based on observation, would be at most only functional for the argumentation of the analysis of certain cognitive processes, that is, it would be deprived of any ontological and epistemological status. Peirce, on the other hand, believed that the distinction should not only be respected but also taken into account more than Kant suggested, because not necessarily, in his view, passivity and spontaneity should have been considered to be speculatively functional to one another. After having attributed to the first part, a kind of unconscious induction, a great fineness, and to the other part, the conscious one, the main role to form a theory of the object of observation, Peirce clearly states that it is the first to represent 'the very most important of all the constituents of practical reasoning'.

A broadly Kantian strategy wants that there must be non-inferential knowledge if there is empirical knowledge at all. According to the Brandom-Sellars model, it is only because we bring to experience a full rule-governed conceptual framework of reason's own making that it is possible for us to produce appropriate observation responses to the world. Starting from

Peirce's works, the aim of this paper is to answer the question: is the observer's response contentful just insofar as it occupies a node in a web of inferential relations? In addition, what is the right kind of content for a basic observation?

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Peirce and distributivity

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In 1880, Peirce stated the following [see 3, p. 33]:

> "E. $(a + b) \times c = (a \times c) + (b \times c) \qquad (a \times b) + c = (a + c) \times (b + c)$.
> These are cases of the *distributive principle*. They are easily proved by [4] and [2], but the proof is too tedious to give",

where [2] and [4] state that $\times$ and $+$ behave as the usual infimum and supremum in a lattice. So, Peirce seems to be saying that every lattice is distributive!

Now, it is very well known that there are non-distributive lattices. The usual examples are the pentagon and the diamond. So, how can we explain Peirce's statement?

In 1890, regarding Peirce's statement that every lattice is distributive, Schröder observed that he could prove that the following hold in any lattice [see 4, p. 280]:

Theorem 25$_\times$) $ab + ac \leq a(b+c)$ and **Theorem 25$_+$**) $a + bc \leq (a+b)(a+c)$.

However, he stated that the given inequalities did not hold the other way round. So, it seems that Schröder has proved Peirce to be wrong. However, he gave an example with 990 equalities! He also stated a restricted version of distributivity for lattices with bottom (*Prinzip* III) and used it, together with some form of negation, in order to prove usual distributivity [see 4, p. 310].

Many years afterwards, Huntington presented a proof of distributivity for "lattices" "borrowed, almost verbatim, from a letter of Mr. C.S. Peirce, dated December 24, 1903". The given proof was very indirect (see [1, pp. 300–302] proving 22a).

In our talk we give many details concerning the question at issue. In particular, we give a very direct proof of 22a, we state an open question regarding arguments by Schröder, and comment on Korselt's counterexample for distributivity presented in [2].

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Peirce on the Identity of Truth and Reality

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In [1] (1904), Peirce claims that 'the purpose of every sign is to express "fact", and by being joined with other signs, to approach as nearly as possible to determining an interpretant which would be the perfect Truth, the absolute Truth, and as such (at least, we may use this language) would be the very Universe' [2, vol. 2, p. 304]. He adds that this "*entelechy*' or 'ideal sign' would be 'quite perfect, and so identical — in such identity as a sign may have — with the very matter denoted united with the very form signified by it' [2, vol. 2, p. 304]. In this paper, I articulate the account of the identity of truth and reality that Peirce defends in this text.

I will restrict my attention in this paper to propositional signs, namely, ones which can be true of false. Given that, we can say that this account of truth is not concerned with the relationship between any proposition and objects or states of affairs, but rather, with an idealisation of the development of propositions in inquiry. That is, it is a way of making sense of the notion of the truth as the ideal 'end of inquiry'. On the interpretation developed in this paper, this notion of truth can only apply to one proposition. This proposition is a identical with 'the Universe' insofar as the latter is a token (or 'replica') of the former. The ideal proposition cannot leave anything out, in Peirce's words, it is 'is not abstracted but complete' [2, vol. 2, p. 304].

I defend this interpretation by means of a close reading of 'New Elements', along with the closely related 'Sketch of Dichotomic Mathematics' (c. 1903). I introduce Peirce's use of the three Aristotelian concepts of form, matter, and entelechy [cf. 1, vol. 4, pp. 293–295] along with his account of 'facts'. I then develop Peirce's distinction between the mode of being of a sign and that of individual objects. I argue that this account is what requires the identity to hold between a replica of the ideal proposition and reality.

I conclude by briefly considering the consequences of taking on this account as a notion of the 'end of inquiry'. One traditional objection to the 'end of inquiry' is to point out the difficulty of using the notion as a test for truth. How, for instance, are we to determine what future inquirers will think about a given question? That is, it is difficult to see how we could have any access to the end of inquiry. However, if the end of inquiry is identical with the reality then it is already present. The test for the truth of a given proposition is simply to apply whatever the relevant methods of inquiry are. I argue that, rather than offering a test for the truth of propositions, the identity account of truth and reality defended in 'New Elements' and related writings is intended to show what it means to take reality to be intelligible.

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Logical Consequence in the Diagrammatic System of Assertive Graphs

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The notion of assertion plays an essential role in logic. It is a key ingredient in most logical systems, either implicitly or explicitly. For instance, Frege's ideographical language of the *Begriffsschrift* introduced a specific sign designating assertion, "$\vdash$", which expresses the acknowledgement of the truth of the content of the assertion. In Peirce's graphical logic of Existential Graphs (EGs), there is no specific sign for assertion, although the notion of assertion is used virtually everywhere in his logical writings. The reason is that making an assertion signals the responsibility that the utterer of the logical statement bears on the truth of the proposition [1]. Indeed Peirce has assertion as a sign that is embedded in the Sheet of Assertion (SA) [2], while SA represents both the logical truth as well as the assertoric nature of those graphical logical formulas that are scribed upon it. In intuitionistic logic, on the other hand, an explicit notion of assertion has been used in order to analyse inference and proof, to explicate the meaning of logical constants, and so on [3].

The idea of the notion of assertion thus appears robustly invariant across a range of logical theories, logical methods, and logical notations. In the light of the existence of such a common and shared character of assertions, we present a new system of graphs that makes the embedded or implicit nature of assertions in logical graphs explicit. We develop a graphical logic of assertions (called "Assertive Graphs", AGs). We will show that it is possible to extend this intuitionistic logic of AGs into a classical graphical logic (ClAG) without a need to introduce polarities. We compare the advantages of these two approaches and point out the nature of a deep inference of their transformation rules. Finally, we discuss implications of AGs to logical consequence. We will point out that logical consequence in AGs is based on some standard aspects of model theory, related to the notion of truth, as well as on the (antirealist) notions of proof and assertion, related to the epistemic acknowledgement of truth.

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A Generic Figures Reconstruction of Peirce's Existential Graphs (Alpha)

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The powerful mathematical tools of category theory and, particularly, the techniques within topos theory for representing and investigating the deep relationships between logic and topology are especially well-suited for examining the iconic and diagrammatic properties of Peirce's system of Existential Graphs. The Existential Graphs remain at once one of the most important contributions of Peirce to modern logic and one of the least studied and most underappreciated aspects of his overall philosophy. The integration of Peirce's graphical system with the contemporary mathematical methods of category theory promises to make Peirce's innovative logical notation accessible to a broader audience of researchers and to open new avenues of inquiry into the Existential Graphs themselves. Furthermore, placing the Existential Graphs in a categorical and topos-theoretical setting may help to suggest new applications and creative extensions of the Existential Graphs at alpha, beta and gamma levels.

*Interest Group in Pure and Applied Logics

We show in particular how Peirce's alpha level of the Existential Graphs may be faithfully reconstructed within the presheaf category of *forests*, that is, the category of contravariant functors from the category of natural numbers (with morphisms corresponding to the usual $\leq$ ordering) into **Set**, the category of sets and functions. The reconstruction proceeds in three stages: first, it is shown how the fragment of Peirce's EG alpha system involving only variable-free "cuts-only" graphs corresponds naturally to finite forests, where the branchings of such forests represent the nestings of EG alpha cuts; secondly, variable-tokens are introduced by treating pairs of cuts-only graphs, one of which is a subobject of the other, in terms of the lattice of intermediate subobjects they induce; finally, individual variable tokens are shown to be representable as tokens of common types through a natural construction via groupoids. Once these graphical syntactic constructions are made, it is straightforward to show how Peirce's logical semantics for the graphs are derivable naturally from the iconic (i.e. structural) properties of the graphs themselves.

Our approach throughout makes use of the generic figures techniques developed by Reyes, Reyes and Zolfaghari in *Generic Figures and Their Glueings: A Constructive Approach to Functor Categories* (Polimetrica, 2008). This way of treating functor categories lends itself naturally to diagrammatic systems of various kinds including directed graphs (in the usual mathematical sense), dynamical systems and other constructions. Thus, this quite general mathematical setting allows for the comparison of the Existential Graphs with a variety of other diagrammatic logical and mathematical systems and provides a milieu for investigating the interplay of iconic syntax and logical semantics in multiple contexts. This approach has not previously been applied to Peirce's Existential Graphs and may be contrasted with alternate analyses of the Existential Graphs that make use of category theory such as that of Brady and Trimble. We conclude by sketching out several paths for future development of this approach to the Existential Graphs, including the adjunction of infinitely nested graphs to EG alpha and the extension of the generic figures approach to EG beta and gamma.

A Peircean Logic of Operations

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> "…De Morgan's *Open Sesame*, the Aladdin matmûrah of relative logic…"
> — C.S. Peirce

Charles Sanders Peirce was the first philosophical logician to develop the mathematics for a comprehensive logic of relations; that is, a logic for the modelling, combining, and manipulating relations of any adicity (valency) whatsoever. The *corazon de corazon* of his logic of relations is a thesis which he characterized as a "remarkable theorem", specifically, the claim that a relationally complete logic requires, but only requires monadic, dyadic, and triadic relations. One of the immediate consequences of this theorem is that there are genuine triadic relations, relations of three relata which cannot be analyzed into combinations of either monadic or dyadic relations. All other n-adic relations can be composed out of combinations of the three elementary species of relations by two elementary logical operations of relative and auto-relative multiplication. Peirce also recognized that binary mathematical operations are special cases of triadic relations; that is, any binary operation $X \circ Y = Z$ is equivalent to some triadic relation $t^{\circ}(X, Y, Z)$. He argued that such operations cannot be analyzed into operations of arity less than two. This realization provides additional grounds for his contention that there are genuine triadic relations.

Peirce's insights are the seeds for a logic of operations as a branch of his logic of relations — seeds he never nurtured to fruition. This essay presents the rudiments of that field of inquiry consonant with his logic of relations. Binary operations in simple algebras such as magmas, semigroups, loops, and groups will be represented by Peirce-inspired directed wye diagrams. These can be grafted together by (auto-)relative multiplication to generate both acyclic (tree) and cyclic graphs to model n-ary operations. This approach can be further applied to rings, fields, bi-algebras, categories, partial orders, lattices and other algebraical systems. This work, when married to Wacław Sierpinski's theorem that every n-ary operation ($n > 2$) can be analyzed into a compound operation consisting exclusively of binary operations, provides a Peircean framework for Universal Algebra.

Frege and Peirce on the signs of generality

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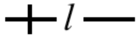
According to Michael Dummett, Frege's discovery of a notation for quantifiers and variables for the expression of generality was one of the most important discoveries in the history of logic, certainly the most important since Aristotle. It was by means of this discovery that Frege solved a problem that had blocked the progress of logic for centuries: the problem of how to treat multiply quantified sentences of the kind of "Everybody loves somebody". Frege's discovery allowed him to treat such sentences (and indeed, every sentence in which signs of generality occur) as being constructed in stages corresponding to the signs of generality occurring in it. Thus the sentence "everybody loves somebody", in which the sign of generality "somebody" occurs within the scope of the sign of generality "everybody", is not obtained by the simultaneous combination of the three components "everybody", "somebody", and "x loves y". Rather, it is obtained in two stages, i.e., by first combining "x loves y" with "somebody", thus obtaining "x loves somebody", and then by combining this with "everybody", thus obtaining the complete sentence. Only under this mode of analysis, Dummett explained, the truth-conditions of a multiply quantified sentence can be satisfactorily determined.

In fact, the discovery is independent of the specific notation that Frege devised. A symptom of this is the fact that Dummett feels no need to present Frege's Begriffsschrift in order to expound Frege's discovery. Frege's insight that sentences are constructed in stages corresponding to the signs of generality occurring in them might equally be represented in notations other than the Begriffsschrift, and in point of fact it has become part and parcel of modern quantificational logic in the guise of the Peanian/Russellian linear notation. The relations of dependence of the signs of generality is represented in this latter notation by the linear ordering of those signs, and this manner is no less effective than the manner in which it is represented in Frege's Begriffsschrift.

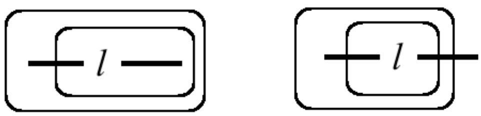
By 1882, on the other side of the ocean, Peirce had made the same discovery as Frege. But unlike Frege, Peirce spent the rest of his logical life to experiment with different and alternative notations for the representation of quantification theory. The first, complete version of the theory is what Peirce would later call the General Algebra of Logic. In the General Algebra, the above multiply quantified sentence would be represented as "$\Pi_i \, \Sigma_j \, l_{i,j}$", of which the contemporary "$\forall x \, \exists y \, \mathrm{L}xy$" is a mere notational variant. But in parallel to the General Algebra, in 1882 Peirce created a system of logical graphs in which the sentence in question would be represented thus:



where the crossed line at the left of the predicate term "l" is the sign of the universal quantifier, the plain line at the right of it the sign of the existential quantifier. But since the sheet on which these graphs are scribed is symmetric and thus *unordered*, not only the system can only express symmetric predicates, but also, the relations of dependence of the signs of generality cannot be represented, as in the General Algebra, by exploiting the linear ordering. The first solution that Peirce found for this problem was to add to the vocabulary: numerical indices are attached to the lines to indicate the order of selection.

The graph on the left would thus represent "$\Pi_i \, \Sigma_j \, l_{i,j}$" ("everybody loves somebody"), while that on the right would represent "$\Sigma_j \, \Pi_i \, l_{i,j}$" ("somebody is loved by everybody").

This must have been highly unsatisfactory to Peirce. In 1896 he invented two systems of graphs, later termed Entitative and Existential Graphs, respectively. The 1896 graphs adopt the 1882 substructure of spots and lines of identity but add to that substructure the "oval". By means of the ovals, the compositional ("endoporeutic") structure of the formulas is immediately represented, and thus also the relations of dependence of the quantifiers.



In the Existential graphs on the left, the first line is less enclosed than the second, and thus its corresponding quantifier has logical precedence; in the graph on the right, the dependence relation is reversed. By means of the endoporeutic structure expressed by the nesting of the ovals, Peirce's logical graphs show in a very perspicuous way how sentences are constructed in stages corresponding to the signs of generality occurring in them.

G. Boole, A. De Morgan and C.S. Peirce at the birth of symbolic logic

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The presentation will show how C.S. Peirce developed his symbolic logic from the works of G. Boole and A. De Morgan. Boole devised a calculus for what he called the algebra of logic to overcome syllogistic. Interpreting categorical propositions as algebraic equations, Boole showed an isomorphism between the calculus of classes and of propositions, being indeed the first to mathematize logic. With a different purport, De Morgan tried to improve on syllogistic, taking it as object of study. With a very unusual system of symbols of his own, De Morgan develops the study of logical relations that are defined by the very operation of signs. Although his logic is not a Boolean algebra of logic, De Morgan defined the central notion of a universe of discourse. Peirce takes a critical and decisive step forward. First, claiming Boole had exaggeratedly submitted logic to mathematics, thus mistaking the nature and the purpose of each discipline to the point of erasing their characteristic differences to the impairment of the first, Peirce emphasizes the normative purport of logic. Second, identifying De Morgan's limitations as a rigid restraint of logic to the study of relations, thus hindering compositions of relations with classes, Peirce develops his own logic of relative terms. Peirce's originalities to be highlighted in the presentation are:

(a) a theory of multiple quantification,
(b) the development of the logic of relatives from it (and not vice-versa),
(c) the calculus for multi-saturated expressions (arity superior to 2),
(d) the contrast between mathematics and logic by their distinct ends and degrees of generality.

In the end, brief considerations on Peirce's relation to Tarski's interpretation of the logic of relatives will be hinted at.

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[*] Centre for Logic, Epistemology and the History of Science

[†] State University of Campinas

[‡] In 1975, Alfred Tarski, invited by Ayda Arruda and Newton da Costa, visited the Institute of Mathematics, Statistics and Scientific Computing at Unicamp. In this bilingual text, it is presented an inedited transcription of the two lectures delivered by him on relation algebras, preceded by a brief introduction, with an update on some open problems mentioned by Tarski.

The Lvov-Warsaw School: Past, Present and Future

This workshop is organized* by

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The beginnings of the Lvov School, later on called the Lvov-Warsaw School, are connected with the person of Kazimierz Twardowski, a disciple of Franz Brentano, and his taking the post of Head of the Chair of Philosophy at Lvov University. It was thanks to Twardowski that a modern school of philosophy was established, which was where a host of outstanding philosophers, logicians, psychologists, university professors and organizers of scholarly life in independent Poland came from. Owing to the activity of the School, multiplicity of attitudes and a variety of represented views, not only philosophical, it was also possible to develop formal logic and mathematics, and the accomplishments of representatives of these disciplines are often included into pioneering and seminal on the global scale.

J. Łukasiewicz and S. Leśniewski were the founders of the world-famous Warsaw School of Logic. The former propagated the idea of applying logical tools to the classical metaphysics. The latter built three systems of logic (protothetic, ontology and mereology), which showed formal values and applications in the spirit of nominalism. Their disciples were, among others, A. Tarski — the author of a pioneering dissertation on semantic theory of truth (1933) and, following World War 2, the founder of the Californian School at Berkeley, S. Jaśkowski, A. Lindenbaum, Cz. Lejewski, B. Sobociński, J. Słupecki and M. Wajsberg.

*The first two organizers are editors of the forthcoming book "The Lvov-Warsaw School: Past and Present".

The works by the following disciples of K. Twardowski can also be considered seminal: K. Ajdukiewicz — in the field of logical theory of language (significant for the so-called mathematical linguistics; Y. Bar-Hillel, N. Chomsky), and also in the area of logical analysis of epistemology; T. Kotarbiński — the founder of reism (nominalistic philosophical conception) as well as praxeology — science of effective action.

Logical theory of science was the subject matter successfully dealt with by T. Czeżowski, Z. Zawirski, I. Dąmbska, M. Kokoszyńska-Lutmanowa, J. Hosiassion-Lindenbaumowa, J. Kotarbińska and H. Mehleberg.

The flourishing of the Polish school of logic and philosophy before the outbreak of WW2 received a lot of attention worldwide. After the War, the Lvov-Warsaw School ceased to exist. Its representatives, who had managed to survive the turmoil of war, went to live in different parts of Poland and all over the world, having left the output, which — despite the communist regime — was able to revive and develop a new Polish logic, owing to continuation of its traditions and strong connections with multiple disciplines: philosophy, mathematics, computer science, linguistics, semiotics and others.

The keynote speakers at this workshop are Jan Woleński (page 165), Kordula Świętorzecka (page 162) and Grzegorz Malinowski (page 138).

Call for papers

We invite submissions on the following topics:

- Historical analyses on what the L-WS phenomenon was
- Philosophical motivations for creation of logical research by representatives of the L-WS
- L-WS, the Vienna Circle and the Berlin circle
- Achievements of the main representatives of the L-WS and their development or continuation
- Influence of results of the L-WS on the development of new fields of knowledge
- Influence of creatively-developing Polish logic (by, among others, S. Jaśkowski, A. Mostowski, J. Słupecki, A. Grzegorczyk, J. Łoś, R. Suszko, L. Borkowski, R. Sikorski, H. Rasiowa, Z. Pawlak and R. Wójcicki) on the level of contemporary philosophy and other domains of science
- Alfred Tarski and L-WS
- Polish Logic in the world today

Abstracts (one page) should be sent by November 15, 2017 via email to skardowska@gmail.com and/or agarrido@mat.uned.es.

Methodological peculiarities of the Lvov-Warsaw School

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The Lvov-Warsaw School is considered as a branch of the twentieth-century analytical movement. Among characteristic features of analytic philosophy, there are: focus on detailed analyses of small problems instead of creating all-embracing general syntheses, the use of logical methods in philosophizing, and the respect for the results of science. The Lvov-Warsaw School had some methodological peculiarities that differentiate it from English analytical school on the one hand, and the Vienna Circle on the other. In the present paper, we indicate these peculiarities.

We will discuss namely the following particular methodological issues:

(1) Kazimierz Twardowski's and his descriptive psychology, originating from Franz Brentano and developed in the direction indicated by the analysis of language;

(2) Jan Łukasiewicz's early conception of analysis and construction of concepts as well as his program of logicism in philosophy;

(3) Tadeusz Kotarbiński's semantic reism as a tool of clarifying definitions and theses;

(4) Tadeusz Czeżowski's concept of analytic description as a paradigmatic analytical procedure of the Lvov-Warsaw School;

(5) Kazimierz Ajdukiewicz's so-called method of paraphrases and his application of categorical grammar to the analysis of the structure of philosophical theses.

These examples certify some distinctive elements of Polish analytical philosophy. Firstly, the conception of analysis in the Lvov-Warsaw School was constructive and did not fall under the paradox of analysis. Secondly, there was a conviction that by the use of broadly understood logical tools the real progress in philosophy may occur. Thirdly, the language analysis was considered as a tool to reach reality and resolve real problems.

On Ludwik Borkowski's philosophico-logical views

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Ludwik Borkowski's vast knowledge of philosophy allowed him to put his logical studies in a philosophical context. As a logician he was one of the Lvov-Warsaw school followers. He dealt with the basic issues of the widely understood logic as well as with those having strong philosophical implications (e.g. non-classical logics, the theory of truth, natural deduction, the theory of consequence). He also worked on the theory of definition and the intuitive interpretation of logical results. For Borkowski logic was an autonomous science which can be used for other service. Although he did not create any philosophical logic works, his whole life research was accompanied with investigating philosophical sources, inspirations and logical consequences.

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Free Ontology as the logic for reism

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Although strong arguments speak for ontological reism, its original version formulated by Kotarbiński encounters serious problems related to the ontology of the domain of mathematics and its set-theoretical foundations. Still worse, the positive thesis of reism is endangered by triviality, and its negative theses with contradiction. It is argued that the problems of the first kind may be overcome by combining reism not with classical nominalism, but with the so-called theory of the respectus. Unfortunately, the latter is not expressible in any extensive logic, in particular, in Leśniewski's Ontology, chosen by Kotarbiński as the logical background for reism.

Next, in order to avoid triviality of the positive thesis of reism, one must express it not in the reistic language, but rather in some multicategorial language. However, in the framework of Leśniewski's Ontology, this implies commitment to the existence of individuals of categories other than the category of things. This, in turn, would make negative theses of reism contradictory.

A remedy for the problems of the first kind is choosing as the logical background for reism, instead of Leśniewski's Ontology, a weaker calculus that may be called Weak Ontology, obtained from it by disabling the rule of extensionality and appropriate weakening of its axiom. A further weakening of the calculus by modifying the quantifier rules in the spirit of free logics results in what may be called Free Ontology, which enables avoiding both kinds of problems. Thus, Free Ontology is preferable to Leśniewski's Ontology as the logical background for reism.

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From Aristotle to Lvov-Warsaw School*

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As we know, logic is the study of the structure and principles of correct reasoning, and more specifically, attempts to establish the principles that guarantee the validity of deductive arguments. The central concept of validity is for logic, because when we affirm the validity of an argument are saying that it is impossible that its conclusion is false if its premises are true.

Propositions are descriptions of the world, that is, are affirmations or denials of events in various possible worlds, of which the "real world" is just one of them. There is a long philosophical tradition of distinguishing between truth necessary (a priori or "logical") and facts "contingent" (a posteriori or "factual").

Both have really led the two concepts of logical truth, without being opposed to each other, are quite different: the conception of truth as coherence, and the conception of truth as correspondence. According to the point of view of consistency, a proposition is true or false depending on their relationship with respect to a given set of propositions, because of the rules of that system. Under the terms of correspondence, a proposition is true or false, if it agrees with reality.

To further enhance the complexity of the problem, not only analyze trueness or falsity of propositions, but also of theories, ideas and models. And so, we allow new and different conception of truth.

The basic idea underlying all these approaches is that of an intrinsic dichotomy between true and false. This opposition implies the validity of two fundamental laws of classical logic:

— *Principle of excluded middle*: Every proposition is true or false, and there is no another possibility.
— *Principle of non-contradiction*: No statement is true and false simultaneously.

Such fundamental ideas produce some series of paradoxes and dissatisfaction that is based on the need to overcome this strict truth-bivalence of classical logic.

*To the memory of Marcin Mostowski, who has recently passed away.

Searching for the origins could lead too far and eventually disperse, which, as we know is not very convenient for a job pretending to be research. So we will refer to these first signs that appear in the East (China, India,...), and then we may analyze the problem of "future contingents", treated by Aristotle in *Peri Hermeneias.*

About Future Contingent Propositions, we must remember that they are statements about states of affairs in the future that are neither necessarily true nor necessarily false. Suppose that a sea-battle will not be fought tomorrow. Then it was also true yesterday (and the week before, and last year) that it will not be fought, since any true statement about the case that will be was also true in the past. But all past truths are now necessary truths; therefore it is now necessarily true that the battle will not be fought, and thus the statement that it will be fought is necessarily false. Therefore it is not possible that the battle will be fought. In general, if something will not be the case, it is not possible for it to be the case.

As we know, although the starting point of Leibniz's "calculus universalis" were Stagirite's theories, Leibniz ends to be dependent from the ideas of Aristotle, to finally develop its own axiomatic system, a more general type, based on applying the Combinatorial Instrument to syllogistic. That issue (Future Contingent's problem, with variations) would be then crucial in medieval times, as during the Scholasticism, with William of Ockham, and Duns Scotus, looked at from different point of views, for its relationships with Determinism and 'Divine Foreknowledge'. Then, this issue is taken up by Spanish Jesuit F. Luis de Molina (and the famous controversy 'De Auxiliis' maintained with the Dominican Fray Domingo Báñez), or Francisco Suarez, and even the great polymath G.W. Leibniz dedicated his time.

At first, Łukasiewicz introduced the three-valued logic and then generalized to the infinite-valued. That possibility modulation can be expressed by a membership function, which is to come all the unit interval [0,1], instead of being reduced to the dichotomy of classical logic: True vs. False, 0 vs. 1, White vs. Black, etc., allowing the treatment of uncertainty and vagueness, important not only from the theoretical point of view, but also for applications. The deep and far connection from Leibniz to Łukasiewicz, and then to Zadeh, crossing through Bernhard Bolzano, Franz Brentano and Kazimierz Twardowski has its progressive justification by Jan Woleński, Roman Murawski, Urszula Wybraniec-Skardowska and Roger Pouivet, among others.

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*Broad Research in Artificial Intelligence and Neuroscience

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Jan Łukasiewicz and Many-Valued Logic*

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The roots of the Lvov-Warsaw School (LWS, by acronym) can be traced back to Aristotle himself. But in later times we better put them into thinking G.W. Leibniz and who somehow inherited many of these ways of thinking, such as the philosopher and mathematician Bernhard Bolzano. Since he would pass the key figure of Franz Brentano, who had as one of his disciples to Kazimierz Twardowski, which starts with the brilliant Polish school of mathematics and philosophy dealt with. Among them, one of the most interesting thinkers must be Jan Łukasiewicz, the father of many-valued logic.

Jan Łukasiewicz (1878–1956) began teaching at the University of Lvov (now Lwiw, former Lemberg, but also Leópolis), and then at Warsaw, but after World War II must to continue in Dublin. Some questions may be very astonishing in the CV of Łukasiewicz. For instance, that a firstly Polish Minister of Education in Paderewski cabinet, into the new Polish Republic, and also Rector for two times at Warsaw University, was awarded with a Doctorate 'Honoris Causa' in spring 1936, at University of Münster, into the maximum of effervescence of Nazism in Germany. The explanation must be their good relation with a very good friend, the former theologian, and then logician, Heinrich Schölz, which was the first Chairman of Mathematical Logic in German universities.

Łukasiewicz firstly studied Law, and then Mathematics and Philosophy in Lvov (then Lemberg). His doctoral supervisor was Kazimierz Twardowski, and in 1902 he obtain his Ph.D. title with a very special mention: 'sub auspiciis Imperatoris' (i.e., under the auspices of the Kaiser). Also he received a doctorate ring with diamonds from the Kaiser of the Austro-Hungarian Empire, Franz Joseph I.

From 1902, Łukasiewicz was employed as a private teacher, and also as a desk in the Universitary Library of Lvov. So it was until 1904 when he obtained a scholarship to study abroad. He defends his 'Habilitationschrift'

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in 1906, entitled "Analysis and construction of the concept of cause". This permits to give university courses. His first lectures were on the *Algebra of Logic*, according to the recent translation to Polish of this book of the French logician Louis Couturat.

Between 1902 and 1906, Łukasiewicz continued his studies in the universities of Berlin and Leuwen (Lovaina). In 1906, by his 'Habilitationschrift', he obtain the qualification as university professor at Lvov. And, in 1911, he was appointed as associate professor in his 'alma mater' (Lemberg).

Jan Łukasiewicz was also very active in historical research on logic, giving a new and up-to-date interpretation of Aristotle's syllogism and of the Stoics' propositional calculus. According to Scholz, the better pages on history of logic are due to him. And also, as Arianna Betti says, "Jan Łukasiewicz is first and foremost associated with the rejection of the Principle of Bivalence and the discovery of Many-Valued Logic."

The discovery of MVL by Łukasiewicz was in 1918, a little earlier than Emil Leon Post. According to Jan Woleński, "although Post's remarks were parenthetical and extremely condensed, Łukasiewicz explained his intuitions and motivations carefully and at length. He was guided by considerations about future contingents and the concept of possibility". So, he introduces, firstly, three-valued logic, then four-valued logic, generalized to logics with an arbitrary finite number of veritative values, and finally, to logics with a countably infinite-valued number of such values.

Very noteworthy is his treatment of the history of logic in the light of the new formal logic (then called Logistics). Thus, not only he addressed the issue of future contingents departing from Aristotle, but also put in value logic of the Stoics, at least so far taken. In fact, Heinrich Scholz said, rightly, that Łukasiewicz had written the most lucid pages on the history of logic.

On Grzegorczyk's Logics of Descriptions and Descriptive Equivalences

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In 2011 Andrzej Grzegorczyk gave birth to Logic of Descriptions (LD), a new logical system in which the classical equivalence has been replaced with the descriptive equivalence which produces a new compound sentence out of two simpler ones asserting that they describe the same.

The main philosophical assumption of Grzegorczyk's standpoint was that in the human description of the cognised world's phenomena, the roles of negation, conjunction, and disjunction differ significantly from those of implication and equivalence. Negation, conjunction, and disjunction are very primitive and have clear intuitive descriptive meanings, while the classical implication and equivalence are derivative and have no intuitively plausible sense. Furthermore, it is exactly implication and equivalence that are responsible for some paradoxical laws of classical logic, such as "false implies everything", "truth is implied by anything" and "all true sentences are logically equivalent to each other".

As a consequence, states Grzegorczyk, we are forced to accept that among all the logical connectives exactly negation, conjunction, and disjunction, together with the equimeaning connective (or descriptive equivalence) expressing the assertion that two descriptions have the same meaning, are well suited as the primitive concepts of a new logic. As descriptions and descriptive equivalences among them have became crucial for Grzegorczyk's approach, he called his new logical system the Logic of Descriptions, or LD for short.

The first exposition of Grzegorczyk's new logic, its philosophical motivations, and assumptions was published in 2011 in [1], where Grzegorczyk proposed a number of axioms and rules that the equimeaning connective (descriptive equivalence), should satisfy. He also posed a number of open problems, in particular whether the new connective is different than the classical equivalence. This problem was solved in [2] by showing that the descriptive equivalence connective is essentially different than the classical one and the logic itself is indeed new. In [2] many other peculiar properties of LD were proved. Further results on LD and some of its extensions and modifications are presented in [3].

In this talk we will present the basics of Grzegorczyk's logic LD and then we survey the recent results on LD.

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On the Notion of Independence

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The notion of independence is closely related to the notion of provability. A formula α is meant to be independent from a set A of formulae if it is impossible to prove α from A. A set A is independent if each formula in A is independent from the set of remaining formulae in A.

Independence is usually investigated in the context of to the set of axioms of a theory; an independent set of axioms is regarded as clear and elegant, because it doesn't contain any dependent axioms (or rules) which, in fact, are redundant. This notion of logical independence, seen as a formalization of the notion of simplicity, was one of four pillars of Hilbert's project. From the theoretical point of view, looking for independent axiomatizations is not crucial, however, it has played an important role in the development of formal logic and universal algebra.

The first formal solution to the problem of independent axiomatization was given by Tarski, who proved that every countable set of formulae is independently axiomatizable. But, in general, proving the independence of a system of axioms is not an easy task. It is usually done by constructing appropriate models, as for example in the case of the proof of independency of the Axiom of Choice, by means of forcing technique, or Łukasiewicz's proof of independency of the system of his axioms of three-valued logic, by means of matrices.

Let us notice that sometimes we consider more general notion of independence; for example, the Axiom of Choice can neither be proved nor refuted from ZFC. This is a special case of a general notion of algebraic independence introduced by Marczewski. In this sense, a set X of elements of an algebra A is independent if the subalgebra generated by X is free in the variety generated by A. This notion was thoroughly investigated by Marczewski, Mycielski, Œwierczkowski, Głazek and many others.

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Stanisław Jaśkowski and the first textbook based on Natural Deduction

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In the history of natural deduction (ND) one may distinguish two problems: when the method was invented and when it started to be practically applied in textbooks as a tool for teaching logic. Stanisław Jaśkowski was one of the founders of ND although his work is not as well recognized as the work of Gerhard Gentzen. However, in the talk we are not going to focus on his role as the inventor of ND but rather on his priority in the field of the application of this new approach to logic.

Quine in "Methods of Logic" claims that the first textbook applying ND is Cooley's "A Primer of Logic" printed in 1942, then reprinted in 1946. In fact, Cooley applies a variety of inference rules, however it is doubtful if there is ND system in his book. Conditional proofs are only briefly described on few pages but not widely used in the text. Moreover, Cooley did not apply any devices for separating subproofs and his rule for elimination of existential quantifier is stated without sufficient restrictions. In Quine's "Methods of Logic" from 1950, ND system is correctly defined but also introduced only in three sections as an illustration rather, not as the main proof system. Quine mentioned also some earlier mimeographed notes of himself and of Rosser which applied ND but I had no possibility to check them and in the light of known textbooks of these logicians it is also doubtful. For example, a well known Rosser"s textbook "Logic for Mathematicians" from 1953 is using axiomatic system and introduces additional ND-like rules only as a metalogical devices for simplification of axiomatic proofs. Undoubtedly, the first widely known textbook which consequently applies ND as the way of doing logic is "Symbolic Logic" of Fitch published in 1952.

However, in 1947, Jaśkowski published in mimeographed form his lecture notes "Elements of Mathematical Logic and Methodology of Deductive Sciences" in Polish. The book consists of 105 pages and is of great importance since it is perhaps the first logic textbook where natural deduction is uniformly used as a method for presentation of logic. It is used from the beginning for proving theorems of logic without any reference to axiomatic systems. Moreover, it is applied also in proofs of metalogical results and even truth-functional semantics is introduced via analysis of ND proofs of selected theses. In the talk I will briefly characterize the main features of ND introduced in this textbook and make a comparison with his original version from the paper published in 1934.

Methodological aspects of research on the Ukrainian branch of the Lvov-Warsaw School

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For many years, Kazimierz Twardowski's philosophical school was considered as a purely Polish formation. However, in the first part of the 20th century, Lvov was a multinational and multicultural city. It was, among others, the biggest center for Ukrainian culture and science. So, the question arises whether there exist an Ukrainian branch of the Lvov-Warsaw School. Recent research gives many evidences for positive answer to this question. Firstly, there were many Ukrainians among direct and indirect students of Twardowski. Secondly, some Ukrainians consciously referred to the results of Twardowski and his school.

In the paper, I will focus on methodological problems connected with the procedure of distinguishing a philosophical school or a branch of it. This methodological approach stems from the definition of the Lvov-Warsaw School, according to which, he term “Twardowski’s student” refers to a person related to the founder of the School twofold: institutionally (i.e. through the relationship with the institutional School’s center, i.e., University of Lvov, University of Warsaw and Polish Philosophical Association in Lvov) and ideologically (i.e. through the use of theoretical ideas of the Master in their own work).

Hence, I will focus on the exhibition of the institutional and ideological bond between Twardowski and his Ukrainian students.

Polish trends in the logic of questions

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There is no doubt that the Lvov-Warsaw School contributed substantially to the development of logic in the 20th century. As in the case of many other logical theories, the heritage of the School gave birth to some vital trends in the logic of questions.

The problems of logical analysis of questions, initiated by Kazimierz Twardowski, were considered already by Kazimierz Ajdukiewicz [1,2]. But the systematic reflection and substantial work on the matter (in the 60s and 70s) was mainly due to Tadeusz Kubiński and Leon Koj. Kubiński’s monograph [4] was probably the first in the world such an extensive account of the logical theory of questions; however, it appeared in Polish, and the English language monograph [5] was published few years after the monograph by Belnap and Steel. Finally, Kubiński and Koj influenced Andrzej Wiśniewski. Inferential Erotetic Logic [8,9] developed by Wiśniewski is nowadays one of the most important paradigms in the field of the logic of questions (next to the paradigm of inquisitive semantics and epistemic approaches to questions). There is also a rich semiotic tradition of erotetics represented today by Jacek Jadacki and Anna Brożek [3].

There are many issues undertaken by the logic of questions conducted by Polish logicians and philosophers. However, one of the distinguishing features of the Polish tradition is the focus on the analysis of relations between questions and/or declaratives. This kind of approach, started by

Kubiński and extended by Wiśniewski, is nowadays used extensively in such diverse areas as the formal analysis and modelling of erotetic reasoning [9], analysis of natural language dialogues [7] and proof theory [6].

The aim of my presentation is to bring the listeners closer to this part of the Polish tradition and its latest achievements.

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Ontology of logic and mathematics in the Lvov-Warsaw School

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The aim of the talk is to present views of representatives of Lvov-Warsaw School concerning the ontological status of objects of mathematics and of logic. In particular views of Jan Łukasiewicz, Stanisław Leśniewski, Alfred Tarski, Tadeusz Kotarbiński and Kazimierz Ajdukiewicz will be considered. Additionally views of Andrzej Mostowski (who belonged to the second generation of the School) as well as of Leon Chwistek (who did not belong

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directly to the School but whose views are interesting) will be presented. The problem whether those philosophical views did influence the technical investigations and results will be discussed.

The application of Cz. Lejewski's Chronology in determining mereological genidentity

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Identity of objects that are subject to changes is called genidentity. The term was introduced to the language of science by Kurt Lewin in 1922 [1], although the problem of continuity and change has been present in philosophy since its inception. Among the various kinds of genidentity there is mereological genidentity, which is connected to the part-whole relationship. The key notion used to describe that kind of genidentity is the notion of temporal part. It is determined by referring to the notions that describe the persistence of objects and their mereological relationships [2,3]. The attempt to describe them using Czesław Lejewski's Chronology seems to be particularly interesting. He built his theory on Stanisław Leśniewski's Mereology [4]. As a young man, Lejewski was a student of Jan Łukasiewicz and Stanisław Leśniewski at the University of Warsaw. A number of his later works were devoted to Leśniewski's systems. In the views of its author, Chronology was supposed to serve as a tool for describing temporal relationships among objects as understood in Leśniewski's Ontology. It was supposed to be a general theory of objects as ordered and extended in time. Lejewski wanted also to construct a theory of objects as distributed and extended in space, which he named Stereology. Together with Protothetic, Ontology and Mereology were supposed to constitute a reist's systematic presentation of the science of being. Chronology itself, although presented in 1986, has not been subject to any in-depth analysis yet, not to mention its application in philosophical investigations. The aim of the present paper is to discuss the possibility of applying it in the description of temporal parts and the mereological genidentity of objects.

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Lvov-Warsaw School and the Artificial Intelligence

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Many ideas and achievements of he contemporary Artificial Intelligence have its roots in the achievements of the Polish mathematicians of the Warsaw School and the logicians of the Lvov-Warsaw School. In my talk I would like to consider some of them.

Jan Łukasiewicz is best known for his concept of multi-valued logic. Application in AI seems to be the most promising of all the possible applications of multi-valued logics. This kind of logic form the basis for the description of vague concepts, which are characteristic of natural language and non-formal reasoning. A many-valued approach to vague notions and commonsense reasoning is the method of expert systems, databases and knowledge-based systems, as well as data and knowledge mining. In the AI the conception of fuzzy logic and fuzzy sets are used. The conception of fuzzy sets was developed in the 1960s by Lofti A. Zadeh. He applied Łukasiewicz's logic to elements of a set, thereby creating an algebra of fuzzy sets. A similar solution in connection with the research on expert systems was worked out in Poland by Z. Pawlak. The theories of fuzzy and rough sets are applied in artificial intelligence and expert systems. They are used for the automation of data and knowledge exploration. Jan Łukasiewicz invented also the parenthesis-free notation known as PN (Polish Notation) and RPN (Reverse Polish Notation). The idea of the notation which avoids the use of parentheses appeared in connection with examining formal systems.

In a contemporary informatics, natural logic is applied first of all in broadly understood issues relating to artificial intelligence. Jaśkowski, independently from Gentzen, created a system of natural deduction which is the basis of systems regarding automatic deduction and theorem proving. From the point of view some of researches Jaśkowski's system is more useful in computer-assisted proof verification, while Gentzen's system is better in computer-assisted proving. Jaśkowski also created a system of paraconsistent logic. Such logics are used in AI.

Kazimierz Ajdukiewicz, with his categorial grammar, participated in the development of formal grammars, the field significant for programming languages. The first attempts of computer translation from English to Russian were based upon the notion of syntactic coherence of a sentence introduced by Ajdukiewicz. These ideas have been developed and used as theoretical foundation for the first translations.

Alfred Tarski is the most famous member of the Lvov-Warsaw School. His works were essential to the foundations of Artificial Intelligence. He created and described a theory of reference and truth value relationships. Modern computer scientist have related this theory to programming languages and other specifications for computing.

Helena Rasiowa was deeply interested in computer science and its applications. Rasiowa initiated intensive investigations on methods of inference under incomplete information, which she called approximate reasoning. At present approximation logics are one of the central topics of research in artificial intelligence.

The Axiom of Choice and the Road Paved by Sierpiński

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Ernst Zermelo used the disastrous reception to his 1904 Well-Ordering Proof as a catalyst for serious inquiry into the requirements of a proper formal axiomatic system for set theory. Presented in 1908, Zermelo's attempt was without doubt inspired by Hilbert's 1899 *Grundlagen der Geometrie*. Of Hilbert's deductive system, Zermelo would retain: (i) the use of a domain of objects with a primitive relation; (ii) the explicitation of implicit assumption and transfiguration into axioms; and (iii) the emphasis on the independence and consistency of these axioms. But, given the overwhelmingly negative immediate reception of both his 1904 and 1908, how did this abstract view of sets come to be canonical by the mid-1930s? Particularly, how did the contentious "general postulate of choice" come to be the widely accepted "axiom of choice" of modern set theory and classical mathematics?

The acceptance of AC can be seen as "a turning point for mathematics (...) symptomatic of a conceptual shift in mathematics" (Kanamori 2012, p. 14). Whilst Western Europe remained quite hostile to this new vision of logic and mathematics, it was in Eastern Europe, at the Warsaw and Lwów Schools of Mathematics (1918–1939) that the seeds of this conceptual shift briefly landed and yielded a cultivar that was to supplant and overtake the Western world. From 1908 until 1916, articles supporting AC or exploring some of its consequences were scant and scarcely concerted. The situation changed dramatically in 1916 when Wacław Sierpiński, a young professor at the Lwów University published a series of articles on AC and revived the dormant debate surrounding AC — albeit on completely different grounds. Eschewing theoretical concerns about the nature and methodology of mathematical practice, he paid little attention to the dominant question as to whether Zermelo's existence postulate could be accepted as a mathematical construction.

Instead, he recentred the discussion towards practical matters (viz., its consequences, its interrelations and degree of necessity within various proofs, as well as its role in obtaining various basically trivial mathematical theorems). Starting in 1918, Sierpiński also rallied the newly formed Polish schools of mathematics around a common programme of research which was to include an in-depth exploration of AC's role in a few select branches of mathematics. Originally adopting an objective stance vis-à-vis AC, his programme was to eventually completely supplant the previous philosophical and methodological debates — and Sierpiński was to become AC's biggest champion since Zermelo. The posterity of AC as we know it would be unimaginable without Sierpiński's efforts: "Since the labours of Mr. Sierpiński and of the Polish School, a revolution has been produced. A certain number of mathematicians have fruitfully used the axiom of choice; things are no longer in the same place" (Lebesgue 1941, p. 109).

Jerzy Łoś' Juvenilia

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Jerzy Łoś (∗1920–1998†) is best known for his works in foundations of mathematics, abstract algebra, probability theory and mathematical economics. Yet, in our talk we will address his two, not sufficiently acknowledged, early works "Podstawy analizy metodologicznej kanonów Milla" (Foundations of methodological analysis of Mill's method) and "Logiki wielowartościowe a formalizacja funkcji intensjonalnych" (Many-valued logics and the formalization of intensional functions). Both papers incorporate pioneering research in positional logics with temporal (the former) or epistemic (the latter) interpretation. We will present Łoś's novel formal solutions and their philosophical assumptions tracing back to the tradition of Lvov-Warsaw School of philosophy.

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Abstraction principles via Leśniewskian definitions: potential infinity and arithmetic

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This paper starts with an explanation of how the logicist research program can be approached within the framework of Leśniewski's systems. One nice feature of the system is that Hume's Principle is derivable in it from an explicit definition of natural numbers. I generalize this result to show that all predicative abstraction principles corresponding to second-level relations, which are provably equivalence relations, are provable. However, the

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†KUL Publisher. KUL is a shorthand for John Paul II Catholic University of Lublin.

system fails, despite being much neater than the construction of Principia Mathematica (PM). One of the key reasons is that, just as in the case of the system of PM, without the assumption that infinitely many objects exist, (renderings of) most of the standard axioms of Peano Arithmetic are not derivable in the system. I prove that introducing modal quantifiers meant to capture the intuitions behind potential infinity results in the (renderings of) axioms of Peano Arithmetic (PA) being valid in all relational models (i.e. Kripke-style models, to be defined later on) of the extended language. The second, historical part of the paper contains a user-friendly description of Leśniewski's own arithmetic and a brief investigation into its properties.

Jerzy Słupecki and the Consequence Operation

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Consider a denumerably infinite language $\mathbf{L}$. Every mapping $f\colon 2^{\mathbf{L}} \to 2^{\mathbf{L}}$ can be considered as a consequence operation. The set of such mappings has the power $\aleph_1$. General conditions (idempotency, monotonicity, etc.) introduced in Tarski's axiomatization of Cn radically reduce the number of consequences. If we add special axioms determining a logic, we obtain a singular mapping $\mathbf{f}$. Tarski's axiomatization of Cn was governed by the idea that Cn is associated with inference preserving truth.

Słupecki, following Łukasiewicz's rejection function, introduced the operation Cn^{-1} (the rejection logical consequence). The main heuristic idea consists in considering Cn^{-1} as related to the process of rejection of a proposition on the base of other propositions. Hence, Cn^{-1} preserves falsity as the distinguished value. Cn^{-1} has the same general properties as Cn with exception of the fact that $\mathrm{Cn}^{-1}\varnothing = \varnothing$, although $\mathrm{Cn}\,\varnothing \neq \varnothing$ (more precisely, $\mathrm{Cn}\,\varnothing = \mathsf{LOGIC}$). On the other hand, Cn^{-1}, supplemented by special axioms, generates concrete rules of rejection, for instance, if $A \vee B$ is rejected, then A is rejected.

Yet is possible to introduce (it was done by Ryszard Wójcicki) other rejection consequence operation, namely dCn (the dual consequence operation), which has all general properties of Cn, including $\mathrm{dCn}\,\varnothing \neq \varnothing$ (more precisely, $\mathrm{dCn}\,\varnothing = \mathsf{dLOGIC}$), and generates the specific rules of rejection, the same as in the cases of Cn^{-1}.

The rejection consequences suggest some formal as well as philosophical issues. For instance, we can ask whether there are mixed consequence operations, that A is asserted relatively to the set of sentences X, if $A \in \mathrm{Cn}\,X$, assuming that all elements of X are asserted, and rejected, if $A \in \mathrm{Cn}^{-1}\,Y$ ($\mathrm{dCn}\,Y$), provided that $Y \subseteq X$ and sentences in Y are rejected. Perhaps this could reduce non-monotonic logic to monotonic one. Other formal problem consists in defining rejection consequences for non-classical logic. In general, the category CONSEQUENCES might do have some relevance for the idea of universal logic. Philosophically speaking, the issue "Why Cn and assertion look as the most natural accounts of inferences?" seems interesting from the epistemological point of view.

The Rasiowa-Pawlak School: From Algebra of Logic to Algebra of Data (and Back)

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The Rasiowa-Pawlak School (formed during the 70s and 80s of the 20th century in Warsaw) may be considered as the contemporary branch of the Lvov-Warsaw School (established by Kazimierz Twardowski in Lvov in 1895), which extends this old methodological tradition on Polish modern computer science.

The most important step towards the Rasiowa-Pawlak school was made in the middle 1980s. In this time Helena Rasiowa started her work in cooperation with Victor Marek and Andrzej Skowron on logics related to rough set theory — the mathematical method of data analysis founded by Zdzisław Pawlak. In the same time, Cecylia Rauszer (the student of Helena Rasiowa) cooperated with Zdzisław Pawlak, Victor Marek, Andrzej Skowron, and Andrzej Jankowski. Intensive investigations conducted by this group led to important research results and to merging two conceptual frameworks and scientific circles created around Helena Rasiowa and Zdzisław Pawlak into a school: the Rasiowa-Pawlak school.

In the talk I would like to return to the theoretical results of Cecylia Rauszer obtained in the 1970s about semi-boolean algebras [1,2], and discuss them against the background of rough set theory and formal concept analysis. Although formal concept analysis can be traced back to almost the same time as rough set theory (in the 1980s Zdzisław Pawlak even met Rudolf

Wille, the founder of formal concept analysis), due to different semantics staying behind the data in these two theories, it seemed that no significant relationships might exist. However, using algebraic methods popularised by Helena Rasiowa, one can merge both theories by means of the results obtained by Cecylia Rauszer in her Ph.D. thesis and the articles published in the 1970s [1,2]. One can even extend these result upon modern extensions of rough set theory as, e.g., dominance based rough sets. It shows, indeed, how fruitful were, or better still *have been*, logical investigations (in the spirit of the Lvov-Warsaw School), and how they — even today — are important for the leading theories of data analysis and knowledge discovery.

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The Lvov-Warsaw School and Indian Logic

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In his seminal work [1], J.M. Bocheński warned that some currents of thought that support opposing ontologies could nevertheless embrace similar logical postulates. And he gave as an example the doctrines developed by the Nyāya and Buddhist schools, both originally from ancient India. This being so, we nonetheless find certain aspects of Buddhist ontology that generate important differences in the field of logic; we refer to the absence of paradoxes and antinomies, alluded to by the Megaric sages and that the Nyāya philosophers also knew how to detect. The purpose of this paper is to analyze why these paradoxes did not appear in the Buddhist ontology and if the Alfred Tarski hypotheses carried out in [2] can explain this singularity in some way.

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How to reply today to the issues raised by Kazimierz Twardowski in his *Images and Concepts* (1898)?

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Images and Concepts, written by Kazimierz Twardowski, was originally published in Polish as a small book in Lviv in 1898. It is a significant work for epistemology and psychology, both for the methodological studies and for the research on concepts and imagining processes. At the methodological level, Twardowski sees the need for making an agreement between epistemology and psychology in such a way that the requirements of epistemology become consistent with the results of psychological research. According to these requirements (in opposition to some classical theories), he develops his own approach to inquiry into cognitive processes. The aim of his research presented in *Images and Concepts* is the search for the general theory of concepts which would include all particular theories of different kinds of concepts. The result of his research is the theory according to which the nature of concepts lies in imagining judgments. This means that concepts have their foundations in images which in some cases can be very complicated. This thesis sounds up-to-date for cognitive linguists and psychologists, although it was put forward within quite a different conceptual framework. Nevertheless, the aim of my paper is to seek some connections between Twardowski's approach and those of contemporary cognitive scientists.

Following Wundt, Twardowski maintains that image (or better to say 'imagery') is a synthesis of impressions (the relation between imageries and impressions is that of a whole to its parts). But asking the question "how do impressions synthesize to create imagery?" he maintains that "psychology has not yet and probably never will answer this question". I intend to discuss the issues raised by Twardowski in the light of current controversies. This will be done by appealing to Stephen Kosslyn's research on the origin of imageries, on the one hand, and to the theory of double coding on the other. In appealing to the current debates, I try to show Twardowski's approach in a new light which can also enrich the philosophical background of cognitive science.

Reflections on Paraconsistency

This workshop is organized by

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Paraconsistent logics form a lively sector of the discipline we call Philosophical Logic. The idea that contradictions are — in some cases or in some way — acceptable without "explosion" of our rational systems has been developed by paraconsistent logicians also with reference to epistemological and metaphysical implications, and there is a wide literature on the theme. However, the arising of contradictions, and the need of coming to terms with them, has also ethical, political and more generally practical implications that sometimes fade into the background.

Not only that, one of the main problems of the philosophy of inconsistency (but this is true also of other fields) is that there are philosophically oriented works with a weak or no technical part and vice versa: some technical paraconsistent works do not take into consideration the philosophical aspects and implications of logical choices.

One aim of our workshop is thus to promote interaction between technical and non-technical works in the field. A second but not secondary aim is to enlarge the view, involving people interested in contradictions, but not exclusively as militant logicians.

We thus encourage contributions able to suggest and treat preliminary questions, sometimes underrated or not extensively studied by logicians, such as:

1. *In metaphysical and truth-theoretic perspective:*
   - Are there contradictory truthmakers? If there are, are they to be intended as two overlapping facts, or only one fact?
   - If the acceptance of contradictions is ruled by truth (as dialetheists hold), can we really renounce the classical exclusive notion of 'T', in virtue of which if 'p' is true then 'not p' must be false?

2. *In epistemological perspective:*
   — Do we really believe the unbelievable?
   — What kinds of epistemic gluts are rationally acceptable?
3. *In ethical and generally practical perspective:*
   — Disagreements and dilemmas are typical contexts in which the occurring of contradictions has political and practical consequences: can the theories of paraconsistent logicians be used to deal with these occurrences?
4. *In meta-theoretical perspective:*
   — Why do we study contradictions?
   — What can we learn from the history of paraconsistency?
   — Is 'philosophy' as such the enterprise that aims at solving or interpreting contradictions, as many authors in the tradition held (see Hegel or Wittgenstein)?

The keynote speakers at this workshop are Jonas R. Becker Arenhart (page 120), Graham Priest (page 158), María del Rosario Martínez-Ordaz (page 139) and Roy Sorensen (page 160).

Call for papers

We invite contributions on all aspects of paraconsistency and contradiction. Topics include:

- paraconsistent systems: respective costs and benefits
- gaps, gluts, and other truth values
- what's so bad about trivialism?
- contradictory truthmakers
- the unbelievable: its role and believability
- contradictions, discussive conflicts and dilemmas
- paradoxes in logic and elsewhere
- what can the historical treatments of contradictions (in Aristotle, Pascal, Hegel, Bergson, etc.) still teach us?
- was Hegel paraconsistentist?
- what was really Aristotle's attitude toward contradictions?
- old and new theories about the square of oppositions
- the philosophical relevance of dialetheism and of other paraconsistent views
- the political relevance of the philosophy of contradiction

Abstracts (one page) should be sent by November 15, 2017 via e-mail to elena.ficara@upb.de.

(De)motivating Gluts

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Semantic paradoxes, like the Liar Paradox, are one of the best-known motivations for the dialetheists' claim that there are true contradictions (dialetheias). Liar-like arguments arise in natural language and dialetheists argue that the Liar sentence is true and false, i.e., a glut. In order to advocate this approach to the paradoxes, some dialetheists, like Graham Priest, advance a dilemma: a semantic theory of English is either inconsistent (glutty) or else incomplete (with respect to its expressive power). Usually, dialetheists choose expressive completeness and urge that glut theory is the only approach able to achieve this ideal without triviality. However, recently in [1] JC Beall argued that, by parallel reasoning, one should also be led from expressive completeness to triviality by validity paradoxes (Curry-style paradoxes involving not a conditional, but the notion of validity). The resulting dilemma produced by Beall is: a semantic theory of English is either *trivial* or else *incomplete*. So, according to Beall, the demand for complete expressive power leads directly to triviality. Validity paradoxes are not dealt with by the adoption of gluts, but rather by other means typically associate with approaches to Curry-style paradoxes, which are not paradoxes directly involving negation and contradiction. In face of these difficulties, Beall have suggested, that "the glutty treatment arises from aesthetic considerations: such target liars simply look like gluts" [1, p. 583]. We shall argue that not even that may be available. Priest in [2] answers Beall's challenge with one typical move against Curry paradoxes: we avoid triviality by avoiding one of the rules involved in the derivation of triviality, which is the rule of absorption. Now, while that move may help one to block Beall's argument from expressive power to triviality, it also seems to pose troubles for the motivation for gluts: some approaches to the semantic paradoxes also bar the derivation of the liar with such restrictions on absorption. As a result, in order to avoid triviality, one may end up having also no gluts. In this talk, we present this dilemma for dialetheists and discuss the resulting trouble for dialetheism and for the adoption of gluts.

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The Role of Paraconsistency in Scientific Change

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According to *the zeroth law of scientific change* [1], at any particular moment, the elements of a given mosaic of theories accepted by a specific epistemic community are *compatible* with each other. Importantly, the law distinguishes the concept of *compatibility* from the logical concept of *consistency.* Two propositions are said to be compatible if they can be part of the same mosaic; they may or may not be mutually consistent in the logical sense. Thus, by the zeroth law, it is possible for two theories to contradict each other and yet be simultaneously accepted by the same community. The compatibility or incompatibility of a given pair of theories within a certain community's mosaic is determined by the *criteria of compatibility* employed by that specific community at that specific time. Naturally, the criteria of compatibility employed by different epistemic communities may or may not be the same: while some communities might be tolerant towards inconsistencies in their mosaics, other communities may impose stricter criteria of compatibility and ban open inconsistencies. Additionally, it is important to appreciate that the criteria of compatibility can change through time: the same epistemic community that was once intolerant towards inconsistencies may at some point alter their compatibility criteria and become tolerant towards inconsistencies. This suggests that a paraconsistent logic can, at least at times, play an important role in the process of scientific change. Multiple historical instances of inconsistency-tolerance in different epistemic communities are the best evidence for this: the history of science knows many epistemic communities that have knowingly accepted contradictions [2].

The goal of this paper is to explain *why* some epistemic communities are inconsistency-tolerant while others are inconsistency-intolerant. On the one hand, by the zeroth law, a community's attitude towards contradictions depends on its *criteria of compatibility*. On the other hand, by *the third law of scientific change* [3], a community's criteria are deductive consequences of their accepted theories. Therefore, it follows that a community's attitude towards contradictions ultimately depends on the specific theories accepted by that community. I show why *fallibilist* communities are normally inconsistency-tolerant, while *infallibilist* communities are normally inconsistency-intolerant. The phenomenon of inconsistency-tolerance is illustrated by means of several examples from the history of the empirical sciences, including the attitude of Newtonian physicists of the 18th and 19th centuries towards anomalous observations, and the attitude of our contemporary physics community towards general relativity and quantum physics. The phenomenon of inconsistency-intolerance is illustrated by several examples from the history of mathematics.

By clarifying the internal mechanism that shapes different communal attitudes towards contradictions, this paper suggests two important questions for future logico-historical research. First, what were the compatibility criteria employed by different epistemic communities throughout history? Second, which specific paraconsistent logics can be said to have been at play in inconsistency-tolerant communities at different times?

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A paraconsistent approach to da Costa's deontic logic: beyond contradictions and triviality

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Chisholm's paradox, Ross's paradox and Sartre's dilemma are good examples of problems that are still "alive and kicking" [1] inside most of deontic logic systems. Despite many good proposals that claimed to have solved one or another of those problems, the fact is that most of them seemed just too shy to save the deontic logic from that high dose of pessimism towards any use in philosophical debate. How can the logician of normative concepts get around this is the real challenge.

Considering Newton da Costa's example in [2], it was interesting to see that, despite the logic presented being very strong, the attempt to represent incomplete actions, like a failed murder, led immediately to a contradiction. In our proposal, a very simple procedure of switching the classical basis for a paraconsistent one showed to be a successful tool to fix the inconsistency. Important to note, however, that providing a paraconsistent view of da Costa's deontic system not only represented a way to get rid of triviality, but almost a mandatory adaptation to extend the language towards a more intuitive and reliable representation of any moral philosophy or legal code. It is shown that non-classical reasoning should be the starting point of any discussion about deontic logic, not just a backup plan when things don't work in the classical field. This particular approach, once accepted, should lead the deontic logic community to change completely the analysis of paradoxes, dilemmas and gluts inside deontic logic.

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On the Possibility of Dialetheic Metaphysics

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Paraconsistent logics that have arisen during the 20th century claim that the law of contradiction is not applicable to certain circumstances. The most radical paraconsistent logic is the dialetheic logic, which argues that some contradictions arising within the limits of thought are true and even real. Dialetheic logic — as Graham Priest and Richard Routley developed — proves to be academically efficient by its metaphysical implications within both at the level of logic and at the level of metaphysics. Priest, claims that some contradictions which are realized in the states of change, motion and time are real. These real contradictions are the subject of metaphysical investigation.

This paper aims to examine the possibility of doing metaphysics based on dialetheic logic. Firstly, after sharing Priest's view on metaphysics of change, motion and time, I will demonstrate that dialetheic logic embodies some important potentials — especially on metaphysics of time, but it does not yet realize these potentials. In the final part, I will argue that the shift from dialethic logic to metaphysics is not sufficiently prepared by Priest and dialetheic metaphysics requires a transcendental point of view.

Paradoxes, Hypodoxes, Hypodox-paradox duality and Hypodoxical Paradoxes

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Paradoxes are a kind of conundrum with more than one good answer [7], but I distinguish among such conundrums: paradoxes, hypodoxes and hypodoxical paradoxes. Paradoxes, like the Liar, are overdetermined, whereas hypodoxes, like the Truth-teller, are underdetermined [2,4]. Being underdetermined explains the Truth-teller phenomenon, which Mackie [6] characterised as consistent but undecidable. Notice that being consistent but undecidable does not explain being underdetermined. The concept of hypodox is a more general concept than Mackie's conception of a Truth-teller counterpart, as I will demonstrate in this paper. While not all paradoxes have hypodoxes, hypodoxes are almost as common. There are hypodoxes of naïve truth, set theory, and time travel to give but a few examples. Only a few types of the ungrounded and ill-founded paradoxes do not have closely associated hypodoxes. With respect to the converse relation, I conjecture that all hypodoxes can be paired with paradoxes by a kind of duality, and demonstrate this for ungrounded or ill-founded hypodoxes [2,3]. Surprisingly, the concepts of paradox and hypodox are not exclusive; I analyse antinomies such as Bertrand's chord paradox and the Ship of Theseus as hypodoxical paradoxes. This explains why it is natural to think of these as paradoxical dilemmas, as Clark [1] does. Hypodoxes are proto-paradoxes in that adding certain principles to a hypodox will generate a paradox [4]. Moreover, many hypodoxes, such as the Truth-teller and the set of all self-membered sets do not use the principles characteristic of their related para-

*International Marketing Institute of Australia doxes, in particular the T-schema and Set Abstraction [4,5]. Finally, I argue that an intermediate resolution of the Liar paradox may reduce it from an apparent dialetheia to a kind of hypodox, a kind that is still distinct from a Truth-teller.

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What is a Contradiction?

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Paraconsistent logics are often characterized as being contradictory tolerant, and dialetheism defined as the thesis that contradictions are true. But what, exactly, are contradictions? Given the important role they play in philosophy and logic, with many believing they form the ultimate black against a theory, it's surprising so little time has been spent considering and evaluating the available definitions of 'contradiction' in the literature. Indeed, with some, such as [6], suggesting that paraconsistent logics and dialetheism are guilty of distorting the meaning of 'contradictories', it is even more important we come to a clear understanding of the concept. This talk shows that many prominent definitions in the literature are non-equivalent, and that this non-equivalence counts against several of the definitions' plausibility.

There are at least four categories of definitions of 'contradiction' available in the literature: Semantic, which define contradictions in terms of semantic properties, such as truth-conditions; syntactic, which define contradictions in formal terms, such as formulae of a certain logical form; pragmatic, which define contradictions in terms of types of speech-acts, such as assertion and denial; and ontological, defining contradictions in terms of physical properties. It is clear, however, that not all instances of these four general categories are equivalent, and thus as equally plausible. After all, each of the categories define contradictions in terms of different entities: semantic in terms of propositions, syntactic in terms of formulae, pragmatic in terms of speech-acts, and ontological in terms of states of affairs.

Nor it is simple to translate between these types of objects to identify the definitions with one another. For example, take a semantic definition of 'contradiction' in terms of truth-conditions, "A proposition is said to be a contradiction when it is logically impossible that it be true", [2, pp. 27–28], and the syntactic definition of contradictions as "wff (well-formed formulae) of the form '$A \wedge \neg A$'" [4, p. 244]. While Detlefsen et al.'s semantic definition [2] proposes that contradictions are logically false propositions, the syntactic definition in [4] mentions not propositions but formulae. Now, given that formulae are not true in themselves, but only true-within-a-valuation-within-a-logic, it doesn't even make sense to apply the property of being (logically) false to Haack's contradictions. Consequently, both accounts fail to pick out the same objects as contradictions, and given that it makes sense at least to say that 'Contradictions are false', this consequence should count against the plausibility of Haack's definition.

Having shown that definitions across the four categories are non-equivalent, and consequently not equally plausible, we move on to demonstrate an even more interesting point — that prevalent definitions within one of the categories of definitions are non-equivalent. We achieve this by looking at three semantic definitions of 'contradiction' dominant in the contemporary literature, which it's not unusual to find cited together as though they were equivalent [1]:

Truth-Conditional: Contradictions are logically false propositions. [2, pp. 27–28]

Structured: Contradictions are conjunctions of propositions and their respective negations. [5, p. 224]

Explosion: Contradictions are sets of propositions that imply every proposition. [3, p. 24]

We argue that while the set of structured contradictions is a subset of both the set of truth-conditional and explosion contradictions, the inverse isn't true, and that this divergence between the definitions demonstrations the structured account's superiority. Firstly, we show that there are many explosion contradictions that are neither truth-conditional nor structured contradictions. Particularly important are those explosive sets of propositions, such as those of the form $\{B, A \to \neg B, A\}$, which the explosion account contains as contradictions. Including these explosive sets as contradictions must count against the definition's plausibility, as they ensure the explosion account cannot effectively distinguish between sets containing contradictions, sets containing contradictory pairs, and sets of propositions that entail contradictions, which are particularly important distinctions to make when discussing theories and beliefs.

Secondly, we show that there are truth-conditional contradictions, such as propositions of the form $\neg(A \to A)$, which though logically false in classical logic, are not structured contradictions. Again, we argue that this divergence from the structured account counts against the truth-conditional account's plausibility, as propositions of the form $\neg(A \to A)$ fail to have two important properties we expect of contradictions: i) While contradictions are formed of contradictories, such propositions have no clear constituents identifiable as contradictories; ii) We are able to derive the negation of an assumption from a contradiction when constructing a formal reductio. However, we are not able within our formal systems to complete a reductio from propositions of the form $\neg(A \to A)$. Thus, to admit propositions of this form as contradictions would be to admit our formal systems fail to recognise certain contradictions as such. The talk, therefore, concludes that not only are three prominent semantic definitions of 'contradiction' non-equivalent, but that in future discussions of paraconsistency and dialetheism, it is the structured account of contradiction we should use.

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How to Compose Programs in Belnapian Dynamic Logic?

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Consistency is not something we can guarantee all the time, but inconsistency, when dealt with properly, has its advantages. In our intellectual activities, if we were to ignore some aspects we would thought of as unimportant, impossible, *inconsistent*, we would risk losing something *new*.

This is why paraconsistency is around and has been getting so much attention lately. In the words of João Marcos in [7], "paraconsistentists have very often flirted with modalities", and the works [1,3,2,8], among others, seem to prove him right.

Our curiosity has been sharpened when we decided to equip hybrid logic with a paraconsistent reasoning at the level of propositions [4], thus being able to define a measure for inconsistency that allows the comparison between two theories that support the information in a certain set. The next step would be to incorporate the ability of dealing with inconsistent information at the level of the accessibility relation; and why not grabbing dynamic logics and view transitions as the execution of programs [5]? Rivieccio, Jung and Jansana [8] introduce a modal logic with Belnapian values for transitions in [8]; Sedlár presents a dynamic logic with Belnapian truth values in [9]. To us, it makes sense to join both of the ideas together.

We aim to discuss the interpretation of the composition of programs, whose associated transitions are four-valued functions, for example the program of choice between α and β, usually denoted by $\alpha \cup \beta$, or what comes out of a sequence between α and β, for α and β *Belnapian programs.* If we consider relations as functions in B4, as in [8], then the program of choice between α and β may not correspond to the union of R_α and R_β in B4. We should investigate this issue also in the context of the work in [6], by Madeira et al., where a method to generate dynamic logics is presented.

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Paraconsistency meets refutation: a case of maximality

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Generally speaking, paraconsistent logics are used for the formal treatment of the inconsistent theories outside the classical approach. The defining feature of all paraconsistent logics is that the so-called explosion law, which holds true in classical logic, is rejected. Arising from the critique of how classical formal logic deals with contradictions, paraconsistent logics form a wide array of systems with these paraconsistent logics that retain as many classical laws as possible being of special interest to the researchers.

The *maximal* paraconsistent logics, as they are known, can also be adequately described in terms of what they reject from classical logic, rather than what they retain. There are many ways, in which such maximality can be defined. We focus on two ways of doing so: maximality of theory and maximality of consequence and consider logics that have both maximal theory and consequence relation. This shift in focus from what is valid to

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what is not, allows one to use the methods from the so-called refutation calculus in order to analyse such systems. Traditionally, refutation dealt with designing anti-axiom systems with rules preserving non-validity (this has been initiated in the context of Aristotle's syllogistic by Łukasiewicz in his seminal [1]).

However, we view refutation in a somehow broader way: it is not the task of designing such systems as such but the change in approaching logical systems that is important here. We describe a specific method used in the past in this context to show maximality of certain (mostly) three-valued paraconsistent logics. Here, logics are described using matrix semantics and certain refutation-related conditions are defined that ensure maximality. The conditions, first described in [2] mean that the problem of maximality is reduced to finding specific substitutions. Our main contribution is to show how this method generalises, allowing one to prove maximality of a number of such logics in one fell swoop, thus providing a simple and unifying account. We also present parts of the work in progress on extending this method even further to cover classes of n-valued paraconsistent logics for $n > 3$, and the results of an initial computational analysis confirming the theoretical results in case $n = 3$.

Surely, maximality of paraconsistent logics has been the topic of research from the very beginning. However, early results were related to showing maximality of specific systems, using a variety of methods. More recently, there were attempts at a more general approach [3], mostly limited to the case of three-valued logics. The approach we propose has the benefit of being arguably the simplest one and thus having a potential to be further generalised to values greater than three (which we attempt to do). And this, according to our knowledge, has not been accomplished yet.

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Reasoning about Complexity Needs Reflections on Paraconsistency

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It is always possible to give complex phenomena a representation which is consistent, simple and false, i.e. non-descriptive. Modeling the "blooming, buzzing confusion" therefore means simplification up to [but not crossing!] the limits of descriptiveness. In many cases this requires pluralism of perspectives, pragmatic coping with various levels of abstraction and with multiple partial explanations. Next all these aspects of our knowledge need to be integrated.

Usually, this will lead us through inconsistent stages of description. As in physics, they may persistently resist all efforts to remove them. So we better care for a methodology ready to cope with inconsistencies [2].

Jaśkowski's so-called discussive logic D_2 is a promising candidate [1]. It was originally designed for controlling inconsistent arguments in law, but the scope of application can be extended to other areas of rational discourse.

The core idea of this non-adjunctive approach is to keep the *ex contradictione quodlibet*-principle, but avoid the *ex falso quodlibet*. Technically, it can be done by blocking adjunction:

$$H, F \not\models H \wedge F.$$

Complexity is the great frontier to be explored by contemporary science. Inconsistency-tolerant reasoning engines seem indispensable for the enterprise.

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On the Possibility of Metaphysical Dialetheism

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All forms of dialetheism share a belief in "dialethas" (i.e. true contradictions). Metaphysical dialetheism is the belief that there are contradictions in the world. I will argue that metaphysical dialetheism is, rightfully understood, the most (and possibly only) controversial form of dialetheism, and further that it remains an open possibility.

I begin by explaining how dialetheism is seen as conflicting with the law of non-contradiction (LNC) and how, despite the widespread acceptance of LNC, there is no universally accepted form of the law. There is potential to define contradictions semantically, syntactically, pragmatically, or metaphysically. Depending on how we define contradiction and LNC, we will develop correspondingly different conceptions of dialetheism. I focus on the two most prevalent forms of dialetheism: semantic and metaphysical. In order to be a semantic dialetheist one only needs to deny a version of LNC based on a syntactic definition of contradiction, but they could accept a version of LNC based on a metaphysical LNC. While few authors have considered the possibility of metaphysical dialetheism, there are some important arguments against it to consider. Frederick Kroon argued in [1] that in so far as we find arguments for dialetheism persuasive, we also have good reason to reject realism. Tuomas Tahko [2] argued against the possibility of metaphysical dialetheism by explicitly arguing for a metaphysical definition of contradiction, and a metaphysical version of LNC. This paper will take up both of these challenges, and defend the possibility of a metaphysical dialetheist position.

I must mention one important caveat of this discussion. Graham Priest rightly points out at the very beginning of his discussion of these metaphysical matters that there is a certain difficulty in speaking of things as being "consistent". He writes "Consistency is a property of sentences (statements, or whatever), not tables, chairs, and people." [3] And he is certainly correct. It may be that the talk of consistency in metaphysics is a category mistake. However, this is a difficulty faced equally by the proponent of a metaphysical LNC or by their opponent.

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Logic, Probability and their Generalizations

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Logic and Probability have a long partnership, having survived together as a legacy from Leibniz, Bernoulli, De Morgan, Boole, Bolzano, Peirce, Keynes, Carnap, Popper and several other contemporary thinkers. According to this tradition, the problem of generalizing logical consequence relations raises questions that transcend both logic and probability, as a consequence of modern logical pluralism. This also leads to a probability pluralism, represented by non-standard theories of probability (i.e., theories of probability based on non-classical logics) that open new avenues and pose novel challenges to theory and to applications.

All such tendencies and areas of investigation are naturally generalized to possibility logics, necessity logics and other credal calculi that extend probability, considering that belief can be regarded as generalized probability or as evidence. This workshop intends to contribute to the state-of-the-art of such research topics, emphasizing the connections between all such topics.

The keynote speaker at this workshop is David Miller (page 143).

Call for papers

Submitted manuscripts should not have been published previously, nor be under consideration for publication elsewhere. All manuscripts will be refereed through a peer-review process. Manuscripts should be submitted in agreement with the UNILOG'2018 guidelines. Papers are invited in the following (non-exclusive) topics:

- Probabilistic generalizations of logic, including non-classical logics
- Generalizations of probability measures (as credal calculi and others)

- Interpretations of probability
- Philosophy of probability
- Probabilistic argumentation and inference, and their generalizations
- Questions on foundations of probability, including conditional probabilities

Abstracts (one page) should be sent by November 15, 2017 via email to walter.carnielli@gmail.com.

Is there any really autonomous proof for the non-existence of probabilistic inductive support?

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In a 2013 interview for the Royal Statistical Society [2], D.V. Lindley pointed out that, in his 2000 article [1], none of the reviewers of his paper made any kind of comment about the presented axioms of subjective probability. Lindley's axioms of subjective probability are equivalent to the relative probability functions for $\mathsf{L_I}$ presented in [7] — which are equivalent to the axiomatic system of Miller and Popper [4,5]. Lindley viewed statistics as the study of uncertainty, combined with the rules of the probability calculus. There seems to be little disagreement, thus, between subjectivists and others concerning the nature of the probabilistic reasoning. Bayes Theorem is true, Popper admits, although warning that such a 'triviality' cannot be claimed to support inductive generalization, or predictions about the future. Using a minimum of propositional logic and the elementary calculus of probability, Popper [6] revisits Miller and Popper [3], a short paper in which they proved that probabilistic support in the sense of the calculus of probability can never be inductive support. Miller and Popper's paper [3] has received a wide audience, with more than hundred other papers directly referring to it, many of them with harsh criticisms. Good part of such criticisms is directed towards the logic contents of [3]. It is interesting to note, however, that Miller and Popper dispose of a powerful autonomous axiom system for relative probability which assumes neither the propositional calculus nor Boolean algebra (albeit allows their derivation). An interesting question would be to know whether the main argument of [3] can be made entirely autonomous, independent of propositional calculus and Boolean algebra. If so, the argument would be much stronger than it seems. If not, however, the argument could be sensible to change of logic, a movement that Pooper and Miller themselves do not dismiss. I intend to discuss this question based on the diverse proofs available for this important result.

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A generalization of Popper’s probability theory

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Part of Popper’s philosophical program is intended to find a common general theory capable of generalizing both notions of logic and probability theory. Such a program involves a study of the logic of probability judgments as well as the logical relations among probability judgments. We refer to it as *Popper’s Probability Theory.* According to Miller in [2] Popper aims at introducing an alternative interpretation of physical probability,

*2013 marks the 250th anniversary of the publication in December 1763 of “An Essay towards solving a Problem in the Doctrine of Chances” in the Philosophical Transactions of the Royal Society of London, based on notes by Thomas Bayes and edited by Richard Price who submitted the Essay posthumously.

the propensity interpretation, and to elicit its significance both for quantum theory and for a new metaphysics of nature. In [3] the *autonomous probability functions* play a central role in the development of the Popper's theory, since they are independent of semantic notions. There are two ways to define autonomous probability functions: in *absolute* (given by unary functions) and in *relative* (given by binary functions) sense.

In this approach the connection between logic and probability is guaranteed by the result that shows that absolute probability theory is a generalization (in some sense) of two-valued propositional logic.

In this paper we intend to extend Popper's Probability Theory by enlarging the scope of the theory so as to include a class of paraconsistent logics based upon the Logics of Formal Inconsistency introduced in [1]. Our strategy is to generalize the autonomous probability functions (in absolute and relative sense) to a class of *paraconsistent autonomous probability functions* and show some technical results which ensure that these functions are indeed autonomous and satisfy the paraconsistent requirements.

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Paraconsistent autonomous probabilities

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Since one of his earlier papers [3], Popper championed an *autonomous* characterization of probability functions, independent of any semantic notion. I call such characterizations *semantically autonomous*, while a more radical characterization, independent of any syntactic property of consequence relations of a logic would be said to be *syntactically autonomous.*

In [1] a semantically (but not syntactically) autonomous characterization of paraconsistent probability is investigated, based upon Ci, a paraconsistent logic member of the family of Logics of Formal Inconsistency (LFIs).

Absolute and relative probabilities are presented by Popper as an abstract calculus, independent of any kind of semantics. Popper and Miller [4, p. 19] have at their side formidable mathematical tools, such as semilattices, distributive lattices and Boolean algebras, which provides them a completely autonomous approach to probability. Such tools, strictly connected to classical logic, are not available for the non-classical logicians, or are very incipient on what regards other logics. Devising autonomous characterizations of probability functions for non-classical logics seems quite a challenging task, therefore. I intend to offer a first step to this task, defining semantically autonomous probability functions for a famous three-valued paraconsistent logic, which has appeared in the literature in various guises, the maximal logic known as J3, LFI 1, MPT or LPT0 [see 2, section 4.4.7], while discussing the obstacles for a syntactically autonomous probability functions for such logics.

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Probability Valuations*

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It is well-known how to extend a truth-value assignment $V(p) \in \{0,1\}$ to the classical valuation $V'(\phi) \in \{0,1\}$ for all PL-formulae. Is there a possibility to generalise this approach to the unit interval? *Multi-valued* or *probability assignments* can be defined by $P(p) \in [0,1]$ indeed. However, $P'(\phi \wedge \psi), P'(\phi \vee \psi) \overset{\exists}{=}$? There is a royal road to extend P suitably if the literals in question are independent, as discovered by Wilfried Buchholz.

Let ϕ be such that $\mathrm{at}(\phi) \subseteq \{p_1, \ldots, p_n\}$. Then there is the full disjunctive normal form $\delta = \bigvee K$ such that $\phi \mathrel{\rightleftharpoons} \delta$, whereby K is some (possibly empty) set of $\kappa = \bigwedge \Lambda$ and each $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ for some $\lambda_k \in \{p_k, \neg p_k\}$. P' can now be calculated as follows:
$P'(\neg p_k) := 1 - P(p_k)$,
$P'(\kappa) := \prod_{\lambda \in \Lambda} P'(\lambda)$,
$P'(\phi) := P'(\delta) := \sum_{\kappa \in K} P'(\kappa)$.

An example is *logical probability*: $P_{\log}(p) := \frac{1}{2}$, whence $P'_{\log}(\neg p) = \frac{1}{2}$, $P'_{\log}(\kappa) = \frac{1}{2^n}$, $P'_{\log}(\delta) = \frac{|K|}{2^n}$, which equals *Wittgenstein-Wajsberg probability*.

We remark that e.g. $P'(\phi \wedge \psi) \neq \min\{P'(\phi), P'(\psi)\}$. This questions basic definitions of "many-valued logic".

Continuous truth-functions can be defined in analogy, whence corresponding *analog switching* can be derived.

Usually, $\pi \geq 0$ is called a *probability function* if $\models \phi$ implies $\pi(\phi) = 1$ and $\phi \perp \psi$ implies $\pi(\phi \vee \psi) = \pi(\phi) + \pi(\psi)$, whence the properties used for the definition of P' can be derived — except for the *literal axiom*:
$\pi(\bigwedge \Lambda) = \prod_{\lambda \in \Lambda} \pi(\lambda)$ for all Λ.

Consequently, *for all* literal-independent *probability functions, there exists P such that* $\pi = P'$, as can be shown for $P(p) := \pi(p)$ via the build-up of δ_ϕ for any ϕ. Conversely, demonstrably *all P' are probability functions, fulfilling the literal axiom.*

How far do the P' reach? Is their (tentative) name suitable? Is the literal axiom true in general? We had to recognise $\pi(p_1 \wedge p_2) \neq \pi(p_1) \cdot \pi(p_2)$ for $p_1 :=$ head, $p_2 :=$ tail. However, this contains the additional information $\pi(p_1 \wedge p_2) = 0$, which logic itself cannot not provide — unless axiomatically.

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On the universality of the probability concepts

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It is known that R. von Mises' frequency interpretation was strongly criticized. In a well-reasoned study, M. Lambalgen has demonstrated that the critical analysis was not related to formal shortcomings, but to the rejection of the foundations of the frequency interpretation, since its critics for example E. Borel and P. Levy, were subjectivists. Less known is the criticism of A. Kolmogorov's requirements for probabilities in the context of their application [1]. For example, Borel and Levy criticized Kolmogorov's requirement that the probability of event and its frequency characteristics should be close, since, in their opinion, it is redundant, because it is the conclusion of Bernoulli's theorem.

We show that Kolmogorov's condition is correct in frequency interpretation. Firstly we introduce a metrological concept, which is an informal finite version of von Mises' frequency interpretation [2]. On the basis of the empirical interpretation of Bernoulli's theorem, we have shown that Kolmogorov's informal requirement is not described by the conclusion of the theorem. This condition determines the geometric proximity of the probability and frequencies. In turn, it is a consequence of the stability of frequencies; this stability is a primary, ontological characteristic of the world to which Bernoulli theorem is applicable.

It is shown that criticism by the subjectivists of Kolmogorov's condition on the basis of Bernoulli's theorem falls short of goal. Is a fruitful synthetic concept possible based on different interpretations? We consider critical arguments by N. Cartwright [3] against the universal character of G. Shafer's approach.

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Paraconsistency, evidence and probability

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An epistemic interpretation of paraconsistent logic in terms of non-conclusive evidence was proposed in [1]. The basic idea is that a logic that is both paraconsistent and paracomplete can represent argumentative contexts where not only truth but also (non-conclusive) evidence is at stake. Evidence may be incomplete and contradictory: in the former case there is no evidence at all for A, nor for $\neg A$; in the latter, there is conflicting evidence, i.e. evidence for both A and $\neg A$. Two formal systems have been proposed in [1], the Basic Logic of Evidence (BLE) and the Logic of Evidence and Truth ($\mathrm{LET_J}$). BLE is a paracomplete and paraconsistent logic designed to express inferences that preserve evidence, instead of truth. Neither excluded middle nor explosion hold in BLE. $\mathrm{LET_J}$ is an extension of BLE that has been conceived to deal simultaneously with evidence and truth. $\mathrm{LET_J}$ is equipped with a recovery operator that recovers classical logic, thus preservation of truth, for propositions for which there is conclusive evidence available. Adequate valuation semantics and decision procedures have been presented for both BLE and $\mathrm{LET_J}$. In these semantics, the attribution of the semantic value 1 and 0 to a proposition A that does not behave classically means, respectively, that there is and there is not evidence for A. Thus, only the existence and absence of evidence for a given proposition can be expressed. However, the idea of formalizing preservation of evidence would be rather improved if the degree of evidence enjoyed by a given proposition could be quantified. In this talk we analyze the prospect of providing a probabilistic semantics that could be able to quantify the evidence enjoyed by a proposition in BLE and $\mathrm{LET_J}$.

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Logical Modalities in Statistical Models

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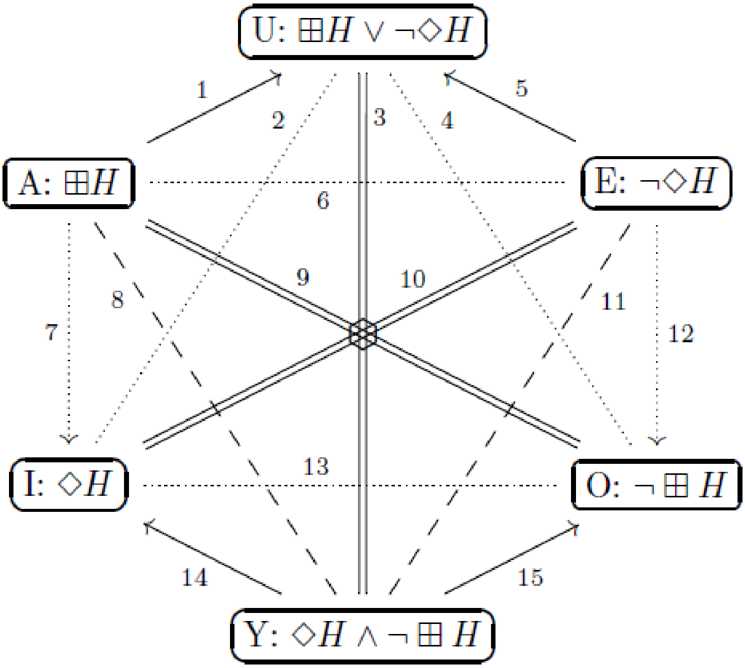

Hexagon of Opposition for Statistical Modalities

In previous work the authors have explored logical conditions for consistent and coherent statistical test of hypothesis, using these conditions to derive the GFBST — Generalized Full Bayesian Significance Test. [1,3,6] explore in detail the mathematical statistics and logical properties of the GFBST. The GFBST generalizes the previously defined non-agnostic version of the test, see [2,4,5]. However, these articles do not provide specific methodologies and concrete examples on how to construct an extended non-sharp version of a sharp hypothesis, a necessary step to apply the GFBST theory in some real statistical modelling situations.

In this paper we explore a method for constructing non-sharp versions of sharp hypotheses, using two simple statistical models as concrete examples, namely, the Hardy-Weinberg equilibrium hypothesis and the constant coefficient of variation hypothesis, as presented in [5]. Such extensions are constructed using techniques of perturbation analysis, based on engineering, instrumentation, observational and another implementation information about the pertinent statistical trial or experimental setting [see 7].

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*Interest Group in Pure and Applied Logics

Logic for Dynamic Real-World Information

This workshop is organized by

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> *In constructing symbolic logic, Frege, Peano and Russell always had their eye on its application to mathematics alone, and they never gave any thought to the representation of real states of affairs.*
> — Wittgenstein's conversation with Waismann, 1929

Over the past fifty years, the fabric of our society has been radically transformed by successful logic-based applications. In today's world, logic chips (i.e., CPUs) and the logic-grounded software that controls them support nearly all socio-economic infrastructure, from banking to defense.

Semantic Information Systems (SIS) are software that process information based on some formal (i.e., logical) understanding of the meaning of that information. SIS typically provide interpretation, representation and/or reasoning capabilities. They include:

1. Data-knowledge bases (or languages) that represent and support querying and calculations over an internal canonical form. For example: Oracle (2016), Teradata (2016), DB2 (2016), CYC (2016), OWL (2016) and RDF (2014).
2. Natural Language Processing (NLP) that converts text into an internal canonical form that expresses facts, rules and definitions. For example: OpenNLP (2016), Stanford parser (2016) and Berkley parser (2016).

Overwhelmingly, SIS are grounded in some (possibly restricted variant of) First Order Logic (FOL). For example, Relational (or, more properly, SQL) databases, which are still the predominant form of data management for organizations around the world, are varyingly faithful implementations of Codd's Relational Model [3,13,11], which itself was explicitly grounded in FOL. NLP also uses FOL as its predominant target representation [5,10]. In this sense, logic provides the abstract material technology from which SIS are engineered. SIS where the semantics are pre-defined and the domain knowledge is static within any particular transaction (e.g., banking and e-commerce applications), form the information backbone of our global economy. They are incredibly reliable. FOL has proven to be an extremely successful abstract material technology for these sorts of software applications.

*Chief Technology Officer

However, for other SIS where the semantics are hard to predefine, or where domain knowledge may need to change within a transaction, what we call "Dynamic Real-World Information Domains", FOL-based SIS's exhibit real world problems. Sometimes the problem is that the software generates incorrect inferences based on the information entered into the system. This happens, for example, when the assertions made represent only some of the domain that needs to be reasoned over and bivalency, tout court, is insufficient [14]. Sometimes the problem is that the software cannot compose a formal representation to interpret what a normal adult can speak or understand [18]. Sometimes the problem is that the software assumes certain attributes of the objects it is representing, and performs analysis based on those assumptions even though there was information ingested about the objects that a human would have understood as signaling that background assumptions about the domain were false [1].

The goal of this workshop is to explore:

- Where and how real-world SIS problems (e.g., those that occur in the analytical information systems of large corporations and governments) can be traced to specific characteristics of the logic (e.g., FOL) upon which they are constructed.
- Modifications to FOL (e.g., the alternative views espoused by Wittgenstein in the Tractatus or by Peirce) that would enable the construction of SIS that can operate more successfully in dynamic information domains.

The keynote speaker at this workshop is David McGoveran (page 142).

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[*] Association for the Advancement of Artificial Intelligence

[†] Empirical Methods in Natural Language Processing

[‡] **P**rinciples **O**f **D**atabase **S**ystems

[§] Association for Computing Machinery

[¶] Special Interest Group on Management Of Data

[‖] Special Interest Group on Algorithms and Computation Theory

[**] Special Interest Group on Artificial Intelligence

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Semantic softwares

- Berkley Parser (2016)
- CYC (2016)
- Db2 (2016)
- OpenNLP (2016)
- Oracle (2016)
- OWL (2016)
- RDF (2014)
- Semeval (2015)
- Stanford Parser (2016)
- Style Syntax (2012)
- TAC (2015)
- Teradata (2016)

Call for papers

Topics of interest to the workshop include but are not limited to:

- Real world information system problems that can be traced to the logic upon which the information system is built:
  - Missing data
  - Non-applicable data
  - Nulls
  - Inaccessible data
  - Word sense disambiguation
  - Semi-autonomous representational layers
  - Autonomous systems
  - Application contexts
  - Data-driven schema updates
  - Multi-agent planning systems with imperfect information

- The justification for specific non-classical logic (features) to solve specific classes of real world information problems such as the representation of missing and meaningless data:
  - Temporal logics
  - Spatial logics
  - Relevance logics
  - Dialetheist logics
  - Mereological/Mereo-topological logics
  - Modal logics
  - Higher order logics

  - Sub-structural logics
  - Tractarian logics
  - Peircean logics

- Principled approaches to specializing/extending logics (constraints, operators) for new domains:
  - Merging specialized features into a new/enhanced multi featured logic
  - Specialized logic blades (akin to specialized RDB blades)
  - Upper ontology (e.g., BFO*) logics
  - Cascading constraint-based processing

- Principled approaches to determining the computational properties of semantic representations:
  - Data-driven approaches
  - Definitional and reasoning-based approaches

- Principled approaches to defining computational properties (e.g., atomic operators) that can be used to support semantic information systems:
  - Well formed types/domains
  - Lambda calculus
  - Types as propositions
  - Martin-Löf (or intuitionistic) inspired type approaches

The proposed papers/talks should keep an eye on both theoretical and real world practical aspects. Abstracts (one page) should be sent by December 1st, 2017 via email to ethomsen@blenderlogic.com.

Impacts of Statistical Learning Theory for Enterprise Software

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*Basic Formal Ontology

Datalog access to real-world web services

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Web services now play an important role in our lives. Both our personal and enterprise related data can now be found in some remote data centers accessible only through third party application programming interfaces (API). This shift from self-controlled database systems to third-party managed database systems has brought forward many research challenges, one of which is the ability to integrate such multiple heterogeneous and autonomous web service API in a transparent manner. Mediation-based data integration approach when extended to web service API on one hand helped to achieve a declarative approach to the problem of extracting desired information from the web services, but on the other hand led to several new open challenges.

Data providing operations of web service API can be considered as relation with access patterns [1], i.e., a relation that takes as input one or more values and returns associated tuple of values. Several currently available web service documentations are only human readable. Hence, one major goal is to reduce the human programming effort of the process of extracting information from web services. Datalog program, including conjunctive queries help to describe API operations as well as query them using query rewriting algorithms like inverse-rules algorithm. However, there are several practical challenges [2] especially considering the strict certain answer semantics of such algorithms that fail to address the problem of incomplete information as well as a large number of spurious calls that need to be made for API operations involving more than one input arguments. These two cases were studied by us while building DaWeS [1], a data warehouse fed with web services.

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Singular reference, dynamic thoughts and spatial representation

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Keywords: singular reference, spatial representation, dynamic thoughts, situated cognition.

Evans attributes to Frege the intuition that it is not possible to capture the thought "this is now Ψ" without a propensity to capture the same thought in the series "this was Ψ a few moments ago", "this will be Ψ in a moment", etc. Here lies the nucleus of a situated and dynamic conception of cognition: to capture modes of presentation of objects located space-temporally requires the exercise of the dynamic ability to follow the trace of the object. Evans develops the point by showing how the understanding of sentences containing singular terms referring to objects requires locating the object through the synthesis between travel-based representations and map-based representations of space. The purpose of this paper is, first, to argue in favor of the Evansian dynamic conception of singular reference. Second, to develop an explanation that accounts on how it is possible to articulate local travel-based representations with global map-based representations, which provide an understanding of space and the possibility of using referential expressions. Third, to extend the explanation on how to identify indexical's and demonstrative's senses to the individuation of senses of proper names and predicates, showing that it is possible to explain the phenomena of semantic flexibility, semantic under-determination and contextual dependence by the same principles.

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Smart, Sentient and Connected: Trends and Directions in Information-Driven Applications

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The development and implementation of universal logic is essential to the direction of software applications and services, and to addressing challenges companies face in using these systems to make informed decisions and take smarter actions. As organizations move toward automated systems with analytics and AI embedded in them and expect to use standard application programming interfaces to flow data and connect disparate analytics components, they are likely to encounter problems that stem from a lack of universal logic.

This talk will offer an industry perspective on directions in applications and data architecture toward the goal of smarter, "sentient" (that is, highly aware and responsive), and connected systems across a disparate, heterogeneous data platforms. It will look what is happening as software and data management technology providers incorporate AI, machine learning, automated analytics, and more into their solutions. It will discuss industry research into the challenges organizations are facing with current applications and data management and how these trends will either help them overcome the challenges or make their problems more difficult to solve, especially if there is no universal logic.

*The Data Warehousing Institute

Logic-Grounded Ontological Fusion of Sensor Data and Natural Language

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Keywords: ontology, ontology-driven information system (ODIS), portion of reality (POR), fusion, sensor data, natural language understanding, thematic roles, reasoning, intelligence analysis, Basic Formal Ontology (BFO).

We describe an approach that is being realized in the construction of a prototype ontology-driven information system (ODIS) that exploits what we call Portion of Reality (POR) representations. The system takes both sensor data and natural language text as inputs and, on this basis, composes logically structured POR assertions that support computational reasoning. Our goal is to represent both natural language and sensor data within a single logic-grounded ontology-based framework that is capable of discovering and representing new kinds of situations (e.g., new kinds of processes and roles) based on new compositions of existing representations. To achieve this goal, all representational elements in the ontological framework must be embedded in a broader, logical computing language that allows for the composition of assertions. This (logical and) ontological grounding applies not just to objects, processes, and attributes, but also thematic roles (such as agent, patient, instrument). We applied our prototype in an intelligence analysis use case to test the hypothesis that a framework of this sort can produce structured information from combined natural language and sensor data inputs. We further tested our hypothesis by adding an enhanced US Air Force ontology framework to our ODIS to: (1) process a collection of sensor data, intel reports, and mission plans; (2) build composite POR representations from these data; and (3) machine analyze the fused results to infer mission threats.

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A Universal (?) Framework for Representing Knowledge about Real World Phenomena

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The research, presented in the talk, is driven by the necessity to represent knowledge about geological phenomena in a formal and computerizable way (the second author is essentially volcanologist) where as well the static as the dynamic aspects of those phenomena should be covered. Our framework has not reached its final stage yet, but seems to be matured enough to present it to a broader audience and to find out to what extent it may serve as a general framework for representing knowledge about real world phenomena.

A highly common experience is that first-order predicate logic and set theory are not adequate to formalize the meaning of sentences like the following, that make quite sense for experts:

> *Sandstones may equally well disintegrate in case of tectonic uplift, or, on the contrary, if downsagging occurs, be fused at depth and become crustal magmatic melt of silicic composition. Similar stuff may originate from melting of granites if they are driven down by tectonic downsagging. The silicic melt, even in crustal conditions, once appeared, unlikely will immediately crystallize back.*

We developed a set of rules to parse those statements. As result of parsing a statement we obtain the context of the statement identifying the predmet's (subjects, things), the primeta's (properties) and their relationships. The analogue of a context in first-order predicate logic would be a many-sorted signature Σ where sorts correspond to predmet's and unary predicates to primeta's. Deviating from set theory we allow predmet's to be "inhabited" by different kinds of "substance": grainy (elements, atoms), lump, or field [1]. Water, lava and sandstone, for example, are considered as substances of kind lump. Inhabited contexts are called environments and are the analogue to Σ-structures in first-order logic.

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The dynamic of a phenomenon is formalized by changes of environments in perfect in analogy to Petri nets, for example, where markings are changing, and evolving algebras (abstract state machines), where algebras are changing. To describe the allowed/observed behaviour of a phenomenon different kinds of formalisms, like activity diagrams for example, can be adapted. The second author developed for this purpose the Event Bush method [2,3,4] that has been applied in various case studies in Geology [2,5].

In the talk we will present the different parts of our framework covering syntax and semantics as well as statics and dynamics. Especially, we intend to discuss similarities and differences to first-order predicate logic and set theory, respectively.

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Naming Logics II

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Blaise Pascal famously claimed in *Les Provinciales*: "Je ne dispute jamais du nom pourvu qu'on m'avertisse du sens qu'on lui donne" (I never quarrel about a name, provided I am apprised of the sense in which it is understood). However to find the right word for the right thing is a sophisticated art.

Modern logic has been qualified by various expressions: "symbolic logic", "formal logic", "mathematical logic", "metamathematics". What does all this mean? For example "mathematical logic" is typically an ambiguous expression since it can mean both logic treated in a mathematical way or/and the logic of mathematics. "Symbolic logic" is also a mixture of different things, it can make reference to the use of some formal mathematical signs, or some true symbols, like Venn's diagrams. "Formal logic" is an expression put forward by Kant but ironically it has been often used to denote modern mathematical logic by opposition to traditional logic. "Metamathematics" was coined by Hilbert and he used it as synonymous to "Proof theory" (Beweistheorie) for him. Although it has been quite popular (cf. the classical book of Kleene, "Introduction to Metamathematics"), it is not much used today, probably because too much related with a specific approach to logic.

Concerning the names of systems of logic, there is also a lot of ambiguity. In which sense "classical logic" is classical, "intuitionistic logic" is intuitive, "linear logic" is linear, "relevant logic" is relevant, "free logic" is free, "intensional logic" is intensional? "Modal logic" encompasses many different systems, in which sense are they all dealing with modalities and what is a modality? "First-order logic" and "second order logic" are expressions which are often used. What do they mean exactly, are the involved qualifiers appropriate? Do they make sense in relation to "third-order logic"? The expression "zero-order logic" is not much used. Does it make sense to use it to qualify propositional logic, or does it correspond to something else?

A careful analysis of names used in logic can provide a fresh look at the different logical systems and/or the concepts and methodologies used to study and develop them. It can clarify what has been done and give some clues for new developments. This is a follow up of the workshop *Naming Logic(s)* organized at the 15th Congress of Logic, Methodology and Philosophy of Science (Helsinki, August 3–8, 2015).

The keynote speaker at this workshop is Göran Sundholm (page 161).

Call for papers

We invite contributions discussing logical terminology:

- in which sense symbolic logic is symbolical?
- in which sense mathematical logic is mathematical?
- in which sense formal logic is formal?
- in which sense classical negation is classical?
- in which sense intensional logic is intensional?
- in which sense minimal logic is minimal?
- in which sense free logic is free?
- in which sense relevant logic is relevant?
- are there many truth values for truth?
- can we put truth in a table?

Abstracts (one page) should be sent by December 1st, 2017 via email to isaac.manuelgustavo@gmail.com.

In which sense *symbolic logic* is symbolic?

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Modern logic was at some point canonized through the expression "symbolic logic", originally due to John Venn, but later on promoted by the *Association for Symbolic Logic* in a different way. This expression has been used in a ambiguous way and the result quite misleading, in particular through the promotion of a line of excessive formalism, and the use of convoluted notations, despite Paul Hamos's warning: "The best notation is no notation".

It is important to recall that the expression "symbolic logic" was inspired by the British School of *symbolic algebra*. And the related field was developed in the same spirit, even going further in abstraction: a sign having *any* interpretation. Tarski pushed this to the limit with *Model Theory*.

In this talk I will discuss these issues, on the basis of a semiotical anaylsis and a critical study of notations in logic from Boole to Tarski, via Frege and Peirce.

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In which sense informal logic is informal?

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In spite of the fact that Informal Logic, as a discipline, has been founded in 1978, its roots go back to the Ancient Philosophy. The primal aim of this paper is to try to identify the place of informal logic in the history of logic while taking the concept of reasoning into consideration. Our second aim is to discuss the relation between informal logic and formal logic through the concepts of form and deductivism. Based especially on their names, a relation of opposition is established between formal and informal logic. We, however, aimed to show the aforementioned relation can be established through unity rather than opposition, intending the concept of reasoning to be the common denominator of that relation.

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Is logic a theory of symbolization?

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Symbolization in logic can have several meanings:

1. development of a formal language, as a system of entities allowing to convey some contents in a faithful, coherent and complete way;
2. use of signs that completely determine the referred entities, allowing for a wholly analyzed construction of the intended objects/structures (like in mathematics);
3. uses of signs to express inferences abstracting from any specified entities to which the signs refer;
4. use of signs that can be substituted by any other signs of the same classes in a proposition.

These different meanings hinge on two crucial ideas:

- *representation*: an entity R ("representation") can present an entity ("content") which, in the non-trivial case, is different from R;
- *abstraction*: an entity can be used abstracting from some of its features.

However, there are some logical puzzles that need to be solved in order to achieve a better understanding of the role of symbols in modern logic:

- what type of representation is a symbol? In particular, what is the logical root of the well-known duality between iconic (analogical/depicting) representations and symbolic (digital) representations ?
- how an entity can present another entity than itself?
- what is the representational function of a symbol?
- what is the logical status of an operation of abstraction that seems to be not represented itself in a logical proposition?
- how the scope of an operation of abstraction can be warranted?

In this contribution, we outline a theory of representation to solve these puzzles. More specifically, we shall propose a definition of symbolic components of representation and show that the basic function of a symbolic system is to access to an universe that extend the immediate presence by acquaintance [1]. The operation of abstraction will also be defined within this framework and we will show that the use of variables can be interpreted as the representation of abstraction [2]. In this framework, functions

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of the traditional propositional connectives can be described as mechanisms of extension of presence; in particular, the symbol of implication can be understood as representing representation. Finally, logical inferences will be interpreted as some abstract laws of representation. Logic itself will appear as the abstract theory of representation, including theory of symbolization but not limited to it.

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Logic, Philosophy and Philosophical Logic

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Among the many field-naming terms used in contemporary logic, some are well established, others are more tentative; some are universal, others are known just in restricted logical circles; some look arbitrary, others seem to be conceptually relevant. "Philosophical Logic" is a term which seems to belong to the second categories just mentioned. While well established in philosophical literature, it is not quite so in computer science and mathematical circles. It provocatively invokes philosophy, the birth-field of logic. From a practical viewpoint, it covers a great deal of subfields and issues in contemporary logic: modal logic in many of its varieties (epistemic logic, temporal logic, deontic, logic of action, for example), many of the so-called non-classical logics (such as paraconsistent logic, relevant logic and intuitionist logic) as well as issues pertaining to the philosophy of logic and metalogic. While it is not difficult to extensively characterize philosophical logic — one has just to take a look at the content of the many textbooks, guides and handbooks available — it is not easy to say what philosophical logic effectively is. First, does it mean something at all? How can a logic, or a logical system, say, be philosophical? Given the concepts traditionally dealt with by logic, does it not seem that all logics (or even all logical issues) are in some sense philosophical? But what is this sense? Or, more generally, in which sense is logic philosophical? My purpose in this talk is to shed some light on these and other related questions pertaining to the meaning and contemporary import of the term "philosophical logic".

Logics and Metalogics

This workshop is organized by

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The focus of the workshop lies in emphasizing the fact that, in the study of logics, there always takes place an interplay between items belonging to object level, meta-level and metameta-level. Making a clear distinction of levels is crucial in logics, particularly in those systems where non-classical views are incorporated in the object language by denying bi-valence and/or denying some classical laws of reasoning. Many-valued logics, fuzzy logics, theory of graded consequence, and similarity based reasoning are a few to name. We shall base greatly on the prescription proposed by Alonzo Church in his book "Introduction to Mathematical Logic", vol. 1, Princeton University Press, 1956.

One of our attempts would be to draw attention to the fact that logic-studies do not usually pay due attention in distinguishing levels, due to which there do arise misconceptions, and even mistakes. Carnap's remark after Tarski's lecture at Vienna Circle meeting in the year 1930 may be recalled: "Of special interest to me was his emphasis that certain concepts used in logical investigations, e.g. consistency of axioms, the provability of theorems in a deductive system and the like are to be expressed not in the language of the axioms (later called the object language), but in metamathematical language (later called meta-language)".

However, in recent times this issue is being raised by some researchers, and, in some cases, though it has not been explicitly mentioned, researchers introduced different languages for different levels of a logic discourse.

The keynote speakers at this workshop are Soma Dutta (page 126) and Rohit Parikh (page 147).

Below we mention a few references.

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Call for papers

Relevant topics include (but are not restricted to):

- the importance and essentiality of making level-distinction in logic
- focusing current researches maintaining this distinction
- scrutinizing logics that ignore this distinction
- the ongoing debate on this issue

Contributed talks should not exceed a duration of 30 minutes including discussion. A one-page abstract should be sent via email before December 1st, 2017 to mihirc4@gmail.com.

Society semantics and meta-levels of many-valued logic

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Collective intelligence is now a buzzword, with several examples and tendencies in many areas. I will briefly survey some relevant aspects of collective intelligence in several formats, such as social software, crowdfunding and convergence, and show that a formal version of this paradigm can also be posed to logic systems, by employing the notion of logic societies and their correlate. group semantics. The paradigm of logical societies has lead to a new notion of distributed semantics, the society semantics, with theoretical advances in defining new forms of n-valued semantics in terms of k-valued semantics, for $k < n$. I summarise the main advances of society semantics, commenting on their general case, the possible-translations semantics pointing to some conceptual points and to some problems and directions still to be explored.

Dialetheic Validity

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I assume for the purposes of this talk that a dialetheic solution to paradoxes of self-reference, such as the liar paradox and Russell's paradox, is correct. In particular, given a language, one should reject the move of elevating its metatheoretic notions to a distinct "metalanguage". This does not mean that one should eschew reasoning about such notions. It means that one should reason about such notions in the language to which those notions apply. And given that the logic of such a language must be a paraconsistent one, this means that we must reason about metatheoretic notions paraconsistently.

A prime metatheoretic notion is *truth (simpliciter)*. Much thought has been put into paraconsistent theories for languages which contain their own truth predicate. How to produce inconsistent but non-trivial theories which contain the T-Schema for sentences of the language of the theory is well known.

Another paradigm metatheoretic notion is the model-theoretic notion of *validity*. The question of what a paraconsistent theory for a language which contains its own validity predicate is like has received much less attention.

The issue runs immediately into difficult questions. The notion of model-theoretic validity (unlike the notion of truth), has to be defined within a set theory. So we face the question of an appropriate paraconsistent theory of sets. Once this has been determined, we can then address a number of important questions concerning validity. Is it itself dialetheic? That is, are some inferences valid and invalid? If so, how widely does this dialetheism spread? Are all valid inferences also invalid? And what is the philosophical import of such inconsistency?

In this talk, I will discuss these issues. Whilst I do not expect to provide definitive answers, I do hope to advance the discussion of such matters.

Formalizing Ontological Disputes of the Systems in Metaphysics by Augmenting First Order Quantificational Logic: A Meta-logical Inquiry

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The objective of this research is to formalize the ontological dispute of the systems in metaphysics. Such formalization is developed by specifying a suitable augmentation of First-Order-Quantificational-Logic (FOQL) by combining it (FOQL) with a criterion of ontological commitment from meta-ontology. Characterization of ontological dispute of the systems in metaphysics happens at meta-level. Hence, the enquiry is meta-logical. This meta-logical enquiry uses techniques from both FOQL and meta-ontology. From meta-ontology Quine's criterion of ontological commitment (QCOC) is used: to be is to be the value of bound variable. QCOC is considered since this criterion takes into account both existential quantifier and the variables bound by the existential quantifier of the FOQL. In order to accommodate the concerns that run across the ontologically disagreeing systems, FOQL is augmented. Method of developing augmented FOQL is dependent on the technique of the creation of the paradox within a metaphysical system. Creation of the paradox within a metaphysical system is used to generate system specific quantifier which are called as Functionally Isomorphic

Quantifier (FIQ). FIQ will be used to formalize the ontological dispute of the systems in metaphysics. Method of developing augmented FOQL is dependent on the technique of the creation of the paradox within a metaphysical system. Creation of the paradox within a system in metaphysics is proposed as a technique which is accomplished by introducing independent-dependent variable distinction in the meta-logic. Following are the questions with regard to the independent variable (IV) and dependent variable (DV) in relation to metaphysical systems.

What is the IV and what is DV in a metaphysical system?

Why certain varieties are regarded as IV and some other varieties are regarded as DV?

How the introduction of IV-DV distinction would be useful in formalizing the ontological disputes?

Sociology and Anthropology of Logic: Past and Present

This workshop is organized by

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The workshop "Sociology and Anthropology of Logic: Past and Present" intends to explore the various ways in which logic can be approached from a sociological or anthropological point of view. We will look into how various actors and peoples concretely define and practice logic. Logic will not be apprehended according to a fixed definition of what it is or what it should be in order to assess their various definitions and practices. Instead, we will analyze their possible plurality.

We will focus on both past and present definitions and practices of logic. Historical investigations are welcome. In particular, we will discuss how philosophy and history of logic might benefit from various methodological approaches developed by historians and sociologists of mathematics and science over the past 40 years.

The organizers have contributed to this endeavor in various ways. In particular, Claude Rosental has been studying contemporary logical demonstrations from a sociological point of point. As for Julie Brumberg-Chaumont, she launched a program called "Homo Logicus, Logic at the Edges of Humanity: Anthropological, Philosophical and Historical Approaches" with Antonella Romano at EHESS** in Paris in 2016, and another program called "Social History of Logic in the Middle Ages" with John Marenbon (Trinity College, Cambridge) in 2017.

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The anthropological dimension of logic may be observed, for example, in the debates that Lévy-Bruhl's notion of "pre-logical mentality" of indigenous peoples has generated for more than a century. Anthropologists and other actors have often referred to logical skills to define the boundaries of humanity. Depending on their more or less open definitions of logic, they have included a limited or a large number of humans within these boundaries. Testing codified logical skills — Aristotelian and traditional logic in the past, thinking skill assessment (TSA) today — has been used since the Middle Ages as a way to select individuals in higher education institutions and/or as a means for excluding "logically disabled" groups in relationship to their so-called "social or racial inferiority". A sociological approach to the history of logic implies that logic is not only a set of theories and doctrines, but also a tool for action that individuals use in different institutional, political, and social settings.

Several authors have contributed to approaching logic this way. For instance, David Bloor's work inspired Irving Anellis and Ivo Grattan-Guiness' criticisms of the notion of "Fregean revolution". The "social history of logic" program developed by Volker Peckaus and Christian Thiel in the 1980s also illustrates this trend.

The keynote speakers at this workshop are Scott L. Pratt (page 156) and Christopher Goodey (page 131).

Call for papers

Papers are expected to cover one of the following topics:

- Logic and the Boundaries of Humanity
- Social Studies of Logic
- Anthropological History of Logic
- Selecting Humans Based on their Logical Skills
- Ethnology and Ethnomathematics
- History of Logic and History of Anthropology

Abstracts (one page) should be sent by November 15, 2017 via email to brumberg@vjf.cnrs.fr.

Pathologies of rationalities and embodied logic: Malebranche's conception of Madness as a case study

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Studies in Early Modern Philosophy take now for granted that the Age of Reason is also the Age of Passions. But the full consequences of that idea are not yet explored for Logic seen as the highest manifestation of Human Rationality.

This link between Reason and Passions is particulary striking in philosophies considering the overthrow of values since the original sin. I aim at analyzing conceptions of Logic within this kind of rationalism and its consequences to understand Boundaries of Humanity. For that purpose, Nicolas Malebranche is a relevant Case Study.

My main finding is a typology of three Pathologies of Rationality:

(i) The fool who is immediately and truly identified as such by every fallen man. This one pronounces incoherent discourses; or discourses obviously contradictory or inadequate to reality;

(ii) The fool who is falsely regarded as the highest expression of rationality by every fallen man. The Stoic Sage, for instance, coherently considers himself and is considered by others, as a God within Men. But this true expression of Monstrousness truly lies on the passions of Glory and Self Esteem;

(iii) The fool who is falsely regarded as fool by every fallen man. Yet this rare Man truly tries to reach the highest Rationality by seing in God's Rationality itself.

In Conclusion, the logical criteria of Coherence in Early Modern Philosophy can't be understood without Anthropological criteria (i.e. a theory of passions) and concrete Reality (i.e. the logical coherence could be mere imagination).

Later Wittgenstein: Logic, Necessity and Social Practice

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The aim of the paper is to sketch a novel philosophical position, at the crossroads of the philosophy of logic and the philosophy of social science. It builds on an overlooked Wittgensteinean insight, namely the idea to regard logic as a special social practice, constitutive to the human form of life. The central goal is to develop a social conception of logic, within a naturalist-anthropological framework. A convincing argument in its favor marks a significant progress in understanding what we actually do when we reason. This is so because it enables us to advance a new approach to the three perennial puzzles about logic, for which there is no agreed-upon solution: what does it mean to say, for instance, that the proposition 'the apple is green' if it can be green or red, but it's not red — is (i) necessary, (ii) certain and (iii) universally applicable (ie., it holds not only for apples)?

The aim is to show that this conception is not only credible in and of itself, but also more plausible when compared to its rivals, the two most influential traditional views according to which logic either is about ethereal conceptual relations holding in a Platonic heaven, or consists in a conventional manipulation of otherwise meaningless signs.

The articulation of this Wittgensteinean conception of logic as a special social practice lays the groundwork for further developments: articulating models of naturalized social practice appropriate for approaching problems in other areas of philosophy, such as the philosophy of social science, aesthetics, and ethics.

The key idea is to argue that it is social aspects that ground the formal-symbolic characteristics of logic. In other words, it is not the case that we first engage in various human activities, and then subsequently discover that some necessary logical truths (which we cannot doubt) apply to them from the outside, as it were, or externally. Rather, logic is intrinsic, already incorporated, silently, within all fundamental human activities; it is how we speak, in most human contexts, that already constitute, at the fundamental level, what later on receives the label 'logic'. To stress, this social aspect is not a consequence of the specialness of logic, but what constitutes it. In a word, we are logical beings — and this is what summarizes the naturalist-anthropological orientation mentioned above. (I here adapt a recent phrase by Hacking [1], who speaks about the "mathematical animal".) It is later Wittgenstein's original ideas about logic that constitutes the starting point of this paper. One promising way to begin explicating the central features of this naturalized social conception of logic is by drawing a contrast between his position and two of the most appealing, traditional views of this discipline mentioned above, Platonism and Formalism. Neither is naturalist, and neither has any place for the social element, but both are still dominant today. The key thought is threefold. Logic is (i) a special type of social practice, which (ii) is inextricably embedded into the human form of life, and thus is natural, immanent, not an historical accident. It is (iii) a way of regulating social interaction, as opposed to a solitary manipulation of abstractions taking place in the heads of individuals. Moreover, if Wittgenstein is right, and the symbols have meaning only in use, then the conception of mathematics as a mere formalism, a conventional manipulation of signs on paper, is extremely problematic too. What accounts for the necessity, certainty and generality of logical propositions is a key — social component, encapsulated within the idea that social interaction is not possible among those who doubt (the universal applicability of) logical truths. Necessity is social, and derived from the commonness of our natural, primitive reactions. Certainty is public, not private. The generality of logic is intrinsic, not conventional.

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Anthropology and Sociology of Logic as a Norm in the Middle Ages

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The second half of the 13th century witnessed two phenomena I would like to bring together: the definition of a strong anthropological ideal, the "philosopher" as the sole "complete human being", where logic is seen as an anthropological norm according to which a "logical scale" of humanity is defined and a (large) group of "logically disable people" is discriminated, and the rise of logic as a social norm for the scholarly worlds, within the newly-born universities as well as outside of them. Logic became then, for the first time, part of the basic European education system for young people, together with grammar. The social history of the practices and theories of concept of logic as a "discipline", in every senses of the term, means that a 'strong' and a 'weak' program for the sociology of philosophical knowledge can be fruitfully combined, and the debates between internalist and externalist approaches to the history of philosophy can be happily overcome. The method followed could be termed a non-normative historical study of the normativity of logic as embedded in practices and theories.

The Grammar of Conflict

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In three recent papers [2,4,5], Cora Diamond has criticized Peter Winch's 1964 paper [10], in which Winch attacked E.E. Evans-Pritchard's classic book [8] for European ethnocentrism. Diamond, in turn, attack's Winch's arguments for, in effect, trying to (use the later Wittgenstein to [11]) lay down what she calls "supergrammatical" laws for what can and can't be done with language, in particular with reference to criticisms of social systems of thought and practice in which one is not a participant. Diamond argues that this kind of argument is not necessarily to be found in the later Wittgenstein's work [11] and, moreover, that it is wrong. While I basically

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agree with Diamond's criticisms [2,4,5], both of Winch's own philosophical positions [9,10] as well as his way of reading Wittgenstein, in what follows I will briefly discuss three implications of her discussions which I suspect may not sit altogether comfortably with her overall views on objectivity. The first is that, while her argument that the mere idea of criticizing from "outside" is logically in order, there nevertheless remains something pragmatically fishy about the result of the argument. My next reservation concerns the fact that according to Diamond's own reasoning, what is internal to our grammar is not that a particular view on a particular question is actually correct. Rather, the grammar contains the bare formal concept of getting things right. Individual claimants in a dispute might gain some solace from this feature of our grammar, but it won't warrant much else in the way of choosing specifics. Finally, the fact that the feature of our grammar that makes criticism from outside possible for us is not a mere given, but rather, as Diamond herself points out, has an historical dimension to it, would suggest that there is no super-grammatical argument showing that this feature of our grammar is immune to change. And this in turn raises the normative question as to whether this feature is worthy of continued support. The very fact of its existence in our grammar now can't by itself be used in any non-circular way to justify our continued reliance on it.

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Hegel on the Naturalness of Logic

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In several passages of his works, Hegel claims that logic is natural and that the forms of thought are the natural element in which human beings live, act and interact. My paper has two parts. In the first I present Hegel's view on the naturalness of logic. I explore his use of the concept of "logic", focusing on the meaning of the expression *das Logische*, coined by Hegel himself, and on Hegel's distinction between *das Logische*, *natürliche Logik* and *die Logik.* In the second I show that Hegel's views on logic's naturalness could profitably join current debates about the sociology and anthropology of logic.

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[*] "ff", in this context, is a shorthand for "and following years".

Pathology of logical thought: Paranoia as a case study

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Logic of psychopathologies is widely studied today from an epistemological and phenomenological perspective in philosophy of psychiatry. How reason could treat apparent unreason? But, only one case of pathology is considered as very problematic with this issue [1]: paranoia as a pathology of logic. How reason could treat apparent reason? Indeed, this rare pathology is a paradoxical "case of a systematic chronic delusion, logically sustained" which challenge the clinical demarcation between the normal and the pathological, neurosis and psychosis [2], but also between the logic and the logical (systematization, autonomy, coherence of interpretation, evidence based). So, how can we establish that the logical reasoning (delusional system without weird delusion or hallucinations, mood disorder, maladjustment) is not consistent with the logic? And what about the logic: a pathological obsession of systematization? More difficult is the moral issue linking responsibility and logical reasoning for forensic evaluation of paranoid persons in case of premeditated murder: how can we establish exemption from criminal responsibility?

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Analyzing the Logic of the Unconscious. Notes on the Work of Ignacio Matte Blanco and its Ramifications

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This contribution aims to discuss the work of the Chilean psychoanalyst Ignacio Matte Blanco. In his work "The Unconscious as Infinite Sets" he proposed an ambitious theoretical reformulation of Freud's metapsychology in order to base an explanation for the operations of the unconscious

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and non-logical aspects of experience on mathematical logic (the principle of symmetry and the principle of generalization). Whereas his work has had important ramifications in clinical work, especially in Italy and Great Britain, the consequences for an articulation of his theory with human and social sciences have received less attention. After an exposition of Matte Blanco's theory in its context, we will discuss its uses, to be found notably in the work of the literary theorist Francesco Orlando and historian Carlo Ginzburg.

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Learning Logic in a Department of Philosophy: An Ethnographical Account

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I will give an ethnographical account of the way logic is taught in the philosophy department of a major university located in the United States. More particularly, I will analyze how logic is constituted as holding by itself, or self-sustaining. I will show how an "artificial" and autonomous language is created around a few isolated words, through several strategies of differentiation. I will analyze in particular how a dichotomy between formal and informal knowledge is formed, and how the categories of logical language, ordinary language and intuition are established. I will then argue that this process of differentiation is supported by an elaborate technology of showing, and that this technology is also used to build demonstrations. Finally, I will examine how doubt is managed, targeted and controlled in front of demonstrations, and how radical doubt is limited by temporal constraints.

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When Science Is and Isn't Paraconsistent

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A reflection on the actual practice of science reveals a seeming paradox: while the rules of classical logic are customarily used by scientists in their everyday practice, at times scientists seem to be tolerant towards inconsistencies in their mosaic of accepted theories. The goal of this paper is to explain *why* in some cases epistemic communities are inconsistency-tolerant and *why* they are inconsistency-intolerant in other cases.

To address this question, we utilize *the zeroth law* of scientific change currently accepted in scientonomy. The law stipulates that at any moment of time, the theories accepted by a certain epistemic community are compatible with each other. Importantly, the law distinguishes the concept of *compatibility* from the concept of *consistency* of classical logic. Two propositions are said to be compatible if they can be part of the same mosaic of theories; they may or may not be mutually consistent in the logical sense. Thus, by the zeroth law, it is possible for two theories to contradict each other and yet be simultaneously accepted by the same community. The compatibility or incompatibility of a given pair of theories within a certain community's mosaic is determined by the *criteria (method) of compatibility* employed by that specific community at that specific time; the criteria of compatibility may differ significantly across different epistemic communities and different time periods. By *the third law* of scientific change a community's criteria (methods) are deductive consequences of the theories accepted by that community. We show that a community's attitude towards an inconsistency between two theories ultimately depends on whether the community accepts these theories as absolutely true descriptions of their domains, or whether these theories are only accepted as approximately true. An epistemic community is tolerant towards an inconsistency, if the theories in question are taken as *approximations.* The phenomenon of inconsistency-tolerance is illustrated by means of several examples from the history of empirical sciences, including 18–19th century attitude of Newtonian physicists towards anomalous observations, and our contemporary attitude towards general relativity and quantum physics. In contrast, an epistemic community is intolerant towards an inconsistency between two theories when it takes these theories to be absolutely true. The phenomenon of inconsistency-intolerance is illustrated by several examples from the history of mathematics.

By clarifying the internal mechanism that shapes different communal attitudes towards inconsistencies, this paper suggests two important questions for future logico-historical research. First, what were the compatibility criteria employed by different epistemic communities throughout the history? Second, which specific paraconsistent logics can be said to have been at play in inconsistency-tolerant communities at different times?

Logic-in-Action? AlphaGo, Surprise Move 37 and Interaction Analysis

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For decades, playing Go at a professional level has counted among those things that "computers still can't do" [cf. 3]. As it seems, this changed dramatically in early March 2016, at the Five-Star Four Seasons Hotel in Seoul, South Korea, when AlphaGo, the most sophisticated computer program in Go to date, beat Lee Sedol, an internationally top-ranked Go professional, by four games to one. A recent documentary movie has captured the unfolding drama [cf. 7]. In turn, this contribution offers a video-based interaction analysis of the second game's "move 37", its surprise delivery by AlphaGo, and the subsequent line of commentary by the attending experts. What kind(s) of mediated expertise happened to be played out and commented upon in situ? What type(s) of "logic-in-action" happened to be achieved and articulated, in and as this particular situation of human-computer interaction? Drawing upon the detailed video analysis of the broadcasted episode, the contribution answers raised questions from an ethnomethodological and conversation analytic perspective [e.g. 1,2,5,8]. In so doing, the contribution offers an empirical analysis of situated logic(s) as a visual production and interactional achievement and, by the same token, reopens

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the conceptual reflection on different notions of logic [e.g. 6] and contrasting pictures of intelligent agency, "artificial" or embodied [e.g. 4]. The outlined contribution is part of a paper series that has charted the implications of Go game analysis for science and technology studies [9] and its "material semiotics" [10], on the one hand, and will do so for alternative approaches in the form of interaction analysis and "artificial intelligence" [11], on the other.

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Hintikka's Logical Thought

This workshop is organized by

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Jaakko Hintikka (1929–2015) was one of the founders of modern logic and philosophy of logic. This workshop is dedicated to the exploration not only of his work but also its future.

The keynote speaker at this workshop is Ahti-Veikko Pietarinen (page 152).

Call for papers

We invite submissions of proposals that advance the work of Hintikka on those areas in which he made significant contributions, including but not limited to:

- Categorical semantics (e.g. topos theory, linear logic, type theory)
- Modal logic
- Epistemic logic
- Possible-worlds semantics
- Game-theoretic semantics
- Model theory
- Interrogative model of inquiry
- Quantifiers
- History of ideas
- Aristotle
- Leibniz
- Kant
- Peirce
- Wittgenstein
- Cross-disciplinary approaches
- Critical analytic philosophy
- Future of logic

Abstracts (one page) should be sent by December 1st, 2017 via e-mail to salouachatti11@gmail.com.

World Lines Semantics and the Contingent *A Priori*

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In their virulent criticism against Kripke [7], Hintikka and Sandu [6] show that the necessity of identity between proper names is question-begging, even if they are rigidly interpreted. Indeed, in the context of world lines semantics, this would amount to assume that world lines never split or merge, i.e. to assume the necessity of identity itself. By contrast, rigidity would restore the validity of existential generalization, which is invalidated it we do not presuppose uniqueness of reference of proper names. This thesis had been initially advocated by Hintikka [4], and deepened in a more systematic study on epistemic logic by Hintikka [5]. In this presentation, I argue that endorsing the validity of existential generalization commits to another paradoxical Kripkean thesis, namely the thesis of the contingent *a priori*. Therefore, in world lines semantics, if we reject the contingent *a priori*, we must also reject the presuppositions of uniqueness of reference. After a critical discussion of the contingent a priori, I propose a modal formulation of the paradox. I conclude with further considerations in relation to Tulenheimo's innovative approach to world lines semantics.

One of Kripke's [7, p. 56] well-known examples is the following: the length referred to by "one metre" is fixed by stipulating its identity with the length of a particular stick (S) at a determinate instant (t), namely the standard metre rod. Since we are fixing that length by stipulation, we know *a priori* (automatically) that "the standard metre rod is one metre long" is true. Once the length of "one metre" has been fixed, "one metre" rigidly refers to that length. The length of S might change, not the length of one metre. Therefore, that S is one metre long at time t is a contingent fact. Even though we know *a priori* that "the standard metre rod is one metre" is true, this statement expresses a contingent truth.

According to Dummett [3, p. 124], this thesis is the sign that something goes wrong with rigidity. By following Donnellan's [1] distinction between attributive and referential uses of definite description, it can be explained that when we know the contingent fact expressed by "the standard metre rod is one metre", we do not know the same that when we know *a priori* that "the standard metre rod is one metre". Indeed, the attributive use consists of introducing "one metre" as a name for the length of S at time t, whatever this length is. By stipulation, we thus know a priori that "the length of S at t = one metre" is true. However, we do not know which this length is. By contrast, the referential use consists of making use of the description with the intention to refer to a determinate length. But this determinate length is an empirical fact that cannot be known *a priori.*

Such a distinction between attributive and referential uses can be expressed in Hintikka's first-order epistemic logic. Attributive use can be phrased in terms of *de dicto* knowledge, i.e.

$$\mathsf{K}(\text{the length of } S \text{ at } t = \text{one metre}),$$

which can express a conceptual knowledge acquired *a priori.* No matter the length of S at t or the length of one metre, we know they are identical, merely by stipulation. The knowledge of a determinate length intended by a referential use would assume a *de re* (*knowing-who*) construction like

$$(\exists x)\,\mathsf{K}(\text{the length of } S \text{ at } t = x).$$

Now, if "one metre" was a rigid designator for that length, the latter knowledge attribution would follow the former, by existential generalization. Therefore, if a priori contingent truths are rejected, existential generalization, and thus rigidity, must be rejected too. For the sake of comparison, Donnellan [2, p. 18] suggests distinguishing between "knowing that a certain sentence expresses a truth and knowing the truth of what is expressed by the sentence", but without emphasizing the role of existential generalization and without rejecting rigidity.

We will conclude by discussing some new insights provided by the distinction of physical and intentional modes of predications put forward by Tulenheimo [8] in the context of world lines semantics. What is the impact of such a distinction with respect to the different kinds of knowledge attributions we have previously referred to? What consequences can be drawn with respect to our analysis of the paradoxical thesis of the contingent a priori?

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Tableau Approach to Epistemic Logic Based on Relating Logics

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Epistemic logic has been traditionally viewed as a certain interpretation of modal logic. Thanks to Hintikka [3], possible worlds semantics turned out to provide standard models not only for alethic and temporal, but also for epistemic logic. From then on, this approach has been adopted to logical folklore, eventually forming something, that can be called a paradigm. Normal modal logics fail to cover epistemic states of non ideal agents (those who are not logically omniscient). This shortcoming, labeled "the logical omniscience problem", appear to stand out as main flaw of epistemic normal modal logic.

In the presentation, we suggest different logics to express propositional attitudes, namely relating logics. First systems of epistemic logic were discovered by Epstein [1,2]. More general description of relating logics can be found in Jarmużek and Kaczkowski [4]. The main ideas of epistemic relating logic are introduced together with their motivations. We choose tableau methods for the proof-theoretic description of our logic, which is founded on the more general theory [5].

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Dialogues and Strategies in Aristotle's Logic: Furthering Hintikka's Insights

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In his 1997 reply to Woods and Hansen, "What was Aristotle Doing in His Early Logic, Anyway?", Jaakko Hintikka clears some misunderstandings concerning his reconstruction, by means of an interrogative model, of Aristotle's logic. He thus explicitly challenges some of the deep-rooted assumptions of Aristotelian scholars and modern logicians: Aristotle's *Analytics*, asserts Hintikka, are not radically separated from his *Topics* and *De Sophisticis Elenchis*, but are rather the pursuit of the same goal at a different level, that of strategies, as opposed to down-to-earth — or "down-to-agora" as he says — dialectical bouts between individual, concrete opponents.

Hintikka justifies the absence of an explicit question-and-answer framework in the *Prior* and *Posterior Analytics* by the strategic principle of anticipation of the answers to one's questions: since the best strategic course of action in a game of questions and answers is to ask only those questions of which you can anticipate the answers, and for which the anticipated answers go your way, then, in a strategic perspective, one can actually do without an answerer. This would thus yield both the presentation of Aristotle's syllogistic framework and Hintikka's interrogative model of it.

Two essential elements of Hintikka's interpretation can thus be outlined: that Aristotle was first and foremost interested in question-and-answer inquiries and in this regard thought like a dialectician; and that this question-and-answer mold for reasoning could be made implicit through a strategic perspective.

The purpose of this talk will be to uphold Hintikka's perspective on Aristotelian logic, which is still not universally accepted among scholars, and further his insights by proposing a new logical framework in which the rules themselves are defined through questions and answers, or, as we

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call them, through challenges and defenses. The dialogical structure of the syllogisms should thus become apparent in the logical framework, with the added benefit that such a framework rests on a rule, the Socratic rule, that directly yields Hintikka's distinction between a justification *ad hominem*, concerning only the dialectical bouts at the agora level, and a justification *ad argumentum*, which also concerns the strategy level.

The path which will be tread in order to defend and illustrate Hintikka's two tenets on Aristotelian logic will not be Hintikka's own path consisting in making the interlocutor implicit, but will rather be the path consisting in making everything *more* explicit, enabling us to emphasize, in the logical framework itself, the structural link between syllogistics (*Analytics*) and dialectics (*Topics*), and to provide a logical rendering of the distinction between *ad hominem* and *ad argumentum* conclusions through the Socratic rule.

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Hintikka on the "Kant-Frege view": A critical assessment

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In [1], Hintikka rejects the "Frege-Russell ambiguity thesis" (i.e., the claim that 'is' is ambiguous) in favor of a contextual account of existence. In addition, he also argues that the "Kant-Frege view" (i.e., the claim that Kant is a forerunner of Frege's treatment of existence) is wrong, for his supporters erroneously assume that also for Kant, 'is' is ambiguous. In my presentation, I will focus on Hintikka's take on the Kant-Frege view. So, I will first critically evaluate his arguments against it and then, in contrast to him, I will attempt to prove that Kant's claim that existence is not a predicate and Frege's claim that existence is a quantifier are in fact logically interdependent.

Reference

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Logic for Children

This workshop is organized by

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When we explain a theorem to children — in the strict sense of the term — we focus on concrete examples, and we avoid generalizations, abstract structures and infinite objects.

When we present something to "children", in a wider sense of the term that means "people without mathematical maturity", or even "people without expertise in a certain area", we usually do something similar: we start from a few motivating examples, and then we generalize.

One of the aims of this workshop is to discuss techniques for *particularization* and *generalization*. Particularization is easy; substituting variables in a general statement is often enough to do the job. Generalization is much harder, and one way to visualize how it works is to regard particularization as a projection: a coil projects a circle-like shadow on the ground, and we can ask for ways to "lift" pieces of that circle to the coil continuously. *Projections* lose dimensions and may collapse things that were originally different; *liftings* try to reconstruct the missing information in a sensible way. There may be several different liftings for a certain part of the circle, or none. Finding good generalizations is somehow like finding good liftings.

The second of our aims is to discuss *diagrams*. For example, in Category Theory statements, definitions and proofs can be often expressed as diagrams, and if we start with a general diagram and particularize it we get a second diagram with the same shape as the first one, and that second diagram can be used as a version "for children" of the general statement and proof. Diagrams were for a long time considered second-class entities in CT literature ([2] discusses some of the reasons), and were omitted; readers who think very visually would feel that part of the work involved in understanding CT papers and books would be to reconstruct the "missing" diagrams from algebraic statements. Particular cases, even when they were the motivation for the general definition, are also treated as somewhat second-class — and this inspires a possible meaning for what can call "Category Theory for Children": to start from the diagrams for particular cases, and then "lift" them to the general case. Note that this can be done outside Category Theory too; [1] is a good example.

Our third aim is to discuss *models*. A standard example is that every topological space is a Heyting Algebra, and so a model for Intuitionistic Predicate Logic, and this lets us explain *visually* some features of IPL. Something similar can be done for some modal and paraconsistent logics; we believe that the figures for that should be considered more important, and be more well-known.

The keynote speakers at this workshop are Bob Coecke (page 124) and Ralf Krömer (page 133).

Some resources related to this workshop are in http://angg.twu.net/logic-for-children-2018.html.

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Call for papers

Topics of interest to the workshop include, but are not limited to:

- Ways to visualize logics or other algebraic structures
- (The many roles of) diagrams in Category Theory
- Categorical semantics (e.g. topos theory, linear logic, type theory)
- Translations between digrammatical languages and formal languages

Contributed talks should not exceed a duration of 30 minutes including discussion. To submit a contribution, please send a one-page abstract by December 1st, 2017 to eduardoochs@gmail.com.

Community of Philosophical Inquiry

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According to Piaget, the first psychologist to study reasoning from a logician point of view, children are not born logical and logical reasoning only appears progressively up to adolescence. His theory of the development of rationality [4] was criticised for diverse reasons. Several studies demonstrated that children have some degree of logical understanding at a very young age [3] and that adults are not optimally logical [6].

David Moshman, a professor of educational psychology at the University of Nebraska-Lincoln, offers a new reading of Piaget's work by understanding the development of rationality at a metalogical level. Following his pluralist rational constructivism theory [2], logical reasoning develops through the increase of metalogical understanding. In order to have a consciousness on ones inference, it is necessary to make it explicit and that process occurs during peer interaction. I argue that Community of Philosophical Inquiry (CPI) used in Philosophy for Children (P4C), if practiced with a special attention on its metacognitive aspects, can constitute the perfect didactic to put into practice Moshman's theory. Furthermore, adding some explicit notions of logic and reflections on logical thinking could transform the CPI method into a logic lesson for children and learners of all ages.

I will present CPI as the practice of dialogue [1] and how this method puts into practice Moshman's theory through intellectual moves performed by the children themselves [5]. My research consists in linking the metacognitive and metalogical strategies with those moves in order to foster the development of logical understanding, transforming CPI in CLI — Community of Logical Inquiry.

The claims I endorse put forward the possibility to build a toolbox for the learning of logical thinking in schools. This work could help teachers' work in providing them the tools they need to develop better teaching methods that they can put into practice in their classroom. Since metacognitive strategies have been proven efficient for all levels learners, this approach could have a major impact in scholar system, in teachers' training and also in a broader social scale.

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Visualization as Restructuring and thus a Source of Logical Paradox

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We survey and systematize the ways our minds organize and visualize thoughts. We then observe their relevance in explaining different kinds of logical paradox. We also show where they arise in math.

We were inspired by educator Kestas Augutis's vision that every high school student write three books (a chronicle, a thesaurus, and an encyclopedia) so as to master three kinds of thinking (sequential, hierarchical, and network) [1]. We thus collected dozens of examples of how we organize our thoughts [2]. Surprisingly, we never use sequences, hierarchies or networks in isolation. Instead, we use them in pairs:

Evolution: A hierarchy (of variations) is restructured with a sequence (of times).

Atlas: A network (of adjacency relations) is restructured with a hierarchy (of global and local views).

Handbook: A sequence (of instructions) is restructured with a network (of loops and branches).

Chronicle: A sequence (of events in time) is restructured with a hierarchy (of time periods).

Catalog: A hierarchy (of concepts) is restructured with a network (of cross-links).

Odyssey: A network (of states) is restructured with a sequence (of steps).

In general, a first, large, comprehensive structure grows so robust that we restructure it with a second, smaller, different structure of multiple vantage points.

In a separate investigation, we listed and grouped paradoxes. This yielded the following six themes:

- Concepts may be inexact. (The paradox of an evolution.) We can't specify exactly at what point in the womb a child becomes conscious, or at what point in evolution two species diverge.
- The whole is not the sum of the parts. (The paradox of an atlas.) If we replace all of the parts of an automobile, and then build a copy with all of the old parts, which is the original?
- Our attention affects what we observe. (The paradox of a handbook.) Achilles can never catch a tortoise if we keep measuring the distance between them.
- There may be a limited contradiction. (The paradox of a chronicle.) How can we reliably learn from one who has ever made a mistake?
- We cannot make explicit all relevant assumptions. (The paradox of a catalog.) $10+4$ may equal 2 if we happen to be thinking about a clock.
- We can choose differently in the same circumstances. (The paradox of an odyssey.) I am lying when I say 'I am lying'.

Each type of paradox brings to light the fundamental gap between the (seemingly infinite) primary comprehensive structure and the (manifestly finite) secondary structure which organizes our vantage points. Our mind visualizes a qualitative but illusory relationship between the two structures.

These same six restructurings arose in a broader investigation which yielded 24 ways of figuring things out in mathematics [3]. We identify the six restructurings with six axioms of set theory: Pairing, Extensionality, Well-Ordering, Power Set, Union and Regularity.

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Elementary introduction to pasting

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The operation of pasting of 2-cells is part of the foundations of 2-category theory [4]. It was introduced by Bénabou in [1], and then further explored in [2]. However its associative property, fundamental aspect that makes it useful to prove theorems, was not proved (or even properly stated) before [4].

The main purpose of the talk is to give some elementary aspects of pasting, giving examples within basic category theory in order to motivate its day-to-day use even in 1-dimensional category theory. These examples intend to demonstrate that, once we assume pasting is well-defined, pasting gives nice ways of understanding and dealing with proofs diagrammatically. For instance, the whiskering and interchange law come for free in proofs using pasting of 2-cells.

If time permits, we finish giving a brief discussion on results that gives another perspective on the well definition/associativity of the operation pasting, relating it with presentation of 2-categories, deficiency of presentations and, hence, topology [3].

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Subjectivism and inferential reasoning on teaching practice

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In this work, we analyse well-succeeded strategies and challenges of teaching principles of decision theory as developed in [2,3,1], for students in secondary school. Among other things, we emphasize the aspects of probability and conditional probability under the subjectivistic interpretation and inferential reasoning based on a Bayesian learning approach [4].

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Categories and Logic

This workshop is organized by

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Category theory and logic interact in many ways. Category theory is used as a general organizational tool for the structures arising in logic, specifically in the study of categories of logics and translations, but also in other ways, via categorical semantics and internal languages of categories, syntactic categories, categorical foundations of mathematics and their relation to set theoretic foundations.

The keynote speakers at this workshop are Pierre Cartier (page 123) and Ingo Blechschmidt (page 121).

Call for papers

We invite contributions on all interactions of category theory and logic. Topics include:

- Categorical semantics (e.g. topos theory, linear logic, type theory)
- Topos theory (also in its not primarily logical aspects)
- Categorical structures arising in logic (e.g. display categories, dagger categories, fibrations)
- Classes of categories arising in logic (e.g. accessible categories, locally presentable categories)
- Particular categories arising in logic (e.g. categories of logics and translations, particular quasivarieties)
- Institution theory
- Categorical algebra for algebraic logic
- Category theoretic accounts of model theoretic constructions (e.g. of ultraproducts, elementary classes)
- Category theoretic foundations for mathematics

Abstracts (one page) should be sent by November 15, 2017 via e-mail to peter.arndt@uni-duesseldorf.de.

Proof Diagrams as Concurrent Syntax for Sequent Calculi

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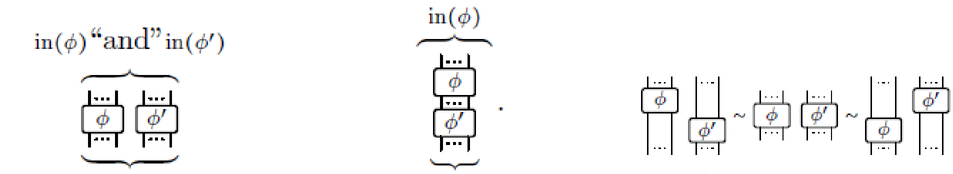
Keywords: string diagrams, linear logic, monoidal categories, proof semantics.

In this presentation we show how the sequent calculus formalism can be replaced by an alternative 2-dimensional version: *proof diagrams*. Proof diagrams are particular kind of string diagrams that allows to capture a more comprehensive notion of proof.

Sequent calculus is a formalism introduced by Gentzen [4] for intuitionistic and first-order classical logic. Since then, sequent calculus has become the standard proof formalism for a wide variety of logics. Sequent calculus main theorem is *cut elimination*, which ensures analiticity of derivations; however, the cut elimination step might not be directly applied to any derivation, since some preliminary permutations of rules inferences could be required.

For this reason the "good" notion of proofs coincides with the equivalence class of derivations for a formula rather then syntactical equivalence. A part of this equivalence is generated by transformations related with syntactical impossibility of sequent calculus formalism to represent concurrent computations.

String diagrams are a formalism for monoidal categories able to capture at the same time the two opposite notions of *causality* and *concurrency*. In this syntax, we have two compositions: the *parallel* one and the *sequential* one, which may interact by the *interchange rule*.



(a) Parallel composition (b) Sequential composition (if in(ϕ′) = out(ϕ)) (c) Interchange rule

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If we consider this rule as an equality, string diagrams are a syntax for *strict monoidal categories*. The string diagrams 2-dimensional representation of terms is able to capture the notion of concurrence in a more intuitive way with respect to traditional "in line" formulas.

Proof diagrams, introduced in [1], are an alternative 2-dimensional syntax for multiplicative linear logic derivations. The syntax allows the definition of a framework with a linear time sequentializability procedure, i.e. a procedure to reconstruct the derivation from a given diagram, able to capture a large part of syntactical proof equivalence (the one related with inference rules concurrency). Moreover, this formalism can be adapted to p-simulate any logical system sequent calculus and to defines for it a non-standard denotational semantics.

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κ-filter pairs and non-finitary logics

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The notion of filter pair was introduced in [1], for creating and analyzing general *finitary* propositional logics and their translation morphisms, expanding the work initiated in [4], that is restricted to the setting of algebraizable logics [2].

Considering the special case of filter pairs $\langle G, i\rangle$ where the functor $G = Co_K$ is given by congruences relative to a class of algebra K, we give criteria when the associated logic is protoalgebraic, equivalential, algebraizable, truth-equational, self-extentional or Lindenbaum algebraizable, just analyzing the relation between Leibniz operator, Suszko operator and Frege operator with the adjoint of i, improving our previous results.

We adjust the notion of filter pair in such a way that we can treat κ-compact logics, for each regular cardinal κ: The corresponding new notion is called κ-filter pair. We show that any κ-filter pair gives rise to a κ-logic and that every κ-logic comes from a κ-filter pair. Taking adequate notions of morphisms, we show that the category of κ-logics and translation morphisms is (isomorphic to) a full reflective subcategory of the category of κ-filter pairs. We use the notion of κ-filter pair to show that logics always admit natural extensions, providing an(other) answer to a question of Cintula and Noguera [3]. We further point out how κ-filter pairs allow to extend standard notions from finitary logics to arbitrary logics, e.g. those of being algebraizable, protoalgebraizable, equivalential or truth-equational.

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Beyond the categorial forms of the Axiom of Choice

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In the sequel of [1], besides to work on new categorial forms of axiom of choice, we explore categorial forms of statements on partial ordered sets such that are equivalent to the axiom of choice, namely Zorn's Lemma, Hausdorff Maximal Principle and the Principle of Cofinality. This categorial forms are different from categorial set-forms of axiom of choice defined in [1]. There, the authors state that a statement φ is a *categorial Set-Form of the Axiom of Choice* if the Axiom of Choice for Sets is equivalent to the statement $\varphi_{\mathbf{Set}}$, since φ declares properties of objects, morphisms and/or constructions in a category, and the relativization of φ with respect to the category C is denoted by φ_C. In the present case, the categorial form is a statement φ such that $\varphi_{(P,\leq)}$ ($(P,\leq)$ is viewed as a category) for any poset $(P,\leq)$ is equivalent to axiom of choice for sets.

In [2], the authors have introduced a notion of categorial Zorn's Lemma that is: "*in a category C, if every filtered diagram has an inductive limit, then C has a quasi terminal object*". We realized that an inductive limit does not translate precisely the notion of "upper bound". So, we introduce another categorial Zorn's Lemma: "*if every filtered diagram has a cocone in C, then it has an almost maximal object*". If $C = (P,\leq)$ is a poset viewed as a category, both categorial notions coincide and are equivalent to Zorn's Lemma on C (this means that the Zorn's Lemma can be considered for any poset $(P,\leq)$ but one concludes that it has maximal element if any chain has upper bound in$(P,\leq)$).

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We also introduce the categorial Hausdorff Maximal Principle, that is: "*the category of filtered subcategories of C has a quasi terminal (almost maximal) object*". A property P on a locally finitely presentable category C has finite character if, for any object c of C has the property P, then c_i has the property P, for all $i \in I$, where c_i is a finitely presentable object, $\{c_i\}_{i \in I}$ is a directed diagram with $c = \mathrm{colim}_{i \in I}\, c_i$. The categorial Teichmüller-Tuchey Principle is considered as: "*For every locally finitely presentable category C and every property P of finite character, there exists a quasi terminal (almost maximal) object with the property P*".

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Boole-Weyl Algebras in a Categorical Context

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We review the construction of the Boole-Weyl algebras, which are the analogue of the algebra of algebraic differential operators on the affine plane over the field with two elements, a proceed to study these algebras from a categorical viewpoint, namely we show that they correspond to the endomorphisms of certain objects in the category of finite dimensional vector spaces over the field with two elements, and study the quantum-like structural properties of this category. This talk is based on [1].

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Logical rules are fractions

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Key: *A logical rule* $\frac{H}{C}$ *is indeed a fraction, but its numerator is* C *and its denominator is* H*.*

The link between inference rules as $\frac{H}{C}$ where H is the hypothesis and C the conclusion, and fractions as $\frac{N}{D}$ where N is the numerator and D the denominator, goes through the categorical notion of fraction. Categories of fractions were introduced by Gabriel and Zisman in [5] as a tool for homotopy theory. The link with logic, using limit sketches, was studied in [3,4]. An application can be found in [1,2].

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Makkai duality, descent and definability

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In his 1963 thesis, Lawvere showed that an algebraic theory T could be reconstructed from its category of models $\mathrm{Mod}(T)$ as functors $\mathrm{Mod}(T) \to \mathrm{Set}$ preserving all limits, filtered colimits and regular epimorphisms. In the '80s, Michael Makkai extended this to first-order theories to show that a first order theory T could be reconstructed from $\mathrm{Mod}(T)$ as Set-valued functors (the "ultrafunctors" preserving all ultraproducts and canonical maps "ultramorphisms" between them.

One thinks of these preservation requirements as descent data for reconstructing, for a given functor F on $\mathrm{Mod}(T)$, an object of the classifying topos of T which realizes F as its evaluation functor.

This talk will have two parts: first, I will construct "exotic pre-ultrafunctors" which show that only considering certain ultramorphisms is not enough to guarantee descent. Second, I will apply these techniques to study internal category theory and internal adjoint functors in the classifying topos of T, and obtain results around an internal general adjoint functor theorem.

Fibrations of contexts beget fibrations of toposes

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The notions of (op)fibration in the 2-category of toposes and geometric morphisms have close connections with topological properties. For example, every local homeomorphism is an opfibration. This connection is in line with the conception of toposes as generalized spaces. Borrowing from work of Ross Street [2], we introduce a syntactic notion of (op)fibration in the 2-category Con of contexts developed in [3]. Among other things Con gives a syntactic presentation of finitary fragment of theory of toposes. It also provides us with good handling of strictness. We establish that every context extension (op)fibration in Con gives rise to an (op)fibration of toposes in the sense of Johnstone [1]. The machinery developed here will produce a large group of examples of toposes over a base classifying internal structures.

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An Abstract Approach to Algebraizable Logics with Quantifiers

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In [2], the authors describe an approach to define categories of logics with good properties, in particular are considered categories of algebraizable logics. They describe a way to algebraize logics though quasi-varieties of algebras in a uniform way, and [3] establishes several connections between the category of algebraizable logics and the category of quasi-varieties.

A natural question is: how to extend (naturally) a category of propositional logics to a category of logics with quantifiers? Historically, several algebrizations of the first-order logic (FOL) have been proposed, like cylindric/polyadic algebras. These algebras are used to establish a particular and *ad doc* algebraic semantics. Others approaches utilizes categorical logic to extend the particular algebraic construct to a suitable category. For example, the propositional intuitionistic logic IPC is (Blok-Pigozzi [1]) related with the variety of Heyting algebras HA; on other hand intuitionistic (higher-order) logic are interpreted in toposes by the slogan "quantifiers as adjoints". Instead to showing a specific construction to understand what is the quantification in a algebraic sense, we want to find what is the best place to "algebraize" logics with quantifiers.

In [4], it appears an abstract notion of "logics with a sets of free variables" as *hierarchy of sets* (H-set), i.e. given a set V, it is a functor

$$F\colon (\mathrm{Parts}(V), \subseteq) \to \mathbf{Set},$$

satisfying adequate coherence conditions: this concept can be used to describe cylindric algebras. Our idea is use this notion to provide a more conceptual approach to abstract quantification that could also extend the idea of Blok-Pigozzi uniform technique to the logic with quantifiers setting. In this proposal, quantifier are *defined* as families of arrows satisfying several commuting diagrams and coherence conditions. These conditions are compatible with some prescriptions of what a quantifier *should be* (see Frege, Russell or Quine).

This new vision of the quantifiers in a H-set can be algebraizable in many ways (Lindenbaum-Tarski, Blok-Pigozzi's algebrization, Czelakowski's protoalgebrization), and since some conditions are satisfied, a (covariant) sheaf-like structure emerges from this H-sets and quantifiers become some morphisms between these algebras that acts like a "deformation retraction".

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Interchangeable formulas and categories of logics

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Keywords: categories of logics, interchangeable formulas, quotient categories.

The study of categories of logics is motivated, among other reasons, by questions such as how to combine logics and when they are equivalent [e.g. 1,2] and the bibliography therein). Categories of logics do not always have good categorical properties such as the existence of finite limits and

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colimits. The existence of these provides natural mechanisms for the factorization and combination of logics. One natural way to obtain from these categories new ones with better properties is to consider the quotient category induced by the interdemonstrability relation, as done in [4]. In this case the existence of finite limits and colimits is only guaranteed when the objects involved are congruential logics.

Following the aforementioned works, we consider the category whose objects are Tarskian logics and whose morphisms are flexible translations that preserve the interchangeability relation between metaformulas. We say that two metaformulas with the same metavariables $\alpha(\xi_1, \ldots, \xi_n)$ and $\beta(\xi_1, \ldots, \xi_n)$ are interchangeable if, for any metaformula $\phi(\xi_1)$ and formulas $\alpha_1, \ldots, \alpha_n$, we have $\phi[\alpha(\alpha_1, \ldots, \alpha_n)]$ and $\phi[\beta(\alpha_1, \ldots, \alpha_n)]$ are interdemonstrable. This relation induces a congruence on the class of morphisms of this category.

In this communication, we study the quotient category induced by interchangeability and some of its categorical properties. For example, the existence of finite products and coproducts, without needing to be restricted to congruential logics. This is of interest since there are relevant examples, such as some logics of formal inconsistency studied in [3] that are not congruential.

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Differential Geometry in Modal Type Theory

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We demonstrate that basic concepts of Differential and Algebraic Geometry may be expressed in a very general and abstract way using Modalities, a notion inspired by Modal Operators. Our framework is built on top of Homotopy Type Theory following ideas of Urs Schreiber.

A Pierce representation theorem for varieties with BFC

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By a *variety with* $\vec{0}$ *and* $\vec{1}$ we understand a variety $\mathcal{V}$ for which there are 0-ary terms $0_1,\dots,0_n,1_1,\dots,1_n$ such that $\mathcal{V} \models \vec{0} \approx \vec{1} \to x \approx y$, where $\vec{0} = (0_1,\dots,0_n)$ and $\vec{1} = (1_1,\dots,1_n)$. If $\vec{a} \in A^n$ and $\vec{b} \in B^n$, we write $[\vec{a},\vec{b}]$ for the n-uple $((a_1,b_1),\dots,(a_n,b_n)) \in (A \times B)^n$. If $A \in \mathcal{V}$, then we say that $\vec{e} = (e_1,\dots,e_n) \in A^n$ is a *central element* of A if there exists an isomorphism $\tau\colon A \to A_1 \times A_2$, such that $\tau(\vec{e}) = [\vec{0},\vec{1}]$. Also, we say that $\vec{e}$ and $\vec{f}$ are a *pair of complementary central elements* of A if there exists an isomorphism $\tau\colon A \to A_1 \times A_2$ such that $\tau(\vec{e}) = [\vec{0},\vec{1}]$ and $\tau(\vec{f}) = [\vec{1},\vec{0}]$. In general, the isomorphism τ is not unique; furthermore, usually the pair $(\vec{e},\vec{f})$ of complementary central elements does not determine the pair of complementary factor congruences $(\ker(\pi_1\tau),\ker(\pi_2\tau))$, where the π_i's are the canonical projections. We call the following property the *determining property* (DP):

(DP) For every pair $(\vec{e},\vec{f})$ of complementary central elements, there is a unique pair (θ,δ) of complementary factor congruences such that, for every $i = 1,\dots,n$,

$$(e_i,0_i) \in \theta \quad \text{and} \quad (e_i,1_i) \in \delta \quad \text{and} \quad (f_i,0_i) \in \delta \quad \text{and} \quad (f_i,1_i) \in \theta.$$

Observe that (DP) is in some sense the most general condition guaranteeing that central elements have all the information about direct product decompositions in the variety. In [1] it is proved that (DP) is equivalent to each one of the following conditions:

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(FD) There is a first order formula $\Psi(x, y, \vec{z})$ such that, for every $A, B \in \mathcal{V}$,

$$A \times B \models \Psi((a,b),(a',b'),[\vec{0},\vec{1}]) \quad \text{iff} \quad b = b'.$$

(BFC) $\mathcal{V}$ has Boolean factor congruences, i.e., the set of factor congruences of any algebra in $\mathcal{V}$ is a Boolean sublattice of its congruence lattice.

Let $\mathcal{V}$ a variety with BFC. If the formula Ψ of (FD) is existential we will say that $\mathcal{V}$ is a variety with exBFC. The aim of this talk is to exhibit a representation theorem for some varieties with exBFC in terms of internal connected models in toposes of sheaves over bounded Boolean algebras. The present work is motivated by the Pierce representation theorem for integral rigs [2] and Lawvere's strategic ideas about the topos-theoretic analysis of coextensive algebraic categories [3].

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Logic, Law and Legal Reasoning

This workshop is organized by

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The workshop will discuss new insights in the interaction between logic and law, and more precisely the study of different answers to the question: What role does logic play in legal reasoning?

It will present both current challenges and historical perspectives in the relation between logic and law. The perspectives to be discussed involve the interface of the following studies:

Foundational studies:
- Logical Principles and Frameworks
- Meaning
- Reasoning in Deontic Contexts

Applications:
- Legal practice and Computer-Based Modelisations
- Argumentation Theory

Historical perspectives:
- Legal reasoning in Ancient Roman, Arabic, Jewish and Far-East contexts
- Others contexts

The keynote speakers at this workshop are Walter Edward Young (page 166) and Matthias Armgardt (page 121).

Call for papers

The submissions should contribute to the development of those perspectives by the discussion of subjects such as:
- Analogical Reasoning in Law
- Deontic Logic and Law
- Non-Monotonic Reasoning and Law

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- Defeasible Reasoning and Law
- Argumentation Theory within legal Contexts
- Epistemic Logics and Law
- Joergensen's Dilemma
- Hypothetical Reasoning and Law
- Proof-Theory and Legal reasoning
- Reliability, Epistemic Logics and Legal Reasoning
- Probability and Legal Reasoning

Contributed talks should not exceed a duration of 30 minutes including discussion. A one-page abstract should be sent via email before November 15, 2017 to nordtveitkvernenes@gmail.com.

The Interaction of Logic and Jurisprudence in the Islamic Tradition: A Genealogy of a Long-Lasting Antagonism

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The Muslim logicians contributed to the consolidation of the relationship between logic and law (Islamic Jurisprudence, *Fikh*), but this relationship had endured turbulence between two contradictory tendencies.

The first one, represented by the jurist Ibn al-Çalāh (13s), who categorically refused the introduction of logic in the field of jurisprudence, not only that, he also with others went so far as to forbid the teaching of logic. What makes this tendency interesting is that it has supporters until today.

The second tendency, represented by the philosophers and jurists Ibn ĥazm (11s), the initiator, who was able to introduce logical analysis within jurisprudence in his eminent work *Al-Taqrīb li-Ḥadd al-Manṭiq wa'l-Madkhal ilayhi bi'l-Alfāẓ al-'Āmmiyya wa'l-Amthila al-Fiqhiyyam*, and Al-Ghazālī (12s), in his work *Al-Mustaçfā fī Uçūl al-Fiqh*, despite their very hostile environment. And even many other philosophers attempted to analyze the statements of jurisprudence from a logical point of view, but their approach was very similar to that of Aristotle. In our contribution we will discuss the arguments of the jurist Ibn al-Salāh and the contribution of philosophers and lawyers such as Al-Fārābī (10s), Ibn ĥazm (on which I will focus), Al-Ghazālī and Ibn al-Nafīs (13s).

Ludics for modelling the role of a judge during legal debates

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Ludics [2] is a Theory of Logic developed in the proof theory framework, when this latter focuses on the proof-as-program paradigm. Even if this theory comes within the context of calculus theoretical study, it provides fruitful concepts for both Logics and Argumentation Theory. Indeed, Ludics arose with the slogan that the core principle of Logics is interaction.

Based on this principle, the authors already used Ludics as a unified framework for analysing both dialogue and reasoning [1]. Namely, dialogues in natural language are modelled in Ludics: in a motto, a dialogue is the interaction between two designs (viewed as strategies, one for each locutor). Moreover, as designs may also be seen as proofs (in fact proof attempts), it is possible to focus, inside the same framework, on logical and rhetorical aspects of dialogues and reasoning. We illustrate our formalization of controversies on the artificial legal debate that Prakken studied. We show in which extent our modelling may account for the actions of a judge during a legal debate. More precisely we interpret the distribution of the burden to prove and the adjudication by means of formal invariants. We account for the judge interventions as if the judge produces and receives interventions in place of the two debaters. In particular, the adjudication consists in closing all still open questions or questionable statements (made explicit in the modelling).

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Textual Discourse Analysis: Towards an Illocutionary-Argumentative Model for the International Legal Discourse

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This research proposes an illocutionary-argumentative theoretical model of analysis for the international legal discourse that serves as a tentative extension of the Textual Discourse Analysis (ATD, in French), as proposed by Jean-Michel Adam [1]. Using it as a stepping-stone, we aim to focus on features of the international legal discourse that the ATD may not seem to address with precision. Such limitations stem mostly from the normative nature of international agreements, which essentially entails instances of legal reasoning and argumentation in the use of imperative sentences and deontic operators.

Thus, we currently investigate how international agreements establish legal commitments through a binding to non-binding force continuum on an illocutionary-argumentative level, and how they convey these meanings based on types and degrees of illocutionary and argumentative forces. In order to do so, we have been following a number of works regarding speech acts and illocutionary logic [2], and argumentative semantics [3], thus reinterpreting Austin's concept of illocutionary acts [4] to integrate argumentative frameworks into the sentence-utterance, establishing a wider spectrum of description. We consider the traditional representation of an illocutionary act, $F(P)$, where P is the propositional content and F its illocutionary force, and reinterpret it with the understanding that P is also charged with argumentative value, and therefore, argumentative force. We propose that such a complementary investigation may be able to lead to a more complete description of normative sentence-utterances, specially considering uses in the field of Linguistics, Argumentation Theory and Law.

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Abū Isḥāq al-Shīrāzī's System of Co-Relational Inferences by Indication

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The notion of *fiqh*, that literally means deep understanding, supposes that legal knowledge is achieved by rational endeavour, the intellectual effort of human being; this is what is meant when the term ijtihād, endeavour of the intellect, is used in Islamic jurisprudence. One of the most remarkable features of the practice of ijtihād is that it presupposes that law is dynamic in nature. Indeed, since the ultimate purpose of such a kind of rational endeavour is to achieve decisions for new circumstances or cases not already established by the juridical sources, the diverse processes conceived within Islamic jurisprudence were aimed at providing tools able to deal with the evolution of the practice of *fiqh*. The dynamic nature of Islamic law, as indicated by Young [2], is put into action by both the dialectical understanding and the dialectical practice of legal reasoning. According to this perspective, the practice of ijtihād takes the form of an interrogative enquiry where the intertwining of giving and asking for reasons features the notion of meaning that grounds legal rationality. More precisely, the conception of legal reasoning developed by Islamic jurisprudence is that it is a combination of deductive moves with hermeneutic and heuristic ones deployed in an epistemic frame.

Several systematic instruments are developed within *ijtihād*, one of them is *qiyās* or co-relational inference. The study is focused on Abū Isḥāq al-Shīrāzī's system of *qiyās* as discussed in his *Mulakhkhaṣ* [1]. He classifies *qiyās* in general into two types: *qiyās* based on the occasioning factor (*qiyās al-'illa*) and *qiyās* based on indication (*qiyās al-dalāla*). The study provides a dialectical meaning-explanation of the main notion of co-relational inference relevant for the development of al-Shīrāzī's system of *qiyās*. In other words, what we are aiming at is to set out a kind of interactive language game that makes apparent the dialectical meaning of the main notions involved

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in these forms of reasoning. The present study is focused particularly on *qiyās al-dalāla*, the second type of *qiyās*.

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A Dialogical Framework for Analogy in European Legal Reasoning

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A challenge in theory of legal reasoning is to understand the procedures that we use when we argue for a certain standpoint based on an older decision. Arguments by analogy have received a lot of attention in the common law tradition. The structure of arguments by analogy that we find in the common law tradition does not seem to be directly transferable to the civil law tradition. More specifically, it does not seem to be transferable to European law. Arguments by analogy are also used in European law, but it has received significantly less attention that its Anglo-American counterpart. Rather than referring to relevant similarities between the source and the target, European law refers to the principle behind the source, ratio legis, and checks if it can also be applied to the target.

Dialogical framework is a kind of game-theoretical semantics and not a logical system in itself. It is rather a universal way to implement or to understand the meaning in a logical system. The main idea is to look at meaning as how a statement can be challenged by another player. It is a semantical framework that seems to fit very well to the situation that we often find in legal reasoning. By explaining the use of analogies in European law in a dialogical framework, we may end up with a very natural and comprehensible way of capturing the actual legal argumentation.

In this on-going project I want to show how we can understand analogical argumentation in a general framework, namely game-theoretical semantics.

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It is an attempt to implement the process of analogical argumentation in a semantical tradition that comes from Wittgenstein and more recently, Hintikka. Based on the actual argumentation deployed in a juridical setting in European law, I would like to examine the insights that the dialogical framework can provide to the different forms of reasoning by analogy. This may both show how we can distinguish different kinds of analogy arguments and how the analogical argumentation in legal reasoning corresponds to the dialogical meaning of analogical argumentation in general.

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Vagueness in the Law and the Sorites Paradox

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It is a common truism to assert that language mediates the relation between our concepts and the world around us. Without the latter, our concepts would have no concrete reference. Conversely, without concepts to describe our experiences, it would be impossible to understand our world. Language, concepts and the empirical world are in constant interaction.

These interactions between language, concepts and the external reality are rarely straightforward. One main issue we face is the philosophical problem of vagueness [2]. The standard definition for vagueness is this: X is vague if and only if X has borderline cases, where X is a concept. Thus, the predicate "tall" is vague; so are "blue", "unjust" and "pretty". One example of this is the Sorites paradox, also known as the paradox of the heap. This paradox arises when there is no clear cut off between borderline cases: when does a hay stack stop being a hay stack and starts simply being strands of hay? At what point does a person become "bald" or "tall"? These questions have been asked for millennia, but no clear answer has managed to establish itself as the dominant position.

In this paper, two main objectives will be pursued. First, I wish to investigate different answers to the Sorites paradox as they have been developed with modern logical tool. Three schools of thought will be presented: many-valued logic, including a few of its variants [1,6,9,14]; supervaluationism and subvaluationism, as the two sides to the same coin [7,11,14]; and contextualism [5,8,12,14].

Secondly, after having examined these possible answers to the Sorites paradox, I will draw parallels between these purely logical solutions and their possible applications to the philosophical problem of vagueness in the law. Philosophical vagueness raises special problems in legal philosophy. Lawmakers often use vague, abstract terms. Some legal scholars have argued that this vagueness is a necessary feature of law [3,4], which cannot be erased or bypassed. They think we should embrace vagueness and use its tools — after all, it does seem to offer some benefits [10]. Others believe that vagueness has no function in law [13], because the philosophical problems it raises are not at issue in practice. This answer highlights a crucial aspect of legal adjudication: unlike a philosopher thinking about hay stacks, a judge cannot simply remain befuddled. She must make a decision that will have direct consequences on the life of other people. The main legal issue concerning vagueness is thus that of pragmatic application.

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Abductive Inference in Legal Reasoning: Reconceiving Res Ipsa Loquitur

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This paper examines the relevance of C.S. Peirce's notion of abductive inference for law and the formation of legal concepts. The importance of this examination comes from the fact that several doctrinal practices in diverse areas of law — including criminal law, torts (delicts), constitutional law, evidence — rely on logical inferences that defy neat explanation under standard forms of deductive or inductive logic. I argue that abduction resolves some of these logical infirmities, providing the logic underlying a number of doctrines or conceptual practices common in modern law. My analysis concentrates on the common law tort maxim res ipsa loquitur ('the thing speaks for itself'). This is for two reasons. First, to at least as great a degree as any other legal concept, res ipsa loquitur manifests the form and conditions of abductive inference. Second, though it entered English common law over 150 years ago and remains today a form of inferential reasoning used in negligence cases throughout most of the common law world, res ipsa loquitur is still highly controversial. Several issues as to its force and effect split courts and prevent it from receiving uniform application across jurisdictions. The issues include: (1) whether it is a necessary condition for res ipsa loquitur that the defendant exerted exclusive control over the injurious 'thing' or instrumentality; (2) whether there must be an entire 'absence of explanation' for the maxim to apply; (3) whether res ipsa loquitur is a species of circumstantial evidence; and (4) whether it authorizes a burden-shifting presumption of negligence or only a permissive evidentiary inference. I argue that the controversy and jurisdictional disagreement over these issues is due, in no small part, to the maxim's little understood logical foundation in abduction. By recognizing that foundation, these issues that mar res ipsa loquitur's consistent application largely fall away.

A Formal Analysis of (Human) Rights and (State) Duties

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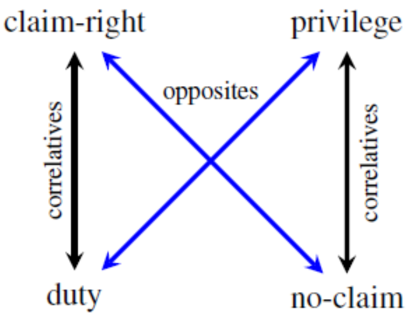
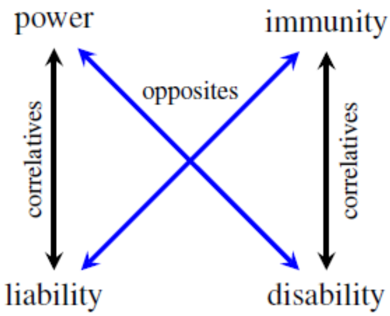
The notion of correlativity — that someone's right always involves someone else's duty, and the other way around — often arises in legal arguments: a duty's acceptability or bizarreness helps us decide whether the correlative (human) right exists or not. One has to see clearly, though, who the involved agents are in these correlative pairs and, if necessary, dissociate them from the State, otherwise the argument fails. W.N. Hohfeld's analysis [3] was the one which provided a compact picture on how correlativity — and opposition — orders the system of different types of rights and duties. The well-known system of correlative pairs of rights and duties [3] built can be reconstructed in the following diagrams:





This theory is decisive in analytical legal theory. Yet a century later, the formalization of his theory remains, in various ways, unresolved. In this talk I provide my own amended version of the Hohfeldian conceptions' formal representation, in which the correlativity plays a crucial role and the involved agents become visible, just like the role of the state. What is more, this role of the state is the point I build on in order to grasp what having a claim-right or duty in a given legal system means. This approach fits to claimant theories, theories that identify a directed obligation's (that is, a duty's) counterparty with the claimant, but while it is usual to involve the notion of power (to initiate a legal action) [see e.g. 9,8] in my formalization claim-right serves as a clue in showing what happens when someone violates a (any kind of) right, and the way judicature is involved in the legal relation.

In the case of power and its group, the formal representation has to give an account the special capacity involved into power and the incapacity to actually do the given action when it lacks. In my approach I call this capacitative feature of power 'potential', captured as borne by power and acts together; and I argue that we can have power only on actions created by constitutive rules.

I follow the classical formalizations authors, S. Kanger and L. Lindahl [4,5,6,7], among others, in using SDL, and a simple 'sees to it that' operator having an axiomatic background that B.F. Chellas [1] called ET (containing only the rule of equivalents' interchangeability and the T axiom in order to have successful actions). But I use this latter operator iterably, enabling us to capture the real action bound by the deontic operators (denoting the rights and duties) that I also agent-indexed, using the notation of directedness, introduced by H. Herrestad and C. Krogh [2], with the relevant modalities in order to emphasize the Hohfeldian conceptions's relationality. Also, to describe the strict operating of a system of legal rights, we need to introduce a legal necessity operator with S5 logic behind.

This formal conceptual analysis shows how a system of legal rights and duties works, expressing the way how new legal facts (statements on rights and duties) arise, concentrating on what legal consequences each type of rights and duties, together with acts and refrainings have. This analysis is also adequate for showing the layered role of the state in ensuring rights, referring to the tasks of judiciary and legislation separately.

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Cohen's Criticisms of the Use of Probability in the Law

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Bolzano [1] was the first to propose explicitly that a probability function $\mathfrak{p}(\mathsf{c} \mid \mathsf{a})$ may be used to measure the degree to which a conclusion c is classically deducible from an assumption (or premise) a. It is central to the *logical interpretation of probability* that the value of $\mathfrak{p}(\mathsf{c}|\mathsf{a})$ is 1 when c is deducible from a, and sinks to 0 when c' is deducible from a; that is, when c contradicts a.

Cohen's *The Probable and the Provable* [2] argues that the other popular interpretations of probability, which are all tied to variants of the axioms of Kolmogorov [4], may also be understood as measures of deducibility, but of deducibility in deductive systems of diverse kinds. It seems to have been the first work to state explicitly that what he labels *mathematical* or *Pascalian probability* is not the only respectable way of grading the deducibility or provability (in a loose sense) of a conclusion c from the evidence (or assumption) a, and to contest the idea that the lowest grade of deducibility must reflect the inconsistency of c with a. According to Cohen, reasoning within incomplete deductive systems, legal reasoning in particular, requires a method of gradation that ranges from the possession of *conclusive evidence* ($\mathsf{a} \vdash \mathsf{c}$) to there being *no evidence at all, one way or the other* ($\mathsf{a} \equiv \top$). The book expounds at length a system of ordinal (non-numerical) comparisons of deducibility called *inductive* or *Baconian probability*.

With regard to legal reasoning, Cohen supports his approach by consideration of half a dozen difficulties that, he maintains, plague the Pascalian construe of probability but are easily accommodated within the Baconian construe. These difficulties are adjudged to show the inadequacy of both the standard product law $\mathfrak{p}(\mathsf{cb}|\mathsf{a}) = \mathfrak{p}(\mathsf{c}|\mathsf{ba})\mathfrak{p}(\mathsf{b}|\mathsf{a})$ and the standard complementation law $\mathfrak{p}(\mathsf{c}|\mathsf{a}) + \mathfrak{p}(\mathsf{c}'|\mathsf{a}) = 1$. The soundness of Cohen's criticisms has been evaluated by lawyers (such as Schum [5]), statisticians (such as Dawid [3]), and others, but not, to my knowledge, from a specifically logical standpoint. A brief logical assessment will be offered here.

In § 37 of [2], discussing 'the difficulty about corroboration and convergence', Cohen observes that this difficulty for the probability function $\mathfrak{p}(\mathsf{c}|\mathsf{a})$ would largely disappear if it were replaced by the function $\mathfrak{q}(\mathsf{c}|\mathsf{a}) = \mathfrak{p}(\mathsf{a}' \mid \mathsf{c}')$, which ranges in value from *deducibility* to *absence of evidence*. But he seems not to have considered whether the function $\mathfrak{q}$ might solve some of the other difficulties that he lays at the door of $\mathfrak{p}$. It turns out that it does just that.

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Judgement based on chance in legal ties

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Imagine two cars have a head-on collision in a desert and a police officer arrives on the scene. She has to decide about the liability of a driver but she has no reason to accuse one and acquit the other driver. Sometimes, in such dilemmas, a moral principle [1] or a legal preference [2] helps the judge to adjudicate the case as some scholars have proposed. However, it is hard to see why a principle or a preference might work for one driver and against the other one. Thus, the question is that: How the judge decides in such cases? Sometimes evidence for and against a verdict is equally strong and based on objective criteria; the judge does not know how to decide. In modern contemporary legal systems, these cases simply ask for a subjective judgement pictured by Roy Sorensen [3, p. 300] and a judge would decide based on her personal interests even if she might not be aware of that. This way of decision-making might cause a great injustice by imposing racial, sex-based or ethnic discriminations, reflected in personal unconscious interests of judges.

I argue that in cases in which there is not enough evidence, it is better to leave the case to a chancy process of judgement instead of deciding it based on personal interests since a random process removes the adverse impact of psychological or political biases on legal decisions. In the next step, firstly, I show that an automatic decision procedure that includes randomness is practically more efficient than a manual procedure like flipping a coin. Secondly, I show that an automatic decision procedure could be achieved by using electric devices such as robot referees.

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Narrations in judiciary fact-finding and the difficulty about conjunction

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According to *Legal Probabilism* (LP), degrees of conviction in juridical fact-finding are to be modeled exactly the way degrees of beliefs are modeled in standard Bayesian epistemology [6]: by means of probabilistic distributions satisfying the standard axioms of probability theory. Quite a few conceptual issues with LP have been raised (mostly by legal scholars) [3,4,7,9,10,11,12], but not many of them have been satisfactorily answered.

In [13] I developed a formal, Bayesian account of the interplay of narrations as used in juridical fact-finding. In [14] I argued that independently motivated features of the framework lead to a fairly natural resolution of the so-called gatecrasher paradox. To put the issue very briefly: the paradox arises when, for instance, 999 out of 1000 participants crashed the gates to attend an event without purchasing the ticket, while 1 participant did buy her ticket. Now, randomly pick one of the participants; their probability of guilt is very high (one can manipulate the thought experiment to obtain any threshold < 1), and yet, intuitively, penalization is not justified. The resolution is quite sensitive to the details of the formulation, but the main gist is that the conviction is not justified because no accusing narration satisfying all the requirements is available.

In this paper I look at how the approach handles the difficulty about conjunction (DAC): em if the claim to be proven is a conjunction, what should we apply the probability threshold to: the conjuncts taken separately, or the conjunction as a whole? A secondary goal is to look at existing approaches to DAC [1,2,3,5,8] and to evaluate them from the perspective of the framework.

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Coping with inconsistencies in legal reasoning

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Law cases are triggered by conflicting opinions on juridical matter. Legal discourse reflects these contradictions. One may "index them away" in logical formalization. That, however, usually leads to unduly abstract — or, under-complex — formal models which thus become non-descriptive, i.e. naive and boring. Wouldn't it be better to accept the real differences in a specific legal dispute and to adequately reflect them in the logical description? There seems to be an obvious counterargument: accepting inconsistencies would conflict the principle of contradiction. This principle is considered the very keystone of (Western) rationality and should therefore be preserved, come what may. It turns out, indeed, that there is no good reason to question the *ex contradictione quodlibet*-principle [2]:

$$H \wedge \neg H \models F,$$

and, what is more, there is no need to do so. It suffices to abandon a similar, but stronger rule: the so-called *ex falso quodlibet*-principle:

$$H, \neg H \models F.$$

In order to differentiate between one and the other, one has to block the rule of adjunction:

$$H, F \not\models H \wedge F.$$

To that aim the Polish logician Stanisław Jaśkowski designed his system $\mathsf{D_2}$ [1]. This non-adjunctive calculus, the first inconsistency-tolerant one in history, is correlated with Lewis' modal logic $\mathsf{S4}$. We demonstrate how $\mathsf{D_2}$ (and alternative systems) provide a promising methodological basis for legal reasoning.

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Logic and Physics

This workshop is organized by

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We invite contributions on all interactions of physics and logic, new or old, modest or bold, hot or cold. However, the keynote talks focus on two brand-new, hot and very bold results.

The first talk concerns a unified account on all forms of physical causality, including the indefinite causal structures that one may expect in quantum gravity, and was first presented this year at IEEE*-LiCS†.

The other one concerns a complete logical calculus for Hilbert space quantum theory, which is the 1st ever of its kind.

We would like the speakers, independent of the particular subject of their contribution, to reflect on how logical accounts on physics could contribute to future theories of physics, rather than merely explanation and/or recasting the existing theories. The failure to have done so may be an explanation for why currently the role of logic in mainstream physics is nowhere near e.g. the role of logic either in mathematics or computer science.

The keynote speakers at this workshop are Sander Uijlen (relativity talk, page 164) and Simon Perdrix (quantum talk, page 151).

Call for papers

We invite contributions on all interactions of physics and logic, for example:

- logic for relativity theory
- logic for quantum theory
- logic for thermodynamics
- logic for 'quantum gravity'

Here, logic should be conceived in the very broad sense as any reasoning aid.

Contributed talks should not exceed a duration of 30 minutes including discussion. A one-page abstract should be sent via email before December 1st, 2017 to bob.coecke@cs.ox.ac.uk.

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From Quantum to Cognition

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For well over a decade, we developed an entirely pictorial (and formally rigorous!) presentation of quantum theory [1]. Meanwhile, the pictorial language has also been successful in the study of natural language [2], and very recently we have started to apply it to model cognition, and in particular how compositional reasoning about human senses can be achieved [3]. We present the key ingredients of the pictorial language as well as their interpretation across disciplines.

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Theory of Forms: a reconstruction of ancient metaphysics applied to the logical foundations of modern physics

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Keywords: Periodic table of subatomic particles, Geometric Algebra, Stoic Logic, operational calculus, sub-quarks.

The problem tackled in this work is to develop from purely rational considerations the foundations and ontology of forms universally applicable to any self-managed autonomous system. The physics universe is a special case of such a system. The approach is fundamentally a priorist and so free of empirical or axiomatically determined structures. Key aspects of the approach are developed from a reconstruction of Stoic natural philosophy and logic.

Leibniz famously introduced a new dimension into this ancient problematic, notably that of developing a theory of the forms of nature in terms of a "geometry without number". Nowadays we see that there are two modern geometric traditions, one analytic (Analytic Geometry [AG] generalized from linear analysis) and the other synthetic (Geometric Algebra [GA]). GA arises from the exterior and geometric products of Grassmann developed further by Cayley and Hamilton and in modern times by David Hestenes. Hestenes and others claim that GA is the fulfillment of Leibniz's dream. GA certainly provides the great simplifications that Leibniz demanded and is free of coordinates. But it is not free of number, nor does it provide an algebra based on "a few letters" that would describe the forms of nature both in the biological and non-biological worlds.

This work is presented as a true fulfillment of Leibniz's dream by developing a more fundamental version of GA which is truly a "geometry without number" and integrating it into a radical reconstruction of Stoic logic and physics.

Since the universe we live in can be considered as a totally autonomous self-managed system, the resulting theory should be applicable to developing the foundations of physics from a fundamental quantum perspective. This turns out to be possible and, unlike String Theory, leads to practical results. One result is the development of a sort of "Periodic Table" of subatomic particles that extends beyond the already known constituents. The theory predicts a lower "sub-quark" level as the primary substratum and. Unlike the Standard Model" does not require quarks with fractional charge. Everything is presented in terms of geometric semantics including such allusive notions as "colour charge."

The end-result can best be understood as "doing a Heaviside" by presenting quantum mechanics in a time independent "non-diachronic" form. This approach is considered as the complementary opposite of the present day standard approach. The tools of Laplace formalize Heaviside's approach and works well for DEs but not for partial DEs. To universally handle the latter, a more powerful formalism is required. The elements of that approach can be found in Stoic logic once properly reconstructed and explained.

Eigenlogic

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This work presents an operational and geometric approach to logic. It starts from the multilinear elective decomposition of binary logical functions in the original form introduced by George Boole [1]. A justification on historical grounds is presented bridging Boole's theory and the use of his arithmetical logical functions with the axioms of Boolean algebra using sets and quantum logic.

It is shown that the algebraic polynomial formulation can be naturally extended to operators in vector spaces. In this way propositional logic can be formalized in linear algebra by using combinations of tensored elementary operators. The original and principal motivation of this work is for applications in the new field of quantum information, differences are outlined with more traditional quantum logic approaches. This formulation is named Eigenlogic [3].

The interesting feature is that the eigenvalues of these operators are the truth values of the corresponding logical connective and the associated eigenvectors correspond to one of the fixed combinations of the inputs (interpretations). The outcome of a "measurement" or "observation" on a logical observable will give the truth value of the associated logical proposition, and becomes "interpretable" when applied to its eigenspace leading to a natural analogy with the measurement postulate in quantum mechanics. The following diagram summarizes this point of view:

$$\text{eigenvalues} \longrightarrow \text{truth values} \quad ; \quad \text{eigenvectors} \longrightarrow \text{interpretations} \quad ;$$
$$\text{logical operators} \longrightarrow \text{connectives.}$$

One can generalize to eigenvalues different from the Boolean binary values $\{0, 1\}$ for example with $\{+1, -1\}$ associated to self-inverse unitary operators [2]. In general one can associate a binary logical operator with whatever couple of distinct eigenvalues $\{\lambda_1, \lambda_2\}$ the corresponding family of logical operators can be found by Lagrange-Cayley-Hamilton matrix interpolation methods. The extension from binary to many-valued logic is then considered by defining specific operators using multivariate interpolation. The interesting property is that a unique seed operator generates the complete logical family of operators for a given m-valued n-arity system.

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This method can be applied to the synthesis of binary and multivalued quantum logical gates. Eigenlogic brings a correspondence between control logic (David Deutsch's quantum logical gate paradigm) and ordinary propositional logic. Several of the logical observables turn out to be well-known quantum gates. It is well known that the 2-quibit entangling *Control-Z* ***Cz*** gate in association with 1-quibit gates is a universal quantum gate set. In Eigenlogic the ***Cz*** gate is the conjunction (AND) self-inverse Eigenlogic operator. Following this approach a new design method of the universal Toffoli gate, using $\boldsymbol{T}$ gates is proposed.

Ternary-logic quantum gates using qutrits lead to less complex circuits, the design of a balanced qutrit arithmetic full-calculator circuit is realized using an Eigenlogic approach [4].

In Eigenlogic all propositional binary and multivalued logic can be built on the basis of a complete family of commuting logical observables. With non-eigenvectors the logical operators are no more diagonal and correspond to propositions with a fuzzy logic interpretation []2: the degree of truth corresponding to the fuzzy membership function defined by the mean value (Born rule) applied on the logical observables. Also when using two maximally incompatible logical families such as those generated by the $\boldsymbol{X}$ and $\boldsymbol{Z}$ gates one gets an interesting outlook: the usual Grover gate turns out to be the self-inverse Eigenlogic inclusive disjunction operator (OR) in the X system and can be interpreted in the Z system as a predicative logical existential connective. This could permit to extend the Eigenlogic approach including first-order logic.

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A Categorical Reconstruction of Quantum Theory

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The Hilbert space formulation of quantum theory has remained difficult to interpret ever since its first description by von Neumann [9]. Over the years this has led to numerous approaches to understanding quantum theory from more basic, operational or formal principles. Recent years have seen numerous 'reconstructions' of quantum theory as a theory of information [5,2,6,10], singling it out via certain operational axioms from within the framework of generalised probabilistic theories.

In parallel, the field of *categorical quantum mechanics* (CQM) [1] has developed another more formal framework for studying general physical theories, by representing them as *monoidal categories*, hugely general mathematical structures in which one may speak of composable processes between systems [4]. Categorical aspects are employed in many of the existing quantum reconstructions, but these all rely on the extra assumption of *finite tomography*, enforcing that the processes of any given type form a finite-dimensional real vector space. This makes them less suitable as a logical formalisation of quantum theory, and there has long been desired a purely-process theoretic reconstruction in the spirit of CQM [3].

In this work, we present such a category-theoretic reconstruction of quantum theory. More precisely, we present axioms which ensure that a *dagger-compact* category C is equivalent to a generalised quantum theory Quant_S over a ring S. Further conditions then ensure that S is the ring $\mathbb{C}$ of complex numbers, yielding the category $\mathsf{Quant}_{\mathbb{C}}$ of finite-dimensional Hilbert spaces and completely positive maps. This result differs from previous reconstructions by being entirely category-theoretic in both its statement and proof, in particular not assuming tomography. As such, it can be seen as akin to other logical formalisation of quantum theory (such as [8]) and of quantum reasoning and computation more generally.

Even from the perspective of probabilistic theories, the reconstruction is of interest. Avoiding tomography allows us to novelly reconstruct quantum theory $\mathsf{Quant}_{\mathbb{R}}$ over real Hilbert spaces as well as $\mathsf{Quant}_{\mathbb{C}}$, and we show these to be the unique theories satisfying our axioms whose *scalars* correspond to (unnormalised) probabilities. Our axioms also have an operational interpretation coming from their similarity to those of the so-called 'Pavia' reconstruction [2], particularly its axioms of 'purification', and also of 'perfect distinguishability' and 'ideal compressions' which we explain to correspond categorically to the existence of *dagger kernels* [7].

Our proof works from these axioms to show that the 'pure' processes in our theory possess *phased coproducts*, a categorical feature we introduce to describe superpositions in general process theories. In fact, we show that any theory possessing these contains a copy of Quant_S for some semi-ring S. This provides a general and highly promising recipe for further reconstructions, beyond our special axioms. In the future, we hope to use this to remove our less operationally motivated axiom of *dagger-compactness*, to provide even simpler and more intuitive categorical axioms for quantum theory.

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Negations and Truth-perspectives pertaining to Qudit based Multi-valued Quantum Computational Logics

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In the present work, certain peculiar aspects of the concept of Negation pertaining to the circuit model of quantum computation are explored in the framework of a continuous t-norms based fuzzy-type representation of qudit based multivalued quantum computational logics.

15 – Sessions

Universal

The keynote speaker at this session is Alexander Paseau (page 149).

Logics as models versus logics as proposals

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Robert Brandom contrasted his inferentialist approach to language with what he called representationalism. While representationalism sees the essence of meaningfulness of our language in the relation between an expression and its denotation, inferentialism sees the normative relations of consequence and incompatibility between statements as primary. This general viewpoint leads to an unorthodox stance on the significance of logic, namely to logical expressivism, which claims that logic is here primarily to make the relations of consequence and incompatibility between sentences of our languages explicit.

We will extend this inferentialist approach also into philosophy of logic. From Brandom's viewpoint, it is obvious that the logic which we use to make inference rules explicit must have always been present in our language in the workings of specifically logical expressions, such as *or*, *not*, *if*, *then*, etc. We did not have to wait for the formal logics of Frege or even Aristotle in order to have the capacity to make inference rules explicit, as Brandom mentions Socrates as the great practitioner of expressive rationality.

Nevertheless, given the many formal logics which exist today, we should wonder how they relate to the logic inherent in natural languages? Clearly, many answers have already been provided, yet a specifically inferentialist one — taking logical expressivism seriously — still has to be found. In fact, there is a temptation for the inferentialists to fall prey to representationalism on a higher level and see the various formal logics as representations of the logic which is implicit in our language and which we use for expression of rules. This temptation, I believe, should be resisted.

First of all, there are good reasons to doubt the point of modelling or representing the logic inherent in our language. Clearly, there are many fascinating phenomena which we endeavour to model in our languages and theories, for example the movements of galaxies etc. Yet what would be so fascinating about modelling the behaviour of a few quite common words of our natural languages? I rather propose to see logics as proposals of modifications of our everyday logical practice of making inference rules explicit.

Any parts of our languages can and in fact do develop in multifarious ways. The development does not primarily happen by our arbitrary decrees to start using this or that expression in this or that sense. Rather, it is typically a spontaneous and organic development. Yet it can always be to some degree marshalled by us, exactly by making some of the problematic rules explicit and therewith enabling discussion about whether they are reasonable. Equally in case of logic, we can gain some limited control over the development by means of various formal logics. As there are typically more ways in which we can develop our logical practice, so there are different logics.They show us new ways the complex of logical expressions could be used. Thus they cultivate our logical capacity and possibly influence it. As for the representationalist view the plurality of logics meant a complication which had to be somehow resolved and explained, for my view this pluralism is seen from the very beginning as an asset. The plurality of logic does not mean that we are unsure about what logical laws really hold but rather that we gain more control over them and therewith attain a new kind of freedom.

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Preservationist Consequence and Logical Pluralism

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The possibility of preservationist treatments of consequence relations arises naturally from Dana Scott's treatment of multiple conclusion consequence relations [4]. In his paper, Scott demonstrated (in the course of defending the honour of modal logic against certain Quinean objections) that any reflexive, monotonic and transitive relation $\vdash$ on a language is determined by the set of 1/0 valuations on the language L such that, for Γ and Δ subsets of L, $\Gamma \vdash \Delta$ holds if and only if $\Gamma \models \Delta$, where $\Gamma \models \Delta$ if and only if every valuation that assigns 1 to every member of Γ also assigns 1 to at least one member of Δ.

This result has a straightforward intuitive reading: some *property* of the sentences of the language is preserved (from left to right) by any such consequence relation, while the complementary property is preserved from right to left. There are many reflexive, monotonic and transitive relations on the sentences of an arbitrary language, all of which can be understood as 'preserving' some property of premises from left to right (and equivalently, as preserving the complement of that property from right to left). However, the 1/0 valuations that determine these relations are only fully expressed in a multiple conclusion consequence relation.

In particular, some criticism of proof-theoretic treatments [3] assume single conclusion $\vdash$ relation in which the union of the consequences of a consistent premise set Γ are the *intersection* of the maximal consistent sets including the premises. But in a multiple-conclusion consequence relation, the minimal 'cross-sections' of the conclusion sets, i.e., the least sets of sentences which intersect every conclusion set–are the maximal sets of sentences including all the premises and having the 'property' preserved by the consequence relation [2]. In the case of classical logic these are the maximal consistent supersets of the premises.

1/0 valuations on a language don't just determine what individual sentences 'follow from' assigning all members of a premise set the value 1: they also capture all the sets of sentences at least one of which is assigned the value 1 when all members of the premise set are assigned the value 1. Similarly, multiple conclusion logics allow proof theory to capture the maximal consistent sets that satisfy the premises, rather than just their intersection. The generality of Scott's result suggests a preservationist approach to consequence relations, while encouraging a broader, pluralistic perspective on them.

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Universal Logic $\stackrel{?}{=}$ Logic of the Universal

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Is "Universal Logic", if any, obligatory the fundamental Logic of the Universal (Universe)? The question will lead to interesting conclusive questions:
— What is the "real power" of Universal Logic and Logic in general?
— Which lessons can be drawn on the supposed functioning of the Universe?

Before that, the initial questioning induces more fundamental questions:
— What does the expression "Universal Logic" mean?
— What does the expression "Logic of the Universal" mean?

So we need a definition of:
— Universe: what do we mean by "Universe"?
— Logic: what do we mean by Logic?

Let us note that, contrary to the appearances, our talk will not concern "embedment" of questioning, even if a full interesting article could be devoted to the topic!

As a starting point of the development of our talk, we suggest to consider the "Systematic Creativity" process, where logic (systematicity) is beat by Divergence (Creativity), as the first fundamental Law of the universe.

Indeed, initially, at the so-called beginning of the universe, no standard logic. Why?

What the initial hypothesis is to explain the origin of the universe, it corresponds to "non-standard logic":
— "Eternity" model, with or without divinity: unary logic (1)
— "Spontaneous generation" model: no logic (Λ)
— "Less than nothing" model: negative logic (-1)

We suggest the "Extended Evolution" process as the second fundamental law of the universe. At the universal level, the law of evolution is characterized by the notion of "qualitative jump".

This is where the concept of "Logico-Divergence" can be introduced.

Binary Logic will appear to be a subset of Systematic Creativity. (Do we see limits in "set" approach? Systematic creativity is not a set, not even a category, this is a law; does it prevent us to use the "set" notion in our descriptions?) In any case, logic appears in a second time. So it is not fundamental.

We will conclude by proposing a new approach of the notion of Universal Logic.

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Characterizing Context-Independent Logical Notions Among the Context-Dependent Ones. The Case of Quantifiers and Inferences

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Context is essential in virtually all human activities. Yet some logical notions seem to be context free. The matter is not that simple as each application of a notion, even a logical one, depends on the context of the application. For example, the universal quantifier refers to all elements of the explicit or intended domain. The domain constitutes its context. However, the nature of the universal quantifier, the very meaning of "all", seems to be independent of the context. Similarly, all real life inferences and also actual mathematical proofs are context dependent because they use many

*Center for the Study of Language and Information assumptions assumed to be true in the context of the specific reasoning. However, purely logical consequence seems to be context independent, and it is sometimes operative in the binding mathematical and also real-life conclusions.

Whereas "all" seems context-free there are many quantifier expressions and some are context-independent while other ones are not. Similarly, sometimes the logical consequence is hidden inside an inference while much more often we encounter strong inferences, good enough for practical purposes but not valid.

The two types of examples suggest a general problem, here applied to logic only: How to characterize the context-free logical concepts in their natural environment, that is in the field of their context-dependent associates. This approach is generally not adopted in logical considerations, even in the philosophy of logic. The focus usually is on the strictest notions, the most context-independent and the easiest to treat formally. To consider the context-free notions as special, maybe extreme, cases in a broader field of related context-dependent notions can hopefully shed light on all these concepts. It would be good to have a general method or approach covering all such situations, but there is no guarantee that a uniform way of characterizing context-independence is possible.

In the paper, the issue of quantifiers is treated in the way presented in the forthcoming [4]. A general thesis is formulated: among all quantifiers, the context-free ones are just those definable by the universal quantifier. The issue of inferences is treated according to [1,2], where, however, the subject is not presented as a study of context (in)dependence. The treatment is not fundamentally novel; yet it does stress the unity of all inferences: the valid ones are an extreme case, the result of disappearance of context-dependence. This idea is especially nicely applied, as mentioned in [3], to an analysis of a form of abduction, called "reductive inference" in Polish literature on logic.

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Intensionality as a unifier: Logic, Language and Philosophy

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Intensional Logic as understood here is a research program based upon the broad presupposition that so-called "intensional contexts" in natural language can be explained semantically by the idea of multiple reference.

Our present contribution is a natural consequence of our previous research in Hybrid Intensional Logic. We have defined several systems of intensional logic from a formal point of view; namely, Hybrid Type Theory (HTT) [1], Equational Hybrid Propositional Type Theory (EHPTT) [2] and Intensional Hybrid Type Theory (IHTT) [3]. During the definition of them we were aware of the fact that our systems were trying to solve a variety of problems that arise not only in humanities but also in science. Moreover, we now believe that intensionality can serve as a unifying tool from a logical perspective as well as from a linguistic and philosophical one. That is the main objective of this research work,i.e., to present Intensional Logic as a unifying tool of research in distinct areas, such as logic, philosophy and linguistics. From a logical perspective, the field of combining logics, has a philosophical interest, and could also be motivated from a practical perspective whose goal is to obtain new logic systems from old, by integrating features and preserving properties to a reasonable extent. We have been working in a special kind of combined logic, Equational Hybrid Propositional Type Theory, which combines three heterogeneous logics, namely propositional type theory, equational logic and hybrid modal logic. The driving force of that combination was intensionality.

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Most of the issues treated in intensional logic have their roots in philosophy. A look to the history reveals that modal notions (like necessity or possibility) as well as the distinctions between *de re* and *de dicto* readings of sentences are present all over.

The treatment of the identity concept and its distinction from the also binary relation of equal denotation between terms of the formal language can also be analyzed from a philosophical perspective.

In particular, we were surprised by the philosophical concept of nominalism. The models we have created in our completeness proofs need only terms of the language and maximal consistent sets of sentences with particular extra properties. We wonder if, following ideas of Leon Henkin, we can dispense with sets and relations in the definition of models by using sets of formulas instead, and limit the existing objects to individuals. When modal, hybrid and intensional logics are taken into account this view gains force as it is in consonance with Carnap's view of worlds as maximal consistent sets of sentences.

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Modular analysis of Hilbert calculi*

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Culminating a long research path, we have recently identified the interaction patterns of combined Hilbert calculi, and established the ingredients of a workable modular semantics for them. In this presentation we shall give an overview of these contributions by means of illustrating examples.

Namely, in [3], we have played a game where we depart from different Hilbert calculi given by subsets of rules for classical implication ($\to$) and bottom ($\neg$), and study the negations defined by the usual abbreviation ($\neg A := A \to \bot$) in each of the given logics. In each case we extract a semantics for the defined $\to$, $\bot$-logics using the general recipes for fibred logics [2,5], and then also for the corresponding $\neg$-only fragment. Using [1] we further obtain upper bounds for the complexity associated to deciding these logics.

In a distinct setting, in [4], we take advantage of the same technical tools and of Post's classification in order to show that classical logic cannot be broken into two disjoint non-functionally complete fragments (except in very extreme circumstances). Using the general recipe for fibring we can now give semantics to the myriad logics obtained by combining different fragments of classical logic.

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Semantics for combined Hilbert calculi†

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Combining Hilbert calculi is well known to correspond to the mechanism of fibring logics, yielding the smallest (Tarskian) logic that extends the components. Moreover, Hilbert calculi are notoriously non-modular, which makes their understanding particularly challenging. In this talk, culminating a long research path, we will finally outline the ingredients of a workable semantics for them [1,2]. The results rely on using possibly partial non-deterministic matrices instead of the most common logical matrices, and on the properties of a straightforward but rich saturation operation. Using them, we show how to directly obtain complete semantics for combined Hilbert calculi by suitably combining the semantics of their components. We illustrate the results with some meaningful examples.

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On Dissent Pluralism and Paradigm-shifts from plural perspectives

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Keywords: Pluralism, Philosophical Logic, History and Philosophy of Logic, History and Philosophy of Science, Paradigm-shift.

We explore the evolution of Logic in the West in terms of paradigm-shifts. Here we propose that paradigms themselves should be understood by the kind of debates in which researchers engage, to wit, by the exemplary questions they choose to investigate or exemplary distinctions they use during a certain era. Some distinctions are crucial for theoretical construction, but are not represented in the theories they underline, or even remain inaccessible for a long time. Paradigm shifts coincide with the introduction of new distinctions that allow researchers to access the other crucial distinctions that had not been accessed previously: this movement occurs as a consequence of production conditions on the research, which includes the effort to solve operational problems accrued against one paradigm. Here, we propose that the prevalence of logical pluralism is the result of such process. Paradigm-shifts involve the non-accumulativeness and incommensurability issues: here we focus the former, whilst the latter will be examined in another work.

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Disjunctive and conjunctive multiple-conclusion consequence relations

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In 1978 [2] (cf. also [3]) a concept of multiple-conclusion consequence relation was introduced. It is a binary relation $\vdash \subseteq \wp(L) \times \wp(L)$ defined on the Boolean algebra of all subsets of a set of formulas L, satisfying for any $X, X', Y, Y' \subseteq L$ the following conditions:
Overlap: $X \cap Y \neq \varnothing \Rightarrow X \vdash Y$,
Dilution: $X \subseteq X', Y \subseteq Y', X \vdash Y \Rightarrow X' \vdash Y'$,
Cut for Sets: $\forall S \subseteq L[\forall Z \subseteq S(X \cup Z \vdash Y \cup (S - Z)) \Rightarrow X \vdash Y]$.

In turn, in [1], a concept of a multiple conclusion consequence relation was introduced as a reflexive and transitive relation $\vdash \subseteq \wp(L) \times \wp(L)$ satisfying the conditions:
$Y \subseteq X \;\Rightarrow\; X \vdash Y$,
$X \vdash \bigcup\{Y \subseteq L : X \vdash Y\}$.

This new multiple conclusion consequence relation can be called conjunctive as a finite set Y of conclusions in case $X \vdash Y$ is equivalent, in a sense, to the conjunction of conclusions. This is in opposition to the former multiple conclusion consequence relation, where a set of conclusions may be interpreted disjunctively.

Both types of consequence relations are generalized to be defined on any complete lattice $(A, \leq)$. The disjunctive relation (i.e., in the sense of Smiley) is associated with the following standard Galois connection

$$f\colon (\wp(A^2), \subseteq) \longrightarrow (\wp(A), \subseteq),\; g\colon (\wp(A), \subseteq) \longrightarrow (\wp(A^2), \subseteq):$$
$$\text{for any } \rho \subseteq A^2, x, y, t \in A \text{ and } T \subseteq A:$$
$$t \in f(\rho) \quad \text{iff} \quad \forall (x, y) \in \rho\, (x \leq t \Rightarrow y \wedge t \neq 0_A),$$
$$(x, y) \in g(T) \quad \text{iff} \quad \forall t \in T\, (x \leq t \Rightarrow y \wedge t \neq 0_A).$$

The counterdomain of the mapping g is just the set of all disjunctive relations defined on $(A, \leq)$. In turn, the conjunctive relation is associated with the Galois connection (f_1, g_1) defined by
$t \in f_1(\rho) \quad \text{iff} \quad \forall (x, y) \in \rho\, (x \leq t \Rightarrow y \leq t)$,
$(x, y) \in g_1(T) \quad \text{iff} \quad \forall t \in T\, (x \leq t \Rightarrow y \leq t)$.

The counterdomain of g_1 is the set of all conjunctive relations defined on $(A, \leq)$.

Some theorems which are consequences of those Galois connections (especially when some enriching conditions are put on the lattice $(A, \leq)$) are established.

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S5 is a semi-bivalent logic, and so is classical logic

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Let $||\varphi|| = X \Rightarrow Y$ an arbitrary *valuation* for a given sentence φ, where X is the input value and Y is the output value of φ.

The Principle of Bivalence (PBV) can be defined as a combination of valuation rules:

(PBV) A proposition is either true or false, not both.

This means that, for any φ, the lack of both truth and falsity is excluded: (PBV_1) $\overline{1} \Rightarrow 0$, and ($\mathrm{PBV}_2$) $\overline{0} \Rightarrow 1$.

This also means that the occurrence of both truth and falsity is also excluded:

(PBV_3) $1 \Rightarrow \overline{0}$, and ($\mathrm{PBV}_4$) $0 \Rightarrow \overline{1}$.

Likewise, sentential negation (NEG) can also be defined by valuation rules:

(NEG) A sentence φ is false if and only if its negation $\neg\varphi$ is true.

This means that the negation of non-falsity yields falsity, and conversely: (NEG_1) $\overline{0} \Rightarrow 0$, and ($\mathrm{NEG}_2$) $0 \Rightarrow \overline{0}$.

This also means that the negation of non-truth yields truth, and conversely:

(NEG_3) $\overline{1} \Rightarrow 1$, and ($\mathrm{NEG}_4$) $1 \Rightarrow \overline{1}$.

Then it can be shown that both (PBV) and (NEG) can be translated into a 4-valued logic AR_4 in the form of *affirmative* operators $[a]$ and *negative* operators $[n]$. We want to defend two main theses about modal logic $\mathsf{S5}$ and classical logic CL, considered as closed sets of theorems:

(1) $\mathsf{S5}$ can be characterized by a *semi-bivalent* affirmative operator $[viii]\varphi = ([ii] \otimes [iii])\varphi$ obeying (PBV_2) and (PBV_3), together with a Morganian and non-Boolean negation obeying none of (NEG_1)–(NEG_4).

(2) CL can be characterized by a *semi-bivalent* affirmative operator $[x]\varphi = ([i] \otimes [iv])\varphi$ obeying (PBV_1) and (PBV_4), together with a Morganian and Boolean negation obeying all of (NEG_1)–(NEG_4).

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Internal Logic of the H-B topos and Universal Metalogic*

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For each topos one can define a language which would be employed as a convenient mean for yielding statements on objects and arrows of the topos in question or even for proving theorems about them. Brief description of the language and zero-order topos logic, formulated in this language, could be found, for example, in [1].

Chris Mortensen, in his book "Inconsistent Mathematics" [2], introduced the notion of complement topos where internal logic is dual to the usual logic of standard topos. A principal peculiarity of complemented topos lies in a presence of complement classifier in the latter. At the same time Mortensen shown that a complement classifier in a topos **Set** is indistinguishable (via categorical methods) from a standard subobject classifier, that they are isomorphic. Thus, in **Set** we always have paraconsistency because of the presence of both types of subobject classifiers. And complement toposes support paraconsistency logic via Brouwerian algebra in a way exactly parallel to the way toposes support intuitionistic logic via Heyting algebras.

Since toposes support intuitionistic logics due to reflecting the Heyting algebra structure by subobject classifier then in a complement topos complement classifier reflects the Brouwerian algebra structure respectively. Hence, to describe an internal logic of complement topos we have to proceed in a dual way. Taking that into account L. Estrada-Gonzáles in [3] presented a sequent calculus for the zero-order complement topos logic.

But it seems that this 'abstract' categorical structure of toposes is principally twofold by its nature. Actually, Heyting logic and Brouwer logic always appear as Siamese twins — if one is given then the second might be reconstructed. So, maybe in this case we should discuss not the standard topos alone and not the complement topos alone but another type of category which, in a sense, contains them both.

The respective an H-B topos would be defined as a topos for which an algebra of subobjects of any object is a *semi-Boolean* algebra [4]. It might be regarded as the join of Heyting and Brouwerian algebras. For an H-B topos one can also define an internal typed language and internal logic which would be employed as a convenient mean for yielding statements on objects and arrows of such topos in question.

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A non-standard example of H-B topos is a category of logical systems $\mathfrak{Log}$ introduced within the framework of Universal Logic [5]. Universal Logic itself should be consider as a general theory of logical systems regarded as a specific kind of mathematical structures the same manner Universal Algebra treats algebraic systems. A category-theoretical approach where logical systems are combined in a category of the special sort provides us with some basis for inquiring the universe of Universal Logic. The category $\mathfrak{Log}$ appears to be, in fact, the H-B topos. Internal logic of $\mathfrak{Log}$ in this case would serve as a convenient tool for obtaining statements about the Universe of Universal Logic being a system of *universal metalogic* — a logic of logical systems and their translations.

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Tarski: Logical Concepts as Invariants

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Tarski's analysis of logical concepts was modelled on Klein's program in the foundations of geometry. Roughly speaking, this program consisted in considering geometrical notions as invariant under definite transformations. Tarski together with Lindenbaum applied a similar idea to logical concepts in the 1930s; they even planned a monograph (in German) devoted to this topic (the book was even announced but never published). In the 1950s,

*Polish Scientific Publishers, formerly **P**olskie **W**ydawnictwo **N**aukowe.

1960s and 1970s, Tarski delivered at least three lectures on the nature of logical concepts, but his related work was posthumously published in 1986 (in History and Philosophy of Logic). According to Tarski, a concept is just logical provided that "it is invariant under all possible one-one transformations of the world onto itself". The adequacy of this characterization depends whether we work in the type theory (the answer is "yes") or axiomatic set theory (the answer is negative). Thus, logicism can be defended in the former scheme, but not in the latter. The problem is how to relate invariance of logical notions to the universality of logic.

Modal

Polynomial Semantics for Normal Modal Logics

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The *Polynomial Ring Calculus* (PRC) introduced in [4] is an algebraic proof method that consists in translating formulas into multivariate polynomials over finite fields and in reducing polynomials in order to determine whether a formula is a theorem or not. The PRC works for every finite-valued logic and even for some paraconsistent logics that cannot be characterised by a finite-valued matrix. The PRC is extended to the modal logic $\mathsf{S5}$ in [1] and to the modal logics K, KD, T and $\mathsf{S4}$ in [2].

PRC is not the only method that proposes to interpret formulas of a logic system into polynomials and to use algebraic properties and algorithms to solve logic questions. In [5], for instance, formulas of finite-valued logics are also translated into polynomials over finite fields and several logic questions are algebraically characterised. Then, it is shown that the theory of Gröbner bases can be used to solve these logic questions.

In [3], it is given an abstract definition of whether a propositional logic is characterised by polynomials and the methods in [4,5] are combined and generalized.

In this work we extend the results in [1,2], following the approach in [3]. In summary, we show that any normal modal logic can be characterised by polynomials and that the theory of Gröbner bases can also be applied to solve logical questions in any of these logics.

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Divided modality for Ockhamists

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The earliest known treatise on consequences to discuss modality belongs to William of Ockham, and is to be found in his *Summa Logicae*. Central to Ockham's account is his distinction between composite and divided modality. Contemporary literature addressing the topic assimilates Ockham's distinction between composite and divided modality to one of scope: in a composite modal proposition, the modality takes wide scope, while it takes narrow scope in a divided one. But this representation renders aspects of Ockham's account incoherent. In this paper, I provide a coherent and complete account of the relations between categorical modal propositions of subject-predicate form on Ockham's approach. I show the Ockhamist distinction between divided and composite modals is orthogonal to Russellian considerations of scope: for Ockham, a modal proposition with syntactically wide scope will always have both a composite and a divided reading. However, the difficulty in representing Ockham's views arises not so much from modality, as the ways in which modality interacts with quantification and negation. After describing these difficulties, the relations between divided Ockhamist modals are catalogued, then illustrated with an example from Ockham's text.

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Hypersequential Argumentation Frameworks: An Instantiation in the Modal Logic S5 *

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Logical argumentation is a branch of argumentation theory in which arguments have a specific structure. This includes formalisms that are based on Tarskian logics [e.g. 3], in which classical logic is the deductive base (the so-called *core logic*). The latter were generalized in [1] to *sequent-based argumentation*, where Gentzen's sequents [5] are incorporated for representing arguments, and attacks are formulated by *sequent elimination rules*. The result is a generic and modular approach to logical argumentation, in which any logic with a corresponding sound and complete sequent calculus can be used as the underlying core logic.

In this work we further extend sequent-based argumentation to *hypersequents* [2]. This is a powerful generalization of Gentzen's sequents which was used for providing cut-free Gentzen-type systems to a variety of non-classical logics such as Gödel-Dummett intermediate logic LC, the modal logic S5, and the relevance logic RM. It allows a high degree of parallelism in constructing proofs and has some applications in the proof theory of fuzzy logics. In the context of argumentation theory, the incorporation of hypersequents allows to split sequents into different components, and so different rationality postulates [4] can be satisfied, some of which are not available otherwise.

The usefulness of hypersequential argumentation is illustrated here on frameworks whose core logic is S5. We show that the hypersequent-based argumentation framework that is based on this well-studied and applicable modal logic yields a defeasible variant of S5 with several desirable properties. We consider some of these properties and discuss the potential applications of this framework for argumentation with conflicting norms, temporal reasoning, multi-agents argumentation, and other forms of reasoning by (dynamic) epistemic logics.

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Intensional: what it is about?

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In general, a formal language consists of a set of well-formed formulas that expresses meanings to which correspond denotations. Formulas might involve logical terms, which can have different denotations corresponding to a single meaning in an intensional logic. As an example, the meaning of a logical term can be "the president of the United States", which corresponds to the denotation "Barack Obama" in 2013 and "Abraham Lincoln" in 1863. The semantics of a formal logical language has to specify how to associate meanings with the well-formed formulas and the terms of that language. In formal logic, meanings are not linguistic entities, they are mathematical entities. Viewed this way, intensional logic provides a mathematical foundation for semantics.

Frege did not give a formal definition of "sense", but Carnap (1947) gave a precise definition of two closely related notions called "extension" and "intension". Moreover, Kripke (1963) gave a new semantics for modal logic by using possible worlds to model possibilities. Montague (1974) brought these ideas together and showed that modal logic could be used to model intensions in natural language. Later, in Lewis (1968), intensions are defined as functions from possible worlds and times to extensions. Montague's program is also related to the program of Church for representing natural language, especially in its use of higher-order logic (type theory). Frege's important distinction between sense and reference, which modern intensional systems translate to intension and extension, has proven to be very successful in the study of natural language semantics, especially in analyzing "intensional contexts". When embedding definite descriptions in such environments their behaviour changes significantly.

Moreover, it often happens that the discussions of the intensional phenomena in natural language and other philosophical or logical contexts, both in relation to definite descriptions, but also in the relation to the treatment of various quantifiers and indexicals, do not benefit from the formal sophistication that recent intensional logics have achieved. On the other hand, it is noticeable that in the existing logical literature on intensional contexts most of the examples contain descriptions; however, the formal treatment of descriptions in this literature is often formally simplistic and unsatisfactory.

Thus, our initial question is: do intensions, despite being a good mathematization of senses, capture their meaning totally? The objective of this work is a revision of the definition and use of the intensional main concepts, like intension, extension, sense, denotation, description or existence, in order to explore and clarify them, and answer the previous question.

Topology and Measure in Logics for Point-free Space

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For a long time, logicians have wondered whether our ways of representing space are in some sense too idealized. Euclidean space is made up of *points*: dimensionless, indivisible regions. These are the smallest parts of space — the atoms beyond which we can divide no further. On an alternative, region-based approach to space, extended regions together with some mereological and topological relations are taken as primitive; points are constructed as mathematical abstractions from regions.

In recent years, simple *formal logics* for region-based theories of space were developed in [2] and further discussed in [3]. As Balbiani et al. [2] show, such logics have an algebraic, relational and topological semantics. In the algebraic semantics, the logics are interpreted in contact algebras, or Boolean algebras with a binary 'contact' relation. In the relational semantics, the logics are interpreted in reflexive, symmetric Kripke frames, or pairs $F = (W, R)$, where W is a non-empty set and R is a reflexive and symmetric relation on W. Surprisingly, every consistent axiomatic extension of the minimal logic for contact algebras, $\mathbb{L}^{\text{cont}}_{\text{min}}$, is weakly complete for the class of frames determined by the given extension — and indeed, for the subclass of finite frames determined by the extension.

In the topological semantics, on the other hand, logics for region based theories of space are interpreted in the Boolean algebra $\mathrm{RC}(X)$ of regular closed subsets of some topology X, together with a contact relation defined on the algebra. Balbiani et al. [2] show that the minimal logic of contact algebras, $\mathbb{L}^{\text{cont}}_{\text{min}}$, is weakly complete for the class of all topological spaces — and indeed, for the smaller class of all compact, semiregular, $T0$, κ-normal topological spaces. The present paper explores the question of completeness of $\mathbb{L}^{\text{cont}}_{\text{min}}$ and its extensions for individual topological spaces of interest: the real line, the rationals, Cantor space, and the infinite binary tree. A second aim is to study a different, algebraic model of logics for region-based theories of space, based on the Lebesgue measure algebra (or algebra of Borel subsets of the real line modulo sets of Lebesgue measure zero). As a model for point-free space, the algebra was first discussed in [1]. The main results of the paper are that $\mathbb{L}^{\text{cont}}_{\text{min}}$ is weakly complete for the rationals and Cantor space; the extension $\mathbb{L}^{\text{cont}}_{\text{min}} + (\text{Con})$ is weakly complete for the real line and the Lebesgue measure contact algebra. We also prove that the logic $\mathbb{L}^{\text{cont}}_{\text{min}} + (\text{Univ})$ is weakly complete for the infinite binary tree.

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Moving from the Opposition of Normal and Non-Normal Modal Logics to Universal Logic: Synthesizing T, S4, Tr, Verum and Falsum systems by the Square and Hexagon

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An axiomatic epistemology system Ξ contains all formulae, axioms and inference-rules of the classical propositional logic. Symbols α and β (belonging to meta-language) stand for any formulae of Ξ. Additional formulae of Ξ are obtained by the following rule: if α is a formula of Ξ then $\Psi\alpha$ is a formula of Ξ as well. The symbol Ψ belonging to meta-language stands for any element of the set of modalities $\{\Box, \mathrm{K}, \mathrm{A}, \mathrm{E}, \mathrm{T}, \mathrm{P}, \mathrm{Z}, \mathrm{G}, \mathrm{O}, \mathrm{S}\}$. Symbol $\Box$ stands for the alethic modality "necessary". Symbols K, A, E, T, P, Z, G, O, S, respectively, stand for modalities "agent *knows* that…", "agent *a-priori knows* that…", "agent *a-posteriori knows* that…", "it is *true* that…", "it is *provable* that…", "there is *an algorithm* (a machine could be constructed) *for deciding* that…", "it is *(morally) good* that…", "it is *obligatory* that…", "under some conditions in some space-and-time a person (immediately or by means of some tools) *sensually* perceives (has *sensual verification*) that…". Meanings of the mentioned symbols are defined by the following schemes of own-axioms of epistemology system Ξ which axioms are added to the axioms of classical propositional logic. Schemes of axioms and inference rules of the classical propositional logic are applicable to all formulae of Ξ (including the additional ones):
Axiom scheme AX-1: $\mathrm{A}\alpha \to (\Box\beta \to \beta)$.
Axiom scheme AX-2: $\mathrm{A}\alpha \to (\Box(\alpha \to \beta) \to (\Box\alpha \to \Box\beta))$.
Axiom scheme AX-3: $\mathrm{A}\alpha \leftrightarrow (\mathrm{K}\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg\mathrm{S}\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta))$.
Axiom scheme AX-4: $\mathrm{E}\alpha \leftrightarrow (\mathrm{K}\alpha \,\&\, (\neg\Box\alpha \vee \neg\Box\neg\mathrm{S}\alpha \vee \neg\Box(\beta \leftrightarrow \Omega\beta)))$.

In AX-3 and AX-4, the symbol Ω (belonging to the meta-language) stands for any element of the set $\{\Box, \mathrm{A}, \mathrm{T}, \mathrm{P}, \mathrm{Z}, \mathrm{G}, \mathrm{O}\}$. Let elements of this set are called "perfection-modalities" or simply "perfections".

If $\mathrm{A}\alpha$, then the inference-rule of $\Box$-elimination is derivable in Ξ, namely; if $\mathrm{A}\alpha \vdash \Box\beta$, then $\mathrm{A}\alpha \vdash \beta$. Moreover, if $\mathrm{A}\alpha$, then Gödel necessitation-rule is derivable in Ξ too, namely; if $\mathrm{A}\alpha \vdash \beta$, then $\mathrm{A}\alpha \vdash \Box\beta$. Consequently, under the condition that $\mathrm{A}\alpha$, the system Ξ contains the normal modal logics T, S4, Tr and Verum. However, in general, both mentioned rules of inference are not valid in Ξ. The applicability domain of these rules is limited to the sphere in which the presumption, that $\mathrm{A}\alpha$, is acceptable.

If Eα, then the two rules of derivation are not valid. But, if Eα, then in Ξ there is a possibility of the *non-normal* modal system called Falsum one. Thus Ξ *unites* the normal and non-normal modal logics in one conceptual scheme modeled by the square-and-hexagon [1].

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The Logic of Change LC enriched by Leibnizian modalities

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The logic of change LC was given by K. Świętorzecka in [1]. It is an extension of the classical sentential logic by the primitive operator C to be read *it changes that...* In [2] the Leibnizian interpretation of LC was given which is our start-point of the proposed analysis.

We add a new kind of necessity to LC which may be associated with a weak version of Leibnizian *conditional necessity*. This modality is characterized by two modal relations: temporal succession and copossibility of possible worlds.

The main subject of our interest is the description of relationships between C and conditional necessity.

We give an axiomatization of the proposed extension of LC and prove the completeness theorem in the assumed semantics.

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Correctness and Strong Completeness for Logic of Time and Knowledge

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Our aim is to provide axiomatization and to prove correctness and the strong completeness for the logic of time, isomorphic to non-negative integers, and knowledge [1].

Let $\mathbb{N}$ be the set of non-negative integers. We denote $\mathbf{N} = \{n_0, \dots, n_{m-1}\}$, where $m \in \mathbb{N}$, and then let $\mathbf{N_1} = \mathbf{N} \cup \{u\}$ be the set of propositional variables. The set For of all formulas is the smallest superset of $\mathbf{N_1}$ which is closed under the formation rules: $\psi \mapsto *\psi$, where $* \in \{\neg, \bigcirc, \bullet, \mathrm{K}_i\}$, and $\langle\phi, \psi\rangle \mapsto \phi * \psi$, where $* \in \{\wedge, \mathrm{U}, \mathrm{S}, \succ, \prec\}$. The remaining logical and temporal connectivities $\vee$, $\rightarrow$, $\leftrightarrow$, F, G, P, H are defined in the usual way. The operators $\succ$ and $\prec$ represent relations successor and predecessor of a node.

We define models as Kripke's structures. A model $\mathcal{M}$ is any tuple $\langle R, W, \pi, \mathcal{K}\rangle$ such that: $R = \{\langle x^j_{i_1}, \dots, x^j_{i_m}\rangle^j \mid j = 0, 1, \dots\}$ is the set of possible runs (r^j), W is the set of time instances with the time flow isomorphic to $\omega, \pi\colon R \times W \times \mathbf{N} \to \{\top, \bot\}$ is the truth assignment, with the restriction: when some x^k_s become $\top$, then for every positive l x^{k+l}_s will never be $\bot$, and $\mathcal{K}_i \subset (R \times W)^2$: $\langle r, t\rangle \mathcal{K}_i \langle r', t'\rangle$ iff $(\langle r, t\rangle \models x_i$ iff $\langle r', t'\rangle \models x_i)$ are the sets of possibility relations. The satisfiability relation $\models$ (formula α is satisfied in a time instance of a run $R \times W \models \alpha$) is defined recursively as usual.

The axioms of our theory are all instances of tautologies, standard axioms of the discrete linear temporal logic, the axiom that takes into consideration specificity of our model and the restriction that when some n_i become $\top$, then it will never be $\bot$, standard axioms for reasoning about knowledge, axioms about successor and predecessor of a node.

The inference rules of our theory are the modus ponens, necessitations for temporal operator $\bigcirc$, and for knowledge operators K_i, and the infinitary inference rules that characterize the Until and Since operators.

Strong completeness theorem. Every consistent set of formulas is satisfiable.

Decidability theorem. Checking the satisfiability of a given formula ψ is decidable.

Using our logic, we can prove the correctness of the maintenance of the ring topology of the Chord protocol [2] with the respect of the fact that nodes are not allowed to departure the system after they join it.

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Modal logics obtained by means of Jaśkowski's model of discussion§

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We consider modal logics obtained by application of Jaśkowski's model of discussion. The original discussive logic $\mathsf{D_2}$ has been formulated [cf. 1,2,3] with the help of the modal logic $\mathsf{S5}$ by means of the following condition:

$$\mathsf{D_2} := \{ A \in \mathrm{For^d} : \ulcorner \Diamond A^{\bullet} \urcorner \in \mathsf{S5} \},$$

where $(-)^{\bullet}$ is Jaśkowski's standard translation of discussive formulae $\mathrm{For^d}$ into the set of modal formulae $\mathrm{For_m}$.

In [4], an extension of $\mathsf{D_2}$ with the help of the modal operators of possibility '$\Diamond^{\mathrm{d}}$' and necessity '$\Box^{\mathrm{d}}$' are considered. The translation used in [4] is an extension $(-)^{\bullet_{\mathrm{n}}}$ of the function $(-)^{\bullet}$ onto the set of modal formulas by means of two additional clauses:
(i) $(\Diamond^{\mathrm{d}} A)^{\bullet_{\mathrm{n}}} = \ulcorner \Diamond A^{\bullet_{\mathrm{n}}} \urcorner$,
(ii) $(\Box^{\mathrm{d}} A)^{\bullet_{\mathrm{n}}} = \ulcorner \Box A^{\bullet_{\mathrm{n}}} \urcorner$.

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The obtained logic is called $\mathsf{S5}_{\mathsf{D}_2}$, and it is meant as the set

$$\{ A \in \mathrm{For}^{\mathrm{d}} : \ulcorner \Diamond A^{\bullet\mathrm{n}} \urcorner \in \mathsf{S5} \} .$$

The formulation of $\mathsf{S5}_{\mathsf{D}_2}$ depends on the normal modal logic $\mathsf{S5}$. Although, it is known [e.g. 5] that the very same logic D_2 can be obtained by means of different modal logics, it is possible to indicate modal logics that give other than D_2 discussive-like logics. Similarly, by appropriately varying the accessibility relation, i.e. the relation that connects participants of a given discussion, we can obtain other than $\mathsf{S5}_{\mathsf{D}_2}$ modal discussive logics. The change of the accessibility relation leads to change of the philosophical interpretation of functors in terms of Jaśkowski's model of discussion.

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A Modal Logics Framework for the Modeling of Human Reasoning

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Many different modal logics have been developed about different modalities (ontic, deontic, temporal, topological, epistemic…). Syntactically, they all share modal operators that affect propositional contents. Semantically, from a Kripkean standpoint, they all can be represented as systems that establish relations between possible worlds or between the actual world and possible worlds. Each modal system has some axioms that define the specific relations that it establishes between these worlds. This way, the specificity of a modal system resides in the presence or the absence of axioms of seriality, reflexivity, density, transitivity, symmetry, asymmetry, euclidianity…

Our talk aims at producing a general metamodal framework for modal logics that can classify existing modal logics within it and that can generate new modal logics from that framework. In such a framework, there are two basic modalities, the strong one (the one true in all possible worlds, like "necessary") and the weak one (true in at least one possible world, like "possible").

The second goal of our talk is to develop hypotheses about the conceptions of different modalities that spontaneous reasoners might apply when they reason with modalities. This way, many modal fallacies can be represented and explained as simplifications of the information occurring in the premises. For example, many spontaneous reasoners consider "not necessary p" ($\sim\Box p$) as equivalent to "possible p" ($\Diamond p$). On the other hand, some ways of reasoning can be different from what the modal system allows, without being a fallacy, but being rather a distinctive philosophical conception of the modality, like avoiding the asymmetry of time ($\mathrm{FH}p \supset p$ and $\mathrm{PG}p \supset p$) in temporal modalities, when holding, for example, a circular conception of time. Of course, some logicians have made relevant contributions on such philosophical conceptions of modalities [see e.g. 1,2], our aim is to suggest a general framework for such contributions.

So, our metamodal framework for modalities allows 1) the generation of new modal systems, 2) the modeling of what can be considered as modal fallacies and 3) the modeling of different philosophical conceptions about the different types (ontic, deontic, temporal…) of modalities.

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Modal approach to region-based theories of space: undecidability of modal definability*

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The region-based theories of space, also known as point-free theories of space, basically study the complete Boolean algebra of the regular closed (or regular open) sets in a topological space with the binary predicate contact — topological contact algebras. Recall that the meet and complement in this algebras are not the corresponding set-theoretic operations, and also that two regular closed sets A and B are in contact if and only if their set-theoretic intersection, $A \cap B$, is non-empty. The Kripke style semantics of the quantifier-free fragment of the first-order language of the Boolean algebras with a binary predicate symbol is proposed and studied in [2,1], in particular, a complete axiomatization of all validities in the class of all topological contact algebras is found.

The atomic formulas of this language are all expressions of the type $(t_1 = t_2)$ and $C(t_1, t_2)$, where t_1, t_2 are Boolean terms. The formulas are all propositional formulas constructed by atomic formulas. A Kripke frame is an ordered pair $\mathcal{F} = \langle W, R \rangle$, where $W \neq \varnothing$ and $R \subseteq W \times W$. A valuation V is a map from the set of all Boolean variables into the power set of W. In a standard way the value $V(t)$ of the terms t is defined. The truth value of atomic formulas is defined in the following way:

$$\mathcal{F}, V \models (t_1 = t_2) \quad \text{iff} \quad V(t_1) = V(t_2)$$

and

$$\mathcal{F}, V \models C(t_1, t_2) \quad \text{iff} \quad (\exists x \in V(t_1))(\exists y \in V(t_2))(xRy).$$

The extension to all propositional connectives is defined in a standard way. A formula is valid in a given frame iff it is true in all valuations in

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the frame. The completeness theorem from [1] says that (1) the formulas valid in all contact algebras are exactly the formulas valid in all Kripke frames $\mathcal{F}$ with reflexive and symmetric accessibility relation R. Moreover, if the contact algebras correspond to the connected topological spaces, then the valid formulas are exactly the valid formulas in the class of Kripke frames with reflexive, symmetric and connected (in the graph theory sense) accessibility relation.

In this talk we will demonstrate undecidability of the modal definability of the first-order sentences over the class of all (1) reflexive and symmetric frames and (2) reflexive, symmetric and connected frames.

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On the modal and first-order definability

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In this talk we will discuss the correspondence problem between modal and first-order sentences in several classes of Kripke frames. For simplicity we consider the basic modal language M, with the standard Kripke semantics and the first-order language with equality and a binary predicate symbol, L. Let $\mathcal{C}$ be a class of Kripke frames. Recall that a sentence A

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from L is called *modally definable in* $\mathcal{C}$ if there is a modal formula φ from M such that, for any frame $\mathcal{F}$ from $\mathcal{C}$, $\mathcal{F}$ is a model of A if and only if φ is valid in $\mathcal{F}$. Analogously, a formula φ from M is called *first-order definable in* $\mathcal{C}$ if there is a sentence A from L such that, for any frame $\mathcal{F}$ from $\mathcal{C}$, φ is valid in $\mathcal{F}$ if and only if $\mathcal{F}$ is a model of A.

Modal $\mathcal{C}$*-definability* is a problem for an arbitrary sentence A to decide whether A is modally definable in $\mathcal{C}$. *First-order* $\mathcal{C}$*-definability* is a problem for an arbitrary modal formula φ to decide whether φ is first-order definable in $\mathcal{C}$. The classical Chagrova's theorems [3] say that, in the case when $\mathcal{C}$ is the class of all Kripke frames, both definability problems are undecidable. Nevertheless, in certain cases, both problems are decidable, for example, if $\mathcal{C}$ is the class of all equivalence relations [2] or $\mathcal{C}$ is the class of all KD45-frames [4]. Let us note that, for the class $\mathcal{C}_{\text{euc}}$ of all Euclidean frames, any modal formula is first-order definable in $\mathcal{C}_{\text{euc}}$, but the modal $\mathcal{C}_{\text{euc}}$-definability is an undecidable problem [1].

The new results presented here concern the class $\mathcal{C}_{\text{S4.3}}$ of all linear transitive and reflexive frames with finitely many clusters.

Theorem.
(i) Any modal formula is first-order definable in $\mathcal{C}_{\text{S4.3}}$.
(ii) First-order $\mathcal{C}_{\text{S4.3}}$-definability is decidable.

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Non-Classical Logics

The keynote speakers at this session are Didier Dubois (page 125), Anna Zamansky (page 167) & David Makinson (page 138).

Abduction for Reconstructing Proto-Languages

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The aim of this paper is to analyze the kind of inference that we use to reconstruct Proto-Languages [7,4,6]. The importance of hypotheses for the process and the whole structure of reasoning make us think that we are in front of a certain kind of reasoning called abduction [5,3,1,8]. We analyze the nature of this concrete abductive reasoning and we specify its nuances and particularities. In fact, what is new in this work is that we center our point in the importance of context. We pay attention to a form of abduction that goes beyond the context, where the scientific work is developed by using other sciences as a contextual frame. So, an inter-contextual chain of inferences can be part of an abductive process, being the resulting hypothesis still provisional. To explain the ways for reconstructing Proto-Languages — where several disciplines play a cognitive role, as archeology, history and linguistic — we need several contexts, not just one.

We use the case of Proto-Semitic language reconstruction from the unexpected fact of the discoveries of Ugarit and Ebla languages. Pointing at Proto-Semitic, we need to presuppose the existence of a proto-Semitic society, just like the most plausible hypothesis. The problem we have is

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that archeological evidence provides limited remaining material. Therefore, we use different hypotheses that take into account the lack of the material sources and the lack of the linguistic evidence. Sometimes, we have an unexpected fact, the discovery of some material sources (maybe with some language sources like the discovery of Ebla or Ugarit). This fact helps us to fill the gap in the relationship between family languages and society changes (mainly migrations and ethnic contact). Other disciplines serve to complete the picture of the history of languages, such us archaeology, prehistory, comparative religion, epigraphy, comparative literature, etc. Otherwise, linguistics help to fill out history, archaeology...

So, by taking into account this process, we try to approach it by using logical tools. First, we use contextual logic [2] to express the contextual relation between disciplines. Then, we add dynamics [9] to approach the whole reasoning process. Our goal is to specify the abductive structure behind Proto-Languages reconstruction. We try to analyze the specific context change and the analogies used to decide the new hypothesis. This will help us to understand better the abductive process and the selection of a certain hypothesis. By analyzing Proto-Language reconstruction as an abductive reasoning, we may also shed light on the use of abduction to explain certain scientific practices.

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Nonmonotonic Reasoning, Context-Sensitivity and Indexicals[†]

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A special type of *context-sensitive* expressions caught philosopher's attention in the past decades: the *indexicals*. Roughly speaking, *indexicals* are linguistic expressions whose reference shifts from context to context. Considering a statement made at a given time t, we would associate both a *content* and a *character*. The content is the specific context-sensitive information within the statement; and the character, on the other hand, is the context-free manner which gives us the content. Thus character is independent of the context while content is dependent both on character and on context. Expressions involving demonstratives will, in general, express different concepts (content) if uttered in different contexts.

Default reasoning, on the other hand, is widely studied by theorists of artificial intelligence, but philosophers are also interested in the subject. One interesting aspect about defeasible reasoning, for instance, is that in some cases the defeasibility is related to the context of the utterance. Other cases might include situations where we derive a conclusion even though we do not know a relevant information, or we infer about ongoing actions (we often use progressive verbs in our everyday language), or even many forms of generalizations. Defeasible reasoning allow us to engage in *hypothetical* reasoning, for instance. Hypothetical thinking is relevant in developing knowledge since we often use *suppositions* in order to prove a point. A supposition is always related to a context; however, it creates a new parallel context with new parameters. Some of those hypothetical thinking might even help in the process of making belief changes.

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We shouldn't ignore the importance of nonmonotonic systems and default systems are not the only ones that we should consider. In everyday reasoning we use propositions with no justification other than we do not have evidences that contradict them. Moreover, we might give a closer look to our beliefs set and update ours preferences or infer new conclusions, specially when they involve a indexical belief. That is why when faced with new information that contradict the previous conclusion, most agents would withdraw the conclusion.

Nonmonotonic Modal Logics can be used to deal with context-sensitive expressions, since they offer a possible interpretation to the Kripke models to be *preferred* or *intended*. That allows us to do many kinds of inferences and deal with belief sets in many ways. These possible interpretations are particularly interesting if we consider the so called *intended context* for demonstratives.

On one hand, indexicals are present in many examples all over philosophical literature and, yet, most philosophers of Language do not agree on how they should be treated within the theories; on the other hand, give up on *monotonicity* is important to deal with many situations such as generalized information confronting new specific information; incomplete information confronting additional ones; doubts concerning the credibility of premisses; and so on. For many reasons, sometimes we try to make a sincere assertion, but when confronted with new information, we are forced to withdraw our conclusions. However, in our daily reasoning, we keep inferring defeasible conclusions — often involving utterances of indexicals/demonstratives in the sentence. In that sense, adding a nonmonotonic feature that is context-sensitive to a system depending on contexts seems to be interesting as contexts shift all the time. Indexicals and default reasoning have very aspects in common: many examples of defeasible reasoning involve occurrences of demonstratives of some form.

Acquiring information in a decision making process, for instance, is quite common and a formal system able to deal with them should be considered as relevant. Thus it is possible to see a relation between knowledge and belief to acquiring new information that might intervene in our decision making process; for instance, many false propositions might be considered as common knowledge and used to make an assertion or even a choice. Moreover, those new information might contain special features and be context-sensitive. Those special cases are as common as they are interesting.

Universal Logic and Generalized Probability Theory

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As we all know, Boolean Algebra is the mathematical basis of mathematical logic (standard logic, rigid logic), which represents certainty reasoning specifically. It is a common view in field mathematician. The rigid logic elements prescribe rigid constraint, which can be described as "either this or that". Based on the principles of mathematical logic, we change them to flexible constraint, like "both this and that". Successfully, we build a propositional universal logic, containing both rigid logic and flexible logic. It is able to describe many uncertainty reasoning, and is widely used.

As an universal logic that includes both standard logic and all of the possible non-standard logics, it has different mathematical basis. We used to regard it as an abstract algebra system, which is called flexible logic algebra. In this paper, we will further discuss that such flexible logic algebra is equal with generalized probability theory. Generalized probability theory is our own original theory. We make use of Schweizer operator in triangular norms to expand and refine classical probability theory. As a result, it can describe not only uncertainty reasoning under universal logic, but also the working principle of flexible neuron.

There are five parts discussing about our theory in this paper:

1. The truth-degree of flexible proposition equals the probability within its feature space.
2. Estimating error is acceptable during probability measure. If error does exist, it is additive measure. Otherwise, it is nonadditive measure.
3. The correlativity between propositions x, y is described by generalized correlation coefficient.
4. We use a partial coefficient $\beta \in [0,1]$ to measure the relative change between propositions x and y. By using this method, we success in adding flexible predicate logic to define each flexible quantifier.

5. Differing from flexible logic, rigid logic does not have transition values between 0 and 1. Consequently, it is workable as soon as it satisfies reliability and completeness, without any abnormal results. However, flexible logic does have transition values between 0 and 1. We cannot use them directly, because abnormal results may appear. To solve this problem, we put forward the condition to the integrity of flexible logic. The basic principles for ensuring logical integrity are:
   (1) The truth of two propositions may be the same, but h is not necessarily 1, and the truth of the same proposition must be the same, and h must be 1.
   (2) As long as k is the same, integrity must be guaranteed.

The Syllogistic System: A Paraconsistent Logic Model for Human Reasoning

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Our first argumentation is that human reasoning with information can be modelled with the moods of the syllogistic system and more expressively with the fuzzy syllogistic system that consists of fuzzy moods, ie moods with intermediate quantifiers, such as few, several, half, most, many, etc and their negations.

Humans memorise knowledge essentially with symbolic concepts and their relationships, which can be represented as quantified propositions. We associate new information as evidence for the quantified proportions that we know. For instance, if we know a proposition like "all sports are suitable for children", but are confronted with evidence like "marathon is not suitable for children", then we would probably relax the universal quantification in our knowledge to "almost all sports are suitable for children". Additionally, we would memorise the evidence as new knowledge and relate it to our existing knowledge, like "marathon is one sport", which has an exact cardinality, rather than a fuzzy linguistic quantifier. The more evidences we encounter for further sorts of sports that are not suitable for children, the more we would relax the quantifier "almost all" further, maybe down until "few". This is because over time we will accumulate only a "few" such evidences for children, whereas we will accumulate "many" such evidences for teenagers. We may also be confronted with exceptional evidences, such that under restricted circumstances, individual children may be suitable for some variations of high-performance sports too. This way we continuously refine our knowledge and create increasingly more complex ontological networks in our mind. That can be modelled with numerous interrelated fuzzy quantified propositions.

Our second argumentation is that, for any new evidence we encounter, we try to discover directly associated propositions in our ontological networks for "recalling" facts and in a "thinking" process we may even find interrelated propositions with reasonable categories for syllogistic reasoning.

Now we need to define, what we mean with syllogistic system and fuzzy syllogistic system. The syllogistic system consists of the well known 256 different moods of categorical syllogisms, of which only 25 are fully true, 103 are more true, 103 are more false and 25 are fully false. The 25 true moods are well studied and understood in the literature. It is also known that the mood barbara is the easiest to understand by everybody, whereas the remaining 24 true moods are not that easy to fully grasp by everybody. This is due to the fact that barbara has only one true syllogistic case. A syllogistic case is a specific combination out of the possible 7 distinct spaces for thee sets in the Venn diagram, which are matched or not by a particular mood. Another 11 of the true moods have also just one true syllogistic case each and therefore may also be understand relative easily, if each one's specific syllogistic case is inspected carefully. Another true mood has 4 true cases, 2 have 5, 1 has 9, 4 have 10 and 5 have 11 true cases. In order to fully understand the truth of a mood, one needs to verify everyone of its cases. Every syllogistic case of a mood can be interpreted as an alternative true state of the mood.

Out of the 103 more true moods, 9 have in total 4 cases, of which 3 are true and 1 is false. 2 moods have as much as 72 cases in total, of which one has 65 true and 7 false and the other has 61 true and 11 false. The remaining 92 moods have varying numbers of cases in between. In order to able to fully understand the truth of such moods, one needs to verify everyone of its true and false cases. This is impossible for humans to do, neither in daily information processing nor in conscious think processes. However it is possible and should make sense in electronic information processing, where logical and hence interesting relationships in complex quantified ontological networks can be discovered and with truth values associated.

Fuzzy syllogistic systems have additionally to the universal and existential quantifiers and their negations, further linguistic quantifier, such as few, several, half, most, many, etc and their negations. Fuzzy moods with linguistic quantifiers can represent more closely logical relationships in information processing than syllogisms with solely "all" or "some" quantified propositions.

Finally we need to explain our understanding of paraconsistency in such logic systems. Basically for every single mood or fuzzy mood that is not fully true, we can define a specific sort of paraconsistency based on its specific true and false syllogistic cases. In the syllogistic system we could potentially identify 125 distinct paraconsistencies, as there are that much distinct groups of identical moods.

A Probabilistic Interpretation for an Intuitionistic Sequent Predicate Calculus with Strong Negation

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In this paper we present a sequent calculus for intuitionistic first-order logic with strong negation (SCISN). The semantic is probabilistic, more precisely, it is based on partial conditional probability functions.

A standard system of sequents for intuitionistic logic is introduced, to which is added left and right rules governing the strong negation "~". For example:

$$\frac{\Gamma \Rightarrow A}{{\sim}A, \Gamma \Rightarrow C}\ \mathrm{L}{\sim}, \qquad \frac{A, \Gamma \Rightarrow C}{{\sim}{\sim}A, \Gamma \Rightarrow C}\ \mathrm{L}{\sim}{\sim}, \qquad \frac{{\sim}A, {\sim}B, \Gamma \Rightarrow C}{{\sim}(A \vee B), \Gamma \Rightarrow C}\ \mathrm{L}{\sim}\vee,$$

$$\frac{\Gamma \Rightarrow A \qquad \Gamma \Rightarrow {\sim}B}{\Gamma \Rightarrow {\sim}(A \supset B)}\ \mathrm{R}{\sim}\supset, \qquad \frac{\Gamma \Rightarrow {\sim}A[t|x]}{\Gamma \Rightarrow {\sim}\forall x A}\ \mathrm{R}{\sim}\forall,$$

and so on.

The notion of partial conditional probability function is introduced. It is any partial function $\mathrm{Pr}\colon \mathrm{WFF} \times 2^{\mathrm{WFF}} \to [0,1]$ satisfying 20 postulates which express constraints. For example, POS.4: If $A \in \Gamma$, then $\mathrm{Pr}(A, \Gamma) = 1$, POS.8: $\mathrm{Pr}(A \supset B, \Gamma) = \mathrm{Pr}(B, \Gamma \cup \{A\})$, and so on. POS.8 is critical: it says that the probability of the conditional in SCISN is the conditional probability (in fact, Lewis' triviality result does not hold in SCISN nor in intuitionistic logic). See [1] and [2].

The definition of probabilistic validity is given: the sequent $\Gamma \Rightarrow A$ is valid iff for any Pr and Δ, $\mathrm{Pr}(A, \Gamma \cup \Delta) = 1$. See [3].

All the rules of SCISN are proved to be sound, i.e. if Pr satisfies all the POS.n, then when the antecedent(s) is (are) valid, the succedent is valid.

For example, if $\Pr(A, \Gamma \cup \Delta) = 1$ for all Δ and $\Pr({\sim}B, \Gamma \cup \Delta) = 1$ for all Δ, then $\Pr({\sim}(A \supset B), \Gamma \cup \Delta) = 1$ for all Δ, so $\mathrm{R}{\sim}{\supset}$ is sound.

SCISN is proved to be complete: if, for any Pr satisfying the POS.n and any Γ, $\Pr(A, \Gamma \cup \Delta) = 1$ for any Δ, then $\Gamma \Rightarrow A$. Completeness is proved by defining a canonical Kripke model using saturated sets.

We argue that SCISN can be viewed as the set of logical constraints on rational beliefs of constructive agents.

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A basic dual intuitionistic logic

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Concerning da Costa's paraconsistent logic C_ω, Richard Sylvan (originally called Routley) notes that "C_ω is in certain respects the dual of intuitionistic logic" [2, p. 48]. This opinion is based upon the following facts [cf. 2, pp. 48-49]: (1) Both C_ω and intuitionistic logic H extend positive logic H_+ (i.e., the positive fragment of intuitionistic logic); (2) H rejects the "Principle of Excluded Middle" (PEM), $A \vee \neg A$, but asserts the "Principle of Non-Contradiction" (PNC), $\neg(A \wedge \neg A)$, while C_ω do the reverse; and (3) H accepts "Double Negation Introduction" (DNI), $A \rightarrow \neg\neg A$, but denies "Double Negation Elimination" (DNE), $\neg\neg A \rightarrow A$, whereas, again, C_ω do the reverse. Sylvan adds [2, p. 49]: "This duality also takes a semantical shape: whereas intuitionism is essentially focused on evidentially incomplete situations excluding inconsistent situations, the C systems admit inconsistent situations but remove incomplete situations".

The aim of this paper is to define a basic dual intuitionistic logic in Sylvan's sense. This basic logic is included in Sylvan's extension of C_ω, CC_ω.

Consider Sylvan and Plumwood's basic logic B_M [cf. 3]. Given a language with conditional, conjunction, disjunction and negation as propositional connectives, it is known that B_M is the minimal logic representable with the Routley-Meyer semantics [cf. 1]. Then, the aims of the paper are the following: (1) to expand B_M with a dual intuitionistic negation of the kind discussed above; (2) to endow this expansion with a Routley-Meyer semantics; (3) to show how to extend the basic logic to CC_ω", and beyond; and (4) to study the relations between the dual intuitionistic negation and the De Morgan negation characteristic of relevance logics in general and B_M in particular.

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Many-Valued Decision Logic for Rough Sets

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Pawlak introduced the theory of rough sets for handling rough information [1]. Rough set theory is used as a mathematical foundation of granularity and vagueness in information systems and is applied to a variety of problems. In applying rough set theory, decision logic was proposed for interpreting information extracted from data tables. However, decision logic adopts the classical two-valued logic semantics. It is known that classical logic is not adequate for reasoning with indefinite and inconsistent information. Moreover, the paradoxes of material implication of classical logic are counterintuitive.

Rough set theory is concerned with the lower and the upper approximation of object sets. This approximation divides sets into three regions, namely, the positive, negative, and boundary regions. Thus, Pawlak rough sets have often been studied in a three-valued logic framework because the third value is thought to correspond to the boundary region of rough sets [2,3].

In this study, we show the relationship between decision logic and three-valued semantics based on partial semantics and propose extended decision logics based on three-valued logics. The formalization of many-valued logic is carried out using a consequence relation based on partial semantics [4]. The basic logic for decision logic is assumed to be many-valued, in particular, three-valued and some of its alternatives [5]. The decision logic will be axiomatized using Gentzen sequent calculi and three-valued semantic relation as basic theory. To apply three-valued logics to decision logic, consequence relations based on partial interpretation are investigated, and sequent calculi of three-valued logics are constructed. Subsequently, three-valued logics with different structures are considered for the deduction system of decision logic. These logics can serve as foundations for reasoning about rough and vague information and we propose some extended decision logics.

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On a correspondence of positive and negative modalities on the basis of some non-normal logics*

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In the paper we consider a connection between modal logics with the standard modalities and those which are expressed by means of negative ones. The initial idea comes from [1], where the negation is read as 'not necessary'. Using translation that transforms propositional formulas without modalities into the modal language one can ask, whether an image of the considered assignment is a thesis of a specific modal logic. In [1], the normal modal logic S5 has been used and a logic Z is obtained by the mentioned translation. Similar question can be applied to other normal modal logics. The idea which led Béziau to invention of Z refers to Jaśkowski's idea of modalising of propositional formulas. Logics obtained by treating negation as a negative modal operator we call Béziau-logics. It appears [see 3,4] that there is an equivalence between family of all normal logics and respective Béziau-logics. One can also search for similar equivalences for the case of non-normal logics, for example regular or quasi-regular. It is possible to obtain such equivalence results for chosen logics from the mentioned classes [e.g. 5]. It this context one can also consider the dual negative operator 'it is not possible'. Such negation (or its equivalent version) is known from the literature [e.g. 2]. Using it one can enrich the object language of a given Béziau-logic and obtain equivalence correspondence results for new cases of non-normal logics, in particular for logics closed under tautological equivalences. In the paper it will be also discussed the issue of finding such translation which allows for obtaining a given Béziau-logic directly as a surjective image — usually to obtain a destined logic, one has to apply the closure on the consequence relation.

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Fuzzy Logic and Communication in a Social Context

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In [1], Bellman and Zadeh point out that many of the terms used in ordinary discourse are fuzzy and that probability theory cannot always be used to address the problems which the fuzziness creates. Here is what they say:

> By decision-making in a fuzzy environment is meant a decision process in which the goals and/or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined.
>
> An example of a fuzzy constraint is: "The cost of A should not be *substantially* higher than α", where α is a specified constant. Similarly, an example of a fuzzy goal is: "x should be in the *vicinity* of x_0", where x_0 is a constant. The italicized words are the sources of fuzziness in these examples.

However, a second issue arises, namely that when terms are fuzzy then social agreement on when the term applies or even what it means can become dilute or even vanish. This point was made by me in a survey I took during some lectures in Sicily [3]. I asked questions like "Is a handkerchief an item of clothing?", "Is Sonia Gandhi an Indian?" or "Is Henry Kissinger an American?" The respondents were allowed to answer with fuzzy values. But even then the answers were quite different. Interestingly "Is Henry Kissinger an American?" got an average value of 0.89 whereas "Is Sonia Gandhi an Indian?" only got a value of 0.53. This is odd, because Sonia Gandhi is a naturalized Indian and Kissinger is a naturalized American and the answers should have been close (another fuzzy term).

The conclusion is that fuzzy logic and fuzzy semantics does not go the whole way towards solving the communication problem in the presence of words with inexact meanings and inexact domains of application, thus the success of [2] and [5] is only partial.

This will be a small problem in a purely decision theoretic context. If I am an Indian citizen and think that Sonia Gandhi is only 0.53 Indian, I might then decide to vote for someone else. But many of our practices are game theoretic or communication theoretic. We often need to work with others who may use a different (fuzzy) semantics. See [4] for a partial solution.

Can we develop a semantics where disagreements about the meaning and application of words are minimized or eliminated? If two people have similar values, but one likes feminism and the other opposes feminism, it could be that they understand the same term "feminism" differently. Can we reconcile them somehow? This is a burning question for modern times, where AI has brought us closer and closer to social issues and not just scientific issues or engineering issues.

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Investigations on the axiomatic presentation of $\mathcal{ALC}$ Description Logic and its formalization in Lean

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Description Logics (DLs) are a family of formalisms used to represent knowledge of a domain. It is equipped with a formal logic-based semantics. Knowledge representation systems based on description logics provide various inference capabilities that deduce implicit knowledge from the explicitly represented knowledge.

In [8] some of the authors investigate the proof theory of DL. Sequent Calculi and Natural Deduction-style deductive systems were proposed for the description logics $\mathcal{ALC}$ and $\mathcal{ALCQ}$. The most important meta-theoretic results about semantics and proofs for these systems were proven: soundness, completeness, cut-elimination and normalization. It is argued that those systems can improve the extraction of computational content from DLs proofs, for proof explanation purposes.

The completeness of the Sequent Calculus for $\mathcal{ALC}$ ($\mathrm{SC}_{\mathcal{ALC}}$) was first presented in [9]. It was shown relative to the axiomatic presentation of $\mathcal{ALC}$ presented by Schild in [10], that is, in order to prove the $\mathrm{SC}_{\mathcal{ALC}}$ completeness, we shown the axioms can be derived in the system.

Nevertheless, soundness and completeness of $\mathcal{ALC}$ was not really presented by Schild. Schild cited [4] and it turns out to be a mistake. Schild was intended to cite [3] since both [4] and [5] address only uni-modal logics, clearly not directly related to description logics. On the other hand, although the syntax translation of $\mathcal{ALC}$ concepts to K_n formulas are considered obvious by many authors, and their intuition described by [10,1], in [3] the authors have presented the K_n multi-modal logic axioms only.

In this work, we aim to start a discussion about a detailed mapping from the K_n multi-modal logic from [3] to the $\mathcal{ALC}$ axiomatic presentation presented by [10].

We aim to discuss the classical presentation of $\mathcal{ALC}$, showing that system $\mathrm{SC}_{\mathcal{ALC}}$ in [7] is sound and complete using two proof methods, filling the gaps not explained in [10]. These results can help to confirm that the axiomatic presentation of $\mathcal{ALC}$ presented by [10] is indeed sound and complete too. But there are unanswered questions about how that axiomatic presentation was obtained from [3]. Some further clarifications about the classical $\mathcal{ALC}$ can come from the works on intuitionistic versions of $\mathcal{ALC}$. In [2], the system $\mathrm{SC}_{\mathcal{ALC}}$ was adapted to an intuitionistic presentation of $\mathcal{ALC}$, the i$\mathcal{ALC}$.

We are also working on the mechanization of the meta-theorems about the calculi for $\mathcal{ALC}$ using the interactive theorem prover Lean [6].

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Expansions of relevance logics with a quasi-Boolean negation of intuitionistic character

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In [4], intuitionistic-type negations are investigated from the point of view of Routley-Meyer semantics, negation being interpreted similarly as in standard binary Kripke semantics: $\neg A$ is true in w iff A is false in w' for all w', w'' such that $Rww'w''$. The aim of the present paper is to study a family of logics only marginally referred to in [4]. Negation will now be interpreted by using the Routley operator, the expedient customarily employed for elucidating negation in Routley-Meyer semantics.

Let L be a relevance logic. A Boolean negation (B-negation) can be introduced in L by adding to it the following axioms and rule [cf. 2,3 and 5,§5.4]:
a1) $\neg\neg A \to A$,
r1) $(A \wedge B) \to \neg C \Rightarrow (A \wedge C) \to \neg B$.

Consider now the following axioms:
a2) $(A \wedge \neg A) \to B$,
a3) $B \to (A \vee \neg A)$.

It will be not difficult to prove that if L contains the positive fragment of Anderson and Belnap's "First Degree Entailment Logic", FDE$_+$, [cf. 1], a1 and r1 are derivable in L plus a2 and a3, whence B-negation can be introduced in L by adding a1 and a3 to it. This way of introducing negation in relevance logics suggests the definition of two families of quasi-Boolean negation (QB-negation) expansions of relevance logics. One of them, intuitionistic in character, has a2 but not a3; the other one, dual intuitionistic in nature, has a3 but lacks a2. The aim of this paper is to investigate the former family of QB-negation expansions of relevance logics.

B-negation expansions of relevance logics are of both logical and philosophical interest [cf. 5, pp. 376ff.]. It is to be expected that QB-negation expansions of the same logics (not considered in the literature, as far as we know) will have a similar logical and philosophical interest.

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A pluralist account of relevant implication and a sequent calculus for classical logic's version

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In this lecture we first develop a logic independent account of relevant implication. We propose a stipulative definition of what it means for a multiset of premises to relevantly L-imply a multiset of conclusions, where L is a Tarskian consequence relation. Premises are said to relevantly imply conclusions iff there is an abstraction of the pair $\langle\text{premises}, \text{conclusions}\rangle$ such that the abstracted conclusions are L-consequences of the abstracted premises and none of the abstracted premises or the abstracted conclusions can be omitted while still maintaining valid L-consequence. In this definition, $\langle\Gamma_1, \Delta_1\rangle$ is an abstraction of $\langle\Gamma_2, \Delta_2\rangle$ iff the latter can be obtained from the former by a series of applications of Uniform Substitution (i.e. replace every occurrence of a sentential letter by the same formula).

Subsequently, we apply this definition to the classical logic (CL) consequence relation to obtain NTR-consequence, i.e. the relevant CL-implication relation in our sense, and develop a sequent calculus that is sound and complete w.r.t. relevant CL-consequence. We present a sound and complete sequent calculus for NTR. In a next step we add rules for an object language relevant implication to the sequent calculus. The object language implication reflects exactly the NTR-consequence relation. One can see the resulting logic $\mathsf{NTR}^{\rightarrow}$ as a relevant logic in the traditional sense of the word.

By means of a translation to the relevant logic R via LR (i.e. R without Distribution), we show that the presented logic $\mathsf{NTR}^{\rightarrow}$ is very close to relevance logics in the Anderson-Belnap-Dunn-Routley-Meyer tradition. However there are important differences. On the one hand, $\mathsf{NTR}^{\rightarrow}$ is decidable for the full language, Disjunctive Syllogism (A and $\neg A \vee B$ relevantly

*The results presented here were obtained in collaboration with Inge De Bal (Ghent University, Belgium) and Aleksandra Samonek (Université Catholique de Louvain, Belgium).

imply B) and Adjunction (A and B relevantly imply $A \wedge B$) are valid. On the other hand, certain versions of Modus Ponens and Cut are not valid in $\mathsf{NTR}^{\rightarrow}$.

Finally, an elegant diagrammatic system for $\mathsf{NTR}^{\rightarrow}$ will be presented. This system nicely visualises certain aspects of goal directed reasoning in classical logic, viz. how one reasons from the candidate conclusion towards the premises that may justify it. We will illustrate the logic by means of some diagrammatic proofs.

S-shape Transconsistent Logic System

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The Diagonal Proof Method has played a dominating role in the mathematics field for a long time. For example, Russell finds the paradox of set theory. Cantor proves that the power set of natural numbers is uncountable and the set of real numbers is uncountable. These propositions are based on the same mathematic method which is praised as Golden Diagonal.

By contrast, this paper manages to prove: the propositions proved by Diagonal Methods are out of domain (extra-field term). The uncloseness in mathematical calculus is an extensive and profound mathematical phenomenon, which leads to contradictions and paradoxes. It is significant that Diagonal Proof Method is indeed an unclosed method, therefore we can find uncloseness in logical thinking calculus. Further, we build transconsistent logic systems S-L and S-K, which are called S-shape Transconsistent Logic for short.

For recent years, some logic systems containing contradictions came up in the world, such as da Costa's Paraconsistent Logic, R. Brandow's Inconsistent Logic, R. Routley's Transconsistent Logic and so on. By restricting effective using scope to law of contradiction, these logic systems make use of Harmless Contradictions to break consistency of classical logic system. As a result, such systems conflict with classical logic system, making them be far away from mathematical practice. Consequently, they are of certain philosophy but not of mathematical value.

S-shape Transconsistent Logic directly analyzes source of paradoxes and contradictions in mathematics. It notices that paradoxes come from unclosed terms (extra-field term). It directly takes extra-field term as logical objects in study, making logical and mathematical property in extra-field increasingly clear and close to mathematics. We believe extra-field term can better guide pioneering mathematical practice. Based on S-shape Transconsistent Logic, we find a large group of extra-field term mathematic propositions in mathematical logic field, such as set theory, recursion theory, etc.

The essence of Diagonal Proof Method is a method to prove constructing paradoxes. The paradoxes are used in proving "reduction to absurdity", which may result in errors. It is not a normal error, but a method error, which will lead to deviation in proposition group. Thus the Diagonal Method and "reduction to absurdity" will spread to specific mathematics field which involves in philosophy, mathematical logics, computer, function theory, measure theory and specific mathematics these fields. Therefore, paradoxes and Diagonal Proof Method deserves further study and more attention.

Universal M-Valued logic

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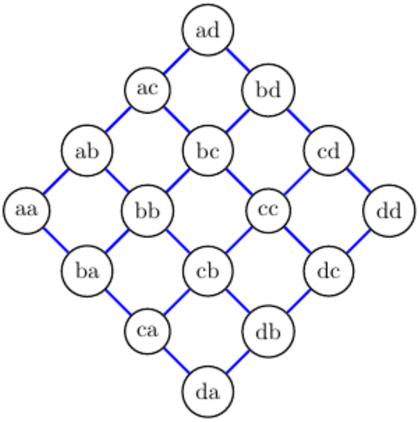
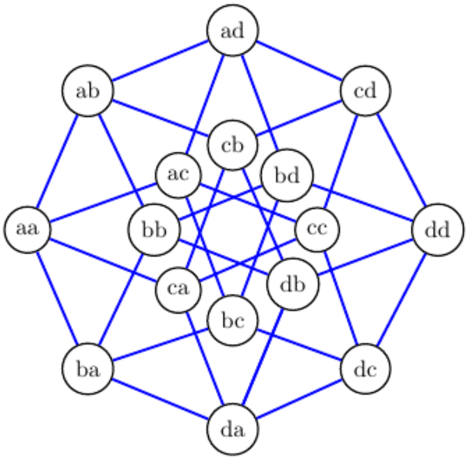
This paper based on M-theory [1] wants to show the correlation of Cartesian product of elements of ordered set with a basic, multi-value logic. Using simple algorithms discovered by M. Šare in the natural laws of electrical networks [3], it is possible to construct logical tables, i.e. grids, for all logic functions of n variables.

$\wedge$	*ba*	*bb*	*aa*	*ab*
ba	**ba**	*bb*	*aa*	**ab**
bb	*bb*	*bb*	*ab*	*ab*
aa	*aa*	*ab*	*aa*	*ab*
ab	**ab**	*ab*	*ab*	**ab**

For example, $n = 2$ gives the Dunn/Belnap B4 logic table (left table), while $n = 4$ gives 'Sweet-sixteen' [2] (figure below). All higher order logics include those below: Sixteen_3 includes Four_2, and Four_2 includes Bool's logic (bold elements in table, where $ba \equiv T$ and $ab \equiv F$).





Each element of the ordered set has its 'semantic domain' in the partition of the total logical space (dashed in the table), e.g. for $n = 2$ domains are 'ba, bb' (for element 'b') and 'aa, ab' (for element 'a'). Unlike fuzzy logic, domains have (hierarchical) structure. By increasing the dimension n, there is a shift of the value (weight) of the logic variable, as already noted in [2].

The paper is accompanied with an interactive M-theory online program written in Python.

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*Interest Group in Pure and Applied Logics

Argumentation

The keynote speakers at this session are Leila Amgoud (page 119), Leon van der Torre (page 163) and Elena Lisanyuk (page 135).

Abduction in Unconceded-Preserving Dialogues

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A lot of divergent interpretations of Peirce's schema of abduction [2, p. 5.189] have been defended. Gabbay and Woods [1] have argued that most of them neglect its pragmatic dimension and the conjectural aspect of the conclusion. By taking this advice as starting point, our proposal consists in an understanding of abduction based on the conditions of use of abductive conjectures in the context of dialogical logic. When abductive conjectures can be introduced in an argumentative dialogue (triggering)? How abductive conjectures are introduced (guessing)? What are we committed to when we introduce abductive conjectures (committing)? According to the GW-model, abduction is triggered by an ignorance problem that acts as a cognitive irritant. That is, an agent faces a surprising fact, something that cannot be explained by his background knowledge. In such a situation, the agent may (among other possibilities) conjecture a hypothesis that allows him to continue a reasoning or an action despite a persisting state of ignorance. That hypothesis can be released in further reasoning. However, it is not stated as a new piece of knowledge and it is defeasible. According to the GW-model, abduction is an ignorance-preserving inference in which what is unknown at the level of the premises remains unknown at the level of the conclusion. Our aim is not to propose a dialogical formalization of the GW-model. It is not to provide criteria to evaluate abduction either. It is rather to think about a dialogical ground for the use of abductive conjectures in order to define a general framework in which a more fine-grained analysis of

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abduction might take place. In this respect, we focus on the notion of concession that plays a central role in deductive dialogues. Indeed, abductive dialogues are triggered by a concession-problem. That is, some participants in a dialogue might want to continue an argumentative interaction despite a blockage provoked by a lack of concessions. In that context, the dialogue becomes non-deductive and the proponent is allowed to introduce an abductive conjecture by means of a conditional move. According to our approach, abductive dialogues are unconceded-preserving dialogues, that is, what has been introduced by the proponent without previous concessions of the opponent remains unconceded when the conjecture is used.

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Analytic Tableaux for Argumentation Frameworks

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Argumentation is a process in which arguments are advanced by contending parties, each one trying to defeat the arguments of the other. We assume the framework of attack relations among arguments of Dung [6], for which several semantics have been proposed [7,2,4,5]. These semantics define the acceptance and rejection of arguments. On the other hand, this can also be determined in terms of proof procedures, as for instance dialogue games [9,3] in which two players, *Pro* and *Con* put forward, sequentially, arguments attacking the previous one, starting with *Pro*. The last one to move wins. If it is *Pro*, the initial argument is accepted; otherwise it is rejected.

In this paper we present an alternative decision method for argument justification, inspired in the method of analytic tableaux [1,8]. The latter allows to decide on the satisfiability of a formula or set of formulas by developing a tree (the *tableau*) in which any formula becomes "expanded" according the truth table of its principal connective. In a finite sequence

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of steps the procedure yields either the satisfiability or not of the formula or formulas. In our case, we want a method to decide if sentences like 'argument a is *in*' or 'argument a is *out*' are satisfiable, combining ideas from proof-theoretic and semantic approaches to argumentation. Starting with an original sentence about the acceptation status of an argument, a tree is build upon the different attack lines. Arguments along them are marked *in* or *out*. In the end, some of the branches remain "open" (i.e. without contradiction) and tracking back we can state whether the original formula is satisfiable or not.

We define notions of satisfiability and validity, based on this tableaux method, for argumentation frameworks. Furthermore, we show how to apply it to capture skeptic/credulous acceptance criteria for Dung's preferred and credulous semantics.

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How not to aggregate reasons

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Assume an arguer forwards two reasons, R1, R2, in support of a claim C. Further, assume the meanings of R1 and R2 neither exhaust, nor fully include, the meaning of C. This makes the argument "R1 and R2, therefore C" an instance of ampliative reasoning, and so projects onto inductive inference.

As an a priori truth, if R1 and R2 provide some positive support for C, then the support that R1 lends to C either does, or it does not, depend on the support that R2 provides, and vice versa. The case where reasons provide negative support (of the undermining or undercutting kind) is analogous. In principle, factual and normative reasons (or evidence) can be treated alike [1].

Abbreviating 'support' by 'P', as in 'probability', we can hence express "How do reasons aggregate?" as "How to define $P(\mathrm{C}|\mathrm{R1} \oplus \mathrm{R2})$?", where '$\oplus$' denotes the concatenation of R1 and R2. Raising this question, then, is to ask how we might specify the function $f(P(\mathrm{R1}), P(\mathrm{R2}))$.

There is no shortage of candidates for this. One candidate is the "weakest link principle" (or Theophrastus' rule), according to which $f(P(\mathrm{R1}), P(\mathrm{R2}))$ outputs the minimum value of $P(\mathrm{R1})$ or $P(\mathrm{R2})$. A second (related) candidate selects the maximum of both values. A third settles for some value below the maximum (e.g., the multiplicative sum, $P(\mathrm{R1}) \times P(\mathrm{R2})$, or the average sum, $(P(\mathrm{R1}) + P(\mathrm{R2}))/2$, or a weighted combination thereof). The fourth candidate, finally, sends $f(P(\mathrm{R1}), P(\mathrm{R2}))$ to values above the maximum.

The four options are exhaustive. Hence, we can express both positive and negative support along whichever scale measures the strength of the 'is a reason for'-relation. Following Spohn's [3] ranking theory [see 4], therefore, we can define the most general scale as:

$$+\infty \geq P(\mathrm{C}|\mathrm{R1} \oplus \mathrm{R2}) = f(P(\mathrm{R1}), P(\mathrm{R2})) = P(\mathrm{C}|\mathrm{R1}) + P(\mathrm{C}|\mathrm{R2}) \times a \geq -\infty,$$

where a (being some rational number) is an "aggregation term" that, if suitably selected, specifies one of the above four candidates.

What many today call a Bayesian (or Pascalian) approach to probabilistically calculating argument strength [e.g. 2], itself defined over the [0,1] interval, is a limiting case of the above Baconian scale $[+\infty, -\infty]$. Of course, important differences between a Pascalian and a Bayesian approach [5] can arise from defining rules that relate $P(\mathrm{C}|\mathrm{R})$ with $P(\neg\mathrm{C}|\mathrm{R})$, which are co-dependent terms in the former approach, but not in the latter.

We briefly review — yet criticize — approaches that define $f(P(\mathrm{R1}), P(\mathrm{R2}))$ according to the structural *linked* vs. *convergent* vs. *serial* distinction. In particular, a probabilistic perspective lets a serial (aka subordinate) structure appear as a non-distinct instance of a linked structure. The two basic structures, then, are the linked and convergent structure.

We also explain why the four aggregation-candidates (rightly) apply to some natural language arguments. Decisive reasons to favor this or that candidate, however, not only cite, but indeed *pivot on* semantic rather than structural information. Which structure properly models reasons R1 and R2, and which candidate properly aggregates them, therefore stands or falls with the dependence or independence of the semantic contents these reasons express.

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Language and Semiotics

How Combinatory Logic can be used to formalize meanings

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Curry's Combinatory Logic (CL) is an applicative formalism in which each expression is constructed by applications of operators on operands. CL is a logical theory (following Curry: an "Ur-Logik") defined from abstract operators (called combinators) used to compose whatever operators (for instance specific operators linked to given domains) or to transform them; the compositions and transformations are executed by intrinsic ways, i.e. without any reference to a domain of interpretations. The combinators can be defined in the general Gentzen's natural deduction system by means of introduction and elimination rules defined for each combinatory [1]. The formalism of CL is not fully equivalent to the Church's lambda-calculus (LC) for three principal reasons: (i) there is an equivalence 'in extension' but not 'in intension' [2]; (ii) CL does not use bound variables as LC necessarily does: the bound variables in LC are necessary linked to a domain of interpretation of variables, hence it is not possible to define intrinsic general properties about operators, as it is possible in CL; and (iii) since CL uses only free variables, the technical complications involved by substitution — with obligatory changes of bound variables in LC — are completely avoided in CL. In Categorial Grammars (CG), the syntactic categories are canonically associated to Church's functional types [3]; in the extension of CG by the Lambek Calculus [4], it is possible to compose functional types and to change a functional type (for instance by a type rising operation). In this communication, we will show that each formal composition of functional types of the Lambek Calculus (defined on functional types) introduces, in a syntactic analysis, some combinator of the 'typed CL'; then, the elimination rule associated to the introduced combinator contributes to construct the applicative organization underlying to the analyzed linguistic expression (for instance a possible sentence). In GRACE (GRammar of Applicative, Cognitive and Enunciative operations) [3], the meanings of lexical and grammatical units are expressed by applicative expressions (schemes) obtained from a composition of a small set of semantico-cognitive primitives. However, in this computational model of Cognitive Semantic, by using combinators, we can explain how these applicative representations (schemes) can be synthesized into lexical predicates and composed with grammatical operators (in a top down approach) and also, how verbal predicates and grammatical units can be decomposed and represented inside of applicative representations of their meanings (in a bottom up approach). The aim of the communication is a presentation of this new logical and computational analysis of natural languages with the help of complete formal treatments of examples in which are explicitly associated, by formal deductions, on one hand, semantic and cognitive representations of grammatical units (tenses and aspects) and representations of the meanings of lexical verbal units (by schemes) and, on another hand, syntagmatic organizations of these linguistic (lexical and grammatical) units in utterances.

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The Structural Unconcious: the Logic of Differences

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F. de Saussure found out an objective reality in "human speech" (fr. langage) totally contingent on the observer's influence. In the Course of General Linguistics [1], he names this reality "language" (fr. langue), opposing it to "speech" (fr. parole). The latter is the individual speaking and the former is systemic. In language, an expression has value in itself not because of what is positively consisted of but because of what makes it different from the other expressions. Therefore, language is a system of differences or oppositions. Linguists from Prague School managed to isolate in language the phoneme, conceived by R. Jakobson as a "bundle of distinctive features" [units of sound] deprived of meaning [2]. In language, as in every communication system, “what makes a sign different is exactly what it is consisted of”. Saussure, thus, announces the project of a new science — Semiology — which object is precisely “the study of the life of signs in social life”. It is through using the phonological model on kinship sociology, that Cl. Lévi-Strauss sorted out the enigma of incest prohibition in Elementary Structures of Kinship [3]. Relating The Gift by M. Mauss to Course in General Linguistics, Strauss established that matrimonial institutions are essentially “reciprocity structures” unconsciously produced by human spirit which make possible the exchange of the “good by excellence”, “the supreme gift’, i.e., the woman. It is important to notice that incest prohibition does not concern intrinsic female characteristics, such as the biological ones, but its belonging to antithetical relations system that make the exchange operative.

What matters is the “alterity sign”, or rather, the woman’s position “as the same or the other” in an “opposition system”. Therefore, there is a clear parallelism between the phoneme and the incest prohibition. In Elementary Structures of Kinship conclusion, Cl. Lévi-Strauss proposes to conceive the woman as a kind of sign. This perspective opens the possibility to think culture as “a set of symbolic systems” and anthropology as a semiological science. The “symbolic function” is, thus, the condition that makes culture possible. “Specifically human”, this function works “in all men according to the same laws”. A symbolic function organ, the structural unconscious, deprived of content, is the basis “of the laws of human thinking”. “Non-reflexive totality, language is a part of human reasoning with reasons that man does not know”, says Cl. Lévi-Strauss, paraphrasing Pascal.

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A Structural Semiotic Study of How We Use Variables in Math and Logic

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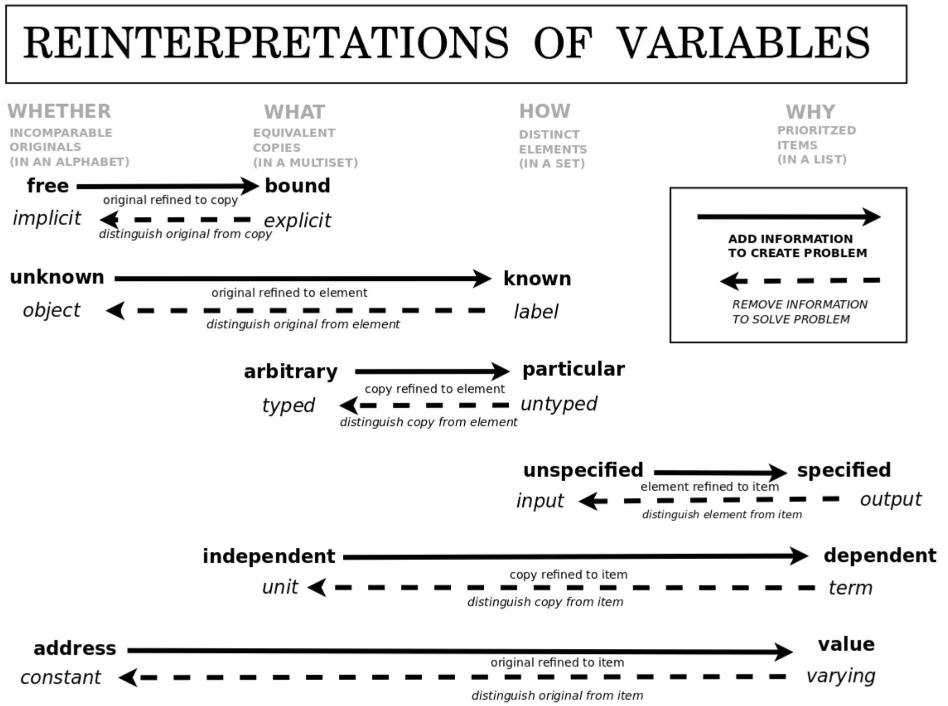
One of the profoundest hurdles for students of mathematics is learning to think of a given mathematical symbol in a variety of ways. The variable X may be thought of as a “constant” which may be “specified” or “unspecified”, and which may “vary” or be “fixed”, and may be “independent” of other variables or “dependent” on them.

The options which our minds have for interpreting a variable may be dictated by a cognitive structure. We collected a variety of terms which characterize variables. Indeed, we found 12 pairs such as “known” and “unknown”, “free” and “bound” or “input” and “output”.

Such pairs indicate how a variable may be reinterpreted. Semiotically, a variable is a sign. What can we say about what different kinds of variables signify? A logical or mathematical expression contains variables from some alphabet A, B, C… If these variables are “free” or “unknown”, then they refer to nothing more and we may think of them as uninterpreted. But if they are “bound” or “arbitrary” or “independent”, then we imagine them indicating a range of possibilities. If they are “known” or “particular” or even “unspecified” then they refer to particular possibilities, which we ourselves may not yet know, however. And if they are “specified” or “dependent” or “values”, then they are explicitly related to other variables so that we may say that we ourselves do know them.

In summary, we may think of variables as referring to unrelated originals (in an alphabet), interchangeable copies (in a multiset), particular elements (in a set) or prioritized items (in a list). They manifest a recurrent cognitive framework of levels of knowledge: whether, what, how and why. Each pair of terms refers to two such levels. Structurally, we find six pairs which our minds use to enrich the content of a variable, for example, to reinterpret a free variable as a bound variable, in order to define a problem by adding information. We also find six pairs which our minds use to emphasize the form of a variable, for example, to reinterpret an output as an input, in order to solve a problem by removing information. The way that our minds recast variables to create and solve problems is meaningful in exploring the cognitive foundations of logic.



The origin of semantics in formal languages

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In [1], María Manzano and Enrique Alonso claimed that talking about semantics before the 1920s is 'a misconception'. For this reason, they propose the word 'proto-semantics' to speak about the collection of algorithmic proceedings which were employed by Post [2], Bernays [3] or Wittgenstein [4] to classify formulas of propositional logic. But contrary to them, we argue that a conscientious analysis of the earliest consistency proofs for the theory of propositional logic reveals that, while Post's procedure of truth tables can be fairly named 'proto-semantics', Bernays was able to identify valid and provable formulas by providing an interpretation to his axioms. What is more, Bernays compared the set of valid formulas with the set of provable formulas, so that he proved soundness and completeness for propositional logic. We conclude our contribution wondering why Hilbert's own consistency proof does not present an independent semantics: since some key concepts are blurred in his texts, syntax and semantics are not precisely delimited.

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Bilingual Logic Based on the Scientific Method System

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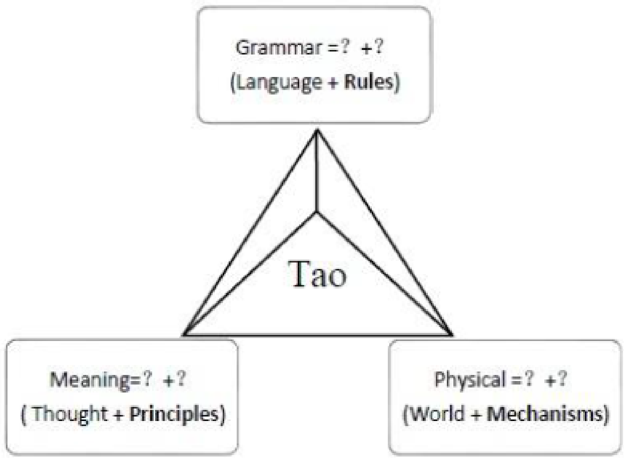
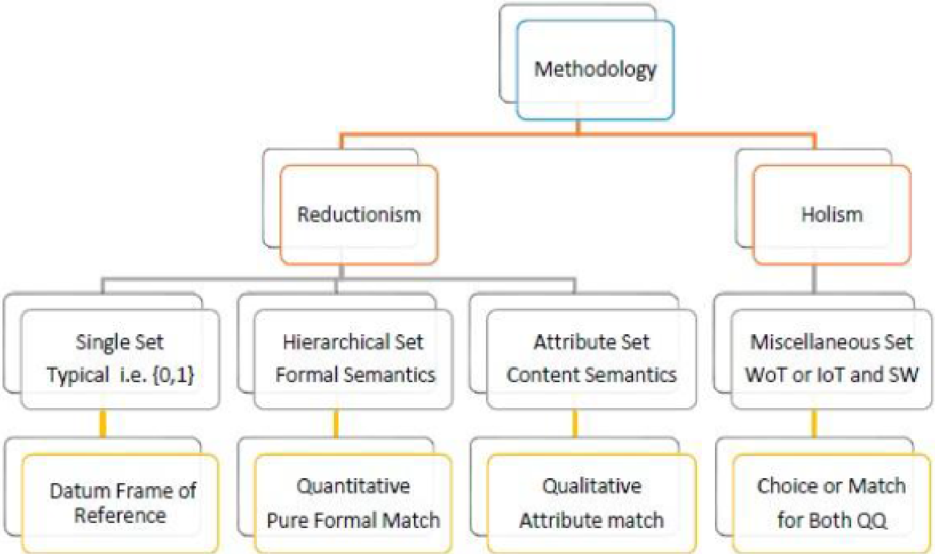
Keywords: logical matrices, tableau, completeness, decidability, bilingual logic, method system.

The purpose of this paper is to start from the basis of formal logic and mathematical logic, to establish the generalized bilingual logic based on the scientific method system. The method is as follows: Step 1, use formal logic with any language or symbol system; Step 2, use mathematical logic with logical matrices and tableaux; Step 3, use logic of sequence and position with the completeness of a single set or a series of finite set from hierarchical sets, and the relative completeness of attribute sets. It is characterized in that the linkage function relation between two series of symbolic systems by using the double list or matrix of both digital and textual, which is based on human-computer interaction system, in both of interaction and batch processing, based on the scientific method system. The result is that the generalized bilingual logic based on the scientific method system, in which the determinability and calculability of formalized system, for that the substantial miscellaneous set which contains three types of sets — single set, hierarchical sets and attribute sets, is the most critical with both language and knowledge. In the dual formalized man-machine system, relying on the logic of sequence and location, and linkage function, interaction process of a series of targeted selection, the advantages of both sides of human brain and computer are played just right, that is, can produce software much easier. Its significance is that conditions of both complete and necessary are available with the foundation for the big production of knowledge, as the time goes, not only formal system engineering of language, knowledge, software and hardware, but also social system engineering of education, management, learning and application, by using of logical matrices, tableaux, completeness and decidability that will be significantly optimized based on the new scientific method system.



(a)



(b)

Basic models of ambiguity analysis:
(a) model of basic views, (b) model of basic methods.

Tools and Results

A second order propositional logic with subtyping

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In this work we propose a variation of second order propositional logic with subtyping over propositions, in a way that can be considered a case of an impredicative higher order type theory [1,2]. We have several subtypes or predicates, all of them being subdomains of the type of all propositions Prop. For example:

- Syntactic subtypes such as At (subtype of atomic propositions).
- Arbitrary semantic subtypes or classes of propositions depending on the nature of the world we are considering.

These subtypes can be considered as predicates over propositions. In this way, we can quantify over propositions of type 1. Thus, we can consider usual propositions as propositions of type 1, and predicates over them as propositions of type 2. This allows us to increase the expressive power of our language, since we can express things such as for any atomic proposition there exists its negation, or if it holds any proposition of class A, then q. Although this work deals with the formalisation and semantics of this language, the most important issues are those that have to be debated, mainly if it is possible and safe the construction of compound propositions that mixes these two types of propositions.

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The generalized probability theory and intelligent information processing

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Generalized probability theory is our own original theory. We make use of Schweizer operator in triangular norms to expand and refine classical probability theory. As a result, it can describe not only uncertainty reasoning under universal logic, but also the working principle of flexible neuron. We believe that it is a indispensable mathematical logic for intelligent science and technology, laying the foundations for uncertainty reasoning and intelligent information processing.

A Natural Deduction System for Leśniewski's Protothetic

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Stanisław Leśniewski's Protothetic is one of the richest systems for propositional logic. It is not only a calculus for the usual connectives, but also a theory for the use of quantifiers binding propositional variables, variables for connectives and, in fact, variables for functions of any order or category (like connectives, properties of connectives, operations on connectives, properties of such operations, etc.). One of the important features of this ambitious system is that its formal language cannot be restricted once and for all to any set of syntactic categories. On the contrary, it must always remain open to the infinitely many new and more complex categories of constants to be further introduced by the way of a procedure of definition, which was explicitly codified by Leśniewski, as a formal inference rule of the system.

Protothetic remains today quite unknown for a large part of logicians, even if it was central in the celebrated Warsaw School. It was studied and used for different purposes by eminent scholars, among them A. Tarski, T. Kotarbiński, K. Ajdukiewics, J. Słupecki, C. Lejewski, A. Prior and P. Simons. More than Leśniewski's death in 1939 and the breaking up of the Warsaw School during the War, the main reason of the current weak following of Protothetic lies probably in the fact that this logic only exists in difficult and old-style axiomatic versions. Teaching Leśniewski's logic is today quite a challenge, for it is often seen as too different from the standard of the subject. Students are accustomed to streamlined methods of working, far from Leśniewski's very meticulous axiomatic method.

Nevertheless, it is known by testimonies and also by examples in the literature that, in everyday work, Leśniewski and his colleagues in Warsaw (among them Łukasiewicz and Tarski) used a convenient method for finding their proofs, very close to nowadays natural deduction. The idea was to draw consequences from assumptions, using derived inference rules and collecting the results into generalized conditional propositions. This is just like our students in logic are accustomed. Jaśkowski's well-known natural deduction system (1934) has clearly been inspired by this uncodified practice. Unfortunately, no one worked after Leśniewski's death on an explicit system of this sort for his logic.

In this talk, we are going to show that a system very close to Fitch's and Jaśkowski's style of natural deduction systems is available for Protothetic. Apart from usual rules for the introduction and elimination of basic logical constants, the system contains a specific and powerful rule for definitions. The main challenge in the conception of such a system is the way to deal with an evolutional notion of well formed formula. Each time a new definition is stated (in particular, when the defined constant is of a new syntactic category) the language and also, possibly, the inferential power of the system increase. These novelties have to be integrated into the proofs to be further constructed. This is indeed an unusual way to proceed in standard formal logic. But it is in fact very common in the developments of scientific theories. Leśniewski's logic is not only a tool for the formalization of a given state of a theory, but a logical machinery the aim of which is to formalize into a single framework the often quite complex definitional expansions through which a theory is developed.

With very elementary propositional examples, we are going to show how this machinery works. We will also make explicit the fact that there exist a way to deal with Protothetic (and later on, to larger Leśniewski's systems) perfectly accessible to everyone acquainted to the basics of propositional standard logic.

Tableau Systems for Epistemic Interpretations of Jerzy Łoś' $\mathcal{R}$-Operator Logics

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Jerzy Łoś is the Polish logician who happens to be one of the pioneers in the field of philosophical logic. Precisely, his works [3,4] covering epistemic and temporal logic, are the very first publications offering systems which formalize topics such as propositional attitudes and time. However, the pathfinder's glory has somehow passed him by. One could point out many causes for such exclusion, but, more than anything, unfortunate times are there to blame. Nonetheless, despite the Iron Curtain, Łoś' ideas became accessible to the western world due to reviews of Henryk Hiż [1] and Roman Suszko [7]. In this respect, Polish thinker's ideas are appreciated in Prior [5] and Rescher [6]. Soon after that, his name is rarely mentioned.

The year 2018 marks the 70th anniversary of publishing [3], which makes a great occasion to bring back some of the Łoś' work. We present general treatment of logics with $\mathcal{R}$-operator logics, as in [2]. Then we give it an epistemic interpretation by selecting certain systems. We construct a tableau-style proof systems for those logics based on the more general theory [8].

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Comparing Classical and Relativistic Kinematics in First-Order Logic

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The aim of this talk is to present a new logic-based understanding of the connection between classical kinematics and relativistic kinematics. These theories are axiomatized as in the tradition of the Andréka-Németi School [see 1].

We show in [2,3] that the axioms of special relativity can be interpreted in the language of classical kinematics. This means that there is a logical translation function from the language of special relativity to the language of classical kinematics which translates the axioms of special relativity into consequences of classical kinematics.

We also show that if we distinguish a class of observers (representing observers stationary with respect to the "ether") in special relativity and exclude the non-slower-than light observers from classical kinematics by an extra axiom, then the two theories become definitionally equivalent (i.e., they become equivalent theories in the sense as the theory of lattices as algebraic structures is the same as the theory of lattices as partially ordered sets).

Furthermore, we show that classical kinematics is definitionally equivalent to classical kinematics with only slower-than-light inertial observers, and hence by transitivity of definitional equivalence that special relativity theory extended with "Ether" is definitionally equivalent to classical kinematics.

So within an axiomatic framework of mathematical logic, we explicitly show that the transition from classical kinematics to relativistic kinematics is the knowledge acquisition that there is no "ether", accompanied by a redefinition of the concepts of time and space.

The above allows us to introduce a metric "conceptual distance" between theories: theories which are equivalent have a conceptual distance of zero, while the distance between non-equivalent theories is the number of concepts which need to be added or subtracted to make them equivalent. Since the only concept which need to be added to relativistic kinematics to make it equivalent to classical kinematics is the "ether", the conceptual distance between both theories is "one".

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Type Theory and the Theory of Forms

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My intention is to formalize in a type-theoretical setting certain metaphysical intuitions concerning intensional objects (properties, concepts or universals, maybe even something like Forms or Ideas) and to describe one possible formal interpretation of the Platonic metaphysical outlook. For that aim, I present a system of cumulative intensional ramified type theory CIRTT, inspired by Russell's ramified type theory from the first edition of Principia Mathematica [1], but with somewhat loosened restrictions given by the Vicious Circle Principle, and guided by a realist interpretation of a ramified type hierarchy. As a formal system, CIRTT is a modification of Fitting's intensional simple type theory with semantics based on an intensional generalisation of Henkin models, as presented in his Types, Tableaus, and Gödel's God [2].

First I explain briefly some of the ideas and intuitions that motivate CIRTT, most notably the reasons for ramification and cumulativeness of type hierarchy, and then sketch formative rules, semantics, and proof procedures. I also describe some basic metatheoretical properties of CIRTT. Lastly, I explain the relations between CIRTT, its intended semantics and the background metaphysics.

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Coloring Venn Diagrams

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Diagrammatic approaches to deductive and formal reasoning [1,2] have seen a resurgence in recent years. We propose a diagrammatic method for deciding whether Boolean equations over set-valued variables are tautologies or not. Conventional diagrammatic approaches to the above decision problem work reasonably well when the total number of sets under consideration is rather small.

However, conventional approaches become cumbersome, if not completely unusable, while dealing with a large number of sets. We devise an algorithm for the above decision problem, and demonstrate that it scales well when the number of set variables in the equations increases rapidly.

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Some mathematical approaches for defining the notion of quasi-topology

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The notion of quasi-topology founded by Jean-Pierre Desclés [1] as a mathematical model must be a model encoding the idea that a set as member of an abstract space can have a strict interior and an interior, a closure and a large closure. This approach comes from the linguistic expressions of the space, the linguistic expression of the time, even from basic notion in law as "legal" opposite to "illegal" or from the social notion of "inhabitant of a city". In the mathematical literature, several approach modeling the notion of strict interior and large closure where de fined and studied. In this paper, we investigate some previously defined mathematical structures which are likely to be more or less closely to the quasi-topology classical topology, rough sets, locology giving to the quasi-topology the status of mathematical model. We study also the quasi-topology induced by each of them in the general case of mathematical space and, in a particular case, when the mathematical space is the extension of a property, Ext(f) [2].

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On Generalized Unified Boolean-Fregean Semantics

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The main aim of this report is to consider the unified Boolean-Fregean semantics and then its generalization.

Let L be the language of the sentential logic with negation ~ and conjunction $\wedge$. Sentential variables: $p, p_1, p_2, \dots$ Formation rules are standard. We will denote the set of variables as *Var*, and the set of wffs as *For*. Metavariables for formulae are P, Q.

George Boole supposed that the truth is represented by 1 and the universe, and the false is represented by 0 and nothing — an empty set (class). 1 and 0 are logical values.

The Boolean semantics, based on the Boolean algebra of subsets, is considered. The valuation function V is a mapping from the set *For* into the set $\{\mathrm{U}, \varnothing\}$ (short $V\colon For \longrightarrow \{\mathrm{U}, \varnothing\}$). Designated value is U. We have:

1. $Var \longrightarrow \{\mathrm{U}, \varnothing\}$;

2.1. $V(\sim P) = \varnothing$, if $V(P) = \mathrm{U}$; 2.2. $V(\sim P) = \mathrm{U}$, if $V(P) = \varnothing$;

3.1. $V(P \wedge Q) = \mathrm{U}$, if $V(P) = \mathrm{U}$ and $V(Q) = \mathrm{U}$; 3.2. $V(P \wedge Q) = \varnothing$, otherwise.

The operations of Boolean logic are associated with the operations of the algebra of sets. Thus, for $\wedge$ and $\cap$ we have that the equality for the valuations $V(P \wedge Q)$ and $V(P) \cap V(Q)$ follows from the above semantic rules, i.e. $V(P \wedge Q) = V(P) \cap V(Q)$.

From the point of view of Fregean semantics, sentences stand for either truth or false. Set-theoretic interpretation of Fregean logic, similar to the set-algebraic interpretation of Boolean logic, we construct as follows:
$V\colon For \longrightarrow \{\{truth\}, \{false\}\}$. Designated value: $\{truth\}$.

It does not hold that $V(P \wedge Q) = V(P) \cap V(Q)$.

The consistent unification of Boolean and Fregean semantics can be constructed by using the set $\{\{truth\}, \varnothing\}$ (or, more abstractly, $\{\{\varnothing\}, \varnothing\}$) as the set of logical values. (This corresponds to the modification of the Fregean semantics by means of rejection of the reference *false* as non-existent.) Now *it does hold that* $V(P \wedge Q) = V(P) \cap V(Q)$.

So, the unified Boolean-Fregean semantics has been obtained.

The proposed approach can be generalized to non-classical cases, for which the bivalence principle doesn't take place. In this case the sentences P, $\sim P$ stand (or doesn't stand) for truth independently. We have four variants:

- $V(P) = \{truth\}$ and $V({\sim}P) = \varnothing$,
- $V(P) = \varnothing$ and $V({\sim}P) = \{truth\}$,
- $V(P) = \{truth\}$ and $V({\sim}P) = \{truth\}$,
- $V(P) = \varnothing$ and $V({\sim}P) = \varnothing$.

Thus, the unified Boolean-Fregean semantics would be extended to the non-classical case too.

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The Rule of Explicit Substitution into (Hyper)intensional Contexts

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Intensional logic employing *possible world semantics* (PWS) resolves the failure of *substitutivity of identicals* (SI) within the scope of belief operators on the proviso that objects of agent's attitudes are rather coarse-grained. Yet objects of belief attitudes are *meanings* of embedded sentences, i.e. fine-grained entities that are beyond the reach of PWS. Thus there is no surprise that there occurs, if one utilizes PWS, a failure of substitution into *hyperintensional contexts* because of non-identity of meanings of *logically equivalent expressions.* As a remedy, several *hyperintensional logics* were proposed, some of them deploy *algorithms* as models of *hyperintensions* (Tichý, Muskens, Moschovakis, Duží et al.).

The present paper utilizes a recent variant of Tichý's proposal [5]. Its basic notion is *construction* — an abstract structured *algorithmic computation* of the denotation of the term expressing it. Constructions are recorded by λ-terms of forms

$$x \mid {}^0X \mid {}^1X \mid {}^2X \mid [CC_1 \ldots C_m] \mid [\lambda x_1 \ldots x_m.C],$$

where X is any construction or non-construction and $C_(i)$ a construction. Tichý's early *natural deduction* system [3,4] has been extended for his *ramified type theory* in [2].

My (SI) rule utilizes achievements of [2]. Let "$C_{(C_i/x)}$" be C that has C_i instead of all hospitable occurrences of x. Sub is a partial mapping from triples $\langle C_2, C_1, C\rangle$ to constructions of form $C_{(C_2/x)}$; $C_{(C_2/x)}$ is v-congruent ($\cong$) with C.

$$\frac{\begin{array}{c} C_{(C_1/x)} \\ [C_1 \cong C_2] \end{array}}{{}^2[\mathrm{Sub}\, {}^0C_2\, {}^0C_1\, {}^0C_{(C_1/x)}]}\ (\mathrm{SI})$$

(SI)'s conclusion is called *execution of explicit substitution*; it is derived from [5], so it precedes the famous proposal in [1]. That this (SI) does not licence the familiar arguments displaying failure of substitutivity, while it retains an expected deductive strength, is demonstrated in the talk.

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Embodiment of some Logical, Computable and Categorical Notions by a Logic of Operators

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Formal systems incarnate abstract ideas on the basis of more or less elementary ones. They are said to be "closed" if the new constructions are of the "same nature", "ground" or "medium", where they do come from. It appears that such kind of mathematical structures integrates their own "internal logic". In the Theory of Categories, Grothendieck's Topos are a Cartesian Closed Category, which is "governed" by the Intuitionist Type Theory and corresponding logic. At a weaker level, in the Typed Illative Combinatory Logic (CL), more or less "abstract" properties (inversion, prolongation, duplication, recursion, abstraction,...) can be embodied into a "medium" by the application of typed syntactic operators (nominal symbols) also called combinators.

This "applicative" embodiment into one operation of application, is isomorphic to the Cartesian product. The application of operators is governed too by a more fundamental logical system (or type system) also called : theory of functionality (without which the applicative system is inconsistent).

Following [3] on ancient texts, we continue on the comparison of CL and Category Theory. They both share the study of processes which are ordinarily carried out by means of variables. We show that the "algebra" of combinators can be embodied into CL itself. In that picture, we rely on Guibert's expression '$\mathsf{BY_0EVAL}$' with the Cartesian product combinator and Church's numerals. The latter is comparable by many aspects to that of exponentiation in Cartesian Closed Categories. Doing such, we embody high level or complex structures into the applicative medium, without having to modify the latter.

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Nested Sequents, Focusing and Synthetic Connectives

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Nested sequent calculus is a generalization of ordinary Gentzen sequent calculus. It has been introduced independently by Kashima, Poggiolesi and Brünnler [1]. The basic idea is that a sequent, which is list of formulas, is generalized to a tree of lists of formulas. This allows for more freedom in the design of inference rules, by still maintaining important properties like cut elimination and the subformula property.

And, indeed, many logics that could not be treated by standard sequent calculus have a concise proof system in nested sequents, for example all logics in the modal S5 cube [1], many intuitionistic modal logics [6], intermediate logics, and substructural logics. This shows, that nested sequents have the potential of becoming a universal tool for universal logics.

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Focusing is a general technique for transforming a sequent proof system into one with a syntactic separation of non-deterministic choices without sacrificing completeness. It has its origin in the foundations of logic programming and is now increasingly relevant in structural proof theory because it improves proof search procedures and because focused proofs have clearly identifiable and semantically meaningful synthetic normal forms.

The full theory of focusing was initially developed for the sequent calculus for linear logic, but it has since been extended to a wide variety of logics and proof systems. This generality suggests that the ability to transform a proof system into a focused form is a good indication of its syntactic quality, in a manner similar to how admissibility of cut shows that a proof system is syntactically consistent. In the talk I will make the case that the focusing theorem is a *universal property of a logic*.

The basic idea behind focusing is to control the non-deterministic choices in a proof, so that a proof can be seen as an alternation of negative phases, where invertible rules are applied eagerly, and positive phases, where applications of the other rules are confined. This, in turn, lets us abstract from the usual unary and binary logical connectives by collapsing whole phases into n-ary *synthetic connectives*.

In this presentation, which is based on joint work with Kaustuv Chaudhuri and Sonia Marin [5,2,3], I will first introduce nested sequents and show how the focusing technique can be applied to a nested sequent system. Then I will use as example the 15 logics in the modal S5 cube and demonstrate the construction of synthetic connectives.

An excellent introduction and overview of the state of the art of focusing for modal logics is Sonia Marin's PhD thesis [4].

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On First-order Mereological Complementation

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Mereology is the theory of the relation "being a part of". The formal language of mereology contains only one binary predicate "P", whose intended meaning is "being a part of". Two additional binary predicates "PP" and "O", whose intended meanings are respectively "being a proper part of" and "overlapping", can be defined as follows. $\mathrm{PP}xy = \mathrm{P}xy \wedge \neg \mathrm{P}yx$ and $\mathrm{O}xy = \exists z\,(\mathrm{P}zx \wedge \mathrm{P}zy)$. The following axiom which can be found in the literature [1] is called by the present writer "global complementation" (GC):

$$\forall x\,(\neg\forall y\,\mathrm{P}yx \to \exists z\,\forall w\,(\mathrm{P}wz \leftrightarrow \neg\mathrm{O}wx)).$$

Such an axiom says that anything which is not the greatest member has a complement. There is another axiom called "local complementation" (LC), which is formulated by the present writer [2] as follows:

$$\forall x\,\forall y\,((\mathrm{PP}xy \wedge \neg\forall z\,\mathrm{P}zy) \\ \to \exists z\,(\mathrm{PP}zy \wedge \neg\mathrm{O}zx \wedge \forall w\,(\mathrm{O}wy \leftrightarrow (\mathrm{O}wx \vee \mathrm{O}wz)))).$$

It says that, for anything which is not the greatest member, any of its proper part must have a complement in respect to that thing. It has been shown that (GC) and (LC) have a lot to do with whether a first-order axiomatizable mereological theory is decidable or not.

However, there can be some kind of complementation located in between, for example, the complement of a member in respect to a definable class of members. Let $\alpha(x)$ be a formula in which x is a free variable (where other variables might also occur free). The axiom of α-complementation (α-C) can be formulated as follows:

$$\forall x\,((\exists y\,(\alpha(y) \wedge \neg \mathrm{O}yx) \wedge \alpha(x)) \rightarrow \exists z\,\forall w\,(\mathrm{O}wz \leftrightarrow \exists u\,(\neg \mathrm{O}ux \wedge \alpha(u) \wedge \mathrm{O}wu))).$$

Other kinds of complementation are still possible and this talk will look into them in a more systematical way but will be confined to those which can be first-order defined. In addition, some metalogical issues, such as completeness or decidability, will also be touched.

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Philosophy

The keynote speaker at this session is Hartry Field (page 128).

Philosophy, Art, Science, Economy (PHASE) of self and internal integrity

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Neuroscientific studies have demonstrated that our brain can be regarded as fragmented, and that increased neuronal synchronization, which is related to cognitive flexibility, attention and self-awareness, can aid in enhancing internal integrity. Several brain-based integrity scales have been developed to measure state of consciousness, and were found to be connected to moral judgments and problem solving. Together, these results emphasize the importance of finding paradigms to elicit efficient neuronal synchronization.

Also logics confirm the complexity of the human brain. However, are we only logical beings? And what does it mean to be logical from a bio-logical, neuro-logical or from a rational point of view? In addition, what role do emotions play in cognitive processes, and how may they change as a result of mental and sensorimotor training?

In this context, neuroscientific research has further demonstrated that neuronal synchronization can be elicited by practicing music and meditation, such as the *Quadrato Motor Training*. QMT was further found to improve emotional regulation, creativity and information processing. In the current talk, we will discuss these results, emphasizing that training can help in enhancing neuronal integrity cognitive and emotional, which are crucial for logical process and moral problem solving.

Is Life Logical? Application of the Peirce-Leśniewski-Tarski Meta-Logics to the Organic Mathematics of the Perplexity of Natural Sorts and Kinds

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Is life logical? Here, I propose a categorical approach to numeric combinations that unite several formal logics for the purpose of scientific analysis of natural sorts and kinds. In order to represent the novelties of organic objects, this logico-scientific strategy generates novel propositional forms for numbers representing natural sorts and kinds. The collective logics of these novel grammatical sentences are graphically represented within the perplex number system [1,2]. These novel propositional forms require both copulative and predicative conceptualization of scientific terms in order to inform imperative numerical relations of addition. The logical compositions of the novel sorts and kinds of numbers stand in one to one correspondence (mappings) with the parts of atoms. From this logico-scientific perspective of natural sorts and kinds, meaningful sentences necessary contain both grammatical and numerical elements in order to generate representations of organic mathematics are empirically represented in propositions relating the denotations and connotations of antecedents and consequences. The predicative symbolic terms must inform the denotive predicative symbolic changes and the copulative symbolic terms must inform the connotative copulative changes. The terms in the propositional equations of organic mathematics must be informed by the abstract scientific units of physics, chemistry, biochemistry, genetics, medicine and other associative disciplines. The guiding compositional principle for validating propositions of organic mathematics is simply stated: *The union of units unites the unity.*

The formal logic of the union of scientific units is constrained by the empirical meanings of the scientific symbol systems used to represent natural sorts and kinds. The several notational systems used to represent natural sorts and kinds emerge coherently from the additive logic of the perplex number system [1,2,3,4]. Perplex numbers have meaning. From this logico-scientific perspective of information, the compositional logic of scientific forms emerges from the physical attributes of electricity and the atomic numbers. (The traditional geometric forms of mathematical physics can be inferred from organic mathematical terms by merely changing the informational content of the representational metric spaces.) But, the scientific meaning of perplex propositions is only achieved by validation of indicative antecedents to generate an imperative consequent.

Within the historical constraints of the scientific symbol systems emerging from empirical scientific observations, a union of the distinctive formal logics of the inorganic sciences inform a set of terms for forming the logical and structural attributes of the organic sorts and kinds. The meaning of perplex numbers is encoded in scientific symbol systems. Within this logico-scientific formalism, the propositional sentences for *forming organic entities* from inorganic entities representing novel electrical relationships among the electrical parts of the whole. The antecedent parts are represented by symbols for mass and electricity and the consequential whole is also represented by mass and electricity. Organic forms emerge from inorganic forms by creating new species of electrical connections. The scientific necessity for conserving the symbolic meanings of both categories of terms infers combining both copulative and predicative logics for all propositions representing natural sorts and kinds. This novel form of scientific logic was named synduction [1,2]. The associated philosophy of science is called perplex systems theory. The essential distinction between synduction and other logics is that the grammar of synductive propositions must relate the copulative and predicative terms representative of the semiotics of the natural entity in order to correspond with empirical scientific symbols deduced from its natural emanations.

Precursors of the combinatorial structure of the perplex logic of synduction were described by C.S. Peirce (∗1839–1914†), S. Leśniewski (∗1886–1939†) and A. Tarski (∗1901–1983†). A rudimentary relational logic for interpreting the emanations of natural sorts and kinds was formulated by Peirce's hypothetical method of constructing propositions from emanations [5, par. 230, p. 136]. The formal logic of perplexity aligns the inorganic attributes of the atomic numbers and the forms of mathematical graph theory as is suggested by the relational logic of parts of wholes as suggested by Leśniewski. The necessity of combining symbolic logics is intrinsic to representing multiple symbol systems as is suggested by the conceptual relations between scientific symbol systems and Tarski's meta-languages. Synductive logic draws on these three conceptual notions of relational logics to create new species of relations in order to organize the parts into wholes (e.g., atoms into molecules). Logical arithmetic operations on the primitive inorganic terms combine predicative terms non-linearly. The combined primitive terms create an emergent whole with the concomitant manifestation of new natural emanations. The copulation of the parts of the whole is expressed as a novel grammatical identity of a natural sort or kind. (For example, the masses of hydrogen and oxygen are combined in the emergent structure of the whole, water, the masses of carbon, hydrogen and oxygen are combined in the emergent structures of carbohydrates and lipids, the masses of carbon, hydrogen, oxygen and nitrogen are combined in the emergent structures of nucleic acid bases and the masses of carbon, hydrogen, oxygen, nitrogen and sulfur are combined in the emergent structures of proteins.) Organic mathematics requires the semantic naming of each unique organic identity *in order to be consistent with the species of emanations, the specific patterns of the isomeric copulative unions and the specific emergent physical predicates that are created by copulative unions* of parts to become wholes.

Consequently, the synductive logic of the natural union of atoms to form molecules differs from B. Russell's abusive metaphor of combining atomic sentences to form molecular sentences. The concept of the natural association of inorganic electrical particles (e.g., atoms) by copulation (e.g., bonding) into molecular patterns parallels logically the concept of association of terms in propositional sentences. The Tarskian difference that makes a difference between Russell's logic and synductive logic is the difference between a simple grammatical conjunction ("and") and a physical combination ("bind to") expressing the logic of stable electrical forces intrinsic to the relations among atomic numbers. Scientifically, the abductive copulative / predicative statements representing the mapping from atoms to molecules are combined into exact equational forms ***that are subject to direct empirical verification***. Scientifically, a three-fold verification of perplex propositions is required. The first verification **denotes** the parts of the whole as a composition of atomic numbers such that these specified parts **connote** the abductive substrates of compositions. The second verification **connotes** graphically the composition of adjacency relations among the parts of the specific emergent identity. The third verification, which is essential for bio-organic molecules, **denotes** the three-dimensional electrical arrangements of all the parts of an entity (e.g., the handedness of amino-acids, carbohydrates, nucleic acids, etc.) These empirical validations inform the change of the verbal propositional mood from terms indicative of parts to an imperative term (or terms).

Perplex logic **links** the scientific disciplines by the composition and decomposition of the anatomies of scientific terms (units). The three-fold logical processes of perplexification associate the connotations of the methodical approach to forming symbolic propositions of C.S. Peirce and logical meta-languages of Tarski with the denotive organic mathematics of the atomic numbers concomitantly with propositional terms consistent with Leśniewski concept of denoting part-whole relations. The mathematical concept of partitioning plays a central role in the scientific symbolic representations of assembly and disassembly of natural sorts and kinds. The units of union and disunion are the perplex numbers and networks of relations.

Deductive propositions can denote the mapping processes that disassemble the anatomies of larger units into smaller units, such as the atomic numbers. The *perplex number **line*** aligns abductively the partitions of all integers with the natural parts of wholes. The *perplex number **line*** associates abductively with each partition a specific set of all possible three-dimensional arrangements of natural sorts and kinds (denoted by electrical particles, atoms, molecules, viruses, cells, organs, organ systems and all higher organisms). The perplex number spine is a line of potential infinite length and a potential infinite number of organic branch points. Each and every organic object associates with an enumerable organization of its electrical parts. Each integer number is associated with a partition of an integer and a perplex mathematical graph that associates the specific electric parts of the whole. The perplex partitions of integers are associated with the labelled bipartite graphs composed exclusively from atomic numbers [1,2]. A living system can be decomposed by partitioning into anatomical structures, each part connotes a semeiotic identity as a term of natural language. Iterative partitioning of natural sorts and kinds decomposes the whole into simple integer units, the atomic numbers. The equivalency of the relational logic paths of composition and decomposition validate the synductive logic of "proof of structures" for entities arranged along the perplex number spine as structural units and as potential emergent dynamic units, under the constraints of the mass and the electrical laws.

The two physical symbolic forms of combinatorial logics of identity (mass and electricity) are used concomitantly to construct deductions from inductive and abductive hypotheses about natural sorts and kinds (scaling). The partitions of anatomical structures of natural sorts and kinds are scaled into collections of forests of the symbolic graphs of perplex numbers. The composition of relations emerges from the electrical relations among the anatomies of the parts. The form of the relations emerges from the electrical forms of the perplex numbers. The number of relations between parts is a function of symbolic language selected to represent the natural sort or kind. **In the case of the very simple chemical symbols, the number of relations gained or lost is a simple exact electrical calculation.** *The interdependencies between the alternative propositional terms for partitions are inferred from semeiotic emanations within the context of the parts* *of the whole.* The propositional terms of different scientific disciplines scale with the identity of the inorganic and organic species, the identity of the ecosystem and the temporal interdependencies among them. **The combinatorial relations among parts of the whole generate the electro-dynamics of natural sorts and kinds (oxidation, reductions, transformations, transpositions, translocations, transfers, transactions, and so forth) as well as the quantum electro-dynamics necessary to represent three-dimensional organic forms (handedness).** The emergence of relations among the parts of the wholes arranges electrical units of partitions into unities to form anatomies of natural sorts and kinds.

In conclusion, the yoga (Sanskrit, union) of the abstract concepts of C.S. Peirce, S. Leśniewski and A. Tarski is the logical ur-root of coherence of multiple scientific symbol systems under the formal numerical constraint: The union of units unite the unity.

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A Formal Representation of Reasoning for Chemistry

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As we know, what is taken to be the underlying logic of an area of research guides and shapes that area of research. For example, there are pointed arguments to the effect that economics should be based on a constructive, or intuitionistic, logic rather than a classical logic.

The chemists studying: quantum chemistry, electro-chemistry, nano-chemistry, the chemistry of acids and bases, the chemistry of metals and so on; quite often are more concerned with properties, relations and sometimes functions, than they are with objects, or even elementary particles (*qua* only particles). The location boundary of a fluid or gas are vague, the purity of a substance is almost always non-existent, the mixing of fluids or gasses often yields emergent properties, chemical reactions depend on context, or 'milieu'.

Standard formal representations of reasoning, assume, in: the grammar, the notation, the operations, that we begin with elements, or objects, or members of a domain. These formal systems representing reasoning are ill-suited to representing reasoning in chemistry.

I propose to give the beginnings of a formal representation of reasoning, based on Lemmon's 1960s development of "a formal logic (sic!) of attributes" and some of the concepts in universal logic that are universal to logic). The notion of 'object' will be secondary and derivative, being merely a collection of properties located within a milieu. In contrast, the notions of 'property' and 'relation' will be central.

Supplying chemists with a formal representation of reasoning tailored to chemists needs and practice, will bring a rigour of reasoning and a clarity of thought. Pluralists in logic, who think of formal representations of reasoning as various ways of regimenting thought in a subject area, might find that the formal system s) developed for chemistry will be useful in other areas of thinking that are more concerned with properties and processes than with objects.

A Common Framework for the Empirical Sciences

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Empirical phenomena result from a collection of interacting *things* that change aspects we observe [1]. The (general) systems concept [2], as collection of interrelated objects, is hence directly applicable in their description. This point of view clarifies the transdisciplinary nature of this concept and its wide applicability. It shows also why it is natural to associate phenomena with a system's *behaviour*, independently of how it is formally expressed. Nevertheless, the systems idea has fundamental shortcomings when used to describe phenomena in W. Weaver's organised complexity class [2]. The difficulties are related to the "structural" invariability of systems and to its inability to incarnate *things*, that is, to serve as an interacting component, as will be seen in this talk.

Explanations about empirical phenomena are constructed iteratively and incrementally in a cycle running through steps of observation gathering, model construction and theory development [3]; where fundamental and more immediate properties of models provide for enunciating axioms supporting the development of theories that unveil hidden, often unexpected, phenomenon properties and behaviour. A first step in constructing models is to handle the complexity of interacting things. To a great extend, things are considered as elements of certain well-known classes: particles, waves, bodies, substances, individuals, and so on. These classes are characterised by aspects and properties important to what is being questioned but cast the subsequent modelling, shaping what can be described and explained about the phenomenon. Next, a collection of aspects attached to things is selected as the system-state and the interrelations and interactions creating the phenomenon are described in respect of this collection. The system concept is cardinal in this last step.

However, the system framework is insufficient to handle systems of variable-structure and the hierarchy inherent in biological phenomena [4], where many *things* are indeed phenomena themselves. In this talk, I shall use interaction graphs to clarify the above statement and argue about its soundness. I shall also show how systems are special cases of organisations [4], that organisations can be typical elements of thing-classes and that its definition comply with biological hierarchy. Organisations relief systems from the insufficiencies above and introduce a brand new perspective in the empirical sciences.

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The Analytic and the Synthetic. From Homology to Heterology

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The distinction between the analytic and the synthetic as well as the problem of the a priori synthetic judgment is an important point in modern history of philosophy and logic. In Kant's Critique [3], the analytic is associated with the a priori and the synthetic with the a posteriori, and the distinction between the two is linked to the criteria of necessity and universality. The a priori synthetic judgment makes it possible to account for the nature of judgments in logic and mathematics, but it appears to many philosophers as a kind of monstrous hybrid. The critics of the analytic / synthetic distinction and of the hybrids it allows for are diverse: some of them assume it, but reject the a priori synthetic judgment [2], while some others does accept it (Husserl); some reject the analytic/synthetic distinction [6] while some others dispute the association between the analytic, a priori and necessity and between the synthetic, a posteriori and contingency [4]. Il would like to suggest that the debate on the analytic / synthetic distinction can be thought over in a new way and framed as a problem of heterological synthesis. It is a fact that the problem of synthesis has hitherto been posited in terms of homology, for instance, when one makes an epistemic synthesis of some theoretical and empirical statements. The outcome of this kind of homological synthesis is well illustrated by an epistemic statement such as P: 'It is possible to clone a human being'. It is quite different if the problem of synthesis is posed in terms of heterology, for instance, when one makes an

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epistemic-ethical combination. This kind of heterological synthesis is well illustrated by an epistemic statement P: 'It is possible to clone a human being' that is related to an ethical statement Q: 'It is not possible to clone a human being'. Here, the modal category of the possible is interpreted in two different senses: on the one hand, an epistemic sense the stake of which is the True; on the other hand, an ethical sense the stake of which is the Good (or some other values such as the Useful or the Beautiful). The question that arises is that of heterological synthesis to be conceived of as a combination of heterologic statements according to some classical patterns of reasoning and inference (disjunction, conjunction, conditional). Unlike the homologic judgment (pariter) that produces a synthesis within a single sphere of rationality, the heterologic judgment (aliter) produces a synthesis between multiple spheres of rationalities. The possibility of heterologic synthesis entails that conditions for the relation to the object and consequently for its construction and regulation are not merely internal and can be also external. It then questions, firstly, the dogma of internal conditions to be seen as a consequence of the Law of Hume that prohibits the derivation of 'Ought' from 'Is'; and, secondly, the possibility of the function $F(x)$ to incorporate in logical terms this extension of statements conditions from the internal (homology) to the external (heterology).

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The logic of content and contentual understanding of sentences

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In our approach, a contentual understanding of sentences means that a content of any sentence (also atomic) can be understood as an intersection of contents of other sentences. Formalization of this approach is possible, among others, by using of the *content implication* connective ":" [1,2]. A classical propositional logic with the content implication connective (*CCl*, the *Contentual Classical Calculus*) is one of possible logics of content. A semantic interpretation of the content implication is the following:

$$v(\alpha : \beta) \in D \quad \text{iff} \quad v(\alpha) = v(\beta) \cap v(\gamma) \text{ for some sentence } \gamma.$$

"*p* says what is said by *q*" or, shortly, "*p* says *q*" is an intended reading of the sentence of the shape "p : q", and unveils a desired meaning of the content implication connective as expressing a fact that a content of some sentence can be a part of the content of other sentence. Obviously, the content implication connective is not truth-functional and a logic strengthened with this connective has several properties, like: it is free from the liar and other liar like paradoxes, it enables an intuitive interpretation of the negation of a sentence as well as justification of the popular in the Middle Ages opinion that *every sentence says of itself that it is true* [3,4], it formalizes Grice's implicatures. It is also possible a "contentual" defining of the truth.

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Stereology

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The tasks put forward by the recent developments in information systems research, especially those connected with the problems of knowledge representation, made the parallel task of advancing a formal ontology for the common sense world — for the world of everyday human experience — of first rate importance. This is, above all, a task of representing the denizens of the naive physics, of the mesoscopic objects and of the quantitative reasoning in space and time in general. By way of addressing this problem, many authors placed great hopes on a variety of new approaches of increasing complexity:

(a) **Set Theory**. To this purpose the resources of the naive set theory are widely used. Unfortunately, set theories — of any kind — fail to do justice to the difference between natural totalities, such as dog, and ad hoc totalities, such as Brezhnev, the sun, the sea.

(b) **Mereology**. This is the discipline investigating the relation between parts and whole. Unfortunately, as it was noted in the literature, in capacity of formal ontology of the mesocosmic objects, traditional mereology has many deficiencies. For example, it neglects the problems of continua and boundaries which are sine qua non when the objective is to represent integral objects of the mesocosmos.

(c) **Mereotopology**. This motivated attempts — made in the recent years mainly by Roberto Casati, Achille Varzi and Barry Smith — to supplement mereology with topology to a mereotopology. Central concepts of mereotopology are boundary, inside/outside, abutting and surrounding. It is based on a simple mereo-topological primitive of connected parthood. Regrettably, mereotopology fails to present the objects in their full integrity as well. This brings some proponents of mereotopology to the idea to supplement it with at least two other disciplines: (1) morphology: theory of qualitative discontinuants, and (2) kinematics and dynamics which explain the behavior of its parts. They must help to represent the shape and behavior of objects, as well as the interaction between them.

(d) **Theory of Granular Partitions**. This new discipline tries to come to terms with the task to represent the results of sorting, classification, dividing into units, counting, parceling out, mapping, listing, pigeonholing, and cataloguing. Its models apply different types of partitions — that are appropriate for different situations — in which the objects under scrutiny can be put in. Partitions themselves are not objects.

In this paper we advance a new type of formal ontology, which suggests more exact models of the experienced reality (or merely treats it in a more exact, full-fledged terms) than set theory, mereology, mereotopology, and the theory of granular partitions. In particular, stereology is clearly different from the theory of granular partitions, since is sees the objects and processes under scrutiny as constructed exactly like their modes — it sees no difference but identity between objects and models.

What Philosophy of Logic are we Teaching?

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The time has come to take a long hard look at the teaching of introductory logic. The very first lesson of the very first logic course contains highly contested material that only those with an interest in the philosophy of logic will ever again re-visit. Teachers say things which are vital to an understanding of logic, right at the point when those things are most likely to be accepted without critical examination, and then never return and re-examine those points. As Stephen Read says, “Teaching in philosophy departments across the world exhibits this schizophrenia, in which the dogmatic approach to the philosophy of logic sits uncomfortably side by side with the ceaseless critical examination which is encourages and demanded in philosophy” [1, p. 2]. This alone is bad enough, but the current state of the teaching of philosophy of logic is *worse*.

The teaching of logic starts in the right place, with a discussion of the nature of the subject matter at hand, which as it happens is also introductory philosophy of logic. Courses typically open with a discussion of what logic is about and a definition of validity, and as Stephen Read says, “The central topic in the philosophy of logic is inference, that is, logical consequence, or what follows correctly from what” [1, p. 1]. So, forcibly excising the teaching of the philosophy of logic from introductory logic course would be the wrong approach because another way to see that very first lesson is not as a lesson at all but merely the sort of sensible introduction to what the student will learn during the course.

There are hard questions in the philosophy of logic, questions that students probably can’t engage with immediately, so the introduction should walk a fine line — setting the stage for a conversation that a student will not be prepared to join until later. But in the very first lesson of the very first logic course students are introduced to the central topic in the philosophy of logic in a manner which is unforgivably contradictory and inexcusably dogmatic. This bad pedagogy is bad enough, but the potential ramifications for the discipline are *worse*. In what has become a pivotal piece in the discussion of the nature of validity MacFarlane says that for addressing the hard questions in the philosophy of logic "The dominant methodology for addressing them involves frequent appeals to our "intuitions" about logical validity. I do not think it should surprise us that this methodology leads different investigators in different directions. For our intuitions about logical validity, such as they are, are largely the products of our logical educations" [2, p. 2]. It is a disservice to philosophical colleges who will not specialise in logic to teach bad philosophy of logic, but it shows a reckless disregard for the advancement of the discipline to introduce a central topic in a way that impairs future investigators.

In this paper, I will present a combination of the results of a text analysis of introductory logic texts along with some exemplars of the ideas that these texts communicate on the subject matter of logic. I break the examples of problematic teaching down into three categories: the pseudo psychological, the normative nonsense, and the over-concentration on argument as if that gets you out of it. I discuss the aspects of the current debate in philosophy of logic that make these teachings problematic.

I finish with a discussion of the connections between logic, psychology, norms, and argument and explain that I am not arguing that these ideas do not belong to the discipline of logic, but their impact on the notion of validity is significant enough that presentation in introductory texts should be conducted with care.

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[*] The Central Division is one of three divisions of the American Philosophical Association, along with the Eastern and Pacific Divisions.

A Reply to Logical Revisionists: Strict Finitism, Feasibility and Structural Rules

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As far as logic is concerned, the conclusion of Michael Dummett's manifestability argument is that Heyting's intuitionistic logic satisfies the core semantic requirement of antirealism. The argument is that since we cannot manifest a grasp of possibly justification-transcendent truth conditions, we must countenance conditions which are such that, at least in principle, we are able to recognize that they are satisfied whenever they are. Intuitionistic logic satisfies the requirement that we should thus constrain the notion of truth by provability in principle [3]. Some philosophers have argued that the traditional antirealist desideratum of decidability in principle is too weak. Semantic antirealism properly construed must be committed to effective decidability. As such, it either leads to strict finitism [4] or to a yet stronger kind of logical revisionism: substructural logics, and in particular linear logics, rather than intuitionistic logic, satisfy the core semantic requirement of strict antirealism [2].

I shall develop two kinds of replies. The first is concerned with the notion of meaning per se and looks to strict finitism directly, although not on the ground that it would lead to soritic paradoxes (the primary focus of discussion in [4]). The second is concerned with the justification of structural and logical rules in Gentzen's natural deduction system where deductive derivations are built out of sequents.

The first kind of reply is that if we jettison the effectively vs. in principle distinction, we end up with an unsatisfactory explanation of how the meaning of logical constants is fixed whenever they occur in statements covering the practically unsurveyable or pro tempora undecided cases. If we have a method which may be used over some small range, then we have determined a way of applying the method everywhere in principle and that this is enough as far as fixing meaning is concerned.

I shall then look at two radical antirealist principles disqualifying structural rules: Token Preservation and Preservation of Local Feasibility. Against Bonnay and Cozic's criticisms of Dubucs and Marion [1], I shall argue that

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(i) some conceptual support may be provided for Token Preservation, which doesn't rely on a causal misreading of the turnstile, and that (ii) the appeal to non feasible ways of doing feasible things is not a good way to argue for Preservation of Local Feasibility.

I shall then assess the merits and limits of radical antirealism and the logic of feasible proofs, whether the radical antirealist merely stipulates what human feasibility amounts to, or dispenses with structural rules by arguing in favour of a curb on the epistemic idealizations they unwarrantedly embed.

It will be noted here that there is a great difference, conceptually speaking, between the rejection of classical logic via the curbing of the epistemic idealizations embedded in structural rules, and the rejection of classical logic via the criticism of introduction and elimination rules which fix the meaning of the classical constants. E.g., the rejection of Weakening and Contraction by way of Token Preservation and Preservation of Local Feasibility doesn't have to rely on arguments in favour of the surveyability of the implementation of decision procedures, or to any particular conception of how one may go from one proof to another by way of deductive rules. The reasons why we should want to narrow the scope of idealization are quite different in each case. One telling case of study in this respect is that of the relation between structural rules and logical rules in the intuitionistic context. The kind of logical revisionism envisaged by intuitionists from Heyting on is in many respects stronger than the one envisaged by advocates of linear logic, should such revisionists ground their arguments on an endorsement of strict antirealism. A clearer philosophical conception is needed of how the rules for the logical connectives in the intuitionistic calculus depend on the structural rules which the radical antirealist wishes to reject.

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On the Status of Questions in the Practice of Science

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Recent developments in erotetic logic have greatly improved our understanding of the nature of questions. However the role of questions in the actual practice of science remains underappreciated. Despite the acknowledgement of the central role played by questions by Popper and Kuhn, discussions of scientific change typically focus on transitions from theory to theory and from method to method. This conception of science is incomplete as it does not include questions as a separate element of scientific change. The evidence from the actual practice of science suggests that just like theories and methods of their evaluation, questions too can be accepted or rejected by epistemic communities. While in the 16th century the question "what is the distance between the earth and the sphere of stars?" was a topic of scientific discourse, it is no longer accepted as a legitimate topic of scientific inquiry. Similarly, the question "what are the properties of phlogiston?" would be accepted in the 18th century, while the question "why are there no observed instances of CP (charge conjunction-parity symmetry) violation in quantum chromodynamics?" is among a plethora of questions pursued by physicists nowadays.

The main goal of this paper is to find out whether questions are reducible to other elements that undergo scientific change — theories or methods — or whether they constitute a separate class of elements. I argue that it is impossible to reduce questions to either methods or theories. I discuss and reject two attempt at reducing questions to either descriptive or normative theories. According to the descriptive-epistemic account, scientific questions can be logically reduced to descriptive propositions [1], while according to the normative-epistemic account, they can be reduced to normative propositions [2,3]. I show that these interpretations are incapable of capturing the propositional content expressed by scientific questions; any possible reduction is carried at the expense of losing the essential characteristic of questions. Further, I find that the attempts to reduce questions to theories introduce an infinite regress, where a theory is an attempt to answer a question, which is itself a theory which answers another question, ad infinitum. Instead, I propose to incorporate the question-answer semantic structure from erotetic logic in which questions constitute a distinct class of elements irreducible to propositions. An acceptance of questions as a separate class of epistemic elements suggests a new avenue of research into the mechanism of question acceptance and rejection, i.e. how epistemic communities come to accept certain questions as legitimate and others as illegitimate.

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Combinations of Interpretations in Universal Logical Hermeneutics

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The method of logical interpretation of a philosophical system according to Bogusław Wolniewicz [1] is aimed to its axiomatization, i.e. to such transformation of a system that it becomes one in which everything (except the axioms themselves) is semantically determinate and deductively complete. There are many obstacles on that way induced by the lack of univocal method of the exact translation from the language of philosophy to the language of logic. And there is one more problem concerning the logical background of a particular philosophical theory. Any theory presupposes a logical system laying in its foundation and Wolniewicz by default supposes that it is a classical one. But from the history of philosophy are perfectly known troubles occurring while we trying to interpret contradictions in particular philosophical theories, e.g. in M. Heidegger's works. Sometimes we can succeed in overcoming such troubles by employing non-classical (e.g. paraconsistent in case of Heidegger) logical systems.

Since *Universal Logic* should be considered as a general theory of logical systems the same manner *Universal Algebra* is a general theory of algebraic systems then it seems that we can think of *Universal Logic Hermeneutics* (modifying Wolniewicz's term of logical hermeneutics) when trying to use one or another logical system for the logical interpretation of philosophical theories. This is especially important in case of logical hermeneutic evaluation of one and the same philosophical system. Logical interpretations may have different validity due to the good or bad choice of the logical background and sometimes simply lead to triviality.

As it was shown in [2] the universe of Universal Logic admits at least the four basic types of combination of logical systems. This allows to obtain respective combinations of logical interpretations while trying to compare competing or disagreeing versions. One type of combinations, fibring of logics, gives us an opportunity to join together two interpretations while generating fibring of interpretations combining them both. Another type of combination, co-fibring or product, leads to the "labelling" of one interpretation with the help of another or to the pairing of interpretations. Exponentiation and co-exponentiation bind two interpretations together admitting just their specific correlations and coordination. All these types of combinations of logics generate the respective combinations of logical interpretations extending the range of possible hermeneutical evaluations of philosophical theories.

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Knowledge, Behavior and Rationality*

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In strategic situations, agents base their behavior on knowledge and beliefs about the circumstances at hand. At the very least, this includes knowledge about the possible strategies of themselves and others and the preferences agents have regarding those profiles, but often agents' preferences rely partly on some other facts beyond the control of those involved. For example, if the weather is nice, one might prefer to go to a park, but otherwise one might prefer to go to a museum. With this in mind, it would be irrational to go to the park knowing that it will rain. Without that

*This joint work extends works previously presented by both authors separately.

knowledge, even if is raining, going to the park might still be considered rational. Put simply, the basis of rationality rests in part on knowledge. Bernheim (1984) and Pearce (1984) each defined the broad game theoretic solution concept of rationalizability, which is built on the premise that rational agents will only take actions that are the best response to some situation that they consider possible. This accounts for the consideration that the other agents are rational as well, which limits the possible actions to which a particular agent must respond, enabling further elimination of strategies until the set stabilizes. This process is fundamentally built on the strategic considerations that underlie behavior. It is natural, then to generalize the notion of rationalizability for games in which payoffs depend not only on the strategies of the players, but on some facts of the world as well. This will enable us to examine the interplay between strategic and knowledge based rationality. We give an account of what it means for an action to be rational relative to a particular state of affairs, and in turn relative to a state of knowledge. Additionally, we present a class of games called Epistemic Messaging Games (EMG) which involve a period of communication between the players prior to the play of the game. This communication stage results in a particular epistemic state among the players which can be quite complex, and thus requires a broad framework to examine. For this purpose we use a version of the kind of history based models presented by Chandy & Misra (1986) and Parikh & Ramanujam (2003), which frames individual knowledge as a local projection of a global history. Using this technique, we present a general account of rationalizability for subclasses of EMG.

Formal intensional semantics of Aczel applied to Bolzanian substantial metaphysics

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Our research concerns a formal representation of Bolzano's original concepts of his substantial metaphysics. The reconstruction is built as an extension of Zalta's theory of abstract objects, describing two types of predication: attribution and representation. Our aim is to show limitations of Bolzano's theory that prevent a contradiction in view of the known suspicions about the contradiction of this theory. We discuss two competing semantics for the proposed theory: Scott's and Aczel's semantics. The first one yields a problematic result, that there are no models for the considered theory, containing a non-empty collection of all substantially funded attributes - the so called adherences. We show that Aczel's semantics does not contain this difficulty. There are described Aczel's models with a non-empty set of all adherences. The self-referentiality of such a collection becomes irrelevant here. Finally, we show that there are Aczel's structures verifying the formula on reloading abstracts and we exclude them from the class of models intended for our theory.

On the universality of the principle of determination

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To contribute to the problematic of the universal logic, we shall ask the question to know which principle can have an universal value. Our answer will be the principle of determination in its various versions.

The standard forms of the principle of determination

Frege introduces the principle of determination for a concept: it must be able to determine unambigously if an object falls or not under him.

Wittgenstein in 1914–1916 defines the determination of a proposition by its bipolarity: b – p – a = false – p – truth [1].

The limits of these standard forms of the principle

The principle of concept determination cannot apply to the fuzzy logic of Zadeh: a concept may determine + or − to its objects.

The principle of determination of the proposition is limited to a bivalent system. Tri, quadri, multivalent, intuitionistic, paraconsistent, etc. systems break the bipolarity of the true and the false.

The universality of the general principle of determination

Other forms of determination, however, apply to any formal system.

A first form was explained by Vasiliev, whose *metalogic* admits the following principle: "A proposition can admit only one value of truth" [2]. We shall show that this applies to any logical system, regardless of the number of truth values mobilized.

In fact, the basic requirement of determination extends to all aspects of a formal system. We can then decline it in its syntactic, semantic and metalogical dimensions. For example, at the syntactic level, a proposition of the system can only be well or badly formed. The analysis of these different declination draws a universal *metalogic* in the sense of Vasiliev.

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Logic does not distinguish any extralogical content

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That logic does not distinguish any extralogical content is a characteristic property of first-logic. Philosophically speaking, this feature displays the fact that logic is topically-neutral. First-order arithmetic is an extension of first-order logic, but this enlargement is not a logic. Consider deontic logic which is an extension of classical propositional calculus by adding deontic operators and suitable axioms. Does this systems does not distinguish extralogical contents? Clearly, the answer is negative. The accessibility relation for deontic frames is not reflexive (A does not imply that is obligatory that A). However, this property is not logical, although it can be formally explicated. This argument can be extended to alethic modal logic as well with exception with respect to the system K — the accessibility relation associated with it has no specific formal properties. The discussed issue may suggest various thoughts related to the concept of logic. If the theorem that logic does not distinguish any extralogical content is taken as basic, it motives a restrictive definition, practically identifying logic with first-order one and its modal counterparts (like K). On the other hand, there are good reasons for considering logic more broadly, that is, as studying various concepts via formal (or logical) methods.

Computation

Is Classical Mathematics Appropriate for Theory of Computation?

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In this talk, we try to show how and why our mathematical framework seems inappropriate to solve some problems in Theory of Computation. More exactly, the concept of turning back in time in paradoxes causes inconsistency in modeling of the concept of "Time" in some semantic situations. As we see in the first chapter, by introducing a version of "Unexpected Hanging Paradox", first we attempt to open a new explanation for some paradoxes. In the second step, by applying this paradox, it is demonstrated that any formalized system for the Theory of Computation based on Classical Logic and Turing Model of Computation leads us to a contradiction. We conclude that our mathematical frame work is inappropriate for Theory of Computation. Furthermore, the result provides us a reason that many problems in Complexity Theory resist to be solved.

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Methodological Principles for Program Logic Construction

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Program logics are main formalisms that are used to prove properties of software systems. Different types of such systems (sequential, distributed, real-time, reactive, embedded, etc.) lead to different types of program logics. Obtained diversity of logics complicates their investigation and usage. In this situation, methodological principles can provide the theoretical underpinning for understanding of program logics construction and investigation.

In this paper we continue our research on developing *methodological basis* for logic construction [1,2] concentrating on *program logics*. The main attention is paid to *the principle of development from abstract to concrete* and to *the principle of integrity of intensional and extensional aspects*.

These principles are applied to the main semantic notions of programs: *data*, *function* and *composition*. Data are specified as *intensionalized data* that represent integrity of data intension and extension [1]. We identify three types of such data: *abstract data*, *nominative sets* and *hierarchical nominative data*. A such classification leads to three types of intensionalized functions: *abstract function*, *quasiary functions* and *hierarchic-ary functions*. Finally, we define three classes of *compositions* over above mentioned classes of functions. This permits to construct various classes of algebras which form the semantic base for *predicate logics* [3] and *program logics of Floyd-Hoare type* [4].

Obtained program logics better represent such program properties as *partiality*, *nondeterminism* and *semistructuring of data*. Such logics generalize conventional program logics and demonstrate usefulness of methodological principles for their construction.

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Turing's Fallacy of Substitution

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In philosophy as well as in mathematics logical formalization is an established practice. However, the rules of this practice are far from clear. Recently, logicians with a philosophical background started to discuss criteria of adequate formalization (cf., e.g., the most recent monograph [1]). However, the discussion mostly focusses on the philosophical practice of formalizing ordinary language propositions. In my talk, I intend to broaden this focus and apply a well-known problem of logical formalization of ordinary propositions to Turing's formalization of Turing machines. It is a well-known problem in the philosophical literature that one cannot infer the logical form from the grammatical form of a proposition. This dictum is one variant of the so-called 'misleading form thesis'. One illustration of this thesis is the fallacy to infer that any formula is a correct formalization of its instances. A necessary condition for the correctness of a logical formalization ϕ of a proposition p is that p must be true if ϕ is provable. However, in so-called 'opaque' ('non-referential', 'non-extensional') contexts, this condition is not satisfied. Examples of such contexts are intensional, meta-linguistic or diagonal contexts. Instances of provable formulas that are not true are 'inadmissible instances'. No general criterion to distinguish admissible from inadmissible instances is available. In particular, no criterion referring to the grammatical or syntactic form of the formalized proposition is at hand. Therefore, the inference that an instance is true because it is an instance of a provable formula is a fallacy. I call this fallacy the 'fallacy of substitution'. Turing's proof of his Lemma 2 in his undecidability proof of first-order logic

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is a prominent example of committing this fallacy. Turing explicitly bases his proof on the following fallacious principle [cf. 2, p. 262]:

> If we substitute any propositional functions for function variables in a provable formula, we obtain a true proposition.

Regarding the problematic application of this principle to propositions involving diagonalization, the question arises whether Turing's application of his principle is valid in the diagonal case such as the logical formalization of a machine that involves a decision machine for logic that, in turn, evaluates its own formalization. I will discuss this problem in my talk.

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Extending Classical Logic with Quasiary Predicates

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Logics are widely used for investigation of software systems. To be successful, such logics should adequately represent systems properties. Among such properties we identify partiality and nondeterminism of software systems (programs), usage of complex data structures etc. In particular, mappings of fixed arity (n-ary mappings) and of flexible arity (quasiary mappings) are used in programs [1]. These properties imply the necessity of construction of program-oriented logics that are based on n-ary and quasiary mappings. In this paper one of such logics — first-order logic of n-ary and quasiary predicates L^{NQ} — is built. We construct this logic as an extension of classical logic with quasiary predicates.

Construction of L^{NQ} consists of several steps:

1. We define semantics of L^{NQ} by special algebras of quasiary predicates. In this case, n-ary predicates can be represented as a subclass of quasiary predicates. The class of compositions (operations) consists of compositions of disjunction, negation, renomination and existential quantification.

2. We define a language of L^{NQ} as the class of terms of the constructed algebras. The logic signature includes sets of n-ary and quasiary predicate symbols. This language being restricted on the class of n-ary predicate symbols corresponds to the first-order language of classical logic; and being restricted on the class of quasiary predicate symbols it coincides with the language of first-order quasiary logic [2].
3. We define formula interpretation mappings as term interpretations in algebras of quasiary predicates.
4. We define the consequence relation as irrefutability.
5. Finally, we construct sequent calculus which is based on the sequent calculus for the logic of quasiary predicates [2].

In the paper we study properties of L^{NQ}. In particular, logical equivalence relations are studied for the introduced consequence relation. Transformations based on such equivalence relations permit to transform formulas to various normal forms. Also, soundness and completeness of the constructed sequent calculus are proved.

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Direct Products on Computing Languages and Models: A preliminary

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A briefing on the mathematical basis for what we began on developing NLP algorithms since [6] we have new applications for compound signatures over product languages. The fragment categories [2] can be addressed at several levels:consider defining a correspondence to an arbitrary infinitary language where the sets you wish to address with respect to the new signatures on structures for languages definable, call that $L \subset \Sigma$. On T_Σ we can take well-defined fragments on the language towards small complete categories and onto creating generic functors [2,3,4] that cradle the new models.

Theorem 1. Let L_1, L_2 be two positive languages. Let $L = \mathrm{L}_1 \cap \mathrm{L}_2$. Suppose T is a complete theory in L and $T \subset T_1$, $T \subset T_2$ are consistent in L_1, L_2, respectively. Suppose there is a model M definable from a positive diagram in the language $L_1 \cup L_2$ such that there are models M_1 and M_2 for T_1 and T_2 where M can be homomorphically embedded in M_1 and M_2.

(i) $T_1 \cup T_2$ is consistent.

(ii) There is model N for $T_1 \cup T_2$ definable from a positive diagram that homomorphically extends that of M_1 and M_2.

Proposition 1. [3,4] There is a small complete category on the infinitary language fragment definable with the $\Sigma \subset i \in \omega$ based on the direct product on $T_\Sigma \subset i \in \omega$.

Theorem 2. [3,4] There is a generic functor on the category the omitting n-types realizing a direct product model.

Application Areas

Defining a category for languages allows us to define lifts, for example, from context. The linguistics abstraction techniques proposed allows us to lift from context. With signature tree languages we can carry on partitions on models for a formal language that factor generic models. Let $L_1, \ldots, L_n$ be language fragments $n \in \omega$. $\Sigma_1, \ldots, \Sigma_n$ are the fragment natural grammar signatures. $T_{\Sigma_1}, \ldots, T_{\Sigma_n}$ are the free trees on the grammar signatures. Each fragment language signature tree $T_\Sigma \subset i \in \omega$ can be assigned a fragment semantics whereby to each $t \in T_{\Sigma_i}$ the free well-formed syntax trees t_i and t_j are congruent, which is denoted $t_i \equiv t_j$, iff there is a context free parse normal form common to both. Let us denote that congruence with $\equiv \subset CTF$ fragment. $L_1, L_2, \ldots, L_n$ are fragments $n < \omega$, i.e. natural numbers.

Proposition 2. $\equiv \subset CTF$ is a Σ-congruence.

Theorem 3. $T_{\Sigma_i} \equiv \subset CTF$ is the initial algebra semantics for the L_i language fragment for $i \in \omega$.

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Rules versus Axioms: a Constructive View of Theories§

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In a Hilbert-style non-logical axiomatic theory the semantics of logical symbols is rigidly fixed, while the interpretation of non-logical symbols usually varies giving rise to different models of the given theory. All non-logical content of such a theory is comprised in its non-logical axioms (e.g. axioms of ZF) while rules, which generate from these axioms new theorems, belong to the logical part of the theory (a.k.a. "underlying logic"). This Hilbertian notion of axiomatic theory and its model has been used by Patrick Suppes and his many followers in their attempts to develop a general formal framework for representing scientific theories belonging to a wide range of disciplines [3].

An alternative approach to axiomatization due to Gentzen amounts to a presentation of formal calculi in the form of systems of rules without axioms. Gentzen did not try to extend his approach to non-logical theories by considering specific non-logical rules as a replacement for non-logical axioms. However the more recent work in Univalent Foundations of Mathematics [2] suggests that the Gentzen-style rule-based approach to formal presentation of theories may have important applications also outside the pure logic.

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A reason why one may prefer a rule-based formal representation is that it is more computer-friendly. This, in particular, motivates the recent work on the "constructive justification" of the Univalence Axiom via the introduction of new operations on types and contexts [1]. Another reason is that such form of the representation allows one to represent formally various extra-logical methods, which play an important role in the justification of scientific theories but are left aside in the standard axiomatic representations.

Using Homotopy Type Theory and the Univalent Foundations as a motivating example I argue that the Gentzen-style rule-based approach provides a viable alternative to the standard axiomatic approach not only in logic but also in science more generally.

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Hypercomputation and Philosophy of Mathematics

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In my talk I will discuss the relationship between mathematical and physical knowledge in the context of hypercomputational models. I argue, that the importance of hypercomputation for the philosophy of mathematics has not been recognized yet. Even if is most probably only a thought experiment, I claim, that is sheds some light on important philosophical problems, in particular:

1. The problem of understanding and explanation in mathematics
2. The problem of empirical elements in mathematics
3. The impact of these models on the realism/antirealism debate

*This is the group pen name of researchers participating in the Univalent Foundations Program at the Institute for Advanced Study in Princeton, NJ, which led to the publication of the monograph entitled *Homotopy Type Theory*.

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Consider first the hypothetical situation, where we have at our disposal a very fast computer (e.g. 2^{1000} times faster than our computers). It works in ZFC and proves new theorems; e.g. it solves Riemann's hypothesis (provided it is not independent from ZFC...). But we do not have any insight into the conceptual structure of the proof, so it no explanatory value. Our supercomputer would just be a kind of (empirical) oracle. The problem of computer-assisted proofs has been discussed extensively in the literature.

But consider also a hypercomputational procedure, which leads to a new mathematical result A (theorem?). One example discussed in the literature is Con(ZFC), a such a device could check by brute force all possible proofs in search of a contradiction (it is usually discussed in the context of the Relativistic Turing Machine). So what is the status of A? It is not a theorem, so perhaps it should be considered to be a new axiom, justified by the "hyperargument"? But the situation is very different from the situation of (hypothetical) new axioms of ZFC, which are justified by metatheoretical considerations (cf. the discussion concerning large cardinal axioms, Woodins Ω-logic providing an answer to the continuum hypothesis, etc.). We do not have any arguments of this kind in favor of this new axiom — just the verdict of an "empirical oracle". Perhaps it is even not mathematical knowledge, but something different?

I argue that hypercomputational models invite us to rethink the traditional model of mathematical knowledge, the notion of explanation in mathematics, the problem of empirical elements in proofs and offer new arguments in the realism-antirealism debate. Being clearly just thought experiments, they offer an important inspiration for the philosophy of mathematics, and deserve a thorough philosophical discussion.

A Universal Language for First-Order Constraints

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Sketches form a category-theoretic analog to the logical concept of a theory and its models. They use the language of arrows, commutative diagrams, limit cones and/or colimit cocones to specify structures on the objects of a category. Generalized sketches have been developed independently by Makkai, motivated by his work on an abstract formulation of Completeness Theorems in logic [1,2,3], and a group in Latvia around Diskin, triggered by their work on data modeling [6]. Later, generalized sketches have been applied and further developed to meet the needs in Model Driven Software Engineering (MDSE) [4,5].

Generalized sketches extend ordinary sketches in two directions: (1) Instead of graphs/categories only, we are allowed to work with objects in any chosen *base category*. In applications in MDSE the base category is usually a certain pre-sheaf topos. (2) Instead of using the "pre-defined" commutativity, limit and colimit predicates, we allow any appropriate predicates. To specify a relation $R \subseteq A \times B$, e.g., we use a predicate *jointly monic*, with its arity given by the graph $(v_1 \overset{e_1}{\leftarrow} v_3 \overset{e_1}{\rightarrow} v_2)$, instead of using an auxiliary product cone plus a monic edge into the product node. Especially, we can use predicates that can not be specified by limits and colimits in the chosen *semantic universe* as the category Rel [see e.g. 5].

To axiomatize arbitrary predicates in arbitrary base categories we need a universal language to define constraints in arbitrary categories. To achieve a proper generalization of sketches, those constraints should be first-order to enable us, at least, to axiomatize universal properties of limits and colimits respectively. Such a universal language is presented in the talk. Following the idea of *sketch axioms* in [3] it further develops the ideas, concepts and results in [4].

At present, we consider only predicates but not operations. We see, however, no principal obstacles to extend our framework by operations [see 6,7]. Following the tradition of first-order logic, we rely here on the semantics-as-interpretation paradigm instead of the semantics-as-instance paradigm as in [4] even if the latter one is more appropriate to formalize meta-modeling [6].

As a sanity check, we show, in generalizing our results in [4], that any appropriate choice of the components of our universal framework in any chosen base category gives rise to an institutions.

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History

The keynote speaker at this session is Anne-Françoise Schmid (page 159).

Logic Functions in the Philosophy of Al-Farabi

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Abu Nasr Muhammad Al-Farabi (870–950 AD), the second outstanding representative of the Muslim peripatetic after al Kindi (801–873 AD), was born in Turkestan about 870 AD.

Al-Farabi's studies commenced in Farab, then he travelled to Baghdad, where he studied logic with a Christian scholar named Yuhanna b. Hailan.

Al-Farabi wrote numerous works dealing with almost every branch of science in the medieval world. In addition to a large number of books on logic and other sciences, he came to be known as the "Second Teacher" (al-Mou'allim al-Thani), Aristotle being the first.

One of Al-Farabi's most important contributions was clarifying the functions of logic as follows:

1. He defined logic and compared it with grammar, and discussed the classification and fundamental principles of science in a unique and useful manner.
2. He made the study of logic easier by dividing it into two categories: Takhayyul (idea) and Thubut (proof).
3. He believed that the objective of logic is to correct faults we may find in ourselves and in others, and faults that others find in us.
4. He said that if we do not comprehend logic, we must either have faith in all people, or mistrust all people, or differentiate between them. Such actions would be undertaken without a basis of evidence or experimentation.

In this paper, I will analyse the functions of logic in Al-Farabi's works, *Enumeration of the Sciences*, *Book on the Syllogism*, *Book on Dialectic*, *Book on Demonstration* and *Ring Stones of Wisdom*, in order to present his contributions in the field of logic.

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A New Method of Demonstration for Aristotle's Ontological Syllogistic

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Since Aristotle's works of logic edited under the title of Organon [1], many logicians prepared and evaluated methods of demonstrating the validity and invalidity of Aristotle's syllogisms that we encounter mainly in the Prior Analytics of Organon. This paper is not a historical evaluation of various such methods and attempts of demonstrations. Instead, I intend to put forward a new and simple method of checking the validity of syllogisms for which I will claim that it has several advantages over some of the previous influential and popular methods. Some of the methods that I am going to evaluate in this paper are either algebraic, geometrical or in modern symbolic fashion. By means of this, I will evaluate Leibniz's linear diagrams, Boole's [2] algebraic treatment of syllogisms, Venn's diagrams and Łucasiewicz [3], Simely and Corcoran [4] types of proofs by natural deduction and compare them with the method I am proposing here.

This paper have two main parts. In the first part I will introduce a new method of demonstration which I will call the hierarchical method of demonstration for the ontological logic of Aristotle. The core reason of introducing this new method is that Aristotle's syllogisms and categories must be read collectively. In other words, I am against the way some logicians take when studying the logic of Aristotle. They mainly separate three headings of Aristotle's logic — categories, syllogisms, and induction — and work them mostly by isolating them from each other. However, I think this is a mistake. Syllogisms is a theory of deduction that Aristotle puts forward for the categories. In other words, these set of deductions is the science of categories. In this sense, the method of this science must reflect this very idea. The hierarchical method is the way to construct the syllogisms in such a way that we can see the genus-species relations for the given premises and to determine whether the relation stated in the conclusion is categorically necessary for the given hierarchy of the premises.

In the second part, I am going to show that the above-mentioned methods are not constructed to relate Categories and Prior Analytics. By showing this, my main objective is not to give an exclusive evaluation of these methods but to critically show their inadequacies. First, whether they are extensional or intensional neither of them are intended to extract the genus-species relations of a given syllogisms. Secondly, they are not capable of showing why exactly an invalid syllogism is invalid. For this, I do not mean the rules of validity but instead I mean the categorical relations that are the sole causes of the (in)validities. Thirdly, these methods lack the power of producing possible valid deductions for a given set of categories. This last point is important for the following reason. By using the notion of Aristotle's categories, we can construct any arbitrary artificial sets of objects that are sorted in a hierarchical order. So, for a n-sized set of objects and with certain preliminary relations we can determine the necessary genus-species relations by which we can determine the ontological export of the given categorical system. By doing this the new method is suitable to be the deductive system for the ontological logic for the categories.

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Does the Metalogic that Underlies the Aristotelian Logic Resemble what Timothy Williamson Calls a "Folk Logic"?

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Etchemendy's [2,3,4] conception of logical consequence as a conditional that always has a model in a non-fixed domain, has been criticised by Mancosu [5,6] and Williamson [8,9,10]. Non-fixed domains, the criticism goes, were not presupposed in Tarski's [7] analysis of logical consequence (this is Mancosu's objection to Etchemendy) and they reflect the idea of logic's being uninformative, which is characteristic of folk logic, not of scientific logic (this is what Williamson argues for). Williamson takes traditional logic to be the main influence of what he — obviously pejoratively — calls "folk logic".

However relevant for Etchemendy's understanding of Tarski, Kaplan's view on the uninformativeness of logic, and Dummett's contempt for abductive methods in logic, Mancosu's and Williamson's criticisms, at least if I am right, leave a great part of traditional logic intangible. Indeed, they leave intangible the most distinctive part of traditional logic: Aristotelian syllogistic [1]. Implicitly, syllogistic does not encourage views akin to what Williamson calls "folk logic" — at least if syllogistic is properly understood. Aristotle not only ignored very much of what we today would take to be features of non-fixed domains: he also employed fixed domains instead.

For example, Aristotle's view of necessity as a feature of valid inference does not involve what contemporary logicians call "logical necessity". It rather involves necessity as a feature of sentences that are true at all times. The fact that he propagated two-valued logic did not make him oversee the deviations he only too briefly discussed in *Peri hermeneias*. His understanding of logical necessity did justice also to these deviations.

Also in the realm of classical logic, Aristotle saw valid inferences as fulfilling some prerequisites that are informative, substantial or (in Etchemendy's sense) extra-logical. I shall show that some of these prerequisites involve the fixity of the domain of discourse. Further, I shall argue that these prerequisites are the motivation for two, in fact well-known features of Aristotelian logic: 1) it does not consider every true implication to be a valid inference; 2) it considers some false implications to be valid inferences.

I shall support my claim that Aristotelian logic is not a folk logic in Williamson's sense by reference to, among others, the following topics of Aristotelian logic: *quaternio terminorum* due to vagueness, the fallacy *a dicto* *simpliciter ad dictum secundum quid*, the subalternation of sentences at the right-hand side of the square of opposition, modal syllogisms with *de re* modalities.

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Interpretations of Chance within the Dialectic

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Given a model of chance M_c, it requires an Interpretation I, formalist, intuitionist, speculative, paraconsistent or empiricist, etc. then its propositional formulae can be Understood U, have different symbolic values. $\langle IU \rangle$ will then be an argument for M_c, but is not synonymous with a Theory of Chance T_c. The latter I will propose also requires a more specialized Knowledge K of a specific concept, so will be represented as $\langle IUK \rangle$; so different method(s) for the likelihood of an event can be instantiated. To satisfy a concept of chance, so it can be in a correspondence to reality, I will presume the method also has a disposition D. This will mean it has relational properties; as being-in-itself, for-itself and or to some other-than-itself. Fulfilling the complete argument $\langle IUK \rangle$ will therefore satisfy the conditions of the dialectic. These are confirmed and advanced by a higher standpoint of system, given true propositions are not self-contradictory [3], as these are contradicted. E.g., 'All propositions are analytical' is self-contradictory, as it refers to itself as being analytical, but is not true by being consistent with a definition of analysis. That is, it is contradicted by 'some propositions are analytical'. What I will try to show is that Lewis' [4,5] credence function is self-contradictory in that it is undermined by the definition of a Dialectical Principle [1,2].

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A justification for Aristotle's Thesis on the basis of the law of non-contradiction

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I am convinced that the *three laws of thought by Aristotle*: Identity, LEM (**l**aw of **e**xcluded **m**iddle) and LNC (**l**aw of **n**on-**c**ontradiction), are some of the fundamental propositions in our belief-set. In this talk I defend the idea that our intuition about **A**ristotle **T**hesis [1,2],

$$\text{AT: } \neg(p \to \neg p) \quad \text{or} \quad \neg(\neg p \to p),$$

is closely connected to the law of non-contradiction,

$$\text{LNC: } \neg(p \wedge \neg p).$$

I work with AT and LNC without negation by virtue of the simplicity of their form. Roughly, I think that:

(i) If we believe in LNC, then we have the intuition that AT is true.
(ii) If we don't believe in LNC, then we don't have the intuition that AT is true.

With the assumption that there is some analogy between our fundamental beliefs and axioms in a logical system, I want show that (i) is true in PA1 [3] and (ii) is true in LP [4].

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The Principle of Excluded Middle in Kant

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In this paper I will examine the principle of excluded middle in the philosophy of Kant from aspects of kinds of judgments and concepts of negation, determination and opposition.

First, I will explain that how the principle of excluded middle in Kant is used to differentiate the kinds of judgment. This principle is the principle of disjunctive and apodictic judgments in Kant's famous table of judgments. For disjunctive judgments this principle is also called the principle of division. "Sphere" is an important concept to elucidate this division.

Then I will examine how the principle of excluded middle is derived from the principle of contradiction according to Kant's understanding of negation. He separates negation of copula, negation of predicate and real negation. A negative proposition with a negative copula indicates that something is not contained under the sphere of a given concept occurs in accordance with the principle of excluded middle.

The principle of excluded middle is a principle of concept not a "thing". This principle assumes two opposite concepts. In this sense Kant calls this principle also as the principle of determination. The determinability of every single concept is the universality of this principle. But determination of a thing needs sum total of all possible predicates and another principle which is not derived from the principle of contradiction.

Lastly, I will enlighten which kind of opposition is related to the principle of the excluded middle [1]. There are three types of opposition, mainly real, logical and dialectical, in Kant. Only logical opposition pertains to the principle of the excluded middle. As merely logical criteria of truth, this principle grounds logical necessity of a cognition. When we necessarily judge, opposite of this judgment is false. When judgments opposed to one another contradictorily, the truth of one of the contradictorily opposed judgments is deduced from the falsehood of the other. This procedure creates apagogical proofs.

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Eristic and the origin of logic

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The term "eristic" *(eristikos)* first appeared during IV century BC to characterise a specific type of *logos* taught to young Athenians as part of their higher education (Isocrates). It was also employed during the same historical period to describe a group of individuals who were engaged in the activity of refutation or *elenchus* (Plato and Aristotle). These individuals, however, seemed to have had a negative reputation in Athens. Plato, for instance, condemned their lack of interest in crucial questions such as truth or forms. Indeed, eristics used refutation to contradict their interlocutors regardless of the veracity or the falsity of the theses defended. As for Aristotle, he criticised the form of eristical arguments — which predominantly rest on verbal trickery and paradoxes — and castigated the eristics' ignorance of what constituted a formal contradiction or a valid refutation.

It is undeniable that eristics had a major influence on the development of logic. Aristotle's analysis of sophisms used by eristics had prompted him to reflect on the validity of arguments and on syllogisms. Some of the logical paradoxes discussed by the Stoics also originated from the school of Megara, a philosophical school that had been repeatedly linked with eristic by ancient commentators and which had developed a different type of logic than Aristotle's. In fact, it is permitted to say that eristics were precursors of logic or, at the very least, instigators of the formal study of arguments and reasoning.

Despite its importance in the "prehistory" of logic, little is known about eristic. Although it was characterised as a form of education *(paideia)* consisting in the teaching of certain types of arguments, it is still unclear if eristic constituted a movement or consisted in a homogenous group of individuals. The identity of eristics is also uncertain. Plato designated two sophists as "eristics" (Euthydemus and Dionysodorus), but nothing is known about them, except that they were real historical figures. Aristotle showed himself equally reticent: although some of the eristical arguments he described can be attributed to members of the school of Megara, he explicitly named no one, except Euthydemus and Bryson. Finally, there remain no texts written by eristics during the IV century BC. Consequently, in order to know what eristic consisted of, it is necessary to rely upon the testimony of its strongest critics, namely Isocrates, Plato and Aristotle. We also find description of arguments used by eristics in works of other authors, but they are usually posterior to IV century BC.

The objectives of the paper are threefold. First, it will aim at giving a thorough description of eristic based on the ancient sources that remain. It will also address the controversial question of the identity of eristics. Secondly, it will analyse the different arguments used by eristics and highlight their originality and philosophical implications. Finally, it will discuss the influence of eristic on the developments of logic and demonstrate its importance.

Nondeterminism and Chinese Traditional Logic

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One of the most basic and difficult problems in Computer Science and Artificial Intelligence is about *human judgment* and *machine decision* as well as their relationship, which concerns *nondeterminism.*

In computability theory, *determinism* is expressed in terms of *computability.* A *deterministic problem (P)* is decidable and solvable by an exact algorithm. The work of Turing [1] shows that the *computability* means that the capacity of algorithm can match the growth of problem size. Moreover, in computational complexity theory, *determinism* is precisely defined by the concept of *polynomial time complexity.* The polynomial time complexity is related to *linear*, while the *exponential time complexity* related to *nonlinear*, which corresponds to *nondeterministic problem (NP).*

On the one hand, the power of computer gives people a conceptual illusion: *NP* may be solved ultimately by exact algorithms as *P*; on the other hand, people still keep a cognitive intuition that *P* is essentially different from *NP*. This situation is expressed as famous *P vs NP* problem, designated as one of seven Millennium Problems by the Clay Mathematics Institute [2].

Although *P* is defined by computability, but *NP* cannot be defined by computability, which is logically *unjudgeable*, thus *undecidable. P vs NP* expresses the inherent difficulty in formal logic due to *self-winding*, the difficult relationship between the pure formal thought and the subjectivity of human cognition.

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The difference and relation between nature languages and formal languages has become an important research direction in the modern philosophy. In Chinese language, Chinese traditional logic focus on human judgment, and has a natural rationality that is consistent with the content of reasoning, so the difficulty of *self-winding* in formal logic can be naturally avoided.

Chinese traditional logic is not separated from natural language to form a special logic system, but combined with Chinese thought, which is integrated into history and classics. It remains a few representative figures and case studies, such as the famous Chinese paradox *white horse is not horse*, proposed by a great Chinese logician Gongsun Long (∗325–250 BC†) [3].

From a modern point of view, Chinese traditional logic is based on the idea of logical hierarchy. Although Chinese traditional logic has a particular style and has not been sufficiently developed, it can bring insight into the most difficult problems involved in modern mathematical logic.

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Should Hegel's theory of the syllogism be included in the history of logic?

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As Redding [6] observes, there is an ongoing debate regarding whether Hegel's logic [4,5] is really a logic in the sense of the term used by analytic philosophers or whether it is a theory of metaphysics. For example, Di Giovanni, in his Introduction to Hegel's *Science of Logic* [4], contends that Hegel's logic primarily contributes to metaphysics, and, along similar lines, Beiser [1] regards Hegel's logic as either metaphysics or ontology. On the other hand, as suggested by Eisnor [3], Hegel's logic is an important rival to

Aristotle's logic, which opens the possibility that it could be regarded as a deviant logic. In *The Science of Logic* [4], Hegel compares and contrasts his three figures of the syllogism (discounting his mathematical syllogism) with Aristotle's three figures of the syllogism, arguing that his figures are actually an inter-related triad where one figure "passes over" into the next figure, whereas in Aristotle's logic, the syllogistic figures are static and simply stand "side by side" [5]. If Hegel's theory of the syllogism is regarded as a logic that is an alternative to Aristotle's theory of the syllogism rather than merely a theory of metaphysics, then there is no reason to exclude it from the history of logic.

Moreover, it could be argued that virtually any system of logic involves a metaphysics and an ontology, so that Hegel's logic is no different in that regard. Hence, it should not be singled out as something to be excluded from the history of logic simply because it involves an ontology. For example, at least in philosophical circles, it is popular to employ a possible worlds semantics for various systems of intensional and hyperintensional logics and such as alethic modal logics, quantified modal logics, doxastic, deontic, epistemic and tense logics as well as intuitionistic and relevance logics. Granted, there have been attempts to de-ontologize the semantics for intensional and hyperintensional logics such as Dunn's truth-value semantics for modal logic [see 2], although even Dunn's semantics makes use of model sets consisting of modal 'facts' along with sets of laws, both of which stand in for possible worlds.

Finally, on the assumption that Hegel's theory of the syllogism can be regarded as an alternative to Aristotle's theory of the syllogism, the possibility of formalizing Hegel's theory of the syllogism will also be investigated.

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Renaissance Analysis as a Solution to the Problem of Induction

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In a gloss on remarks by Euclid on analysis, the ancient geometer Pappus appears to endorse the view that it is possible to discover the axioms necessary to prove a theorem by working backwards from the theorem to the required assumptions by a deductive argument — that you can deduce axioms from the theorems. In the Renaissance Jacobo Zabarella, in his work on method, seized upon the text as a solution to the problem of induction. If Pappus was right, Zabarella reasoned, it would be possible to deduce natural laws from the empirical observations that follow from them as theorems. He found the key in Aristotle's distinction between *demonstratio quia* and *demonstratio propter quid*, a distinction that had been preserved and developed in medieval logic. The method was accepted throughout the 16th and 17th centuries and is endorsed in the Port Royal Logic. This paper explains Zabarella's method and logical flaws. It turns out to be closely linked to the medieval topic of "from the cause to the effect" and "from the effect to the cause".

Alsteed's Encyclopedy

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After my presentation of Alsted's *Praecognitiones theologicae* in his *Methodus sacrosanctae theologiae*, at the 2nd World Congress of Logic and Religion, I propose to present and comment the systematization of his *Praecognita disciplinarum, libri quatuor* of his *Encyclopaedia* (1630) in relation to his *Scientiarum omnium encyclopediae*, since 1649 (vol. I) and his *Logicae systema harmonicum* (1628). The discussion of Alsted's Method will be about its logical construction, elaborated in the conjunction of platonism and aristotelism as cognitive procedures from ignorance (Plato, *Republic* VII) to knowledge in a demonstrative optimization which contains logical questions of epistemological integration. The order and cognitive chains, proposed as deductive, are in the center of the interest.

Two Syllogisms in the *Mozi*: Chinese Logic and Language

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This paper examines two syllogistic arguments presented and contrasted in the ancient Chinese book *Mozi* [2], which expounds doctrines of the Mohist school of philosophers. We can use English plural constructions to formulate the arguments as follows:

1. Any things that are one horse are horses. Any things that are horses have four feet. ∴ Any things that are one horse have four feet.
2. Any things that are two horses are horses. Any things that are horses have four feet. ∴ Any things that are two horses have four feet.

Both arguments involve sentences of the *plural* A *form* (or A^*), the plural cousin of the standard universal affirmative form (or A) of categorical sentences:

> A^* (*the plural* A *form*): Any things that are-P are-Q, where 'are-P' and 'are-Q' stand in for predicates of the plural form (e.g., 'are horses', 'are white', 'run fast').

And the arguments seem instances of the plural cousin of the standard, singular form of Barbara:

> *Plural Barbara*: Any things that are-P are-Q. Any things that are-Q are-R. ∴ Any things that are-P are-R.

One can get (2) from (1) by replacing 'are one horse' with 'are two horses'. While (1) is a valid argument, however, (2) is not. Although its premisses are true, its conclusion is false — two horses have eight feet (with four each), not four.

Some scholars of Chinese philosophy (e.g., [1] suggest that argument (2) shows that Plural Barbara (or its Chinese counterpart) is not a valid form. Like Barbara, however, Plural Barbara is a valid form. The paper argues that (2) is not a genuine instance of the form because it involves equivocal uses of the predicate 'have four feet'. This predicate has the distributive/non-distributive ambiguity (and so does its Chinese counterpart). This shows the importance of the distributive/non-distributive distinction in studies of logic and semantics of constructions that go beyond the confines of singular constructions.

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Algebra and Category

A Constructive Proof of Coherence Theorem for Symmetric Monoidal Category

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Keywords: String diagrams rewriting, Algebraic Theories, Coherence, Monoidal Categories.

An algebraic theory is traditionally described in terms of a logical first-order syntax with function symbols and equational laws. In his thesis, Lawvere [5] gives a description of such (finitary) theories by means of categories with finite products where their operations are represented by morphisms. An higher dimensional point of view on such theories [2] allows us to consider in the same higher dimensional category all the constituent of such theories: objects (1-cells), operations (2-cells), equalities (3-cells) and coherence conditions (4-cells).

In this presentation we detail the proof given in [1] for coherence in symmetric monoidal categories. In order to directly use the confluence of Lafont's string diagram rewriting system [4] which describe the underlying algebraic theory, our proof adapts the method proposed by Guiraud and Malbos in [3]. The resulting proof gives a constructive method to decompose any diagram in a symmetric monoidal category by means of smaller diagrams corresponding to the elementary coherence conditions.

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Filter pairs: A new way of presenting logics

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In this work we introduce the notion of filter pair as a tool for creating and analyzing logics. A filter pair over a signature Σ is a structured collection of lattice homomorphisms $L_A \to \wp(A)$, where A runs over all Σ-structures, L_A is some algebraic lattice and $\wp(-)$ the power set. Taking A to be the formula algebra, the image of this homomorphism can then be taken as the lattice of theories of a logic. Thus a filter pair can be seen as a *presentation* of a logic, different from the usual style of presentation by axioms and derivation rules.

The case of interest for this work is $L_A = Co_K(A)$, the lattice of congruences whose quotient lies in some quasivariety K.

Theorem 1. Let K be a quasivariety, and $\tau = \langle \epsilon, \delta \rangle$ a set of equations (i.e. pairs of unary formulas in the signature of K). Then the collection of maps

$$Co_K(A) \ni \theta \mapsto \{a \in A \mid \epsilon(a) = \delta(a) \text{ in } A/\theta\} \in \wp(A)$$

defines a filter pair.

One can show that the class of logics arising in this way is the class of logics having an algebraic semantics in the technical sense of [2], a huge class containing all algebraizable logics, but also logics that are neither protoalgebraic nor truth-equational.

Theorem 2. Let L be a logic presented by a congruence filter pair as above. If the quasivariety K has the amalgamation property, then L has the Craig interpolation property.

This vastly generalizes one half of a landmark theorem of Czelakowski, saying that an *algebraizable* logic with an additional property has the Craig interpolation property *if and only if* its corresponding quasivariety has the amalgamation property.

In the talk we will motivate and introduce the notion of filter pair and show how it supplies structure which can be used to prove the second theorem. We will further offer a point of view on filter pairs as being an approach to algebraizing logics which is dual to the one via the Leibniz operator.

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Swap Structures and non-deterministic algebraization of logics

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Multialgebras (also known as *hyperalgebras*, or *non-deterministic algebras*), have been very much studied in Mathematics and in Computer Science. In 2016, Carnielli and Coniglio proposed in [3, chapter 6] a semantics based on an special kind of multialgebra called *swap structure* as a semantic framework for dealing with several logics of formal inconsistency (or LFIs) which cannot be semantically characterized by a single finite matrix. In particular, these LFIs are not algebraizable by the standard tools of abstract algebraic logic. The swap structure semantics generalize the characterization results of LFIs by means of finite Nmatrices due to Avron [see, e.g., 1]. Moreover, this semantics allows soundness and completeness theorems by means of a very natural generalization of the well-known Lindenbaum-Tarski process.

In this talk some advances towards a theory of non-deterministic algebraization of logics by swap structures will be described, following the results recently obtained in [4]. It will be developed a formal study of swap structures for LFIs, by adapting concepts of universal algebra to multialgebras in a suitable way. A decomposition theorem similar to Birkhoff's representation theorem will be described for each class of swap structures. It will be shown that, when applied to the 3-valued algebraizable logic J3, the usual class of algebraic models is recovered by means of this technique, and so the swap structures semantics became twist-structures semantics (as introduced by Fidel [6] and Vakarelov [7]). The twist-structures semantics produces semisimple Nelson algebras, which are polynomially equivalent to the variety of MV-algebras of order 3 [see 5]. From this, our representation theorem coincides with the original Birkhoff's representation theorem in the case of the algebraizable 3-valued logic J3. This fact, together with the existence of a functor from the category of Boolean algebras to the category of swap structures for each LFI, which is closely connected with Kalman's functor, strongly suggests that swap structures can be considered as non-deterministic twist structures.

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Analogies of meaning across logic and categories

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A deep understanding of a theorem, a logical proposition or more widely an internal sense of a text, is not always restricted to the sequence of the meanings of the unities. It is also constructed on more global structures allowed by signifying organizations, toward semantic representations. Analogical relations, underlying to syntactic unities and participating of the global structure, create a comparison order and take part in the understanding.

Can we logically highlight some abstract representations from internal relations in such utterances or logical propositions? Can we enter into the logical properties of linguistic "objects", rather than logical or mathematical "objects"? A comparable way does exist in mathematics since Eilenberg and MacLane (1945). Mathematics have a level of representation of an issue (category, topos, set), but can linguistics build one or multiple ones? Can the essence of lexical unities of this language, such as an operator, or a simple verb, be formally represented, as certain mathematical properties are? The approach aims to enter into the internal logic of the construction and to formally bind some analog meanings. The question is the application of a logical calculus of typed operators, to underline some comparisons in a combinatory.

A mathematical approach is proposed in order to show a logical organization of semantic properties. Considering examples of analogical meanings in natural languages as ancient Hebrew, Greek and Latin, a verb can participate to the construction of comparative relations, and the properties of a representation be calculated from signifying unities, to highlight an internal sense.

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Dual Logic Concepts based on Mathematical Morphology in Stratified Institutions: Applications to Spatial Reasoning

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To cope with the explosion of logics, a categorical abstract model-theory, the theory of institutions [6,8], has been proposed, that generalizes Barwise's "Translation Axiom" [2]. Institutions then define both syntax and semantics of logics at an abstract level, independently of commitment to any particular logic. Later, institutions have been extended to propose a syntactic approach to truth [8]. For the sake of generalization, in institutions signatures are simply defined as objects of a category and formulas built over signatures are simply required to form a set. All other contingencies such as inductive definition of formulas are not considered. However, the reasoning (both syntactic and semantic) is defined by induction on the structure of formulas. Indeed, usually, formulas are built from "atomic" formulas by applying iteratively operators such as connectives, quantifiers

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or modalities. What we can then observe is that most of these logical operators come through dual pairs (conjunction $\wedge$ and disjunction $\vee$, quantifiers $\forall$ and $\exists$, modalities $\Box$ and $\Diamond$).

When looking at the algebraic properties of mathematical morphology [4] on the one hand, and of all these dual operators on the other hand, several similarities can be shown, and suggest that links between institutions and mathematical morphology are worth to be investigated. This has already been done in the restricted framework of modal propositional logic [3]. It was shown that modalities $\Box$ and $\Diamond$ can be defined as morphological erosion and dilation. The interest is, based on properties of morphological operators, that this leads to a set of axioms and inference rules which are de facto sound. In this communication, we propose to extend this work by defining, at the abstract level of institutions, a pair of abstract operators as morphological erosion and dilation. We will then show how to obtain standard quantifiers and modalities from these two abstract operators.

In mathematical morphology, erosion and dilation are operations that work on lattices, for instance on sets. Thus, they can be applied to formulas by identifying formulas with sets. We have two ways of doing this, either given a model M identifying a formula φ by the set of states η that satisfy φ and classically noted $M \models_{\eta} \varphi$, or identifying φ by the set of models that satisfy it. As usual in logic, our abstract dual operators based on morphological erosion and dilation will be studied both on sets of states and sets of models. The problem is that institutions do not explicit, given a model M, its set of states. This is why we will define our abstract logical dual operators based on erosion and dilation in an extension of institutions, the stratified institutions [1]. Stratified institutions have been defined in [1] as an extension of institutions to take into account the notion of open sentences, the satisfaction of which is parameterized by sets of states. For instance, in first-order logic, the satisfaction is parameterized by the valuation of unbound variables, while in modal logics it is further parameterized by possible worlds. Hence, stratified institutions allow for a uniform treatment of such parameterizations of the satisfaction relation within the abstract setting of logics as institutions.

Another interest of the proposed approach is that mathematical morphology provides tools for spatial reasoning. Inspired by the work that was done in [3, 5] in the propositional and modal logic framework, we show how logical connectives based on morphological operators can be used for symbolic representations of spatial relations. Indeed, spatial relations are a main component of spatial reasoning, and several frameworks have been proposed to model spatial relations and reason about them in logical frameworks.

Since it is usual to introduce uncertainty in qualitative spatial reasoning, we propose to extend our abstract logical connectives based on erosion and dilation to the fuzzy case. This first requires to develop fuzzy reasoning in stratified institutions. Fuzzy (or many-valued) reasoning has an institutional semantics [7]. The approach proposed here is substantially similar to that proposed in [7], although developed in stratified institutions.

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Categorical semantics for a variation of subjective logic

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Jøsang's subjective logic [2,3] is a relatively new probabilistic logic system which considers not only probability but also uncertainty, similar to Dempster-Shafer theory. In this work we study an epistemic propositional logic system based on these approaches focusing on its categorical semantics. We explore the categorical semantics of a probabilistic modal operator

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with a degree of uncertainty. Akin to Giry monads [1], we consider an ordered tuple of rational numbers in the interval $[0, 1]$ for the degrees of belief and uncertainty. From this point, we examine categorically several aspects of interest for the development of the system. The first one is the set of rules that represent the formal conditions for the probabilistic coherence of the system that allow to meet Kolmogorov's axioms. The second one is the study of the operators of subjective logic. Although most of them are generalisations of usual binary operators, some of them such as abduction or a Bayesian operator do not have a direct equivalence to classical operators [3]. We aim to find adequate categorical interpretations of them in our work. A third issue is the relationship between categorical and Kripkean semantics of this system. We suggest a variation of Kripke frames with the corresponding degrees of belief and uncertainty. In the last section of our work we want to sketch future improvements of the system, such as the adding of polymorphism.

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Finite Strong Standard Completeness of IUL plus t ⇔ f *

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Hahn's structure theorem [2] states that totally ordered Abelian groups can be embedded in the lexicographic product of real groups. Residuated lattices are semigroups only, and are algebraic counterparts of substructural logics [1]. Involutive commutative residuated chains (a.k.a. involutive FLe-chains) form an algebraic counterpart of the logic IUL [3]. The focus of our investigation is a subclass of them, called commutative group-like residuated chains, that is, totally ordered, involutive commutative residuated lattices such that the unit of the monoidal operation coincides with the constant that defines the involution. These algebras are algebraic counterparts of IUL plus t ⇔ f. We shall present a representation theorem for the finitely generated algebras of this class, by using only totally ordered Abelian groups as building blocks, and a here-defined construction, called partial-lexicographic product. As a corollary, we shall extend Hahn's embedding theorem to this class, using partial-lexicographic products instead of lexicographic ones. Its corollary is the finite strong standard completeness of IUL plus t ⇔ f.

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Prime and maximal filters for the free algebra in the subvariety of BL-algebras generated by $[0,1]_{\mathsf{MV}} \oplus H$

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BL-algebras were introduced by Hájek [see 4] to formalize fuzzy logics in which the conjunction is interpreted by continuous t-norms over the real interval $[0,1]$. These algebras form a variety, usually called $\mathcal{BL}$. In this work we will concentrate in the subvariety $\mathcal{MS} \subseteq \mathcal{BL}$ generated by the ordinal sum of the algebra $[0,1]_{\mathsf{MV}}$ and a hoop H, that is, generated by $\mathbf{S} = [0,1]_{\mathsf{MV}} \oplus H$. The main advantage of this approach, is that unlike the work done in [2] and [1], when the number n of generators of the free algebra increase the generating chain remains fixed. This provides a clear insight of the role of the two main blocks of the generating chain in the description of the functions in the free algebra: the role of the regular elements and the role of the dense elements.

We have a functional representation for the free algebra $\mathrm{Free}_{\mathcal{MS}}(n)$. To define this functions we need to decompose the domain $\mathsf{S}^n = ([0,1]_{\mathbf{MV}} \oplus H)^n$ in a finite number of pieces. In each piece a function $F \in \mathrm{Free}_{\mathcal{MS}}(n)$ coincides either with McNaughton functions or functions of $\mathrm{Free}_{\mathcal{H}}(n)$.

We study the filters in $\mathrm{Free}_{\mathcal{MS}}(n)$. In $\mathcal{BL}$-algebras the implicative filters characterize the congruences, and on the contrary, if $\equiv$ is a congruence relation on A, then the set $\{x \in A : x \equiv 1\}$ is an implicative filter. Then the correspondence $F \mapsto \equiv_F$ is a bijection from the set of implicative filters on A on the set of congruences of A.

We will show a characterization of prime filters in terms of prime filters in $\mathrm{Free}_{\mathcal{MV}}(n)$ (which we describe as prime ideals were studied in [3]) and prime filters in $\mathrm{Free}_{\mathcal{H}}(n)$.

We also give a description of the maximal filters and a correspondence between maximal filters on $\mathrm{Free}_{\mathcal{MS}}(n)$ and the points of the cube $[0,1]_{\mathsf{MV}}^n$.

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On Two Mutually Inverse Isomorphisms between NEmHC and NEK4.Grz

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We continue our analysis [5] of the latices NEmHC and NEK4.Grz of the normal extensions of two calculi, NEmHC and NEK4.Grz, which were defined by Leo Esakia [4] as follows:

$$\mathsf{mHC} := \mathsf{Int}^{\Box} + \Box(p_1 \to p_2) \to (\Box p_1 \to \Box p_2) + p_1 \to \Box p_1 \\ + \Box p_1 \to (p_2 \vee (p_2 \to p_1)),$$

and

$$\mathsf{K4.Grz} := \mathsf{Cl}^{\Box} + \Box(p_1 \to p_2) \to (\Box p_1 \to \Box p_2) + \Box p_1 \to \Box\Box p_1 \\ + \Box(\Box(p_1 \to \Box p_1) \to p_1) \to \Box p_1 + \alpha/\Box\alpha.$$

As established in [5], the maps $\tau(\mathsf{mHC} + \Gamma) := \mathsf{K4.Grz} + \Gamma^{st}$ and $\rho(\mathsf{K4.Grz} + \Gamma) := \{\alpha \mid \mathsf{K4.Grz} + \Gamma \vdash st(\alpha)\}$ define mutually inverse isomorphisms $\tau\colon \mathrm{NE}\mathsf{mHC} \longrightarrow \mathrm{NE}\mathsf{K4.Grz}$ and $\rho\colon \mathrm{NE}\mathsf{K4.Grz} \longrightarrow \mathrm{NE}\mathsf{mHC}$, where st is an embedding operation; see [5, section 3].

A frame (W, R) is called a K4.Grz-*frame* if (a) R is transitive, (b) there is no nontrivial R-cluster, and (c) there is no infinite set $\{a_i\}_{i<\omega} \subseteq W$ such that $a_0 R a_1 R a_2 \ldots$ Given a transitive (merely) frame $\mathfrak{F} = (W, R)$, one can define two kinds of modal frame algebras, the Boolean algebra $\mathsf{B}(\mathfrak{F})$ of the subsets of W, equipped with operation

$$\Box X := \{x \in W \mid \forall y\,(xRy \Longrightarrow y \in X)\},$$

and the Heyting algebra $\mathsf{H}(\mathfrak{F})$ of all S_R-upward closed subsets of W, where

$$S_R := R \cup \{(x,x) \mid x \in W\},$$

also equipped with the same $\Box X$. It has been shown [1] that $\mathsf{B}(\mathfrak{F})$ validates $\mathsf{K4.Grz}$ iff $\mathfrak{F}$ is a $\mathsf{K4.Grz}$-frame. It is easy to check that for any transitive frame $\mathfrak{F}$, $\mathsf{H}(\mathfrak{F})$ validates mHC. Given $L \in \mathrm{NE}\mathsf{K4.Grz}$, L is called *Boolean complete* if $L = L\mathsf{B}(\mathfrak{F})$. Given $L \in \mathrm{NE}\mathsf{mHC}$, L is called *Heyting complete* if $L = L\mathsf{H}(\mathfrak{F})$. Following [2], given $L \in \mathrm{NE}\mathsf{K4.Grz}$, we denote by $\mathfrak{F}^c_L = (W^c_L, R^c_L)$ the "classical" canonical frame for L; and, following [3], given $L \in \mathrm{NE}\mathsf{mHC}$, we denote by $\mathfrak{F}^i_L = (W^i_L, R^i_L)$ the "intuitionistic" canonical frame for L. It is easy to see that R^i_{mHC} is transitive.

We claim:

(I) *τ does not preserve canonicity*, since mHC *is canonical*, but $\mathsf{K4.Grz}$ is not [1];

(II) *for any $L \in \mathrm{NE}\mathsf{K4.Grz}$, L is Boolean complete iff $\rho(L)$ is Heyting complete with respect to a $\mathsf{K4.Grz}$-frame*;

(III) *the logic $\mathsf{mHC} + (\Box p_1 \to p_1) \to p_1 + \neg\neg p_1 \to p_1$ is a canonical normal extension of $\mathsf{K4.Grz}$*;

(IV) *for any $L \in \mathrm{NE}\mathsf{K4.Grz}$, if L is canonical, so is $\rho(L)$*;

(V) *for any $L \in \mathrm{NE}\mathsf{K4.Grz}$, L admits the rule $\Box\alpha/\alpha$ iff so does $\rho(L)$*;

(VI) $\mathsf{K4.Grz}$ *admits the rule $\Box\alpha/\alpha$* and hence mHC *does it too.*

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Visualizing Geometric Morphisms

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Different people have different ways of remembering theorems. A person with a very visual mind may remember a theorem in Category Theory mainly by the shape of a diagram and the order in which its objects are constructed. For such a person most books on Category Theory feel as if they have lots of missing diagrams, that she has to reconstruct if she wants to understand the subject.

The shape of a categorical diagram remains the same if we specialize it to a particular case — and this means that we can sometimes remember a general diagram, and the theorems associated to it, from the diagram of a particular case.

In this talk we will present the general technique above and one application: reconstructing the statements, and some of the proofs, of two factorizations of geometric morphisms between toposes described in section A4 of [1], from particular cases that are easy to draw explicitly — in which our toposes are of the form $\mathsf{Set}^{\mathsf{A}}$, where A is a finite category whose objects are certain points of $\mathbb{Z}^2$. The tricks for visualizing sheaves on these '$\mathsf{Set}^{\mathsf{A}}$'s are described in [2].

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Semantic Construction for Hilbert's Category

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The main objective of this study is to provide the basis for the elaboration of a semantics for the Hilbert Spaces Category (Hilb), which requires, in our view, the use of an algebra, necessary to interpret certain properties of bifunctors and isomorphisms specific in (Hilb). Initially designed to deal with certain problems characteristic of Quantum Computing, this category later proved to be also effective in dealing with certain objects of Quantum Mechanics. Thus, in addition to its notorious practical possibilities, this same category possesses an enormous theoretical potential which, it seems to us, deserves to be explored extensively, especially as this allows integrating syntax, semantics and pragmatics of a category through which one can easily and effectively transit both Mathematics and Physics, as well as Logics. Preliminarily, we expose the syntax of (Hilb) based on the works of Baez [1], Coecke [2] and Heunen [3]; then, we will make an interpretation of certain properties of (Hilb) and at the end we will present a sketch of the semantics that we are constructing for this category.

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Philosophy of Mathematics

Universality and intersubjectivity of mathematical constructions. *Toward a dialogical reading of Brouwer's proof of the bar theorem*

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Brouwer's proof of the bar theorem is based on a particular understanding of what an implication is, leading to semantic puzzles. In his 1927 paper [2], in order to set the validity of the implication: "if B is a bar, then it is a well-ordered bar", Brouwer does not start from the hypothesis that B is a bar, but from the hypothesis that a demonstration of B being a bar is actually given (to an epistemic subject). Let us assume that this understanding of what is an implication is sound; then we are immediately confronted with the following difficulty: how can such mental constructions be acknowledged by other minds?

Following [1] we can read Brouwer's argument as a transcendental one (in the sense of Husserl). Such a phenomenological reading provides an interesting perspective on subjectivity: the transcendental reduction yields subjectivity as giving access to a kind of necessity pertaining more to intentional acts than to objective properties of mathematical entities. Yet, the phenomenological framework does not seem to allow transcendental arguments to count as mathematical ones, transcendental reduction being rather an effort toward the purity of a transcendental core the expression of which always remains provisory. Husserl's claim about the impossibility to conclusively formalize transcendental researches could nonetheless be considered relative to a *static* understanding of formality: if the dynamical openness necessary for the acceptance of rules can be expressed within a formal framework, then such a reading of Brouwer's argument could gain a precise logical status.

Our claim is that the dialogical framework offers such means, especially through the distinction it provides between several levels of rules. Our purpose then is to substitute the concept of "mental" with the concept of "intersubjective interaction" in order to bring together universality and subjective construction.

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Discovery in mathematics from a heuristic perspective: the case of the calculus and its development

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I address the question of discovery in mathematics, meant as a practice evolving through time, and I propose an account of how new items are discovered, and I claim that such processes can be properly described in terms of a particular logic, i.e. heuristics.

Scientific discovery is a rather debated issue: on the one side, we find a "psychological" approach according to which discovery is necessarily linked to the character of genius and to intuition; on the other, a "heuristic" approach according to which discovery can be rightfully appraised as a process subject to logical and rational inquiry. The difference between these acceptations of discovery is summarized by the dichotomies individuality/intersubjectivity and intuition/hypothesis formulation.

The first step in my argumentation consists in providing a "deflationary" definition of mathematical objects in terms of a particular kind of hypotheses [see 2] and a general description of mathematics in terms of a highly complex problem-solving activity [see 3,4]. Within a conception of mathematics as a problem- solving practice, the introduction of new hypotheses undergoes a trial-and-error process in order to be accepted, thus being rationally analyzable at each step; including their introduction. Hypotheses as "cognitive efforts" differ from any psychological enterprise because of their non- individualistic and non- idiosyncratic nature and therefore are more apt to describe the mechanisms of discovery in the practice of mathematics. Their very acceptation relies on how satisfactorily they resist to trial-and-error processes brought on by single mathematicians as well as by mathematical communities in different places and different times.

Finally, I propose the historical example of the development of the Calculus [see 1] showing how an actual discovery has entered the corpus of mathematics through a series of changes and improvements of successive hypotheses.

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What is Law?: the perception of Category Theory

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Law controls our lives. We live under the rule of Law. We are everywhere also subject to the laws of science. Law in one way or another affects every facet of human life. We need therefore to understand better the way that Law operates. Do we even know the nature of law? To understand it we need to examine its roots. For they lie in logic. There is *prima facie* a distinction between the scientific law and human civil and criminal laws in that the former are always strictly obeyed whereas the latter may not be observed. However delving deeper we find that scientific law is not always universally obeyed. On the other hand it may come as a surprise to find that there is a sense in that human law always does.

Law whether scientific or human operates from a higher level. Current mainstream mathematics that derives from set theory cannot deal directly with separate levels but only collapse them into a model of first order. Alfred North Whitehead (1860–1947) who had established 20th Century mathematics subsequently drew attention to the limitations of first order models and advocated a move to Process at the higher level of metaphysics [1]. Unfortunately Whitehead did not have a formal metaphysical language to replace the flat mathematics of set theory.

Today Category Theory now fills that gap. It is a formal language that operates across four levels recursively with features like adjointness between universal limits and co-limits that were not appreciated until the 1970's.

All Laws arise from this natural adjointness as underlying functors [2]. But there is the caveat that the current representation of Category Theory only holds up to the natural isomorphism of the axioms of set theory. Applied Category Theory needs to hold up to the natural isomorphism of Physics.

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Structural investigation of the categorial logic-geometrical system

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Mathematics includes itself the fields subjected to both the quantitative and the qualitative research. With the aim of making mathematics a tool for philosophical investigations, Benedict Bornstein (1880–1948) implemented the logical algebra developed by George Boole and the existing projective geometry. He imparted a categorial form to these fields and simultaneously brought them closer to the philosophical investigations.

Bornstein's transformation of the multitude projected geometry into the categorial one, all the categories of locations and directions were derived on the plane. He also introduced the categorial structure of algebra logic, based on a system of axioms listed by Edward Huntington in 1904. He connected categorial algebra logic with categorial projected geometry and obtained a system for the categorial geometrical logic (logo-topics) or in other words the categorial logical geometry (topo-logic). Both dimensions of this system constitute one field ($\lambda\acute{o}\gamma o\zeta$ — $\tau\acute{o}\pi o\zeta$), examined in two aspects. It appeared that this system possesses the profound philosophical meaning (it is like Universal algebra of Whitehead's). With the help of it, ontology and categories of being become the science in quality of mathematics. The logic-geometrical system revealed a reality point in terms of quality and enabled one to develop the general theory of being.

Bornstein's area of research has been presented considering the possibility of their continuation. Algebra, logical and geometrical investigations that involve the categorial and ontological analysis, constitute an original suggestion for a description and qualitative explanation for the structure of the reality. The investigation includes many innovative ideas (topology, theory of category), which remain still inspiring for modern analysis undertaken in the field of formal ontology.

Visual Images and Non-Deductive Rules in Mathematical Discovery

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Frege's view that there cannot be a logic of discovery but only a logic of justification is widely accepted. Even if there is no logic of discovery, there still may be a dynamics of discovery, or at least systematic accounts of ways that mathematical discoveries are made. As Lakatos [3] emphasised, some account of discovery in mathematics is essential for understanding the nature mathematical progress.

One (possibly) common aspect, to which philosophers nowadays pay more attention, is the role of visual thinking in mathematical discovery. Cellucci [1, chs. 20–21] gives rules of discovery. He illustrates use of non-deductive rules by means of several diagrams and claims that diagrams are "not only auxiliary means, they are also an important source of non-deductive inferences on their own" [1, p. 357]. Indeed, mathematical sentential non-visual thinking is normally used in conjunction with visual thinking. The latter usually combines thinking with external visual representations (e.g., diagrams, symbol arrays, kinematic computer images) and thinking with internal visual imagery, and often involves imagining a certain spatial transformation of an object represented by a diagram on paper or on screen. Possible epistemic roles include contributions to evidence, proof, understanding and grasp of concepts.

Though philosophical discussion of visual thinking in mathematics has concentrated on its role in proof, visual thinking may be more valuable for discovery than proof. Giaquinto [2] distinguishes three types of discovery important in mathematical practice: (1) discovering a truth, (2) discovering a proof-strategy, and (3) discovering a property or kind of mathematical entity. He illustrates visual discovery of these kinds using examples and infers that "It is hard to see how properties in these examples would have been discovered without the use of visual resources" (ibid.). This claim raises questions: images may be useful in mathematical discovery, but can they play a crucial role? And if so, how? What properties make images useful even in current mathematics (since some of his examples are relatively recent and advanced)?

This talk is an attempt to go deeper into some of such examples focusing on the third kind of discovery from Giaquinto's list, with an aim to identify possible systematic elements of discovery involving visualisation. Are there any repeated uses of visualisations in discovery? What are their key components? Do they stabilise in visual practices? How do they influence shaping the new concepts? These and related questions are to be discussed.

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Explanation and Existence in Mathematics

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The notion of explanation is widely discussed in philosophy and methodology of natural sciences. The situation is quite different in the case of mathematics: the notion of explanation is widely used in informal discussions among mathematicians, but only recently it received more attention of philosophers of mathematics, and it still has not been analyzed as thoroughly as it deserves. However, it is of great importance for the In the last years, for the realism-antirealism debate in mathematics, in particular in the famous indispensability argument. The original indispensability argument rests on two premises:

(1) we are committed to the existence of those entities, which are indispensable to our best scientific theories;
(2) mathematical objects are such entities.

The discussion has recently been given new impulses by the EIA ("enhanced indispensability argument"), where several cases from biology of physics have been discussed, and the (possible) explanatory role of mathematics has been the subject of analysis (the most famous examples are the: periodical life-cycle of cicadas (and the fact, that they are prime numbers); the honeycomb conjecture, the Borsuk-Ulam theorem — and many others). These examples seem to be clearly non-causal, as mathematical facts and properties are present in the *explanans*. The topic is much discussed, and the problem, which constitutes the special character of mathematical explanations in science, is acute.

The problem of explanation within mathematics, as well as the problem of the explanatory role of mathematics has quite different aspects, but some specific examples of the cases where the notion of explanation can be applied are:

(a) the problem of explanatory role of mathematical proofs (in particular the interplay between the real and formalized versions of mathematical proofs);
(b) the problem of explanation in the context of justifying axioms.
(c) the problem of "local" versus "global" character of explanation within mathematics (in particular the problem of extraneous elements in mathematical proofs; explaining one single mathematical fact versus providing a suitable conceptual framework for a particular branch of mathematics);
(d) the problem of mathematical explanations in empirical science, especially in the context of the enhanced indispensability argument.

In the talk, I will focus on the last problem, and in particular present some new examples inspired by the independence results (in arithmetic and set theory). These results invite us to rethink the explanatory role of mathematics in natural science, as their status is quite different from the well-known examples. I claim, that their analysis in the realism-antirealism debate will give new, important insights (as the problem of the strength of necessary "background resources" becomes more acute).

16 – Contest Logic Prizes

In the previous editions of UNILOG, we had a prize based on a contest with a theme, for this edition this has evolved in the contest Logic Prizes (see details in the paper *Universal Logic: Evolution of a Project*), and, to understand how the prize works, have a look at the page A Prize of Logic in Every Country!. Each winner of a prize will present his/her work in 30mn, including discussion, on Sunday 24 afternoon.

The jury will then give the Universal Logic Prize to the best of them. Besides receiving this honorific prize, the winner will be an invited speaker at the next UNILOG and will receive a purchase voucher of Birkhäuser/Springer-Nature.

Jury Members:

- Hartry Field
  Department of Philosophy, New York University, USA
- Michèle Friend
  Department of Philosophy, Columbian College of Arts and Sciences, George Washington University, USA
- Grzegorz Malinowski
  Department of Logic, University of Łódź, Poland
- Ahti-Veikko Pietarinen
  Tallinn University of Technology, Estonia
  Nazarbayev University, Astana, Kazakhstan
- Peter Schroeder-Heister
  Department of Computer Science, University of Tübingen, Germany
- Göran Sundholm
  Department of Philosophy, University of Leiden, The Netherlands
- Leon van der Torre
  Computer Science and Communication Lab,
  University of Luxembourg, Luxembourg

Organizer of the Contest:
Jean-Yves Beziau
University of Brazil, Rio de Janeiro, Brazil
École Normale Supérieure, Paris, France

The 2018 Winner of Universal Logic Prize is Ivan Varzinczak (Université d'Artois, Lens, France) for his paper "A note on a description logic of concept and role typicality" (page 575).

A prize of Logic in every country!

Logic is a fundamental field of research. Among the most influential intellectuals of the 20th century, at least four are directly connected to logic: Kurt Gödel, Bertrand Russell, Alan Turing and Ludwig Wittgenstein. However generally at a university there is no department of Logic. People working in Logic are spread in various institutes: philosophy, mathematics, computer sciences, linguistics, cognitive science... In a sense this spreading of logic is good. But it is also important to reinforce the interaction between logicians working in one university, in one country, in the world. This is the spirit of logic prizes.

The Idea of a Logic Prize in a Given Country, say Smurfland, is:

(1) To encourage the development of logical research in Smurfland.
(2) To foster interaction between people having interest for logic in Smurfland.
(3) To make better known logic among researchers of all fields in Smurfland.
(4) To make the work of logicians in Smurfland better known outside of Smurfland.
(5) To develop, promote and make better known logic in the world.

How to organize a Prize of Logic in Smurfland:

(1) Decide who will organize the prize and find a name of a famous logician of Smurfland for the prize.
(2) Name a Jury of important logicians representative of the various tendencies and geographical locations of Smurfland.
(3) Circulate a Call for Papers in Smurfland with a deadline, not too close but not too far, e.g. March 31, 2018.
(4) The Jury of Smurfland chooses the winner
(5) Attribution of the prize: publication of the paper of the winner in *Logica Universalis* + participation to UNILOG'2018 in Vichy.

The prize of logic in Smurfland is open to anybody living and working in Smurfland. There are no restrictions of age, sex, race, nationality. The contestants only need to live in Smurfland and be affiliated to a University (or other educational institution) in Smurfland.

If there is a logical association in Smurfland, this association can organize the prize. If there is no such association of logic in Smurfland, it is a good opportunity to create one. If it is too complicated to do it immediately the prize can be organized to start by an already existing group of logicians of Smurfland. Financial support: Smurfland will support the travel of the winner to Vichy for UNILOG'2018. UNILOG will support the accomodation of the winner of the logic prize of Smurfland at UNILOG'2018 in Vichy and will waive the registration fee for the winner.

Logic Prizes around the World

Newton da Costa Logic Prize, in Brazil

Organizers:

- Jean-Yves Beziau (University of Brazil)
- Itala D'Ottaviano (State University of Campinas)

Jury for 2018:

- Francisco Antônio Dória (Federal University of Rio de Janeiro) — Chair
- Andreas Brunner (Federal University of Bahia) — Mathematical Logic
- Osvaldo Pessoa (University of São Paulo) — Philosophy of Science
- Mario Benevides (Federal University of Rio de Janeiro) — Logic and Computation
- Abílio Rodrigues (Federal University of Minas Gerais) — Philosophical Logic

Newton da Costa was First President of the Brazilian Society of Logic, and is President of Honor of the Brazilian Academy of Philosophy.

Da Costa's international recognition came especially through his work on paraconsistent logic and its application to various fields such as philosophy, law, computing and artificial intelligence. He is one of the founders of this non-classical logic. In addition, he constructed the theory of quasi-truth that constitutes a generalization of Alfred Tarski's theory of truth, and applied it to the foundations of science.

The 2018 Winner is Jonas R. Becker Arenhart (Federal University of Santa Catarina, Florianópolis, Brazil) for his paper "New logics for quantum non-individuals?" (page 566).

Schotch-Jennings Logic Prize, in Canada

Organizers:

- John Woods (University of British Columbia)
- François Lepage (University of Montréal)

Jury for 2018:

- Alasdair Urquhart (University of Toronto) — General Logic (Chair)
- Wendy MacCaull (Saint Francis Xavier University) — Logic and Computer Science
- Jean-Pierre Marquis (University of Montréal) — Foundations of Logic
- Sandra Lapointe (McMaster University) — History and Philosophy of Logic
- Bryson Brown (University of Lethbridge) — Non-Classical Logics

Peter K. Schotch is Emeritus Munro Professor of Metaphysics at Dalhousie University in Halifax, Nova Scotia, Canada. He is co-founder of the Canadian School of paraconsistency known as preservationism, and has written papers in many areas of philosophical logic, particularly many valued logic and epistemic and deontic logic.

Ray Jennings is Professor Emeritus of Philosophy at Simon Fraser University.

The 2018 Winners are Allen P. Hazen & Francis Jeffry Pelletier (University of Alberta, Canada) for their paper "FDE, Ł3, K3, RM3, LP: Making Many-Valued Logic Work" (page 570).

Georgius Benignus Logic Prize, in Croatia

Organizers:

- Croatian Logic Association
- Centre for Logic, Methodology, and Decision Theory (University of Rijeka)
- Institute of Philosophy in Zagreb
- Research Centre for Logic, Epistemology and Philosophy of Science (University of Split)

Jury for 2018:

- Zvonimir Šikić (President of the Croatian Logic Association, University of Zagreb)
- Srećko Kovač (Institute of Philosophy in Zagreb)
- Nenad Smokrović (University of Rijeka)

- Mladen Vuković (University of Zagreb)
- †Berislav Žarnić (University of Split)

Georgius Benignus (∗circa 1445–1520†) was a Croatian humanist, philosopher, Franciscan, archbishop and theologian. He is the author of several philosophical-theological works in Latin.

The 2018 Winner is Tin Perkov (University of Zagreb, Croatia) for his paper “Abstract logical constants” (page 571).

Vasiliev Logic Prize, in Russia

Organizer: Elena Lisanyuk (Department of Logic, Institute of Philosophy, Saint Petersburg State University)

Jury for 2018:

- Valentin Bazhanov (Ulyanovsk State University)
- Vladimir Vasyukov (Institute of Philosophy, Russian Academy of Sciences, Moscow)
- Alexey Kislov (Ural Boris Yeltsin Federal University, Yekaterinburg)
- Ivan Mikirtumov (Chair, Saint Petersburg State University)

Nicolai Alexandrovich Vasiliev (∗1880–1940†) was a Russian logician, philosopher, psychologist and poet. His ‘imaginary non-Aristotelian logic’, proposed in 1910–1914, was a forerunner of paraconsistent and multi-valued logic.

The 2018 Winner is Yaroslav Petrukhin (Lomonosov Moscow State University, Moscow, Russia) for his paper “Natural Deduction for Regular Three-Valued Logics and their Four-Valued Analogues” (page 573).

Bimal Krishna Matilal Logic Prize, in India

Organizer: Calcutta Logic Circle

Jury for 2018:

- Amita Chatterjee (Jadavpur University) — Philosophical Logic, Indian Logic and Cognitive Science
- Rohit Parikh (City University of New York) — Mathematical Logic, Logic for Computer Science and Social Software
- Raja Natarajan (Tata Institute of Fundamental Research) — Logic and Computer Science, and Automated Theorem Proving
- Hanamantagouda P. Sankappanavar (State University of New York at New Paltz) — Algebraic Logic and Universal Algebra

- Mihir K. Chakraborty (Chair, Jadavpur University) — Non-classical logics, Foundations of Mathematics and Logical Diagrams

Bimal Krishna Matilal (∗1935–1991†) was at the same time an exponent of Indian logic and well conversant with modern/Western logical theories. He studied with Quine in the early 1960s and from 1977 to 1991 he was the Spalding Professor of Eastern Religion and Ethics at University of Oxford. He was the founder editor of the Journal of Indian Philosophy.

The 2018 Winner is Jolly Thomas (International Institute of Information Technology, Hyderabad, India) for his paper "Developing Metalogic to Formalize Ontological Disputes of the Systems in Metaphysics by Introducing the Notion of Functionally Isomorphic Quantifiers" (page 574).

SILFS Italian Logic Prize, in Italy

Organizer: Società Italiana di Logica e Filosofia delle Scienze (SILFS)

Jury for 2018:
- Giovanna Corsi (University of Bologna)
- Silvio Ghilardi (University of Milan)
- Roberto Giuntini (University of Cagliari)
- Hykel Hosni (University of Milan)
- Pierluigi Minari (University of Florence)

The 2018 Winner is Stefano Bonzio (Marche Polytechnic University, Ancona, Italy) for his paper "Logics of variable inclusion and Płonka sums of matrices" (page 567).

Aristotle Logic Prize, in Greece

The 2018 Winner is Takis Hartonas (University of Applied Sciences of Thessaly, Greece) for his paper "Canonical Extensions and Kripke-Galois Semantics for Non-Distributive Propositional Logics" (page 570).

Alfred Tarski Logic Prize, in Poland

Organizers:
- Andrzej Indrzejczak (University of Łódź)
- Andrzej Pietruszczak & Marek Nasieniewski (Nicolaus Copernicus University in Toruń)

Jury for 2018:
- Tomasz Bigaj (University of Warsaw) — Philosophy of Science
- Janusz Czelakowski (University of Opole) — Mathematical Logic

- Jacek Malinowski (Chair)(Polish Academy of Sciences in Warsaw, Editor-in-Chief of Studia Logica) — Philosophical Logic
- Roman Murawski (Adam Mickiewicz University of Poznań) — Logic and Computation, and History of Logic
- Tomasz Połacik (University of Silesia in Katowice) — Formal Semantics

Alfred Tarski (∗1901–1983†) is one of the most important figures in the history of logic. Born in Warsaw, he was a student of Leśniewski and Łukasiewicz and soon became a central member of the Lwów-Warsaw school of logic. He then moved to the USA and founded a major group of logic at the University of California, Berkeley. His contributions touch nearly all the areas of logic: set theory, model theory (he coined the expression and was the main developer of this field), many-valued logic, the theory of consequence operator (founder), the theory of truth (creator), Boolean algebra and other fields.

The 2018 Winner is Zalán Gyenis (Jagiellonian University, Kraków, Poland) for his paper "On the modal logic of Jeffrey conditionalization" (page 569).

Louis Couturat Logic Prize, in France

Organizers:
- Jean-Yves Beziau (University of Brazil, École Normale Supérieure)
- Christophe Rey (CNRS[*], LIMOS[†], University of Clermont Auvergne)

Jury for 2018:
- Julie Brumberg-Chaumont (LEM[‡], CNRS[§], PSL[¶] Research University) — History of Logic
- Jean-Pierre Desclés (Paris-Sorbonne University) — Logic and Linguistics
- Gilles Dowek (LSV[‖], INRIA[**], École Normale Supérieure Paris-Saclay) — Computational Logic
- Didier Dubois (IRIT[*])— Non-Classical Logics and Artificial Intelligence

[*] Centre National de la Recherche Scientifique

[†] Laboratoire d'Informatique, de Modélisation et d'Optimisation des Systèmes

[‡] Laboratoire d'Études sur les Monothéismes

[§] Centre National de la Recherche Scientifique

[¶] Paris Sciences & Lettres

[‖] Laboratoire Spécification et Vérification

[**] Institut National de Recherche en Informatique et en Automatique

[*] Institut de Recherche en Informatique de Toulouse

- Pierre Joray (Department of Philosophy, University of Rennes) — Philosophical Logic
- Bruno Poizat (Department of Mathematics, University of Lyon I) — Model Theory and Mathematical Logic
- Francesca Poggiolesi (CNRS†, IHPST‡) — Proof Theory

Louis Couturat (∗1868–1914†) was the main promoter of symbolic logic in France at the beginning of the 20th century, author in particular of a book on the algebra of logic. He was a good friend of Bertrand Russell with whom he had a long correspondence which was recently published. He rediscovered and made worldwide known the work of Leibniz on logic. He promoted the artificial language Ido.

More information about Couturat can be found in the tutorial on Louis Couturat that will be presented by Oliver Schlaudt (page 87) at the 6th World School on Universal Logic, June 16–20, 2018, and in the talk by Anne-Françoise Schmid (page 159) at the 6th World Congress on Universal Logic, June 21–26, 2018, in Vichy.

The 2018 Winner is Ivan Varzinczak (Université d'Artois, Lens, France) for his paper "A note on a description logic of concept and role typicality" (page 575).

Talks of Contest Logic Prizes

New logics for quantum non-individuals?

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According to a very widespread interpretation of the metaphysical nature of quantum entities, the so-called *Received View on quantum non-individuality*, quantum entities are non-individuals [see 1]. Still according to this understanding, non-individuals are entities for which identity is restricted or else does not apply at all. As a consequence, it is said, such approach to quantum mechanics would require that classical logic be revised, given that it is somehow committed with the unrestricted validity of identity. In this paper we examine the arguments presented in [1] to the

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§2018 Winner of Newton da Costa Logic Prize, in Brazil (page 561).

inadequacy of classical logic to deal with non-individuals, as previously defined, and argue that they fail to make a good case for logical revision. In fact, classical logic may accommodate non-individuals too. What is more pressing, it seems, is not a revision of logic, but rather a more adequate metaphysical characterization of such entities.

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Logics of variable inclusion and Płonka sums of matrices

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It is always possible to associate with an arbitrary propositional logic $\vdash$, two substitution-invariant consequence relations $\vDash^{l}$ and $\vDash^{r}$, which satisfies, respectively, a *left* and a *right variable inclusion constraint*, as follows:

$$\Gamma \vDash^{l} \varphi \iff \text{there is } \Delta \subseteq \Gamma \text{ such that } Var(\Delta) \subseteq Var(\varphi) \text{ and } \Delta \vdash \varphi;$$

$$\Gamma \vDash^{r} \varphi \iff \begin{cases} \Gamma \vdash \varphi \quad \text{and} \quad Var(\varphi) \subseteq Var(\Gamma), \text{ or} \\ \Sigma \subseteq \Gamma, \end{cases}$$

where Σ is a set of inconsistency terms for $\vdash$. Accordingly, we say that the logics $\vDash^{l}$ and $\vDash^{r}$ are respectively the *left* and the *right variable inclusion companion* of $\vdash$.

Prototypical examples of variable inclusion companions are found in the realm of three-valued logics. For instance, the left and the right variable inclusion companions of classical (propositional) logic are, respectively, *paraconsistent weak Kleene logic* — PWK for short [6] — and *Bochvar logic* [1].

Recent work [3] linked PWK to the algebraic theory of regular varieties, i.e. equational classes axiomatized by equations $\varphi \approx \psi$ such that $Var(\varphi) = Var(\psi)$. The representation theory of regular varieties is largely due to the pioneering work of Płonka [7], and is tightly related to a special class-operator $\mathcal{P}_{\mathrm{l}}(\cdot)$, nowadays called *Płonka sum*.

*2018 Winner of SILFS Italian Logic Prize, in Italy (page 564).

One of the main results of [3] states that the algebraic counterpart of PWK is the class of Płonka sum of Boolean algebras. This observation led us to investigate the relations between left and right variable inclusion companions and Płonka sums in full generality. Our study is carried on in the conceptual framework of abstract algebraic logic [5].

The starting point consists in generalizing the construction of Płonka sums from algebras to logical matrices. This allows us to characterize the matrix models for variable inclusion logics by performing appropriate Płonka sums over direct systems of models of $\vdash$. As a matter of fact, variable inclusion companions are especially well-behaved in case the original logic $\vdash$ has a specific kind of *partition function* [7,8], a feature shared by the vast majority of non-pathological logics in the literature.

The use of the mentioned algebraic tools allow, on the one hand, to produce a general method to transform every Hilbert-style calculus for a finitary logic $\vdash$ with a partition function into complete Hilbert-style calculi for both $\vdash^{l}$ and $\vdash^{r}$. On the other hand, partition functions can be exploited to tame the structure of the matrix semantics $\mathsf{Mod}^{\mathrm{Su}}\left(\vdash^{l}\right)$ and $\mathsf{Mod}^{\mathrm{Su}}\left(\vdash^{r}\right)$, given by the so-called Suszko reduced models of $\vdash^{l}$ and $\vdash^{r}$. As a byproduct of our analysis, this formalism allows to provide topological dualities for the algebraic counterpart of variable inclusion logics [4,2].

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On the modal logic of Jeffrey conditionalization

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In this talk we introduce modal logics to study the logical properties of statistical inference (Bayesian belief revision) based on Bayes and Jeffrey conditionalization.

Suppose $(X, \mathcal{B}, p)$ is a probability space where the probability measure p describes knowledge of statistical information of elements of $\mathcal{B}$. In the terminology of probabilistic belief revision one says that elements in $\mathcal{B}$ stand for the propositions that an agent regards as possible statements about the world, and the probability measure p represents an agent's *prior* degree of beliefs in the truth of these propositions. Belief revision is about to learn new pieces of information: Learning proposition $A \in \mathcal{B}$ to be true, the agent revises his prior p on the basis of this evidence and replaces p with some new probability measure q (often called *posterior*) that can be regarded as the probability measure that the agent *infers* from p on the basis of the information (evidence) that A is true. This transition from p to q is what is called statistical inference. We say in this situation that "q *can* be learned from p" and that "it is *possible* to obtain/learn q from p". This clearly is a modal talk and calls for a logical modeling in terms of concepts of modal logic. Indeed, the core idea is to look statistical inference as an accessibility relation between probability measures: the probability measure q can be accessed from the probability measure p if for some evidence A we can infer from p to q.

We take the standard Bayes model of probabilistic belief revision, where the Bayesian agent can infer to a new probability given an evidence by conditionalizing using the Bayes or Jeffrey rule. We define modal logics that capture the general principles that Bayesian updating satisfies and clarify the containment relations among these modal logics. In particular we show that the logic of Bayes and Jeffrey updating are very close and that the modal logic of belief revision determined by probabilities on a finite or countably infinite set of elementary propositions is *not finitely axiomatizable*.

The significance of this result is that it clearly indicates that axiomatic approaches to belief revision might be severely limited.

*2018 Winner of Alfred Tarski Logic Prize, in Poland (page 564).

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Canonical Extensions and Kripke-Galois Semantics for Non-Distributive Propositional Logics

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This article presents an approach to the semantics of non-distributive propositional logics that is based on a lattice representation (and duality) theorem that delivers a canonical extension of the lattice. Unlike the framework of generalized Kripke frames (RS-frames), proposed with a similar intension, the semantic approach presented in this article is suitable for modeling applied logics (such as temporal, or dynamic), as it respects the intended interpretation of the logical operators. This is made possible by restricting admissible interpretations.

FDE, Ł3, K3, RM3, LP: Making Many-Valued Logic Work

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We investigate some well-known (and a few not-so-well-known) many-valued logics [1,7] that have a small number (3 or 4) of truth values. For some of them we complain that they do not have any *logical* use (despite their perhaps having some intuitive semantic interest [6]) and we look at

*2018 Winner of Aristotle Logic Prize, in Greece (page 564).

†2018 Winners of Schotch-Jennings Logic Prize, in Canada (page 562).

ways to add features so as to make them useful, while retaining their intuitive appeal. At the end, we show some surprising results in the system FDE [2,3], and its relationships with features of other logics. We close with some new examples of "synonymous logics" [4]. An Appendix contains a natural deduction system for our augmented FDE, and proofs of soundness and completeness.

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Abstract logical constants*

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A possibility of defining logical constants within abstract logical frameworks is discussed, in relation to abstract definition of logical consequence. We propose using duals as a general method of applying the idea of invariance under replacement as a criterion for logicality. The question of

*This work has been supported by Croatian Science Foundation (HRZZ) under the project UIP-05-2017-9219.

†2018 Winner of Georgius Benignus Logic Prize, in Croatia (page 562).

logicality is one of fundamental questions which are rarely answered with consensus and often lead to controversies, but trying to answer such questions turns out to be very fruitful for developing useful theories, with sometimes unintended applications. We focus on a recent development [1], which explores a close relation between logical constants and logical consequence.

We can have in mind the following goals:

- ambitious: find the proper notion of logical constants — probably no answer
- less ambitious: understand how a *choice* of constants generates a consequence relation, and vice versa.

For the latter converse goal, we use the idea of consequence extraction as presented by Bonnay and Westerståhl in [1]. Given a language and a consequence relation, they consider a symbol to be a constant if replacing it with another symbol of the same category (categories being e.g. binary Boolean connectives, unary modal operators, first-order quantifiers and so on) makes at least one valid inference of that consequence relation to fail.

The idea of this paper is very simple, but fairly general: we assume that any symbol s such as connective, quantifier, modal operator and so on, has the dual s' present in the language. Given a consequence relation $\Rightarrow$, duality means that we have valid inferences of the forms

$$s'(\varphi_1, \varphi_2, \dots) \Leftrightarrow \neg s(\neg\varphi_1, \neg\varphi_2, \dots)$$

and

$$s(\varphi_1, \varphi_2, \dots) \Leftrightarrow \neg s'(\neg\varphi_1, \neg\varphi_2, \dots).$$

If s is not self–dual, then at least one of these inferences fails if we replace s with s'. Therefore, s is a constant. As for self-duals, we use the following trick: replace with some other symbol of the same category which is not self–dual, thus making at least one of the inferences which express self-duality to fail. Another goal of the paper is to outline possible generalizations of techniques from [1], in particular to the framework of abstract logic [cf. e.g. 2]. We propose a refined definition of abstract logic to encompass the previously presented ideas.

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Natural Deduction for Regular Three-Valued Logics and their Four-Valued Analogues

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In this report, we introduce natural deduction systems for Kleene's [2] regular three-valued logics as well as their four-valued analogues presented by Fitting [1] and Tomova [7]. The class of regular three-valued logics includes such well-known systems as K_3 (Kleene's strong logic), $\mathsf{K}_3^{\mathsf{w}}$ (Kleene's weak logic), and LP (Asenjo's & Priest's logic of paradox [6]). Natural deduction systems for K_3 and LP were presented by Priest [6]. We introduce natural deduction systems for the rest of regular three-valued logics. Besides, we present natural deduction systems for Fitting's [1] generalizations of regular three-valued logics. Moreover, Tomova [7] discovered the class of regular four-valued logics. She considers also monotonic logics and calculated that there are six both regular and monotonic four-valued logics. We formalize all of them via natural deduction systems. Last, but not least, in our presentation of these results, we follow our papers [3,4,5].

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*2018 Winner of Vasiliev Logic Prize, in Russia (page 563).

Developing Metalogic to Formalize Ontological Disputes of the Systems in Metaphysics by Introducing the Notion of Functionally Isomorphic Quantifiers

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A general metalogical theory is developed by considering ontological disputes in the systems of metaphysics. The usefulness of this general metalogical theory is demonstrated by considering the case of the ontological dispute between the metaphysical systems of Lewis Modal Realism and Terence Parsons Meinongianism. Using Quine's criterion of ontological commitments and his views on ontological disagreement, three principles of metalogic is formulated. Based on the three principles of metalogic, the notions of independent variable and dependent variable are introduced. Then, the ontological dispute between Lewis Modal Realism and Terence Parsons Meinongianism are restated in the light of the principles of metalogic. After the restatement, independent and dependent variables are fixed in both Lewis Modal Realism and Terence Parsons Meinongianism to resolve the dispute. Subsequently, a new variety of quantifiers are introduced which is known as functionally isomorphic quantifiers to provide a formal representation of the resolution of the dispute. The specific functionally isomorphic quantifier which is developed in this work is known as st-quantifier. It is indicated that how st-quantifier which is one of the functionally isomorphic quantifiers can function like existential quantifier.

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*2018 Winner of Bimal Krishna Matilal Logic Prize, in India (page 563).

A note on a description logic of concept and role typicality

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Description Logics (DLs) [1] are a family of logic-based knowledge representation formalisms with useful computational properties and a variety of applications in artificial intelligence and in databases. In particular, DLs are well-suited for representing and reasoning about terminological knowledge and constitute the formal foundations of semantic-web ontologies. Technically, DLs correspond to decidable fragments of first-order logic and are closely related to modal logics [4].

Notwithstanding their good trade-off between expressive power and computational complexity, DLs remain fundamentally classical formalisms and therefore are not suitable for modelling and reasoning about aspects that are ubiquitous in human quotidian reasoning. Examples of these are exceptions to general rules, incomplete knowledge, and many others, characterising the type of reasoning usually known under the broad term *defeasible reasoning*. In this regard, endowing DLs and their associated reasoning services with the ability to cope with defeasibility is a natural step in their development.

In this work, we propose a meaningful extension of description logics for non-monotonic reasoning. We introduce $\mathcal{ALCH}^\bullet$, a logic allowing for the representation of and reasoning about both typical class-membership and typical instances of a relation. We propose a preferential semantics *à la* KLM [2,3] for $\mathcal{ALCH}^\bullet$ in terms of partially-ordered DL interpretations which intuitively captures the notions of typicality we are interested in. We define a tableau-based algorithm for checking $\mathcal{ALCH}^\bullet$ knowledge-base consistency and show that it is sound and complete with respect to our preferential semantics. The general framework we here propose can serve as the foundation for further exploration of non-monotonic reasoning in description logics and similarly structured logics.

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*2018 Winner of Louis Couturat Logic Prize, in France (page 565), and of Universal Logic Prize (page 559).

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