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Rio de Janeiro

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Edited by
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1 Organizers of UNILOG'13

1.1 Scientific Committee

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- Joham van Benthem – University of Amsterdam, Stanford University, The Netherlands and USA
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- Miray Yazgan – Istanbul University, Turkey

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- Alexandre Garcia – IME, Rio de Janeiro, Brazil.
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2 Aim of the event

In the same way that universal algebra is a general theory of algebraic structures, universal logic is a general theory of logical structures. During the 20th century, numerous logics have been created: intuitionistic logic, deontic logic, many-valued logic, relevant logic, linear logic, non monotonic logic, etc. Universal logic is not a new logic, it is a way of unifying this multiplicity of logics by developing general tools and concepts that can be applied to all logics.

One aim of universal logic is to determine the domain of validity of such and such metatheorem (e.g. the completeness theorem) and to give general formulations of metatheorems. This is very useful for applications and helps to make the distinction between what is really essential to a particular logic and what is not, and thus gives a better understanding of this particular logic. Universal logic can also be seen as a toolkit for producing a specific logic required for a given situation, e.g. a paraconsistent deontic temporal logic.

This is the fourth edition of a world event dedicated to universal logic, after very successful editions in Switzerland (2005), China (2007) and Portugal (2010). This event is a combination of a school and a congress. The school offers tutorials on a wide range of subjects. The congress will follow with invited talks and contributed talks organized in many workshops. There will also be a contest.

This event is intended to be a major event in logic, providing a platform for future research guidelines. Such an event is of interest for all people dealing with logic in one way or another: pure logicians, mathematicians, computer scientists, AI researchers, linguists, psychologists, philosophers, etc.

The whole event will happen at the feet of the Sugar Loaf in *Rio de Janeiro*, Brazil, known as *The Wonder City*.

UNILOG'2013: A logical way of living!

3 4th World School on Universal Logic

3.1 Aim of the School

This school is on universal logic. Basically this means that tutorials will present general techniques useful for a comprehensive study of the numerous existing systems of logic and useful also for building and developing new ones.

For PhD students, postdoctoral students and young researchers interested in logic, artificial intelligence, mathematics, philosophy, linguistics and related fields, this will be a unique opportunity to get a solid background for their future researches.

The school is intended to complement some very successful interdisciplinary summer schools which have been organized in Europe and the USA in recent years: The ESSLLI (European Summer School on Logic, Language and Information) in Europe and the NASSLLI (North American Summer School on Logic, Language and Information).

The difference is that our school will be more focused on logic, there will be less students (these events gather several hundreds of students) and a better interaction of advanced students and researchers through the combination of the school and the congress (Participants of the School are strongly encouraged to submit a paper for the Congress). We also decided to schedule our event in Spring in order not to overlap with these big events.

3.2 Tutorials

3.2.1 Why Study Logic?

Newton C. A. da Costa
Federal University of Santa Catarina – Brazil

Jean-Yves Béziau
Federal University of Rio de Janeiro – Brazil

Graham Priest
City University of New York – USA

Daniele Mundici
University of Florence – Italy

João Marcos
Federal University of Rio Grande do Norte – Brazil

This topic will be discussed by a philosopher, a mathematician and a computer scientist in a round table animated by J.-Y. Béziau, UFRJ & CNPq, organizer of the School of Universal Logic since 2005, and Newton da Costa, UFSC & CNPq, the Godfather of logic in Brazil.

3.2.2 How to get your Logic Article or Book published in English

Ties Nijssen
Springer, Netherlands

Ties Nijssens, a Springer's logic acquisition editor, will give his perspective on academic publishing. He will explain that while writing quality science is the precondition for getting published, there are also tips & tricks that can help to get a quick reaction from publishers. He will also discuss impact factors and other metric tools in academic publishing. Before the event, you can already send questions to him about this topic here.

This will be followed by a round table discussion with

- Jean-Yves Béziau, founder and Editor-in-Chief of the journal *Logica Universalis* and the book series *Studies in Universal Logic*, member of the editorial board of the *Journal of Logic and Computation*, Editor of the Series *Cadernos de Lógica e Filosofia*;
- Otávio Bueno, Editor-in-Chief of *Synthese*;
- Walter Carnielli, Member of the editorial boards of *Journal of Applied Non-Classical Logics*, *Logic and Logical Philosophy* and *Logica Universalis*;
- Gila Sher, Editor-in-Chief of *Synthese*;

- Heinrich Wansing, Editor-in-Chief of *Trends in Logic*, Managing Editor of *Studia Logica*, editorial board member of *Reports on Mathematical Logic*, *Journal of Applied Logic*, *Journal of Logic and Computation*, *Logica Universalis*, *Theoria: A Swedish Journal of Philosophy*, and *Journal of Philosophical Logic*.

3.2.3 Non-Deterministic Semantics

Arnon Avron
University of Tel Aviv – Israel

Anna Zamansky
TU Wien – Austria

The principle of truth functionality (or compositionality) is a basic principle in many-valued logic in general, and in classical logic in particular. According to this principle, the truth-value of a complex formula is uniquely determined by the truth-values of its subformulas. However, real-world information is inescapably incomplete, uncertain, vague, imprecise or inconsistent, and these phenomena are in an obvious conflict with the principle of truth-functionality. One possible solution to this problem is to relax this principle by borrowing from automata and computability theory the idea of non-deterministic computations, and apply it in evaluations of truth-values of formulas. This has led to the introduction of non-deterministic matrices (Nmatrices) in [6, 7] (see also [10] for a survey). These structures form a natural generalization of ordinary multi-valued matrices, in which the truth-value of a complex formula can be chosen non-deterministically out of some non-empty set of options.

Although various types of non-truth-functional semantics were proposed before (such as bivaluations semantics and possible translations semantics, see [13, 14]), the novelty of Nmatrices is in sharing a very important property with many-valued matrices: (semantic) analyticity. A semantics is analytic if to determine whether φ follows from T it always suffices to check only partial models involving solely subformulas of $T \cup \varphi$. This naturally induces a decidability procedure for any logic characterized by a finite Nmatrix.

Nmatrices have proved to be a powerful tool, the use of which preserves all the advantages of ordinary propositional many-valued matrices (analyticity, decidability, compactness), but is applicable to a much wider range of logics. Indeed, there are many useful (propositional) non-classical logics, which have no finite many-valued characteristic matrices, but do have finite Nmatrices, and thus are decidable. Nmatrices have also another attractive property (not shared by standard matrices) - modularity, which means that in many natural cases, the semantic effect of a syntactic rule can be separately analyzed, and the semantics of a system can be straightforwardly obtained by combining the semantic effects of each of its rules.

In this tutorial we will present the framework of Nmatrices and describe their various applications in reasoning under uncertainty and proof theory. In particular, we plan to cover the following topics:

1. Introduction: We will describe the motivation for introducing non-determinism into truth-tables of logical connectives, provide the basic definitions of the framework of Nmatrices and discuss their properties.

2. Proof Theory: Non-deterministic semantics is a useful tool for characterizing syntactic properties of proof systems, such as (syntactic) analyticity (By syntactic analyticity of a calculus G we mean intuitively that whenever a proposition s is provable in G from a set of assumptions S, then it is also possible to prove s from S in G using only the "syntactic material" available in S and s), cut-admissibility, invertibility of rules, etc. We present several classes of proof systems for which this tool can be successfully applied. One such example is canonical sequent calculi, which in addition to the standard axioms and structural rules have only logical rules in which exactly one occurrence of a connective is introduced and no other connective is mentioned. Cut-elimination in these systems is fully characterized by a simple constructive criterion called coherence. Moreover, there is a remarkable correspondence in these systems between the criterion of coherence, cut-elimination, analyticity and a semantic characterization of these systems in terms of two-valued Nmatrices. Another interesting link is between invertibility of logical rules in such calculi and the determinism of the corresponding two-valued Nmatrix. We will also examine other examples of systems for which the tool of non-determinism (combined in many cases with Kripke-style semantics) can be applied to characterize syntactic properties. These include (i) canonical labelled calculi ([19, 11, 12]), (ii) quasicanonical sequent calculi ([1]) (iii) basic sequent calculi ([4]), which include calculi for intuitionistic and modal logics, and (iv) canonical hypersequent calculi ([5]), which include the standard hypersequent calculus for Gödel logic.

3. Paraconsistent Reasoning: Paraconsistent logic is a logic for handling inconsistent information. One of the oldest and best known approaches to paraconsistency is da Costa's approach ([16, 17, 18]), which seeks to allow the use of classical logic whenever it is safe to do so, but behaves completely differently when contradictions are involved. This approach has led to the introduction of C-systems ([13, 14]), which employ a special unary connective for referring to consistency of propositions in the object language. We will demonstrate how the framework of Nmatrices can be used to provide simple, modular and analytic semantics for practically all the propositional C-systems considered in the literature (for the systems with finite-valued semantics, an algorithm for constructing such semantics was implemented in PROLOG [15]). Moreover, we describe an algorithm

for a systematic generation of cut-free sequent calculi out of the obtained semantics for all these logics ([2], [3]).

4. The First-Order Case and Beyond: We show how the framework of Nmatrices can be extended to languages with quantifiers and discuss shortly the encountered problems that are not evident on the propositional level. We show the applications of the first-order framework for paraconsistent logics ([8]) and proof theory ([9, 19]).

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3.2.4 Logic for the Blind as a Stimulus for the Design of Innovative Teaching Materials

Laurence Goldstein
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I. The value of logic. Learning logic via the extended mind. The relevance of Howard Gardner's theory of multiple intelligences.

W.V. Quine famously said "Logic is an old subject, and since 1879 it has been a great one" (p.vii of the preface to the first two editions of *Methods of Logic*). Quine was wrong and, to his credit, withdrew that remark from subsequent editions of that text. Not only (as Putnam pointed out) is the remark a slight to Boole; it also betrays ignorance of a wonderful mediaeval tradition that included Bradwardine, Buridan and Ockham and, most importantly, ignores Aristotle, the father of logic, who had the sensational idea that, like plants and animals, the content of what comes out of people's mouths when they are debating, or reasoning to a scientific conclusion, can be taxonomized. And, further, that bad arguments can, with mathematical precision, be objectively shown to be such, thereby decisively settling disputes. Aristotle, of course, discussed only a rather narrow range of arguments, but his work held sway and was rightly revered for over two thousand years. The subject that Aristotle inaugurated is among those like History, Literature and Physics, that are not only of deep interest, but are totally absorbing and can shape a person's personality and outlook on the world.

Those of us who are practising logicians know the extent to which we are reliant on writing things down and on manipulating symbols. In the jargon of a now fashionable view, we extend our minds because such manipulation that occurs outside the cranium is indispensable to developing ideas and proving results. In this way, the discipline is different from music, where great composers like Mozart can create whole symphonies in their heads, and juggling notes on paper is not the typical method of composing. In logic, we devise notations that are concise, economical and not unwieldy. Frege defends his two-dimensional concept-script (Begriffsschrift) on the grounds of its perspicuity. He argues that the separate contents are clearly separated from each other, and yet their logical relations are easily visible at a glance (CN: 97; Kanterian 51-3). He further argues that written signs last longer than sounds, have sharp boundaries and are thus excellent tools for precise inference. Sounds are temporal and so do not reflect logical relations which, according to Frege are best displayed pictorially, invoking spatial intuition (Kanterian: 53). All this, of course, is bad news for the blind logician. But, reverting to our comparison with music, Beethoven, when deaf, was able to hear the music in the written score. Conversely, one might hope that the blind logician may be able to utilize a modality other than sight for doing logic. In the terminology of Howard Gardner's theory of multiple intelligences, this would be a matter of invoking another intelligence to reduce reliance on the spatial-visual intelligence.

II. An illustration: A device by means of which blind people learn syllogistic: rationale, design, testing for effectiveness. Capturing beauty.

I shall demonstrate a device of my own design, built in the Haking Wong Engineering workshop of the *University of Hong Kong*, that is a tactile counterpart of the Venn-diagrammatic method of testing syllogisms for validity. Because there is an effective method for determining the validity of an arbitrary syllogism, it is easy enough to devise a computer program such that the blind user could type in 3 sentences (the premises and conclusion), hit a button, and be supplied with the verdict 'valid' or 'invalid'. The educational value of this to the user would be close to zero, because the user would learn nothing about associating such sentences to relations between classes, or of existential import, or of the notion of the containment of a conclusion within the premises etc.. In other words, the user would not come to understand what is fascinating and beautiful about syllogistic. Any device that one invents needs to inculcate deep (as opposed to superficial) learning, while also being easy to use. Almost inevitably, prototypes will turn out to be defective or non-optimal in certain respects, and there may be a lengthy process of refining the design through repeated testing. The same is true of the users manual.

III. Extension of the dog-legged approach to the design of innovatory teaching materials.

Having invested a lot of thought into the design of teaching some aspect of logic to the blind user, some obvious questions present themselves. Will the device be a suitable learning instrument for the sighted user; in other words, is it more effective than traditional methods for teaching this particular aspect of logic? Is there some way of incorporating what one has learned from designing for the blind into a new device to be used by the sighted? How far can this two-stage or dog-legged design methodology be extended to different modalities or 'intelligences', to different areas of learning, to learners across the age spectrum? In summary, the dog-legged design process is this: 1) Identify some part of the syllabus that is taught by some traditional means, e.g. by book learning, where you feel the traditional teaching methods to be stodgy or ineffective; 2) Construct learning material X (it may be a piece of apparatus, a competitive or collaborative activity for two or more students, an interactive computer game, etc.) for a target group of students that suffers some real or imaginary cognitive deficit. The use of X will engage a range of intelligences different from that invoked by the traditional teaching method; 3) Construct a new apparatus, son-of-X that preserves all the pedagogical advantages of X but which also features elements that enhance the learning experience of students who do not suffer the cognitive deficit mentioned in 2; 4)Test the effectiveness of the new apparatus against traditional methods of teaching. Effectiveness is measured not just by the speed at which the student solves various problems, but by the depth of the knowledge imparted. (Testing for depth of learning is a by no means trivial task.)

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3.2.5 Hybrid Logics

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First Steps in Hybrid Logics

These lectures introduce hybrid logics, a family of modal logics in which it is possible to name states (or times, or worlds, or situations, or nodes in parse trees, or people – indeed, whatever it is that the elements of the model are taken to represent).

The course has three major goals. The first is to provide an introduction to modal logics and then convey, as clearly as possible, the ideas and intuitions that have guided the development of hybrid logics. The second is to introduce a concrete skill: tableau-based hybrid deduction. The third is to say a little about the history of the subject, link it to philosophical work of Arthur Prior, and discuss its connections with classical first-order logic. No previous knowledge of hybrid logics is assumed, but I will do assume basic knowledge of propositional and first-order logic.

The lecture outline is as follows:

- **Lecture 1:** From Modal to Hybrid logics. What are Modal Logics. Names in a propositional language. How hybrid logics were born. Syntax, semantics, expressive power. (A bit about) Complexity.
- **Lecture 2:** Hybrid deduction. Propositional Tableaux. Dealing with multiple states. Labeled and Hybrid Tableaux Soundness and Completeness.

- **Lecture 3:** Prior and the connection with first-order logic. The foundational work of Prior Modal and Hybrid Logics as first-order fragments.

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3.2.6 Psychology of Reasoning

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This introductory course will present the main lines of empirical research on the psychology of reasoning of the last decades, by focusing on the points of contact with concepts and themes currently prominent in logic, and by making use of logical tools to clarify the psychological results and discussions. The main topics covered are: reasoning with conditionals, syllogistic reasoning, monotonic vs. non-monotonic reasoning. After attending the course, participants will be familiar with the main themes and empirical results on human reasoning, as investigated by experimental psychologists. It presupposes no more than basic knowledge of propositional logic, syllogistic, and the model-theoretic definition of logical consequence: as such, it is accessible to a wide audience, and requires no previous knowledge of research in psychology.

An often repeated slogan is that logic is the science of correct reasoning. But how does logic in fact relate to human reasoning? Does logic, or particular logical systems, offer an accurate descriptive account of how humans reason? Or is logic supposed to have a prescriptive import for human reasoning? These are some of the most fundamental philosophical questions pertaining to logic. Now, since the 1960s, experimental psychologists have been conducting extensive empirical studies of how human agents reason, and it seems clear that logicians and philosophers of logic have much to benefit from familiarity with this body of literature. After all, the phenomena being analyzed are very much the same: arguments, inferences, the concept of validity etc. And yet, contacts between psychologists, on the one hand, and logicians and philosophers, on the other hand, have been scarce.

One of the main themes having emerged from research on the psychology of reasoning of the last decades is the marked discrepancy between participants' performances in reasoning tasks during the experiments and the normative responses to these tasks, as determined by the canons of traditional (deductive) logic (Evans 2002). These deviances from the 'norm' are conceptualized in terms of the concept of 'reasoning biases', which are systematic reasoning tendencies such as e.g. to take into account the believability of the conclusion to evaluate the correctness of an argument. These results have important implications both for logicians and for philosophers: what is (classical) logic about, if it (arguably) does not describe how people in fact reason? And given these results, in what sense is logic prescriptive for reasoning? Is it possible to develop logical systems which would provide a more accurate picture of human reasoning?

The proposed course, intended as an introductory course, will present the main lines of research in the psychology of reasoning since the 1960s by focusing on the points of contact with some concepts and themes currently prominent in logic and philosophy, and making use of logical tools to clarify the psychological results and discussions. After attending the course, participants will be familiar with the main themes and empirical results on human reasoning.

The course will cover the main lines of investigation in the psychology of reasoning of the last decades: reasoning with conditionals, in particular variations of the famous Wason selection task (Evans 2002; Stenning and van Lambalgen 2008; Counihan 2008); syllogistic reasoning, in particular the studies on some so-called reasoning biases such as belief bias and matching bias (Evans 2002; Dutilh Novaes 2013; Counihan 2008); reasoning with abstract or contentual material (Dutilh Novaes 2013); defeasible vs. indefeasible reasoning, and the related concepts of non-monotonicity and monotonicity (Stenning and van Lambalgen 2008; Dutilh Novaes 2013).

Session (1) Historical and philosophical introduction to the relations between logic, argumentation, reasoning, thinking, human cognition and rationality.

Session (2) Reasoning with conditionals: the many different variations of the Wason selection task, descriptive vs. deontic conditionals, matching bias, the suppression task, conditionals as defeasible.

Session (3) Syllogistic reasoning: the effects of believability on the evaluation and production of arguments, a conceptualization of belief bias in terms of non-monotonicity.

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3.2.7 Truth-Values

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This tutorial provides a detailed introduction into the conception of truth values, an important notion of modern logical semantics and philosophy of logic, explicitly introduced by Gottlob Frege. Frege conceived this notion as a natural component of his language analysis where sentences, being saturated expressions, are interpreted as a special kind of names referring to a special kind of objects: the True (*das Wahre*) and the False (*das Falsche*). These are essentially the truth values of classical logic, which obey the principle of bivalence saying that there may exist only two distinct logical values. Truth values have been put to quite different uses in philosophy and logic and have been characterized, for example, as:

1. primitive abstract objects denoted by sentences in natural and formal languages,
2. abstract entities hypostatized as the equivalence classes of sentences,
3. what is aimed at in judgements,
4. values indicating the degree of truth of sentences,
5. entities that can be used to explain the vagueness of concepts,
6. values that are preserved in valid inferences,
7. values that convey information concerning a given proposition.

Depending on their particular use, truth values can be treated as unanalyzed, as defined, as unstructured, or as structured entities. Moreover, the classical conception of truth values can be developed further and generalized in various ways. One way is to give up the principle of bivalence, and to proceed to many-valued logics dealing with more than two truth values. Another way is to generalize the very notion of a truth value by reconstructing them as complex units with an elaborate nature of their own.

In fact, the idea of truth values as compound entities nicely conforms with the modelling of truth values in some many-valued systems, such as three-valued (Kleene, Priest) and four-valued (Belnap) logics, as certain subsets of the set of classical truth values. The latter approach is essentially due to J. Michael Dunn, who proposed to generalize the notion of a classical truth-value function in order to represent the so-called underdetermined and overdetermined valuations. Namely, Dunn considers a valuation to be a function not from sentences to elements of the set the True, the False but from sentences to subsets of this set. By developing this idea, one arrives at the concept of a generalized truth value function, which is a function from sentences into the subsets of some basic set of truth values. The values of generalized truth value functions can be called generalized truth values.

The tutorial consists of three sessions, in the course of which we unfold step by step the idea of generalized truth values and demonstrate its fruitfulness for an analysis of many logical and philosophical problems.

Session 1. The notion of a truth value and the ways of its generalization In the first lecture we explain how Gottlob Frege's notion of a truth value has become part of the standard philosophical and logical terminology. This notion is an indispensable instrument of realistic, model-theoretic approaches to logical semantics. Moreover, there exist well-motivated theories of generalized truth values that lead far beyond Frege's classical the True and the False. We discuss the possibility of generalizing the notion of a truth value by conceiving them as complex units which possess a ramified inner structure. We explicate some approaches to truth values as structured entities and summarize this point in the notion of a generalized truth value understood as a subset of some basic set of initial truth values of a lower degree. It turns out that this generalization is well-motivated and leads to the notion of a truth value multilattice. In particular, one can proceed from the bilattice *F OU R2* with both an information and a truth-and-falsity ordering to another algebraic structure, namely the trilattice *SI X T EEN3* with an information ordering together with a truth ordering and a (distinct) falsity ordering.

Session 2. Logics of generalized truth values In this lecture we present various approaches to the construction of logical sys- tems related to truth value multilattices. More concretely, we investigate the logics generated by the algebraic operations under the truth order and under the falsity order in bilattices and trilattices, as well as various interrelations between them. It is also rather natural to formulate the logical systems in the language obtained by combining the vocabulary of the logic of the truth order and the falsity order. We consider the corresponding first-degree consequence systems, Hilbert-style axiomatizations and Gentzen-style sequent calculi for the multilattice-logics.

Session 3. Generalized truth-values: logical and philosophical applications Besides its purely logical impact, the idea of truth values has induced a radical rethinking of some central issues in ontology, epistemology and the philosophy of logic, including: the categorial status of truth and falsehood, the theory of abstract objects, the subject-matter of logic and its ontological foundations, and the concept of a logical system. In the third lecture we demonstrate the wealth of philosophical problems, which can be analyzed by means of the apparatus of truth values. Among these problems are the liar paradox and the notion of hyper-contradiction, the famous slingshot-argument, Suszko thesis, harmonious many-valued logics, and some others.

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3.2.8 The Origin of Indian Logic and Indian Syllogism

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India, which has a different culture and language, is one of the oldest civilizations of the world history today. And there is no doubt that predominant way of thinking in India is closely related with occurring philosophical thinking and logic in India.

Today, when we say Indian Philosophy and Indian Logic the first question we think about is, if there is a Western way of philosophical thinking in India and study of logic connected to it. Some say there is not. And some say there is. Eventually, this question obviously shows us that the study of different cultures and their ideologies is very important for understanding the interaction between civilizations and to have chance to find the differences, similarities and parallelism between their ways of thinking.

In this study we will discuss the The Origin of Indian Logic and Indian Syllogism. The importance of this study for us is the belief that, this study will be an important source in the future field of the studies of ours on comparing philosophical thoughts and logic.

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3.2.9 Logical Forms

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Frege generalized the basic intuition of traditional logic that every sentence is of subject-predicate form through two main moves: the analysis of sentences in terms of function and arguments, and the analysis of quantification as higher-order predication. Combining these two ideas, every linguistic assertion, and every logical formula, can be interpreted in different ways as a predication involving one or more subjects. This has important applications to theories of truth and falsity, theories of description, as well as to other logical and philosophical issues. In the three tutorials I will discuss the following subjects:
I. Predicates and properties. States of affairs. Truth and falsity.
II. Logical properties. Logical forms. Logical truth and logical states of affairs.
III. Descriptive terms and descriptive properties. Senses and propositions. Truth as instantiation.

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3.2.10 An Introduction to Arabic Logic

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In this tutorial, I will present an analysis of Arabic Logic by showing some of its multiple aspects and topics. However, due to the lack of space and time, and given the wide scope of this field, I will focus on three main logicians who are Al Farabi (873-950, AD), Avicenna (980-1037, AD) and Averroes (1126-1198, AD) and will present their works by comparing their different views on three main topics, namely syllogistic, propositional logic and modal logic. Nevertheless, I will introduce briefly in the first session the other logicians and will present a chronology of the main authors and schools, following Nicolas Rescher's *The development of Arabic Logic* (1964, Arabic translation, 1985) and Tony Street's *Arabic Logic* (2004). The Arabic treatises were very influenced by Aristotle's texts, and their Greek commentaries, which makes N. Rescher say that Arabic logic is not eastern at all but western. Their works were mainly commentaries of the Aristotelian treatises (a notable exception is Avicenna, who presented his system without focusing on the Aristotelian corpus, although he did know it). They were familiar with Porphyry's text, which they usually comment in the introduction of their own treatises, and also with the Stoïc Logic that they generally include inside their correspondents of the Prior Analytics (called in general Al Qiyas = literally: *The Syllogism*). Many of them focused a lot on Modal Logic because of its relation with Metaphysics. However, in this field and in others, we find many differences between our main logicians, for Avicenna's logic seems to be different from Al Farabi's and Aristotle's ones because Avicenna introduced many original linguistic analyses and distinctions. These new subtleties were rejected by Averroes who tried, in his own work, to return back to the original Aristotelian text and to be as faithful as possible to it. While focusing on these topics and on the three authors chosen, I will try to determine the characteristics of this logical tradition and its relations to Ancient logic as well as to Medieval Logic. I will leave out, though, the study of the inductive and analogical arguments, which were parts of their logical works. The main questions that I raise are the following: What are the Arabic Logicians' contributions, whether in propositional logic or in syllogistic or in modal logic? How are the main logical connectives (and logical notions in general) defined? To what extent are these systems formal? What sense of formality is privileged?

Session 1. A brief historical presentation of the Arabic logicians, followed by a study of their syllogistics and their characteristics.

Session 2. Propositional logic in the three systems chosen and analysis of the definitions of the logical constants and of the hypothetical syllogisms presented in each of them.

Session 3. The modal logics in the three systems; links and differences between these systems and Aristotle's one as well as the Medieval ones.

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3.2.11 Quantum Cognition

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Quantum mechanics is one of the most successful theories ever developed in science. However, ever since it was initially developed, about one hundred years ago, its meaning is still the subject of intense debate. But despite its interpretationalproblems, recently researchers started successfully applying the apparatus of quantum mechanics to the social sciences, such as quantum cognition and quantum finances.

In this tutorial we will start the first day with a quick overview of the quantum mechanical formalism. Then, in the second day we will discuss what makes quantum mechanics "quantum", i.e. how it departs from classical physics, and we will show how it violates Kolmogorov's axioms of probability, bringing even deeper conceptual and philosophical issues than some of the problems raised by the founders of quantum mechanics. Finally, in the last section we will present quantum models of cognition and finances, show how they can better fit empirical data, and end with somediscussions of how to think of quantum cognition as a theory of limited irrationality.

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3.2.12 Towards a General Theory of Classifications

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Classification problems are one of the basic topics of scientific research: in mathematics and physics, as in natural sciences in general, in social sciences and, of course, in the domain of library and information sciences, taxonomies are very useful to organize an exponentially increase of knowledge and to perform information retrieval. But, from a strictly mathematical viewpoint, classes are also concrete sets that need a general theory, whose foundation might be different from that of usual sets.

The main purpose of this tutorial is not to provide a complete exposition of a perfect mathematical theory of classifications, that is, a general theory which would be available to any kind of them: hierarchical or not hierarchical, ordinary or fuzzy, overlapping or not overlapping, finite or infinite, and so on, founding all possible divisions of the real world. For the moment, such a theory is but a dream.

Our aim is essentially to expose the state of art of this moving field. We shall speak of some advances made in the last century, discuss a few tricky problems that remain to be solved, and, above all, show the very ways open for those who do not wish to stay any longer on the wrong track.

The three parts of the tutorial are the following ones:

1. History of classifications and epistemological problems. Here we shall begin with a historical overview of the whole domain, recalling the long history of classifications, from the Greek (Plato and Aristotle) to the most modern forms of clustering analysis, through the advances made in the 18th century (Kant's logic of natural classifications) and 19th century (librarian classifications), until the emergence of the idea of a general theory (by Auguste Comte).

2. Exposition of some formal models and search for a unified language. In this second session, we shall introduce the mathematics of finite classifications, which is based, since G. Birkhoff, on the concepts of order theory (partitions, chains of partitions, semilattice of chains...). We shall study, before all, the well-known domains of hierarchical classifications (Barbut, Monjardet, Benzecri, Lerman), but also have a look at some kinds of overlapping ones (Barthelemy-Brucker). In the end, we shall try to show how the construction of a "metaclassification", i.e. the representation of all kinds of classifications as ellipsoids in a plane, may give a common basis to depict any of them, even if,

unfortunately, the underlying topology does not bring any useful metric to go further.

3. Towards a general theory of classifications. After the impossibility theorem (Kleinberg 2002), it has become obvious that the solution of classification problems cannot lie in some empirical clustering analysis or computer science but in a true algebra of classifications and their transformations. One of the main problems we meet there is the fact that an algebra of classifications is, in principle, a commutative but nonassociative algebra. Of course, we can find some models in reverse polish notation (Lukasiewicz) or in new forms of parenthesized products (Wedderburn-Etherington). Also, some new kinds of associative algebras (dendriform algebras) could possibly apply in the case of trees. But no strict algebra of classifications actually exist until now. Another problem for a general theory of classifications would be to connect what usually appears as completely distinct domains (clustering analysis, logics (classification theory) and pure mathematics (category theory). For that, we have to consider not only finite but also infinite classifications, and so, meet tricky set-theoretical problems. A conjecture of Neuville is that we can construct the continuum of real numbers from the infinite set of classifications.

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3.2.13 Connecting Logics

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This tutorial will give a gentle introduction to the field of combining logics, with a specific focus on the technique of E-connections.

E-connections is a methodology for combining logics with a rather intuitive semantics, being inspired by counterpart theory. It moreover is quite well-behaved computationally in the sense that the combination of decidable formalisms is again decidable, and which, nonetheless, allows for non-trivial interaction between the combined logics.

We begin by briefly outlining some of the more well-known techniques for combining or extending logics, namely fusions, products, fibrings, and concrete domains. We then outline the basic ideas behind E-connections, in which a finite number of formalisms are connected by relations relating entities across different domains, intended to capture different aspects or representations of the 'same object'. For instance, an 'abstract' object of a description logic can be related via a relation R to its life-span in a temporal logic as well as to its spatial extension in a spatial logic.

We discuss the basic differences to the other combination methodologies introduced and then proceed to present E-connections in more technical detail. In particular, we introduce the framework of 'abstract description systems', a specific 'lightweight' form of abstract logic generalising the basic syntactic and semantic features of many modal and descprition logics. This allows us to study general properties of E-connections in a logic independent way. We show how this abstract presentation of E-connections can be used to prove general decidability preservation results and finally illustrate the usefulness of the framework in several application areas, including modularising web ontologies and combining spatio-temporal with conceptual modelling and reasoning.

Three Sessions

I. Combining Logics: Fusions, Products, Fibrings, Concrete Domains, E-connections.

II. E-connections of abstract description systems.

III. Computational properties and applications of E-connections.

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3.2.14 Relativity of Mathematical Concepts

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PUC-RJ – Brazil

Naturality and Universality are key concepts in Mathematics. Category Theory CT was created with the aim of providing means to precisely define naturality in mathematics. Universal properties play a distinguished role in CT. This concept seems to be firstly formalized by Pierre Samuel in 1948 and used in Bourbaki afterwards, although the Bourbaki group did not use Category Theory in their publications. Almost every important categorical construction is defined by means of an universal property: products, sums, limits and co-limits in general, exponentials, subobject classifiers and adjunctions. Regarding this last construction, the well-known "adjoint situations arise everywhere" shows how worth Naturality and Universality are in CT.

Usually the mathematical discourse is ontologically based on parts of Set Theory. The aim of this tutorial is twofold. One goal is to briefly show how the use of universal properties can induce natural equivalences and how both provide interesting logical constructions that can be done inside a class of categories defining what is known as internal logic. The class of categories that having a rich enough internal logical can serve as alternative ontological basis for the mathematical discourse. These categories are known as Topoi. The second goal of this tutorial is to show how an apparently intuitive mathematical concepts, the concept of finiteness, may not be equivalent under equivalent definitions known from Set Theory.

A further discussion on the distinction between external and internal logic on categories follows through some examples. The use of Category Theory to provide semantics for logical systems is lastly discussed as appear.

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3.2.15 Undecidability and Incompleteness are Everywhere

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We prove a version of Rice's Theorem for the language of classical analysis. Main points are a construction of explicit expressions for the halting function (the function that settles the halting problem) in the language of classical analysis, and extensions of those results to all complete arithmetic degrees. We extend these results to incompleteness results for several axiomatic systems.

Main topics to be covered:

Suppes predicate axiomatics for portions of physics. Solution of Hirsch's Problem: is there an algorithmic decision procedure for chaotic systems? Solution to Arnol'd's 1974 Hilbert Symposium Problems: is there a decision procedure for the nature of equilibrium - stable or unstable - for autonomous polynomial dynamical systems? Proof of the undecidability of Nash games and applications to economics.

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3.2.16 Logic, Algebra and Implication

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Carles Noguera
Academy of Sciences of the Czech Republic – Czech Republic

Abstract Algebraic Logic is a relatively new subfield of Mathematical Logic. It is a natural evolution of Algebraic Logic, a branch of Mathematical Logic studying logical systems by giving them an algebra-based semantics. It can be traced back to George Boole and his study of classical propositional logic by means of a two-element algebra that became its canonical semantics. Other non-classical logics enjoy a strong connection with algebras as well intuitionistic logic and Heyting algebras, substructural logic and residuated lattices, etc.). Abstract Algebraic Logic (AAL) was born as the natural next step to be taken in this evolution: the abstract study of logical systems through their interplay with algebraic semantics.

One of the starting points of AAL is the book by Helena Rasiowa [6] where she studies logics possessing a reasonable implication connective. Her approach was later gradually generalized into a genuinely abstract theory [1,4,5]. A crucial technical notion used is this process is the Leibniz operator, which maps any theory of a logic to the congruence relation of the formulae which are provably equivalent in the presence of such theory. Logics were classified by means of properties of Leibniz operator, which gave rise to the two-dimensional Leibniz hierarchy. This classification, starting with the biggest class of protoalgebraic logics, became the core theory of AAL due to its robustness, the characterizations of its classes, and their usefulness for obtaining bridge theorems, i.e. results connecting logical properties to equivalent algebraic properties in the semantics.

The aim of this course is to present a self-contained introduction to AAL. For didactic reasons we present the full Leibniz hierarchy at the end of the tutorial only and, for most of the time, we simplify our account to yet another generalization of Rasiowa approach: the class of weakly implicative logics [2,3]. Although this can be viewed as a rather minor generalization, it provides a relatively simple framework which allows us to describe the arguably more important dimension of Leibniz hierarchy and to demonstrate the strength of existing abstract results.

Session 1 Basic notions of algebraic logic: formulae, proofs, logical matrices, filters. Completeness theorem w.r.t. the class of all models. Implications and order relations in matrices. Lindenbaum-Tarski method for weakly implicative logics: Leibniz congruence, reduced matrices, and completeness theorem w.r.t. the class of reduced models.

Session 2 Advanced semantical notions: closure operators, closure systems, Schmidt Theorem, abstract Lindenbaum Lemma, operators on classes of matrices, relatively (finitely) subdirectly irreducible matrices(RFSI). Completeness theorem w.r.t. RFSI reduced models. Algebraizability and order algebraizability. Examples.

Session 3 Leibniz operator on arbitrary logics. Leibniz hierarchy protoalgebraic, equivalential and (weakly) algebraizable logics. Regularity and finiteness conditions. Alternative characterizations of the classes in the hierarchy. Bridge theorems (deduction theorems, Craig interpolation, Beth definability).

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3.2.17 Hypersequents and Applications

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Hypersequents constitute a natural generalization of ordinary sequents. They may be thought as finite multisets (or sequences) of usual sequents. Obviously, every standard sequential calculus is also a hypersequential calculus. Hypersequents were first considered by G. Pottinger in 1983 an independently by A. Avron in 1987.

In Gentzen systems, the cutelimination theorem is the central result which establishes the importance of this kind of systems. Intuitively, the cutelimination theorem states that any assertion that possesses a proof in the (hyper)sequential calculus that makes use of the cut rule also possesses a cutfree proof, that is a proof that does not make use of the cut rule. A of cutelimination theorem in a (hyper)sequent calculus for a given logic is desirable because of its important consequences such as the consistency of the logic and interpolation properties. Hypersequent showed to be a very suitable tool for presenting cutfree Gentzen type formulations of logics. The elimination for some systems of other rules such as contraction was already studied in the literature.

Session 1. We will present an introduction to hypersequents. Definitions and basic properties will be presented.

Session 2. Some examples of hypersequential calculi will be analyzed. In particular, hypersequents for the fourvalued logic T ML (tetravalent modal logic) will be presented as well as a cutelimination theorem for this calculus.

Session 3. Formal hypersequents will be studied. Departing from a formal presentation of hypersequent in which meta-variables for contexts (i.e., sequents) are introduced in the language, hipersequential calculus will be presented as objects within a suitable category, along with its morphisms. Applications to this formal view of hypersequents will be exhibit.

Prerequisistes. This is an introductory course and as such will be highly self-contained. Students will only be assumed to have a basic knowledge of classical propositional logic and some rudiments of category theory.

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3.2.18 Introduction to Modern Metamathematics

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This tutorial will consist of three lectures on metamathematics. Metamathematics is the study of what is possible and what is impossible in mathematics, the study of unprovability, algorithmic undecidability, limits of mathematical methods and "truth".

I will go through the history of metamathematics and explain what people believed in different eras of the history of metamathematics: pre-Gödelean history, and at various stages of post-Gödelean history.

I will start with four old scenarios that a metamathematical result may follow ("Parallel Worlds", "Insufficient Instruments", "Absence of a Uniform Solution" and "Needed Objects Don't Yet Exist").

Then I will talk about Gödel's theorems and modern developments: the Paris-Harrington Principle, Harvey Friedman's machinery, Andreas Weiermann's Phase Transition Programme and my own recent results, some joint with Michiel De Smet and Zachiri McKenzie. I will give some sketches of proofs but will not overload the lectures with technical details. Instead I will concentrate on new and old ideas in modern unprovability theory and explanations about the methods of finding and proving unprovability.

Here are some important questions that will guide us throughout this tutorial. How does one prove that something is unprovable? What are the reasons for unprovability? What are the sources of unprovability in mathematics? Is it possible to argue in one way and get the answer "yes" to a mathematical question and then reason in another, incompatible way and get the answer "no"?

I am planning to make these lectures very simple and accessible to the widest possible audience.

3.2.19 Erotetic Logics

Andrzej Wiśniewski
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The term *erotetic logic* is often understood as synonymous to the logic of questions. There is no common agreement as to what erotetic logic should be. The most developed proposals will be overviewed and compared. Then a general setting, the Minimal Erotetic Semantics (MES), will be presented;

kinds of answers to questions, types of their presuppositions, basic relations between questions, and certain normative concepts pertaining to questions will be characterized in terms of MES. Next, conceptual foundations of Inferential Erotetic Logic (IEL) will be discussed. IEL focuses its attention on erotetic inferences, that is, roughly, inferences which have questions as 'conclusions'. Some of these inferences are intuitively valid; we will show how IEL explicates the relevant concept of validity.

We will also address some more technical issues. First, we will consider models of problem decomposition, offered by Hintikka's Interrogative Model of Inquiry and by IEL. Second, a certain proof method grounded in IEL, the Socratic proofs method, will be presented.

Finally, the idea of erotetic logic as a theory of internal question processing will be discussed.

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3.2.20 History of Paraconsistent Logic

Evandro Luís Gomes
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In this tutorial, we will present some research results concerning the history of paraconsistent logic. In order to develop such discussion, we will focus on the history of the Principle of Non-Contradiction and also on that of the ex falso sequitur quodlibet rule. Such two issues are strictly connected and its analysis has offered a valid ground to a logical history of paraconsistent positions all along western thought tradition.

The outline of this tutorial is as follows.

First, we will study some passages from the Ancient Greek logic legacy in which we have found several theoretical positions, inference schemata and the logical rules usage, which can be interpreted today as being part of paraconsistent approach. We will analyze some classical reductio ad absurdum inference schemata used by Zeno of Elea, Plato and Aristotle. Such classical approach contrast with paraconsistent positions found in Heraclitus and even in Aristotle. We also present that ex falso sequitur quodlibet, a classical thesis related to trivialization, as far as we know, although could not be deduced in Stoic logic, it seems coming from this tradition.

Second, we will introduce textual evidence concerning mediaeval logic which can fix some misunderstandings still extant in some historical studies on paraconsistent logic. We will give special attention to claims of Peter Abelard, Adam of Balsham, William of Soissons, Petrus Hispanus and William of Ockham. All these authors seem supporting paraconsistent positions. Some of them work in a full-fledged logical perspective; others work on behalf of preserving theological matters of falsity and trivialization. The medieval theory of consequences is the theoretical setting in which such disputations took place.

Third, we will outline contemporary history of paraconsistent logic in order to rescue the important role played by some forerunners and especially by the founders of this logical field of study. On the basis of new historiographical foundation, we intend to show that a pure chronological way of thinking the history of the paraconsistent logic leads to equivocal conclusions. We intend to present that such approach, supported in historical context of contemporary logic, is not only more accurate but also more full and reasonable.

Section 1 Ancient Greek Logic and Paraconsistency.

Section 2 Some Paraconsistent Positions in Medieval Logic.

Section 3 A Concise History of Contemporary Paraconsistency.

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3.2.21 Institutions

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Institution theory is a major model theoretic trend of universal logic that formalizes within category theory the intuitive notion of a logical system, including syntax, semantics, and the satisfaction relation between them. It arose within computing science, especially specification theory [1], as a response to the population explosion of logics there and where it has become the most important foundational theory Later on institution theory has been succesfully used in pure logic studies in the spirit of universal logic. This means the development of model and proof theory in the very abstract setting of arbitrary institutions, free of commitement to a particular logical system [2]. In this way we gain freedom to live without concrete models, sentences satisfaction, and so on, we gain another level of abstraction and generality and a deeper understanding of model theoretic phenomena not hindered by the largely irrelevant details of a particular logical system, but guided by structurally clean causality. The latter aspect is based upon the fact that concepts come naturally as presumed features that a "logic" might exhibit or not and are defined at the most appropriate level of abstraction; hypotheses are kept as general as possible and introduced on a by-need basis, and thus results and proofs are modular and easy to track down regardless of their depth. The continuous interplay between the specific and the general in institution theory brings a large array of new results for particular non-conventional, unifies several known results, produces new results in well-studied conventional areas, reveals previously unknown causality relations, and dismantles some which are usually assumed as natural. Access to highly non-trivial results is also considerably facilitated. The dynamic role played by institution theory within the wider universal logic project is illustrated by the fact that institution theory papers have come second and first, respectively, in the contests of the Montreux (2005) and Xi'and (2007) UNILOG.

In this tutorial we will start with a brief explanation of the historical and philosophical origins of institution theory, followed by a presentation of its basic mathematical concepts. We will also have a trip through the rather rich body of methods and results of the institution theoretic approach to logic and model theory. Although institution theory is primarily a model theoretic approach we will also discuss recent proof theoretic developments in the area. However our real emphasis will be not on the actual mathematical developments but on the non-substantialist way of thinking and the top-down methodologies promoted by institution theory, that contrast sharply the substantialist view and the bottom-up methodologies that pervade and underly conventional logic, this being the most profound message of institution theory as a universal logic trend.

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3.2.22 Description Logics

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This tutorial is an introduction to Description Logics in the context of knowledge representation and reasoning. Description Logics (DLs) are a family of logic-based knowledge representation formalisms with interesting computational properties and a variety of applications. In particular, DLs are well-suited for representing and reasoning about terminological knowledge and constitute the formal foundations of semantic web ontologies. Technically, DLs correspond to decidable fragments of first-order logic and are closely related to modal logics. There are many different flavors of description logics with specific expressiveness and applications, an example of which is ALC and on which we shall focus in this tutorial.

The outline of the tutorial is as follows: We start with an introduction to the area of Knowledge Representation and Reasoning (KRR) and the need for representing and reasoning with terminological knowledge, which stands as the main motivation behind the development of DLs. We then present the description logic ALC, its syntax, semantics, logical properties and proof methods. In particular we make explicit the aforementioned relationship between ALC and other logical frameworks. Finally we illustrate the usefulness of DLs with the popular Protégé ontology editor, a tool allowing for both the design of DL-based ontologies and the ability to perform reasoning tasks with them.

Lecture 1: Introduction to KRR and DLs; Introduction to the description logic ALC

Lecture 2: The description logic ALC

Lecture 3: Formal ontologies in Protégé

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3.2.23 Ideospheres

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Le Laboratoire d'Informatique, de Robotique et de Microélectronique de Montpellier – France

This tutorial will present formal structures which often structure the development of ideospheres in Human Sciences. An ideosphere (Barthes77) is initiated by a founding speech and helps establish correspondences to other such speeches and take commitment and refusal positions in a system that is inachieved. In pedopsychiatry, an ideosphere is focus on early interactions between the infant and his environment, and examine the processes of semiotization, as well as the use of representation abilities as a means to communicate (Golse 99,07) (Dor 02).

These structures and their organization within a general system can be formalized with category theory, as is done for instance when modeling computation systems and relating different models. We show under what conditions they correspond to a formalization within modal logic of the system in use; at this point we will make a comparison with what is done with categorial models which relate various logics. Finally we develop the concepts of autonomy and learning, and use them to illustrate the presented mathematical tools and methods; this will help model zig-zag processes between various formalizations.

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3.2.24 Mathematical Fuzzy Logic

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Originating as an attempt to provide solid logical foundations for fuzzy set theory, and motivated also by philosophical and computational problems of vagueness and imprecision, Mathematical Fuzzy Logic has become a significant subfield of mathematical logic. Research in this area focuses on many-valued logics with linearly ordered truth values [2] and has yielded elegant and deep mathematical theories and challenging problems, thus continuing to attract an ever increasing number of researchers [5].

Fuzzy logics emerged from Zadeh's fuzzy set theory, which has become extremely popular in computer science and engineering, giving rise to a broad area of research, with countless applications. At the beginning of the 1990s, Petr Hájek started a "tour de force" to provide solid logical foundations for fuzzy logic. In his approach, soon followed by numerous researchers in mathematical logic, fuzzy logics were taken as non-classical many-valued deductive systems with a semantics given by totally ordered algebraic structures (typically based on t-norms on the real unit interval). Hajek's monograph [1] started the study of t-norm-based fuzzy logics by the methods of algebraic logic, thus giving birth to Mathematical Fuzzy Logic. The last decade has witnessed a significant development of Mathematical Fuzzy Logic, summarized in the new Handbook [3], and a proliferation of various systems of fuzzy logic.

The tutorial will follow Chapter I of the recently published Handbook of Mathematical Fuzzy Logic [4]. The electronic version of the chapter will be made available to the participants of the tutorial.

The tutorial will cover the following topics:

* Propositional logics of continuous t-norms: standard and general semantics, axiomatic systems, completeness theorems.

* Variations of basic propositional fuzzy logics: adding or discarding axioms or connectives.

* Families of fuzzy logics in the logical landscape: fuzzy logics among substructural logics, core fuzzy logics, fuzzy logics as algebraically implicative semilinear logics.

* Metamathematics of propositional fuzzy logics: completeness theorems, functional representation, proof theory, computational complexity.

* Predicate fuzzy logics: syntax, semantics, completeness, notable axiomatic theories.

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3.2.25 Logics of Plurality

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The correct logical analysis of plural terms such as the trees in the trees are similar or the trees are green is at the center of an important debate both in formal semantics and in philosophical logic. Two fundamentally distinct approaches can be distinguished, one on which the trees refers to a single collective entity, a plurality of trees, and one on which the trees refers plurally to the various individual trees. The first tradition is linked to the work of Link and related mereological approaches, the second to the work of Boolos and subsequent work in that tradition (Oliver, Yi, Rayo and others). This course will give an overview over the two kinds of approaches to the logical analysis of plural terms with its various developments and discusses the crucial linguistic empirical and conceptual motivations for the two kinds of approaches.

Session 1:

Reference to a plurality: The mereological approach

This session discusses the motivations and the development of the mereological approach such as that of Link and others. It presents a range of potential empirical and conceptual problems for that approach.

Session 2:

Plural Reference: The second-order approach

This session will discuss the seminal work of Boolos and subsequent developments such as the work of Oliver, Rayo, Yi. It focuses on the formal and conceptual aspects of that approach.

Session 3:

This session discusses potential extensions of the second approach, such as to to mass terms like courage, as in courage is admirable. It also discusses various ramifications of the plural reference approach and the challenges it faces from the point of view of natural language.

Vagueness being an important area of investigation in many fields (e.g. logic, philosophy, computer science) nowadays, various logics have emerged during the past few decades to address the topic e.g., fuzzy logics, rough logics and the theory of graded consequence. But in most of the existing literature, be it of logic or philosophy or computer applications vagueness is thought to occur at the level of object-language only. The metalogical notions are usually considered to be crisp. There was a claim made by Pelta in [11] that many-valuedness at the metalevel had been first explicitly considered by Marraud in [10]. But this claim is incorrect. There is a series of papers viz. [2,3,4,5] dealing with the theory of gradation of metalinguistic concepts like *consequence*, *consistency*, *tautologihood*, *completeness* and the like.

The notion was further generalized in [5] by the use of the product operator of a residuated lattice as the and-ing operator instead of the usual lattice meet. In the same paper, the notion of graded consistency was introduced. The idea of partial consistency or consistency to a degree is frequent in everyday encounters. The interface between the notions viz. graded consequence and graded consistency has been investigated.

The conceptual difference of the two makes significant difference in organizing a comprehensive theory of vagueness. Those who have followed and further developed the line of thought of Pavelka e.g. Hajek [9], Novak [6], Godo et al. [9] etc. have not taken up this issue either, except perhaps Gerla [7] who devoted a section on graded consequence in his book and Gottwald [12] who moved closer to this idea.

The present tutorial is an attempt to remove these confusions by

- clarifying the three levels,
- using appropriate symbolisms (viz. quotation mark for naming at level I and quotation mark   for naming at level II) to distinguish the levels,
- identifying the predicates, functions, connectives and quantifiers required at each level,
- probing into the interrelation, both syntactic and semantic, between the levels.

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3.2.26 Graded Consequence

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Graded Consequence

Fuzziness being an important area of investigation in many fields (e.g. logic, philosophy, computer science) nowadays, various logics, e.g., fuzzy logics, rough logics and the theory of graded consequence, have emerged during the past few decades to address the topic. But in most of the existing literature, be it of logic or philosophy or computer applications fuzziness is thought to occur at the level of object-language only. The metalogical notions are usually considered to be crisp. Since 1986, there is a series of papers viz. [1, 2, 3, 4, 5, 6] dealing with the theory of gradation of metalinguistic concepts like 'consequence', 'consistency', 'tautologihood', 'completeness' and the like. That this idea of keeping fuzziness away from its natural flow from object level to meta level would not properly capture a logic of imprecise information, was felt by a few more researchers [14, 12, 16, 18]. Following is a Gentzen-type axiomatization for the consequence relation, which was introduced in [3], in the context of imprecise information.

A graded consequence relation $|\!\sim$ is a fuzzy relation from $P(F)$, the power set of F to F satisfying conditions
$(GC1)$ If $\alpha \in X$ then $gr(X \mid\!\sim \alpha) = 1$,
$(GC2)$ If $X \subseteq Y$ then $gr(X \mid\!\sim \alpha) \leq gr(Y \mid\!\sim \alpha)$ and
$(GC3)$ $\inf_{\beta \in Z} gr(X \mid\!\sim \beta) * gr(X \cup Z \mid\!\sim \alpha) \leq gr(X \mid\!\sim \alpha)$,
where $gr(X \mid\!\sim \alpha)$ denotes the degree to which α is a consequence of X and, $*$ and inf, the monoidal composition and infimum operation of a complete residuated lattice are used to compute the meta-linguistic 'and' and 'for all' respectively.

In the same paper, the notion of graded (in)consistency was introduced. The idea of partial (in)consistency or (in)consistency to a degree is frequent in everyday encounters. The axioms of graded inconsistency are taken as:

- If $X \subseteq Y$ then $Incons(X) \leq Incons(Y)$
- $Incons(X \cup \{\neg\alpha\}) * Incons(X \cup Y) \leq Incons(X)$ for any $\alpha \in Y$.
- There is some $k > 0$ such that $\inf_\alpha Incons(\{\alpha, \neg\alpha\}) = k$,

where $INCONS$ is a fuzzy subset over X, an arbitrary set of formulae, denoting 'the degree of inconsistency of X'.

Assuming the presence of negation in the language the (GC) axioms also have been extended by adding the following two axioms.
$(GC4)$ There is some $k > 0$ such that $\inf_{\alpha,\beta} gr(\{\alpha, \neg\alpha\} \mid\!\sim \beta) = k$.
$(GC5)$ $gr(X \cup \{\alpha\} \mid\!\sim \beta) * gr(X \cup \{\neg\alpha\} \mid\!\sim \beta) \leq gr(X \mid\!\sim \beta)$.

The interface between the notions viz. graded consequence and graded inconsistency has been investigated.

Although in Pavelka's seminal work [15] there are traces of these ideas, the main theme of his work being different, the above issues did not surface. Pavelka axiomatized consequence operator C in the Tarskian tradition in the following way:
A consequence operator C in fuzzy context is a function from the set of all fuzzy subsets $(\mathcal{F}(F))$ over F to $\mathcal{F}(F)$ satisfying the following conditions.
1. $X \subseteq C(X)$,
2. if $X \subseteq Y$ then $C(X) \subseteq C(Y)$ and
3. $C(C(X)) = C(X)$,

where X is a fuzzy subset of premises and $C(X)$ is the fuzzy subset of consequences of X.

The question, however, is "What does the value $C(X)(\alpha)$ indicate? Is it the degree to which α follows from X? That is, is it a value of the process of derivation or is it a value that is assigned to α through the derivation process? C is after all a crisp function on the domain of fuzzy subsets."

The conceptual difference of the two makes significant difference in organizing a comprehensive theory of vagueness. Those who have followed and further developed the line of thought of Pavelka e.g. Haj́ek [10], Novak [13], Godo et al. [11] etc. have not taken up this issue either, except perhaps Gerla [8] who devoted a section on graded consequence in his book and Gottwald [9] who moved closer to this idea.

A comparison between existing fuzzy logic approaches (Pavelka, Hajek et. al) and the approach of graded consequence, and a scrutiny of the two reveal that
1) these are not identical methods,
2) there are confusions and arbitrariness in the usage of algebraic operators for evaluating values of meta-level expressions, and more importantly,
3) three levels are involved in the discourse of such logics, viz.
a) object level, comprising of well formed formulae with terms interpreted generally but not necessarily, as vague notions;
b) meta level I, required to talk about object level items, the predicates etc. of this level being in general vague as well;
c) meta level II, required to talk about and/or define the notions of meta level I.
These three levels usually appear blurred in the discourses.

The present tutorial is an attempt to remove these confusions by
- clarifying the three levels, using appropriate symbolisms (viz. quotation mark ‘ ’ for naming at level I and quotation mark $\langle\ \rangle$ for naming at level II) to distinguish the levels,
- identifying the predicates, functions, connectives and quantifiers required at each level,
- probing into the interrelation, both syntactic and semantic, between the levels.

The set of axioms for graded consequence under such symbolism takes the form
(i) $gr(^{\langle}$ ‘X’ $\in$ ‘α’ $^{\rangle}) \leq gr(^{\langle}$ ‘X’ $|\!\sim$ ‘α’ $^{\rangle})$.
(ii) $gr((^{\langle}$ ‘X’ $\subseteq$ ‘Y’ &‘X’ $|\!\sim$ ‘α’$^{\rangle}) \leq gr((^{\langle}$ ‘Y’ $|\!\sim$ ‘α’$^{\rangle})$.
(iii) $gr(^{\langle}\forall x(x \in$ ‘Z’ $\rightarrow$ ‘X’ $|\!\sim x)$&‘$X \cup Z$’ $|\!\sim$ ‘α’$^{\rangle}) \leq gr(^{\langle}$‘$X$’ $|\!\sim$ ‘α’$^{\rangle})$.

The expressions with the mark ‘ ’ are terms of meta level I while expressions with the mark $\langle\ \rangle$ are names of wffs of level I in level II. In order to write the version of $(GC4)$ names of (truth) values such as $^{\langle}\ k\ ^{\rangle}$or $^{\langle}\ 0\ ^{\rangle}$are also required at meta level II but could be dropped for simplicity in writing the above axioms. Similar is the case for the sign $\leq$. The complete list of symbols used at each level shall be presented in the full text of the paper.

Besides dispelling of above mentioned confusions, this study helps in the assessment of the two approaches viz. [15, 10] and [1, 2, 3, 4, 6] with respect to the incorporation of degrees at the meta levels. At the metalevel II, all the notions are crisp. The study also suggests a general methodology for construction of logics with several tiers in which sentences of one tier ‘speak’ about objects of the next lower tier. And a detailed study on proof theoretic results maintaining this distinction between levels of logic and their corresponding algebraic structures reveals a new way of looking at the interpretation of proof theory of a logic.

This also gives rise to a way where a decision maker would be able to choose her own logic to compute the grades of the sentences like, ‘α’ follows from ‘X’, with the help of a database filled up by experts' opinion, which is again free to choose its own logic, on the basic sentences of the object language.

The tutorial is planned as follows :

Session 1:Introducing the notion, motivation, and a detailed comparison with existing fuzzy logics.

Session 2:Introducing other metalogical notions like inconsistency, consistency, and their interface in the graded context.

Session 3:Introducing proof theory in the graded context, and illustrative examples with indications towards applications.

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3.2.27 The Notions of Empathy and Transcendence in Quine's Philosophical System

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Our objective in this tutorial is to show some of our contributions to the exegesis of Quine's philosophy, emphasizing the role played by notions such as empathy and transcendence in his philosophical system.

We sustain the hipotheses that Quine is a rigorous systematic philosopher and that, in order to understand each one of his theses and critiques, we need to analyse them within the totality of his philosophical investigations.

We believe that his system derives from what we call his epistemological project and that all his philosophical theses are founded in his theory of language learning.

We also maintein that Quine's philosophy shows a very strong pragmatic aspect more closely related to Wittgenstein's tradition and that one of speech acts theorists than to skinnerian tradition with which Quine is often associated.

We also sustain that the overall position of his theses has been presented in Word and Object and that all his subsequent works served to clarify or complement his ideas, not to change them. Thus, we do not agree with those who argues that Quine changed his philosophical positions in the course of time.

In this tutorial, we plan to cover the following topics:

– Quine's epistemological project and his theory of language and language learning;

– About the notions of empathy and transcendence;

– The theses of indeterminacy of translation, reference and theories;

– The thesis of epistemological holism and semantic holism;

– The theses of naturalized epistemology;

– The theses of ontological relativity and the ontological commitment;

– The theses about the truth value of sentences

– The criticism about the notions of meaning, attributes, entification of propositions, synonymity, analyticity.

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3.2.28 Logic, Inquiry, and Discovery: Peirce's vision of Logic

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The interest in Peirce's ideas about logic and scientific inquiry is constantly increasing. Acknowledged as the "father of pragmatism" and as one of the founders of predicate calculus, simultaneously with and independently of Frege, his ideas have known more and more interest. But, even if Peirce's conception of logic as "semiotic? is often quoted as innovative and heterodox, they are more often still misunderstood and misapplied. This tutorial aims at introducing Peirce's views of logic as semiotic within the broader context of his philosophy, in order to highlight the originality and peculiarity of his views.

According to Peirce, logic, or semiotic, is the necessary, or quasi-formal study of signs. As such, it is one of the three normative sciences - esthetics, ethics, and logic - and its main branches are: A) Speculative Grammar, or the study of the nature and meanings of all means of expression; B) Critic or Speculative rhetoric, or the study of the classification and determination of the validity and force of all arguments; and C) Methodeutic, or the study of methods of investigation.

In order to clarify such distinctions, this tutorial will present:

1) Peirce's classification of the sciences and the place of logic within it;

2) An introduction to Peirce's semiotic, with special attention to speculative rhetoric and methodeutic. The main themes to be dealt with are:

2.1) Peirce's early critique of psychologism within the broader context of his theory of signs; 2.2) Peirce's conception of the method of science as an intertwinement among Deduction, Induction, and Abduction; 2.3) Peirce's pragmatism as the logic of abduction.

3) Semiotic, pragmatism and the possibility of metaphysics.

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3.2.29 Graph calculi for relational reasoning

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Traditionally, formulas are written on a single line. S. Curtis and G. Lowe suggested a more visually appealing alternative for the case of binary relations: using graphs for expressing properties and reasoning about relations in a natural way. We extended their approach to diagrams that are sets of graphs.

More specifically, in this setting, diagrams corresponds to sentences and transformations on graphs diagrams correspond to inference rules, that can be used to infer a diagram from a set of diagram taken as hypothesis. The basic intuitions are quite simple, leading to playful and powerful systems. Our systems treat positive, negative, and intuitionistic information.

In this minicourse we summarize these achievements, presenting proper formulations of these systems as logical calculi and discussing soundness, completeness and decidability.

The course has no pre-requisites besides some familiarity with formal reasoning and with the basic logic ideas on the syntax and semantics of formal systems. Besides, all the necessary background will be presented as necessary.

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3.2.30 Continuous-valued Logic Algebra

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Several non-standard logics were proposed to satisfy the need of uncertainty reasoning in intelligent information processing. They have characters of mathematical dialectical logic. Some of them are continuous-valued logic. Logical algebra is the foundation for constructing mathematical dialectical logic, just as Boolean algebra plays an important role in standard logic. The continuous-valued logic needs as foundation its logical algebra.

In this tutorial, complete continuous-valued logic algebra is proposed, and the definitions of seven kinds of integrity cluster of logical operation model that may exist in continuous-valued propositional logic are given.

Continuous-valued propositional logical operator models can directly include and deal with four uncertainties (dialectical contradictions). They are the uncertainty of propositional true degree arising from true /false (dialectical) contradictions, the uncertainty of logical operator model arising from enemy / friends (dialectical) contradictions, the uncertainty of logical operator model arising from loose/strict (dialectical) contradictions, and the uncertainty of logical operator model arising from familiar/unfamiliar (dialectical) contradictions. Using continuous-valued logic algebra can improve all kinds of existing continuous-valued propositional logical system, which is the important basis for further establishment of continuous-valued of dialectical logic.

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4 4th World Congress on Universal Logic

4.1 Invited Keynote Speakers

4.1.1 Irving H. Anellis

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Peirce's Role in the History of Logic: Lingua universalis and Calculus ratiocinator

Jean van Heijenoort divided the history of logic into three phases; the earliest, in which the algebraic logicians from Boole through Schröder played a part, consisted in the conception of logic as a calculus; the second, prominently played by Frege and Russell, and to a lesser extent Peano, sought to develop logic as a formal language; the third, through the efforts of Löwenheim, Skolem, and Herbrand, united the two conceptions of logic, and, with Hilbert standing partially in both camps, developed modern mathematical logic as first-order predicate calculus being both a calculus and a language. Charles Sanders Peirce (1839-1914) has historically been placed in the camp of the algebraic logicians. However, his own concept of logic was fluid, and developed over time, and there are elements of both calculus and language in his evolving conception. In the broader aspect, he regarded *logic* as semiotic, or theory of signs, which itself was divided into three branches: syntactics, semantics and pragmatics. Corresponding to this three-fold division, semiotics for Peirce included speculative grammar, methodeutic, and critic. The latter, Peirce understood in the same sense as the original Aristotelian *analytic*, namely, the formal theory of deduction. In the narrow usage of logic, Peirce understood *critic*. In attempting to disentangle Peirce's equivocal use of the term "logic", we suggest that, for Peirce, logic was, after all, both a language and a calculus.

4.1.2 Arnon Avron

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A New Approach to Predicative Set Theory

The predicativist program for the foundations of mathematics, initiated by Poincaré, and developed by Weyl and Feferman, seeks to establish certainty in mathematics without necessarily revolutionizing it. The program as is usually conceived nowadays is based on the following two basic principles:

(PRE) Higher order constructs, such as sets or functions, are acceptable only when introduced through definitions. These definitions cannot be circular. Hence in defining a new construct one can only refer to constructs which were introduced by previous definitions.

(NAT) The natural-numbers sequence is a basic well understood mathematical concept, and as a totality it constitutes a set.

The main goal of this paper is to suggest a new framework for the Weyl-Feferman predicativist program by constructing an absolutely (at least in our opinion) reliable predicative *pure set theory* PZF whose language is type-free, and from a platonic point of view, the universe V of ZF (whatever this universe is) is a model of it.

Our basic idea is that principle (PRE) means that the predicatively acceptable instances of the comprehension schema are those which determine the collections they define in an absolute way, independent of the extension of the "surrounding universe". This idea is implemented using a syntactic safety relation between formulas and sets of variables. This safety relation is obtained as a common generalization of syntactic approximations of the notion of domain-independence used in database theory, and syntactic approximations of Gödel's notion of absoluteness used in set theory.

One important feature of our framework is that it requires us to make an extensive use of abstraction terms. In fact the main axiom of PZF is the comprehension schema $\forall x(x \in \{x|\varphi\} \leftrightarrow \varphi)$, where φ is syntactically safe with respect to $\{x\}$. Unlike the official language of ZF, this well reflects the real mathematical practice of working with sets. Still, this does not involve an essential departure from first-order language. in contrast, in order to implement also Principle (NAT) within our framework, we find it necessary to really go beyond first-order languages. This is done by using *ancestral* logic, which is stronger than first-order logic, but much weaker then full second-order logic.

Another important feature of our framework is that it is not committed to any particular underlying logic. It is possible (and makes sense) to use it together with classical logic, but it equally makes sense to use it in combination with some non-classical logic, especially (but not only) intuitionistic logic.

4.1.3 Gianluigi Bellin

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Disambiguating bi-intuitionism

Bi-intuitionistic logic, as defined by C.Rauszer 1974, 1977 and others, lacks the "rich proof-theory" which is characteristic of intuitionism: its categorical (and topological) models are degenerate, in the sense that they reduce to partial orders. But already for co-intuitionistic logic categorical models do not exist in the category Set, since the coproduct is disjoint union and the co-exponent of two non-empty sets cannot be defined, as shown by T.Crolard 2001. Such problems vanish if we "polarise" bi-intuitionistic expressions as either "assertive" or "hypothetical"; namely, if intuitionism is a logic of assertions, as M.Dummett held, then co- intuitionism is a logic of hypotheses; if "conclusive evidence" for a proposition p justifies the assertion that p, then a "scintilla of evidence" for p justifies the hypothesis that p. Strong intuitionistic and weak co-intuitionistic negation ("doubt") are mutually inverse and relate the dual sides: thus a "scintilla of evidence" against an assertion justifies a doubt, but only "conclusive evidence" against a hypothesis justifies its refutation. Moreover co-intuitionistic disjunction is similar to Girard's par rather than to intuitionistic disjunction. In fact by "linearizing" co-intuitionism one can produce categorical models of co-intuitionistic logic by dualizing the construction by Benton, Bierman, Hyland and de Paiva 1993 and also Benton 1995; here the term assignment to co-intuitionistic proofs is based on T.Crolard's calculus of "safe coroutines" (2004). Finally, using strong and weak negations one can define intuitionistic modalities that change the side. If we also define a "conjecture" as the "possibility of an assertion" and an "expectation" as the "necessity of a hypothesis", then Parigot's lambda-mu-calculus can be typed in an intuitionistic "logic of expectations", an intermediate logic where the law of double negation holds but the law of excluded middle does not. Our "disambiguated" version of bi-intuitionism is translated into classical S4, extending Godel, McKinsey and Tarski's modal interpretation, rather than into Rauszer's "tensed S4", as shown by Bellin and Biasi (2004). The classical S4 translation of our "logic of expectations" is the "Diamond Box" interpretation of classical logic (see Smullyan and Fitting 1996). One expects that using bi-modal S4 a similar treatment can be given of Fairtlough and Mendler's Lax Logic (1997) and Alechina, Mender, DePaiva and Ritter's Constructive S4 (2001) in our bi- intuitionistic setting.

4.1.4 Otávio Bueno

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Logic and Rationality

What is the connection between logic and rationality? A number of paradoxes in economics, psychology, and probability theory have been used to challenge the alleged link between rationality and logic. From the voting paradox (Craven [1992]) to fallacies in probabilistic reasoning (Kahneman [2011], and Tversky and Kahneman [1974]), deeply held assumptions about rationality have been questioned. However, rather than challenging logic, these paradoxes presuppose a particular logic in order to be obtained. If one changes the underlying logic, they simply do not arise. This means that it is still possible to maintain a significant connection between logic and rationality as long as it is understood that neither logic nor rationality are unique. Just as there is a plurality of logics, each appropriate for a given domain, there is a plurality of rationalities, each similarly domain-dependent. In this paper, I present and defend this pluralism about both rationalities and logics. Central to the proposal is the recognition that inconsistencies do not threaten rationality, as long as the underlying logic is paraconsistent (da Costa, Krause, and Bueno [2007]). Finally, I examine the distinctions between inconsistency and contradiction (Carnielli, Coniglio, and Marcos [2007]), and its implications for pluralism about rationality.

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4.1.5 Hans Burkhardt

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The Leibnizian logic as link between traditional and modern logic

Gottfried Wilhelm Leibniz (1646-1716) has developed different systems of logical calculi. One of the contents of these calculi is traditional syllogistic. During his whole life he reflected upon new formulations of syllogistic systems. The highlight of his research is the arithmetical representation of syllogistic.

Besides this he was the first to write down the basic structure of the logic of predicates in the form of his theory of indeterminate concepts. The difference between Leibniz and the logic of the 19th and 20th century is that Leibniz has conceived the logic of terms and not the logic of propositions. The most important step in this concept of logic is the transition form consistent or inconsistent composed concepts to true or false propositions, in Leibniz terms from (AB) est to A est B, for example from homo-animal is possible or consistent to homo est animal. In the case of a false proposition, for example the case of the composed concept homo-petrus is impossible or inconsistent and therefore the proposition homo est petrus is false. Since Frege there exists a controversy discussion on this structure about Leibniz as the founder of the Boolean algebra.

Another important aspect of the Leibnizian logic consists in the fact that all propositions are analytic, necessary proposition are finite analytic and contingent proposition infinite or virtual analytic. In a true proposition the predicate always is in the subject, or more exactly the predicate concept is a part-concept of the subject concept.

Another important aspect of Leibniz's modal logic is his deontic logic. He has developed different systems of deontic modal logic and he also had formulated the rejection of the modal peiorem for deontic syllogistic. The modal peiorem asserts that in a syllogism with modal propositions the conclusion has the strength of the weakest premise. Aristotle rejects this rule, holding that a necessary conclusion may follow from necessary and assertoric premises provided only that the necessary premise is the major. That Leibniz extends the rejection of the Aristotelian modal peiorem to syllogisms with deontic propositions has hitherto gone unnoticed.

4.1.6 Manuela Busaniche

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Residuated lattices represented by twist-products

A *commutative residuated lattice* is an algebra $\mathbf{A} = (A, \vee, \wedge, *, \to, e)$ of type $(2,2,2,2,0)$ such that $(A, \vee, \wedge)$ is a lattice, $(A, *, e)$ is a *commutative* monoid and the following residuation condition is satisfied:

$$x * y \leq z \text{ if and only if } x \leq y \to z, \tag{1}$$

where x, y, z denote arbitrary elements of A and $\leq$ is the order given by the lattice structure. An *e-involutive commutative residuated lattice* (*e-lattice* for short) is a commutative residuated lattice that satisfies the equation:

$$(x \to e) \to e = x. \tag{2}$$

Let $\mathbf{L} = (L, \vee, \wedge, *, \to, e)$ be an integral commutative residuated lattice. Then $\mathbf{K}(\mathbf{L}) = (L \times L, \cup, \cap, \cdot, \to, (e, e))$ with the operations $\cup, \cap, \cdot, \to$ given by

$$(a, b) \cup (c, d) = (a \vee c, b \wedge d) \tag{3}$$

$$(a, b) \cap (c, d) = (a \wedge c, b \vee d) \tag{4}$$

$$(a, b) \cdot (c, d) = (a * c, (a \to d) \wedge (c \to b)) \tag{5}$$

$$(a, b) \to (c, d) = ((a \to c) \wedge (d \to b), a * d) \tag{6}$$

is an e-lattice, that we call the *full twist-product* obtained from $\mathbf{L}$. Every subalgebra $\mathbf{A}$ of $\mathbf{K}(\mathbf{L})$ containing the set $\{(a,e) : a \in L\}$ is called *twist-product* obtained from $\mathbf{L}$.

In the present talk we investigate the class of e-involutive residuated lattices that can be represented by twist-products. We prove that they form a variety of commutative residuated lattices. Then we consider a variety $\mathcal{V}$ of bounded commutative residuated lattices that satisfies a Glivenko equation, and for each $\mathbf{L} \in \mathcal{V}$ we study the twist-products obtained from $\mathbf{L}$. We establish a correspondence among twist-products obtained from $\mathbf{L}$ and a set of lattice filters of $\mathbf{L}$..

4.1.7 Carlos Caleiro

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Combining Logics, Cryptofibred Semantics and Completeness Preservation

The study of combined logics is certainly a key issue of the general theory of universal logic. Fibring is a very powerful and appealing mechanism for combining logics. As proposed by Dov Gabbay, fibring should "combine $\mathcal{L}_1$ and $\mathcal{L}_2$ into a system which is the smallest logical system for the combined language which is a conservative extension of both $\mathcal{L}_1$ and $\mathcal{L}_2$". Of course, a conservative extension of two given logics does not always exist, but in that case one should aim for being as conservative as possible.

In abstract terms, fibring is well understood. Given consequence relations for $\mathcal{L}_1$ and $\mathcal{L}_2$, one just wants the smallest consequence relation over the combined language that extends the two. This corresponds precisely to putting together the axiomatizations of the two logics. However, if $\mathcal{L}_1$ and $\mathcal{L}_2$ are given in semantic terms, setting up exactly the semantic presentation of the combined logic is not a trivial task. The cryptofibring semantics was introduced by Caleiro and Ramos in order to overcome the serious difficulties of the original fibred semantics that led to the famous *collapsing problem*. Using a more relaxed relationship between combined models and models of the logical systems being combined, cryptofibring allows in general many more combined models, and has been used to obtain several interesting conservativeness results, including the development of a meaningful combined classical and intuitionistic logic.

Due to its richness, it was to be expected that much more useful and universal completeness preservation results could be obtained with respect to cryptofibred semantics. However, with the exception of the results already known about fibred semantics using the heavy assumption of fullness, there has not been much progress in this area. Herein, we will show that the conjectured unconditional preservation of completeness by cryptofibring fails. The problem is better understood when combining logics which are uniform and whose language includes constants, but it is in general a consequence of the fact that, despite its freeness, cryptofibring carries along a mild form of algebraic interaction between truth-values. We will illustrate these facts, and discuss roundabout characterizations of these emerging properties in the combined logics, as well as alternative closure assumptions on the classes of models of the logics being combined (Joint work with Sérgio Marcelino (SQIG-Instituto de Telecomunicaes, IST-UTL - Portugal).

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4.1.8 Roberto Casati

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Blueprint for knot logics

Logic is to natural language what knot theory is to natural knots. Logic explains some cognitive performances; in particular, some natural language inferences are captured by various types of calculi (propositional, predicate, modal, deontic, free, quantum.), which in turn may generate inferences that are arguably beyond natural logic abilities, or non-well synchronized therewith (eg. ex falso quodlibet, material implication). Mathematical knot theory accounts for some abilities - such as recognizing sameness or differences of some knots, and in turn generates a formalism for distinctions that common sense is blind to. Logic has proven useful in linguistics and in accounting for some aspects of reasoning, but which knotting performaces are there, over and beyond some intuitive discriminating abilities, that may require extensions or restrictions of the normative calculus of knots? Are they amenable to mathematical treatment? And what role is played in the game by mental representations? I shall draw from a corpus of techniques and practices to show to what extent compositionality, lexical and normative elements are present in natural knots, thus allowing us to formally explore an area of human competence that interfaces thought, perception and action in a complex fabric.

4.1.9 Roberto Marcondes Cesar Jr.

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Graph-based pattern recognition and applications

Structural pattern recognition plays a central role in many applications. Recent advances include new theoretical results, methods and successful applications. In the present talk, some recent graph-based methods for shape analysis will be shown. The presented methods include a new representation for graph-matching-based interactive segmentation and models for the analysis of spatial relations between objects. Applications will be presented and discussed.

4.1.10 Agata Ciabattoni

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Analytic calculi for non-classical logics: The Baha'i Method

A central task of logic in computer science is to provide an automated introduction of *analytic calculi* for a wide range of non-classical logics. In this talk I will present a general method for doing that. The resulting calculi are obtained by transforming Hilbert axioms or semantic conditions into suitable structural rules in various formalisms. Our method applies to many different formalisms, including hypersequent calculus, labelled deductive systems and display calculus, and sheds some light on their relations and expressive power.

4.1.11 Bob Coecke

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The logic of quantum mechanics – take II

It is now exactly 75 years ago that John von Neumann denounced his own Hilbert space formalism: "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space no more." (sic) [1] His reason was that Hilbert space does not elucidate in any direct manner the key quantum behaviors. One year later, together with Birkhoff, they published "The logic of quantum mechanics". However, it is fair to say that this program was never successful nor does it have anything to do with logic. So what is logic? We will conceive logic in two manners: (1) Something which captures the mathematical content of language (cf 'and', 'or', 'no', 'if ... then' are captured by Boolean algebra); (2) something that can be encoded in a 'machine' and enables it to reason.

Recently we have proposed a new kind of 'logic of quantum mechanics'. It follows Schrodinger in that the behavior of compound quantum systems, described by the tensor product [2, again 75 years ago], that captures the quantum behaviors. Over the past couple of years we have played the following game: how much quantum phenomena can be derived from 'composition + epsilon'. It turned out that epsilon can be taken to be 'very little', surely not involving anything like continuum, fields, vector spaces, but merely a 'two-dimensional space' of temporal composition (cf 'and then') and compoundness (cf 'while'), together with some very natural purely operational assertion. In a very short time, this radically different approach has produced a universal graphical language for quantum theory which helped to resolve some open problems.

Most importantly, it paved the way to automate quantum reasoning [3], and also enables to model meaning for natural languages [4]. That is, we are truly taking 'quantum logic' now! If time permits, we also discuss how this logical view has helped to solve concrete problems in quantum information.

Details can be found in [5, 6].

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4.1.12 Simon Colton

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Shape, Composition and Juxtaposition in The Painting Fool

The Painting Fool is software that we hope will one day be taken seriously as a creative artist in its own right. To that end, we are giving it abilities to exhibit behaviours which might be called skilful, appreciative and imaginative, as described at www.thepaintingfool.com. At its heart, The Painting Fool is a graphics program which gathers, manipulates and renders shapes for artistic effect. In the talk, I will give details of how the software can invent its own painting styles by invention of schema for segmenting images (from photographs and 3D models, and generated synthetically, for instance by context free design grammars) into shape regions, abstracting the shape outlines, assigning colours from palettes, choosing simulations of natural media such as pencils, paints and pastels, and finally simulating the usage of the media to render the final artworks. As discussed in the talk, this is part of a bigger picture of how The Painting Fool collates and composes source material for collages based on texts, which itself is part of the bigger context of the potential for Computational Creativity research to lead to fully autonomous, creative agents.

4.1.13 Newton C. A. da Costa

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Structures in Science and Metaphysics

This paper has three main goals: (1) It discusses some aspects of the general theory of set-theoretic structures as presented in da Costa and French [2003]. (However, our discussion is in principle independent of the contents of this book, being in good part based on the paper da Costa and Rodrigues [2007].) The theory of structures examined here can be employed in the axiomatization of scientific theories, especially in physics. The theory is related to that of Bourbaki [1968], but in contrast to Bourbaki, the emphasis is on the semantic dimension of the subject rather than on the syntactic treatment that Bourbaki provides. Set-theoretic structures are important, among other things, because they constitute the basis for a philosophy of science (see Costa and French [2003] and Suppes [2002]). As an illustration, we consider some aspects of geometry in order to make clear how the central notions of the theory can be applied to higher-order structures found in science. The case of geometry is significant, since pure geometric structures can be envisaged not only as mathematical abstract constructs, but also as essential tools for the construction of physical theories, in particular the theories of space and time. (2) The paper also shows how ideas of Caulton and Butterfield [2012] can be extended to what may be called higher-order metaphysics. In this metaphysics, there exists not only first-order objects, but also their corresponding properties and relations of any type of a convenient type hierarchy. (3) Finally, the paper outlines the first steps of the formalization of a metaphysics of structures, or structural metaphysics, inspired by ideas of French and Ladyman (see, for example, Ladyman [1998]). This kind of metaphysics may be seen as an adaptation of the higher-order metaphysics of structures implicit in Caulton and Butterfield [2012].

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4.1.14 Dennis Dieks

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The Logic of Identity: Identical Particles in Quantum Mechanics

There has been a lot of attention, in recent years, for the question of whether identical particles in quantum mechanics violate Leibniz's Principle of the Identity of Indiscernibles. There seems to be an emerging consensus that this is not the case, provided that we allow relational properties in Leiniz's Principle, and realize that identical quantum particles are *weakly discernible entities.* In the talk we shall criticize the assumptions of this consensus. As we shall argue, in many cases identical quantum particles can be considered to be *absolutely* discernible, whereas in other cases it is doubtful whether the concept of a particle is applicable at all.

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4.1.15 Itala Maria Loffredo D'Ottaviano

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On the Prehistory of Paraconsistency and the History of Paraconsistent Logic

Nowadays, paraconsistent logic has a highly developed theoretical framework in which it is possible to analyze how and by whom each achievement was introduced. Recent historical studies of paraconsistent logic and paraconsistency have allowed scholars to assess the contributions of philosophers, mathematicians and logicians with regard to the possible trivialization of deductive theories in the face of contradiction. The answers to these questions have lead scholars down different paths, some toward a paraconsistent position and others toward a classical one. Our ongoing research in collaboration with Evandro Gomes has found evidence in support of paraconsistent positions throughout the history of Western logic. Contributions related to paraconsistency before the appearance of contemporary logic may be considered the prehistory of paraconsistent logic. Sometimes we cannot formally identify a strict paraconsistent position by means of explicit formulations in logical theories. In such cases, we analyze paraconsistent suggestions in a more general way, examining the range of paraconsistency positions which tolerate theoretical contradiction and which were much more common in the era before contemporary logic. The discussion focuses on the history of the principle of non-contradiction and also on that of the ex falso sequitur quodlibet rule in ancient Greek logic and in medieval logic. We outline the contemporary history of paraconsistent logic in order to recall the important role played by some its forerunners and, especially, by the founders of this logical field of study. We analyze the general development of paraconsistent logic, emphasizing the history of the paraconsistent logical systems of Newton da Costa and his contribution to the creation of this important field of inquiry in the 20th century. In this context, we assess paraconsistency as presented by da Costa and compare the relevance of his work with contributions made by other contemporary paraconsistent logicians.

4.1.16 J. Michael Dunn

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The Third Place is the Charm: Ternary Accessibility Relations

B. J. Copeland and R. Goldblatt have each written excellent accounts of the beginnings of what is often called the Kripke semantics for modal logic, which famously uses a binary accessibility relation between possible worlds. I believe that the origins of what is known as the Routley-Meyer semantics for non-classical logics (especially relevance/relevant logics) are not nearly as well understood. I will give a brief but relatively fulsome account of the history of the use of a ternary accessibility relation. I will discuss the "prehistory" including Peirce's use of his idea of Thirdness to explain laws, and the Jónsson's and Tarski (1951-52) papers "Boolean Algebras with Operators I-II". It seems to be accepted that his last anticipated the so-called Kripke semantics for modal logic. I shall show it is far less clear that the Routley-Meyer semantics fits as neatly under the general representation theorem of Jónsson-Tarski. Richard Routley and Robert K. Meyer published three papers and an abstract on the "Semantics of Entailment" in the years 1972 and 1973. Curiously Routley and Meyer did not in fact publish the semantics for Anderson and Belnap's system E of entailment until it appeared in 1982 as an appendix to their book with V. Plumwood and R. T. Brady: *Relevant Logics and their Rivals, Part I, The Basic Philosophical and Semantical Theory*, Ridgeview Publishing Company. The appendix explains that it was written as a paper and submitted in 1972 to The Journal of Symbolic Logic, and accepted subject to revisions, which were never completed in a timely fashion. In this same appendix Routley described the context of his and Meyer's discovery thusly: "As with other intellectual break-throughs, grand or small (e.g., modal logic semantics, the infinitesimal calculus, relativity theory, etc.), so with semantics for relevant logics, the time was ripe, e.g., requisite techniques were sufficiently available, and several people somewhat independently reached similar results at around the same time". These "several people" included Alasdair Urquhart, Kit Fine, Larisa Maksimova, Dov Gabbay, and I shall do my best to describe both the commonalities and distinctiveness of their contributions. I shall go on to discuss more recent uses and interpretations of the ternary accessibility relation by Meyer, Routley, of course myself, and many others, and expound on the recent paper with "uncountably many" authors: Beall, Jc, Brady, R., Dunn, J. M., Hazen, A. P., Mares, E., Meyer, R. K., Priest, G., Restall, G., Ripley, R., Slaney, J., Sylvan, R. (2012), "On the Ternary Relation and Conditionality", *Journal of Philosophical Logic*, **41:3**, 595–612.

4.1.17 Hector Freytes

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The phase space in intuitionistic quantum logic: a logic-algebraic approach

Recently, several authors [1, 2, 3] have paid attention to the study of categorical approach to quantum mechanics. In particular, in [2] a categorical model for quantum mechanics is given where the phase space is represented by a frame. Frames are a complete Heyting algebras and the premier motivating example of a frame is a topology. In this approach, phase spaces are intuitionistic structures. The aim of this work is to study some general properties about frames that allow to develop logical systems associated to the phase space in the sense of [2].

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4.1.18 André Fuhrmann

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Unfitching Knowability: Possible vs. Potential Knowledge

The thesis that every truth is knowable is usually glossed by decomposing knowability into possibility and knowledge. Under elementary assumptions about possibility and knowledge, considered as modal operators, the thesis collapses the distinction between truth and knowledge (as shown by the so-called Fitch-argument). As far as a purely logical refutation of the knowability thesis comes as a surprise, the Fitch-argument is paradoxical. We show that there is a more plausible, non-decomposing way of interpreting knowability such that the Fitch-argument does not apply. We call this the potential knowledge-interpretation of knowability. We compare our interpretation with the rephrasal of knowability proposed by Edgington and Rabinowicz and Segerberg, inserting an actuality-operator. This proposal shares some key features with ours but suffers from requiring specific transworld-knowledge. We observe that potential knowledge involves no transworld-knowledge. We describe the logic of potential knowledge by providing models for interpreting the new operator. Finally we show that potential knowledge cannot be fitched: The knowability thesis can be added to elementary conditions on potential knowledge without collapsing modal distinctions.

4.1.19 Anthony Galton

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Prolegomena to an Ontology of Shape

Influenced by the four-category ontology of Aristotle, many modern ontologies treat shapes as accidental particulars which (a) are specifically dependent on the substantial particulars which act as their bearers, and (b) instantiate accidental universals which are exemplified by those bearers. It is also common to distinguish between, on the one hand, these physical shapes which form part of the empirical world and, on the other, ideal geometrical shapes which belong to the abstract realm of mathematics. Shapes of the former kind are often said to approximate, but never to exactly instantiate, shapes of the latter kind. Following a suggestion of Frege, ideal mathematical shapes can be given precise definitions as equivalence classes under the relation of geometrical similarity. One might, analogously, attempt to define physical shape universals as equivalence classes under a relation of physical similarity, but this fails because physical similarity is not an equivalence relation. In this talk I will examine the implications of this for the ontology of shape and in particular for the relationship between

mathematical shapes and the shapes we attribute to physical objects.

4.1.20 Jonathan Ginzburg

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Quotation via Dialogical Interaction

Quotation has often been viewed as a device requiring unorthodox semantic/logical treatment. In this talk, I will show that, in fact, once one adopts a semantic perspective rooted in dialogue, many of the recaltricant issues associated with quotation dissolve. One of the central issues in the semantic analysis of dialogue is how to model the process of metacommunicative interaction that is constantly in the background of a conversation, as participants monitor each other for mutual understanding and, more generally, for compatible classification of the events that they perceive, linguistic and otherwise (*Grounding*, Clark 1996). Modelling this process requires a theory of types that classify events, linguistic and otherwise, as well as a means of explaining the inference process that occurs when the participants need to engage in repair interaction, when they realize that they are somehow misaligned. I will show how to model aspects of metacommunicative interaction in the dialogue famework KoS (Ginzburg, 2012), logically underpinned using the framework of Type Theory with Records (TTR) (Cooper, 2012)—a model theoretic outgrowth of Martin-Löf Type Theory (Ranta, 1994). One of the distinctive aspects of TTR is that it provides types and their witnesses simultaneously as first class citizens of the semantic ontology, in particular allowing for speech events and their types (grammatical signs). This will enable us to construct the semantic entities needed for metacommunicative interaction and, ultimately, also for quotation.

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4.1.21 Edward Hermann Haeusler

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Universality, Naturality and Logical Systems

Naturality and Universality are key concepts in Mathematics. Category Theory **CT** was created with the aim of providing means to precisely define naturality in mathematics. Universal properties play a distinguished role in **CT**. Almost every important categorical construction is defined by means of an universal property: products, sums, limits and co-limits in general, exponentials, subobject classifiers and adjunctions. Regarding this last construction, the well-known "adjoint situations arise everywhere" shows how worth Naturality and Universality are in **CT**.

Logical Systems are in almost every subject in Formal Sciences. There are some ways of defining a Logical System: (1) By means of a satisfaction relation between models and sentences; (2) By means of a proof-theoretical relation between sets of sentences and a sentence, and finally; (3) By means of a closure operation on sets of sentences. In [1], it is defined the concept of *Indexed Frame* on top of an indexed natural situation. The approaches (1) and (2) above can be viewed as instances of *Indexed Frames.* If we consider Lax naturality instead of naturality we obtain *Indexed Closure Operators* that have (3) also as instances.

One of the positive points of the approach described in [1] is its ability to deal with denotational semantics, in the style of Lambek categorical semantics, at the same abstraction level that Tarskian based semantics. It works even when theories in the logic cannot be seen as forming a pre-ordered category, as it is the case with some sub-structural logics. This talk aims to critically discuss the role of naturality and universality in logic and formal sciences.

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4.1.22 Yuri Gurevich

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Plenary Closing Lecture:
The intrinsic logic of information

Infon logic was discovered and developed as part of our investigation of policy and trust management. Infons are items of information communicated between principals (or agents) in a distributed system. Infons are also used by the principals for deduction.

The relation "y is at least as informative as x", in short $y \geq x$, gives rise to a semilattice, with the binary operation of union (or conjunction), In addition there are unary operations "p said x" where p ranges over the principals. This leads to Primal Infon Logic with an interesting balance of expressivity (common policies are expessible) and efficiency: the propositional fragment is decidable in linear time. In primal infon logic, an implication $a \to b$ is a solution of constraints

$$(a \wedge x) \geq b \geq x.$$

The two constraints give rise to a watershed divide between weaker and stronger infon logics. The existence of a least solution of these constraints (presumably that is what a principal means saying that a implies b) results in an extension of constructive (also known as intuitionistic) logic.

We discuss the problem whether the extension is conservative. The problem is rather familiar to intuitionists and constructivists except that they traditionally deal with mathematical propositions. The framework of infons is richer.

Another problem that we discuss is that constructive logic is not sufficiently constructive for the intended applications of infon logic.

The report reflects joint work with a number of collaborators, in particular Lev Beklemishev, Andreas Blass and Itay Neeman.

Yuri Gurevich

What, if anything, can be done in linear time?

The answer to the title question seems to be "Not much". Even sorting n items takes $n.log(n)$ swaps. It turns out, however, that quite a bit can be done in linear time. We start with a brief discussion of the importance of linear time in the age of big data and with illustration of linear-time techniques. Then we sketch a linear-time decision algorithm for propositional primal infon logic (PPIL). Finally we mention some extensions of PPIL with linear-time decidable derivability problem.

4.1.23 Zhitao He

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Universal Logic Description of the Dynamic Evolution of Complex Systems
An important turning point in the development of artificial intelligence depth is the artificial intelligence theory crisis in the mid-1980s, which clearly shows that many systems in the real world is not simply a determined system, but a complex dynamic evolving system. Such as cosmic system, ecological systems, biological systems, climate systems, social systems, economic systems, network systems, language systems, human systems and thinking systems and so on. In principle, these complex systems either can't be described by binary logic or can't be resolved by simple reductionist approach.

So in terms of logic, the emergence of a variety of non-standard logic, reasoning and commonsense reasoning in incomplete information case laid the ideological foundation for the birth of the Universal Logic. In the area of artificial intelligence technology, the intelligent computing methods, such as genetic algorithms, evolutionary algorithms, artificial life, cellular automata and particle swarm optimization emerge in specifically for the dynamic evolution of complex systems. Complexity science has been rising quietly in system theory. These developments indicate that the information age has left from simple mechanical system oriented certain information processing stage and entered into the dynamic evolution of complex system oriented uncertain information processing stage.

The goal of this study is to explore how to combine the flexible reasoning ability of universal logic and intelligent computing methods, and to simulate system dynamic evolution, and to enrich the research ways and means of complexity science.

4.1.24 Beata Konikowska

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From non-deterministic semantics to ordinary Gentzen sequent calculi
A major generalization of ordinary logical matrices are non-deterministic matrices (Nmatrices) — multiple-valued structures, in which the value assigned by a valuation to a complex formula is chosen non-deterministically out of a certain nonempty set of options. Using Nmatrices, we can provide finite-valued semantics for many important logics which cannot be characterized by a finite ordinary matrix. They include, among others: all logics obtained from classical

logic by deleting some rule(s) from its standard sequent calculus, and thousands of paraconsistent logics known as LFIs (Logics of Formal Inconsistency). Logics with a finite characteristic Nmatrix enjoy the main good properties possessed by logics with an ordinary (deterministic) finite characteristic matrix – e.g., they are decidable and finitary.

A converse task is to provide proof systems for logics with semantics based on finite Nmatrices. We describe a general method (based on a weakened version of Rasiowa-Sikorski (R-S) decomposition methodology) for developing sound and complete, cut-free n-sequent systems for all propositional logics with semantics based on Nmatrices. If the logic in question satisfies a certain minimal expressiveness condition, we show that above n-sequent calculus can be further translated into an equivalent sound and complete calculus of ordinary sequents. The expressiveness condition is that we must be able to identify the truth-value of any formula φ by determining whether certain formulas uniformly constructed from φ are satisfied (i.e.: have designated values) or not. As any language which does not satisfy this condition can be extended to one which does, so the procedure is quite general. An important consequence of this method was the development, in a modular and uniform way, of analytic proof systems for thousands of LFIs.

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4.1.25 Arnold Koslow

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Beth's Implicit Definitions and the Uniqueness of the Logical Operators and First- and Second-Order Quantifiers

We consider the use of E.Beth's definition of "implicit definition" to show the uniqueness not only of all the familiar logical operators, but for first and second-order quantification as well. Part of the demonstration involves the recasting of some ideas of Gentzen, and this suggests a new way of looking at the logical quantifiers and whether even the second-order quantifiers are logical in the same sense in which the more usual ones are.

4.1.26 Tamar Lando

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The topology of gunk

Space as we conceive of it in mathematics and the natural sciences is composed of dimensionless points. Over the years, however, some have denied that points, or point-sized parts are genuine parts of space. Space, on an alternative view, is 'gunky': every part of space has a strictly smaller subpart. If this thesis is true, how should we model space mathematically? The traditional answer to this question is most famously associated with A.N. Whitehead, who developed a mathematics of pointless geometry that Tarski later modeled in regular open algebras. More recently, however, Whiteheadian space has come under attack. In addition to talking about the mereology of space, the gunk theorist must tell us something about the size or *measure* of different regions of space. But regular open algebras of the kind Tarski investigated do not admit of natural, countably additive measures. A newer and better approach to the mathematics of gunk, advanced by F. Arntzenius, J. Hawthorne, and J.S. Russell, models space via the Lebesgue measure algebra, or algebra of (Lebesgue) measurable subsets of Euclidean space modulo sets of measure zero. But problems arise on this newer, measure-theoretic approach when it comes to doing *topology*. According to Arntzenius, the standard topological distinction between 'open' and 'closed' regions "is exactly the kind of distinction that we do not believe exists if reality is pointless." Those who advocate the measure-theoretic approach to gunk have universally turned to non-standard topological primitives. In this paper I argue that the turn to non-standard topology in the measure-theoretic setting rests on a mistake. Recent advances in modal logic show that standard topological structure *can* be defined for the Lebesgue measure algebra, and for a wider class of Borel measure algebras. Once this is pointed out, the

measure-theoretic approach to gunk can claim two important advantages over the traditional approach: it allows the gunk lover to talk about size *and* topology – both in perfectly standard ways.

4.1.27 Vincenzo Marra

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Łukasiewicz logic as a logic of vague propositions, revisited

Jan Łukasiewicz introduced the non-classical logical system that now goes by his name in the Twenties of the last century. His motivations were philosophical—Łukasiewicz intended to address through his formal system the Aristotelian problem of future contingents ("There will be a sea-battle tomorrow" must have no classical truth value now, lest the future be determined). As far as I know, he never thought of his logic as being related to vague propositions. Starting from the Seventies, analytic philosophers interested in theories of predicates whose extensions lack sharp boundaries considered Łukasiewicz' system as a candidate for a formalisation of the logic of vagueness, usually to reject it as unsuitable. Major objections are that (i) Vagueness is not truth-functional (K. Fine), (ii) Vague predicates do not admit a formalisation within a non-vague, classical meta-theory (T. Williamson), and (iii) There is no convincing explanation of why a given vague proposition should receive a truth degree as precise as a real number (R. Keefe). These objections notwithstanding, I argue here that Łukasiewicz logic indeed is <u>the</u> logic of vague propositions (of a certain type). It will transpire that a full justification of this claim requires a mixture of philosophical, logical, and mathematical arguments, drawing on the major advances in our mathematical understanding of Łukasiewicz logic that have taken place in the last three decades.

4.1.28 Daniele Mundici

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Recent developments in Łukasiewicz logic

Just as boolean propositional variables stand for yes-no observables, and the boolean connectives yield complex expressions involving these observables, similarly the Łukasiewicz propositional variable stands for an arbitrary, suitably normalized, (continuous spectrum, bounded, physical) observable. The Łukasiewicz conjunction $\max(0, x + y - 1)$ and negation $1 - x$ yield complex expressions in these observables. We will discuss the special status of Łukasiewicz propositional logic among all many-valued logics. Thus, e.g., among all possible conjunctions $*\colon [0,1]^2 \to [0,1]$ satisfying the usual continuity and algebraic properties, the

Łukasiewicz conjunction is the only one whose adjoint implication connective is continuous. Answers in the game of Twenty Questions are taken care of by boolean conjunction just as Łukasiewicz conjunction does for answers in the Rény-Ulam game of Twenty Questions With Lies—a chapter in Berlekamp's communication theory with feedback. Also, Łukasiewicz logic has a universal role for the representation of events and possible worlds in de Finetti's approach to probability based on the notion of coherent (no-Dutch-book) bet. Boolean algebras stand to boolean logic as (C.C.Chang) MV-algebras stand to Łukasiewicz logic. Up to categorical equivalence, MV-algebras are lattice-ordered abelian groups equipped with a distinguished archimedean element. Finitely presented MV-algebras are categorically dual to rational polyhedra, i.e., finite unions of simplexes with rational vertices. A crucial role in the theory of Łukasiewicz logic is played by functional analysis and measure theory, e.g., in the Kroupa-Panti theorem showing that finitely additive measures (known as *states*) on MV-algebras are in one-one correspondence with regular Borel probability measures on their maximal spectral spaces. Among others, this result shows that de Finetti's approach actually encompasses sigma-additive (Kolmogorov) probability. Also algebraic topology has a crucial role in the analysis of projectives, unification and admissibility in Łukasiewicz logic. For background see the recent monograph by the present author *Advanced Łukasiewicz calculus and MV-algebras,* Springer 2011, which is a continuation of the Kluwer 2000 monograph *Algebraic Foundations of Many-valued Reasoning,* jointly written with R.Cignoli and I. D'Ottaviano.

4.1.29 Sara Negri

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Unifying the search of proofs and countermodels in non-classical logics

Proofs and countermodels are the two sides of completeness proofs, but, in general, failure to find one does not automatically give the other. The limitation is encountered also for decidable non-classical logics in traditional completeness proofs based on Henkin's method of maximal consistent sets of formulas. The powerful formalism of labelled sequent calculi, on the other hand, makes it possible to establish completeness in a direct way: For any given sequent either a proof in the given logical system or a countermodel in the corresponding frame class is found. Proof-search in a basic system also gives a heuristic for finding frame correspondents of modal axioms, and thus, ultimately, for finding complete sequent calculi for axiomatically presented logics. A number of examples will illustrate the method, its subtleties, challenges, and present scope.

4.1.30 Hiroakira Ono

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Translations results in substructural logics

We know many basic translation results between classical logic and intuitionistic one, or between intuitionistic logic and modal logic. Typical examples are Glivenko's theorem, Kolmogorov's negative translation and Gödel-McKinsey-Tarski translation. It will be interesting to see how far these translations can be extended and to understand what are essential points for them to work well in a more general setting. Substructural logics will offer us a suitable framework for our purpose. In fact, these translations results hold farther than we expected. For example, it is shown that classical logic is embedded by Glivenko's translation into a quite weak substructural logic, the one without any of exchange, weakening and contraction rules, and that intuitionistic logic is embedded by Gödel translation into a modal substructural logic with S4 modality which is a conservative extension of full Lambek calculus. Our technical tools are both algebraic and proof-theoretic. This is a survey of my joint works with N. Galatos, H. Farahani, W. Young and some others.

4.1.31 Stephen Read

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Signification, Closure and Indirect Speech Reports

Thomas Bradwardine's solution to the insolubles (the logical paradoxes) depends on the thesis of multiplicity of meaning, that every proposition signifies many things, and any proposition's truth depends on things' being wholly as it signifies. This multiplicity of meaning is underpinned by Bradwardine's claim that signification is closed under entailment, that is, that a proposition signifies everything that follows from anything it signifies. The thesis of multiplicity of meaning has been gaining currency across philosophy of language and semantics in recent years, in, e.g., Cappelen and Lepore's "speech act pluralism" and Cian Dorr's "propositional profusion". But the idea that signification, or meaning, is closed under entailment appears too strong and open-ended, just as logical omniscience is resisted in epistemology. What is desirable is a more restricted claim of closure under some limited form of consequence. The clue can be found in speech act pluralism, based on the idea of indirect speech reports. Such reports allow considerable latitude in describing what was said. For example, if Jack said that Justine bought a picture and Justine was French, then it is not wrong to report Jack as having said that someone bought a picture, and even that a French woman bought the picture despite the fact that Jack might not know that Justine is French. But it would be wrong to report Jack as having said that everyone who came into the shop was French, even if Justine was the only customer that morning, and thus it follows from what Jack said. He certainly didn't say everything that follows from what he said, but nonetheless, indirect speech reports are closed under some restricted form of consequence. One way of getting to the bottom of what this form of consequence is results from examination of Walter Burley's notion of the real proposition (*propositio in re*), the ultimate significate of the spoken proposition. One sense of formal consequence for Burley, which Bradwardine arguably inherited, holds by reason of the form of the constituent terms, from inferior to superior. Using the real proposition to account for the plasticity of indirect speech reports then gives us a handle on the concept of signification which when fed into the account of truth, diagnoses and provides a solution to the semantic paradoxes.

4.1.32 Giovanni Sambin

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Unification of logics by reflection

With the birth of intuitionistic logic at the beginning of last century, logic ceases to be singular. Since them, a multitude of logics have been introduced, with conceptual or applicative purposes. Finding some unifying perspectives has become a pressing need. The principle of reflection provides one such unifying perspective. A logical constant is said to be obtained from the principle of reflection if its inference rules follow from (actually, are equivalent to) a specific equivalence linking object-language with meta-language. One can show that all logical constant in classical, intuitionistic, linear, paraconsistent logic are obtained from the principle of reflection. Moreover, each of these logics is an extension by structural rules of a single system, called basic logic.

4.1.33 Jonathan Seldin

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Logical Algebras as Formal Systems: H. B. Curry's Approach to Algebraic Logic

Nowadays, the usual approach to algebras in mathematics, including algebras of logic, is to postulate a set of objects with operations and relations on them which satisfy certain postulates. With this approach, one uses the general principles of logic in writing proofs, and one assumes the general properties of sets from set theory. This was not the approach taken by H. B. Curry in [1] and [2], Chapter 4. He took algebras to be formal systems of a certain kind, and he did not assume either set theory or the 'rules of logic'. I have not seen this approach followed by anybody else. The purpose of this paper is to explain Curry's approach.

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4.1.34 Gila Sher

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The Foundational Problem of Logic

The construction of a systematic philosophical foundation for logic is a notoriously difficult problem. In Part One I suggest that the problem is in large part methodological, having to do with the common philosophical conception of "providing a foundation". I offer an alternative to the common methodology, "Foundational Holism", which combines a strong foundational requirement (veridical justification) with the use of non-traditional, holistic tools to achieve this result. In Part Two I delineate an outline of a foundation for logic, employing the new methodology. The outline is based on an investigation of why logic requires a veridical justification, one involving the world and not just the mind.

Logic, the investigation suggests, is grounded in the formal aspect of reality, and the outline focuses on the nature of this aspect, the way it both constrains and enables logic, logic's role in our overall system of knowledge, the relation between logic and mathematics, the normativity of logic, the characteristic traits of logic, and error and revision in logic.

4.1.35 Sun-Joo Shin

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The universalism of multimodal logic

The talk invites us to extend the scope of formalism in logic. I am not introducing any new concept of logic, deductive reasoning, or formalization. On the contrary, being faithful to the straightforward concept of valid reasoning, I will show that it is not only legitimate but also natural and urgent to expand the set of media adopted in formal systems.

The essence of deductive reasoning is simple: to extract a certain piece of information from a given piece of information. On the other hand, the practice of deductive reasoning is rather complicated, ranging from mysteries to disasters. Sometimes it is not clear whether a piece of information is implied by the given information. To change a strong conjecture to a theorem takes time, effort, and, most importantly, ingenuity. Given information could be mistakenly taken to be consistent while it is not, and in that case any extraction process would produce pseudo-information. Sometimes we make an error in the extraction process.

A gap between the crystal-clear goal of deductive reasoning and its messy practice has puzzled and challenged many of us. The invention of formal systems has been one way to meet the challenge. Formalism makes a system more mechanical so that it may prevent errors and delineate the scope of consequences in a more accurate way. Historically, witnessing tragic blows in logic and mathematics at the turn of the 20th century and at the same time being equipped with a more powerful logic, mathematicians and logicians were more than ready to fully embrace formalism as a solution. Frege's and Peirce's quantificational logic, attempts at axiomatization of arithmetic, and Hilbert's arithmetization of geometry are prime examples of brilliant responses from brilliant minds.

Here is a catch: The frenzied enthusiasm for formalization has been channeled to only one type of medium – symbols. Hence, formalization has almost been equated with symbolization. This almost-accepted equation tells us how successful symbolic formalization has been, but the triumph of symbolization has come with a price tag: Logical investigation has been mainly limited to symbolic systems. It is time to step out of the comfortable zone of symbolic logical systems and to meet new demands of our era – the computer age and the era of visualization. The demand of efficiency and visualization has been louder and louder. Hence, I propose that remarkable advances made in symbolic formalization could and should be expanded beyond symbolic systems for a more fruitful harvest.

Valid reasoning is carried out by careful manipulations of information. If diagrammatic representation can be formalized and pieces of visual information could be transformed from one to another, we have no principled reason to insist on symbolic logical systems only. I will present a case study to show that diagrammatic formalization is actual and, moreover, it has its own strengths compared with symbolic formalization. If so, we should embrace different forms of logical systems and venture into a multi-modal system. I hope the talk will take us closer to the beginning of the significant and ambitious enterprise – the logic of multi-modal reasoning.

4.1.36 Barbara Tversky

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Lines, Shapes, and Meaning

Lines are the paths we walk in the world, the inscriptions we put on a page, the gestures we draw in the air, the contours the eye discerns. Lines form patterns and make shapes. Lines are the paths we take in life, the connections we make in the mind, the social relations we form. Lines, like the shapes and patterns they form, make meaning.

4.1.37 Safak Ural

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Square of opposition, or circle of opposition? Redefinition of the "Meaning"

It is traditionally accepted that every meaningful term has a denotation and a connotation. Now let me ask whether the term square could denote something logical, or could there be something logical which denotes the geometrical figure square? The concept of square of opposition presupposes that there is something logical that indicates a square. What kind of a relationship could there be between a logical concept and a geometrical figure? I think the traditional relationship between logic and geometry veils some linguistic handicaps. I will try to define the concept of meaning, and to elaborate the presupposed correlation between logic and geometry.

4.1.38 Luca Viganò

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Formal Methods for the Security of the Internet of Services

In the Internet of Services (IoS), systems and applications are no longer the result of programming components in the traditional meaning but are built by composing services that are distributed over the network and reconfigured and consumed dynamically in a demand-driven, flexible way. However, composing services leads to new, subtle and dangerous, vulnerabilities due to interference between component services and policies, the shared communication layer, and application functionality.

I will present the AVANTSSAR Platform [1] and the SPaCIoS Tool [6], two integrated toolsets for the formal specification and automated validation of trust and security of applications in the IoS at design time and run time, respectively. (Both have been developed in the context of FP7 projects that I have been coordinating.) I will focus on two particular results.

First, I will discuss a compositionality result [5] that formalizes conditions for vertical composition of security protocols, i.e., when an application protocol (e.g., a banking service) runs over a channel established by another protocol (e.g., a secure channel provided by TLS). This is interesting and useful as protocol composition can lead to attacks even when the individual protocols are all secure in isolation.

Second, I will discuss how although computer security typically revolves around threats, attacks and defenses, the sub-field of security protocol analysis has so far focused almost exclusively on the notion of attack. I will motivate that there is room for a fruitful notion of defense for vulnerable protocols and that the conceptual bridge lies in the notion of multiple non-collaborating attackers and interference between simultaneous attack procedures [2,3,4].

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4.1.39 Heinrich Wansing

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Falsification, Natural Deduction, and Bi-Intuitionistic Logic

A kind of a bi-intuitionistic propositional logic is introduced that combines verification and its dual. This logic, **2Int**, is motivated by a certain dualization of the natural deduction rules for intuitionistic propositional logic, **Int**. It is shown that **2Int** can be faithfully embedded into **Int** with respect to validity. Moreover, **2Int** can be faithfully embedded into dual intuitionistic logic, **DualInt**, with respect to a notion of dual validity. In **2Int**, from a falsificationist perspective, intuitionistic negation internalizes support of truth, whereas from the verificationist perspective, co-negation internalizes support of falsity.

4.1.40 Andrzej Wiśniewski

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Effectiveness of Question-Answer Systems

The aims of the talk are twofold. A conceptual setting which enables a clarification of some computational issues concerning questions and answers is introduced. Some already obtained formal results are presented and their implications for the areas of question answering and dialogue analysis are discussed.

The intuitive notion of a question-answer system is explicated by:

Definition 1. *A question-answer system is an ordered triple* $\langle \Upsilon, \Theta, \mathcal{R} \rangle$*, where:*

1. Υ *is the set of well-formed expressions of a language,*
2. Θ *is a non-empty set of questions of the language,*
3. Θ *is a proper subset of* Υ*,*
4. Υ *includes a non-empty set of declaratives of the language,*
5. $\mathcal{R} \subseteq \Theta \times \Upsilon$*, where* $\mathcal{R} \neq \emptyset$*, is the answerhood relation, i.e. the set of ordered pairs* $\langle Q, \psi \rangle$ *such that* ψ *is a nominal principal possible answer (hereafter: ppa) to* Q*.*

The following clauses characterize highly desirable properties of question-answer systems:

- if an expression is a question, this can be effectively established/computed,
- if an expression is a ppa to a question, this can be effectively established/computed,
- the set of declaratives is decidable.

The next definition expresses the above requirements in the proposed conceptual setting.

Definition 2. *A question-answer system* $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ *is effective iff*

1. *the set of questions* Θ *of the system is r.e.,*
2. *the answerhood relation* $\mathcal{R}$ *of the system is r.e., and*
3. *the set of declaratives of the system is recursive.*

By an ω-question we mean a question that fulfils the following conditions: (1) each ppa to it is a declarative; (2) the set of ppa's to the question is denumerable, i.e. countably infinite.

Theorem 3. *Let $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ be an effective question-answer system such that the set of declaratives of the system is denumerable, and Θ consists of ω-questions. There exists a denumerable family of infinite recursive sets of declaratives of the system such that no element of the family is the set of ppa's to a question of the system.*

Theorem 3 strengthens Harrah's (1969) incompleteness theorem. We can even go further.

Theorem 4. *Let $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ be an effective question-answer system that fulfils the following conditions:*

1. *the set of declaratives of the system is denumerable, and*
2. *the set of ω-questions of the system is r.e.*

There exists a denumerable family of infinite recursive sets of declaratives of the system such that no element of the family is the set of ppa's to a question of the system.

As a consequence of Theorem 4 one gets:

Theorem 5. *If $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ is a question-answer system such that:*

1. *the set of declaratives of the system is denumerable and recursive,*
2. *the set of questions of the system is r.e., and*
3. *each infinite recursive set of declaratives of the language of the system is the set of ppa's to a question of the system*

then the set of ω-questions of the system is not r.e. or the answerhood relation $\mathcal{R}$ of the system is not r.e.

So there exist, somewhat unexpected, limits of effectiveness of question-answer systems rich enough. The issue of their importance will be discussed.

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4.1.41 Edward N. Zalta

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Models of Object Theory and the Analysis of Mathematics

The axioms of object theory are stated in a "second-order" syntax and the minimal models of these axioms are described. Though object theory uses second-order syntax, it can be interpreted in models that aren't fully second-order: the domain of the variable "for all F" is not the full power set of the domain of the variable "for all x". Thus, object theory uses second-order syntax but can be given a representation on which it is only first-order in strength. A extension of object theory expressed using "third-order" syntax is then described and used to analyze mathematical objects and mathematical relations. Minimal models for this extension of object theory, built in joint work with Hannes Leitgeb, are described and are shown *not* to be fully third-order. Thus a relatively weak theory suffices for the analysis of mathematical objects and relations.

4.1.42 Secret Speaker

University of ???

4.2 Workshops

4.2.1 Scope of Logic through History
What is/was logic? Historical Perspectives

This workshop is organized by

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Amirouche Moktefi
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Throughout most of the history of Western philosophy, there has been a closely related (sub-)discipline called *logic*. However, the common name should not conceal the marked differences among what counted as logic at different times. In other words, despite the stable name, logic as a discipline is not characterized by a stable scope throughout its history. True enough, the historical influence of Aristotelian logic over the centuries is something of a common denominator, but even within the Aristotelian tradition there is significant variability. Furthermore, as is well known, in the 19th century logic as a discipline underwent a radical modification, with the birth of mathematical logic. The current situation is of logic having strong connections with multiple disciplines – philosophy, mathematics, computer science, linguistics – which again illustrates its multifaceted nature.

The changing scope of logic through its history also has important philosophical implications: is there such a thing as the essence of logic, permeating all these different developments? Or is the unity of logic as a discipline an illusion? What can the study of the changing scope of logic through its history tell us about the nature of logic as such? What do the different languages used for logical inquiry ? regimented natural languages, diagrams, logical formalisms ? mean for the practices and results obtained?

The invited keynote speakers of this workshop are Irving H. Anellis (page 57) and Hans Burkhardt (page 60).

Contributed Talks

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Aristotle on Deduction: Inferential Necessity vs. Logical Validity

Aristotle's Prior Analytics deals with deductions simpliciter, which are neither demonstrative nor dialectical. A deduction simpliciter is the necessary inference of a conclusion from given premises (A, 1, 24b18-20). If no conclusion "results of necessity" from the premises, the account cannot be a deduction.

Nowadays, it is standard to identify Aristotle's inferential necessity with logical validity. For instance, Keyt (2009: 36) writes: "the conclusion of a syllogism follows 'of necessity' from its premises: only valid arguments are syllogisms". An argument is valid if its premises logically entail its conclusion. However, this paper shows that logical validity fails to provide a full rendering of Aristotle's notion of inferential necessity. The two main reasons are as follows. First, Aristotle distinguishes inferential necessity from predicative necessity. By focusing on logical validity, we lose sight of this distinction; and we reduce predicative necessity to the modality of a deduction, without realizing that, for Aristotle, a necessary account can be either deductive (with respect to inferential necessity) or not deductive (with respect to predicative necessity). Second and more importantly, logicians interpret Aristotle's complete deduction (as opposed to incomplete ones) by adding the term "obvious" or "transparent" to logical validity, and then criticize Aristotle's position for being unclear. Yet, Aristotle's view does not require such additional terms, since inferential necessity amounts to complete deducibility by definition, and has to be distinguished from incomplete deducibility, whose completeness is only potential. The notion of potentiality is incompatible with logical validity, unless we assume a new concept of potential validity as being distinct from both validity and invalidity. In that respect, modern logicians are confronted with Aristotle's distinction between complete and incomplete deductions, without having the tools to understand its full relevance.

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Irreflexivity and Aristotle's Syllogismos

Aristotle's definition of 'syllogismos' at An. Pr. 24b18 specifies that syllogistic entailment is an irreflexive relation: the conclusion must be different from each premise. This is strange, because if an entailment relation is necessarily truth preserving, then it ought to be reflexive: there is no inference more certain than 'p, therefore p'. Previous explanations of the presence of irreflexivity have taken it to be a logical condition introduced because of the pragmatic contexts in which syllogismoi were used: either epistemic or dialectical. I argue, however, that Aristotle does not formulate a distinction between logical and pragmatic conditions on syllogismoi. I then go on to show that in each context Aristotle envisions, didactic, dialectical, eristic and periastic, irreflexivity is the correct pragmatic condition, and hence could be seen by Aristotle as logically constitutive as well. Aristotle includes irreflexivity for different reasons in each context, but this suggests that there is less distinction between kinds of syllogismoi than has been previously thought.

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Development of the encyclopedia and elaboration of Formal logic: The meaning of Leibniz's project
It is widely recognized that Leibniz made an essential contribution to the development of formal logic. However, this was not his original project. What he aimed at was at first to develop a caracteristica, a lingua philosophica, an ideal language whose characters would express accurately the content - and not only the form - of our knowledge of reality. I would like to describe Leibniz's evolution from the project to create a perfect language to another project, which is more modest but also probably more valuable in the eyes of contemporary logicians: the elaboration of his spécieuse générale, a general science of forms. My approach won't be purely historical, since I intend to examine questions whose impact exceeds the philosophy of 17th century, such as (1) the status of cogitatio caeca, which leads to the opposition between intuitionism and formalism and (2) the question of the existence of simple, primitive ideas which could provide an "alphabet of human thoughts", which leads to revisit the opposition between foundationalism and coherentism.

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Charles S. Peirce and Friedrich A. Lange On Diagrammatic Thinking
It is commonly and justly held that Peirce's theory of diagrammatical reasoning derived from the doctrine of mathematical construction contained in the Disziplin der reinen Vernunft im domgatischen Gebrauch at of Kant's first Kritik. Kant thought that mathematical knowledge involved forming constructions in pure intuition; Peirce thought that mathematicians must reason upon diagrams constructed on paper or in the imagination. But while for Kant only mathematics is constructive/synthetic, logic being instead discursive/analytic, for Peirce the entire domain of necessary reasoning is diagrammatic, i.e. constructive in Kant's sense. Less known is that this shift from limiting constructivity to mathematical knowledge to extending it to logic too was influenced, as Peirce himself repeatedly states, by Friedrich Albert Lange's Logische Studien, posthumously published in 1877 by Hermann Cohen. Lange's book was indeed based upon the affirmation that spatial intuition (rumliche Anschauung) is the source if the apodictic character of logical reasoning. Although hitherto neglected, there is strong evidence that Peirce read Lange's book, meditated on it, and was very much influenced by it. The present paper documents such evidence.

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Realist consequence, epistemic inference, computational correctness
Over the centuries, the notion of *valid logical consequence* has been considered the pivotal turn-around for understanding the philosophy behind our logical systems.

Realist approaches to logical relations understand the notion of consequence as the relation between propositions as truth-bearers, expressing (the obtaining of) corresponding states of affairs. This tradition stems from the Bolzano-Frege-Quine-Tarski school.

A different understanding came from the assertoric perspective on the formulas of logical systems, typically endorsed by anti-realist approaches to logic since Brouwer's intuitionism. The crucial notion here is the one of *correct logical inference* from known premises to a known conclusion.

The foundations of computing, logically represented by functional languages and (modern versions of) type theories, have provided in the last 40 years a revolution in the understanding of logical correctness. In this novel paradigmatic foundation of logic, based on a practical computational setting, there are new models of correctness and validity at stake, influenced by the application of logical relations in the domain of computer science.

In this paper, we will focus on the evolution and definition of validity that originates in theories with the underlying conceptual identity between proofs and programs, known as the Curry-Howard isomorphism. Here validity is intended as syntactic computational correctness. Our main thesis is that in this new and extended sense, correctness differs from both the realistic and anti-realistic viewpoints mentioned above and it is rather satisfied by a more practical aspect of term resolution, which we will present by addressing the following questions:

1. Are conditions for execution and termination admissible?

2. Are resources reachable?

3. Where are processes valid?

4. How is non-validity of a process resolved?

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Medieval formal logic as a practical art of argumentation
There is no doubt that medieval logic was logic in the same sense as we now use the word. It is evident even that it was formal in quite the same sense as we now use the word. Although logical rules, principles and examples were given in Latin, the language was in this context highly technical and regimented. As

scholars have noticed decades ago, using artificial logical language to formalize medieval logic is a relatively straightforward enterprise despite the widespread differences to the current standard structures of formal logic. But when medieval logicians describe philosophically what they are doing, they do not speak of construction of logical systems. On the one hand, logic was taken to study rational structures embedded in language (Kilwardby). It thus belonged to the group of linguistic sciences (scientiae sermonicales) together with grammar and rhetoric. On the other hand, logic was characterized as a practical science guiding human behaviour in relation to understanding linguistic discourse, evaluating truth-values and constructing arguments (Ockham). These characterizations seem to fit a practical art of argumentation better than logic as a formal enterprise. The aim of this paper is to consider this apparent contradiction in order to give an account of what medieval logicians thought they were studying: i.e. what was medieval logic about?

4.2.2 Logic and Metaphysics

This workshop is organized by

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Logic and Metaphysics are closely related disciplines from the very beginning. They are like twin brothers: always fighting, disputing, but, deep in their souls, they like each other and come to help when the other is in trouble. After all, both share a common nature. They are very ambitious concerning scope: absolute generality. But their strategy is different: logic gets generality by topic neutrality, metaphysics by substantive all inclusiveness.

Interestingly enough, for different reasons both become modest in twenty century. Logic was fragmented in a plurality of systems, and no system venture to claim to be about ?everything?. Metaphysics was putted in the shadow of semantics and epistemology. But both recall their old vitality in recent development. Universal logic is the rediscovery of the logic?s destiny to generality, and metaphysics flourishes today free from any epistemic and linguistic constrains.

Old and new questions concerning Logic and Metaphysics will be welcome topic in this workshop: a) Existence and Existential Import; b) Absolute Generality; c) Modal Logic and Possible Worlds; d) Predication and Instantiation; e)Logical Objects.

The invited keynote speaker of this workshop is Safak Ural (page 84).

Contributed Talks

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Point-Free Formalizations of Whitehead's Metaphysics of Space and Time

Alfred Whitehead is well-known as a co-author with Bertrand Russell of the famous book "Principia Mathematica". They intended to write a special part of the book related to the foundation of geometry, but due to some disagreements between them this part has not been written. Later on Whitehead formulated his own program for a new, relational theory of space and time ([3] page 195). This theory should be *point-free* in a double sense, that neither the notion of space point, nor the notion of time moment should be taken as primitives. Instead they have to be defined by a more realistic primitive notions related to the existing things in reality, (like the notion of a *region* as a formal analog of spatial body) and some relations between them (like the mereological relations *part-of*, *overlap* and the mereotopological relation *contact*). In [4] Whitehead presented a detailed program of how to build a point-free mathematical theory of space (see [1] for a survey of contemporary results in this direction). He claimed also that the theory of time should not be separated from the theory of space, but unfortunately, he did not present an analogous program for its mathematical formalization. In [2] we presented an attempt of building such an integrated point-free theory of space and time. However, the (definable) time structure in [2] contains only pure time moments without any internal structure between them. The system in [2] is based on two dynamic versions of the contact relation called *stable contact* – $aC^{\forall}b$, and *unstable contact* – $aC^{\exists}b$. Intuitively $aC^{\forall}b$ means that a and b are always in a contact, and $aC^{\exists}b$ means that a and b are *sometimes* in a contact. In the present talk we extend the approach from [2] studying the following spatio-temporal relations between changing regions: *spatial contact* – $aC^{s}b$ (coinciding in the new context with $C^{\exists}$) , *temporal contact* – $aC^{t}b$, and *precedence relation* – $a\mathcal{B}b$. Intuitively $aC^{t}b$ means that there is a moment of time when a and b exist simultaneously. This relation is a formal analog of the Whitehead's notion of *simultaneity*. Intuitively $a\mathcal{B}b$ means that there exists a moment of time t_a in which *a exists* and a moment of time t_b in which b exists such that *t_a is before t_b* (which fact assumes the *before-after* relation between (definable) time moments). The main result is a point-free axiomatization of the mentioned three relations and a representation theorem into a desirable point-based dynamic model of space and time.

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The Metaphysical Source of Logic by way of Phenomenology
In this work, I set my self the task of providing convincing reasons to consider the possibility of a metaphysical source of logic in the context of phenomenological analysis. My scope will be essentially carried out based on two succeeding steps of reduction: the first one will be the demonstration of 'existence' of an inherent temporal factor conditioning formal predicative discourse, and the second one a supplementary reduction of objective temporality to its time constituting origin which has, by necessity, to be assumed as a non-temporal, transcendental subjectivity and for that reason the ultimate metaphysical basis of pure logic. In the development of the argumentation and taking account, to a significant extent, of W.v.O. Quine's views in his well-known *Word and Object*, a special emphasis will be given to the fundamentally temporal character of universal and existential predicate forms, to their status in logical theories in general, and also to their underlying role in generating an inherent vagueness of continuity reflected, in turn, in the formal undecidability of certain infinity statements in formal mathematical theories. This kind of analysis concerns also metatheorems of such vital importance in mathematical foundations as Gödel's incompleteness theorems. Moreover in the course of discussion, the quest for the ultimate limits of predication, will lead to the notion of ultimate individuals-substrates, taken essentially as the irreducible non-analytic nuclei-contents within predicative logical structures.

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Modal vs. many-valued and partial approaches for future contingents
The well-known problem of future contingents arises from considerations about truth, time and modality. Although there are several formulations of the problem, the most famous and often-debated example is the sea-battle problem discussed by Aristotle. One of the first contemporary attempt to model Aristotle's view in a formal way is Lukasiewicz's three-valued logic, in which a third truth-value, Indeterminate, is added to truth and falsity. However, as it has been pointed out many times, the system L3 is not satisfactory in many aspects, especially because the law of excluded-middle, $p \vee \neg p$, is not a validity of the

system. But one does certainly want the excluded-middle to hold even for the future, because, although there may or may not be a sea-battle tomorrow, it seems already true that either there will or won't be one. For this reason, the problem has been settled in terms of modal logics, especially since A.N. Prior's works in the framework of tense logic. Prior worked on two solutions to the problem, and one especially, called the ockhamist solution, has been seen as a very appealing one. Another aspect of this approach is the notion of a branching model of time, often opposed to a strict possible worlds interpretation. One of the main reason for discussing the problem of future contingents within the framework of modal logic is the importance of the notion of historical necessity, or necessity per accidens, as medieval logicians used to say: the core of the problem seems indeed to rely on the ontological difference between a past which is now necessary, because it cannot change, and a future which is still open, i.e. in which some events may happen or not happen. However, one can wonder if those intuitions must be in terms of modalities or in terms of partiality. Indeed, we shall investigate a position which differs form the traditional treatment of the problem, and argue that historical necessity and possibility boil down to completeness and partiality of the notion of actuality, and that interpretations of the possibility for an event to happen in the future as the happening of that event in a possible world is misguided on certain aspects. We shall study the view that an indeterministic world is in fact a partial world, i.e. a world in which the notion of actuality is not entirely defined at a moment t of time, and wonder if some light upon the problem of future contingents can be shed by partial logics and non-deterministic semantics. Logics with truth-value gaps already play an important role in the debate, especially since the supervaluationist interpretation of the ockhamist solution proposed by Richmond Thomason, together with the recent semantic relativism championed by John MacFarlane; however, those views still accept and rely on a possible world or branching time treatment of the problem; we will wonder if one does not have to go one step further, and study the possibility of partial rather than complete worlds through the technical and philosophical consequences of that position.

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What is not Frege's Julius Caesar Problem?
Section 10 of *Grundgesetze der Aritmetik* is the place where Frege aims to solve the indeterminacy of referent of value-range names. His famous Basic Law V, as he notices, is unable to prove or refute mixed-identity statements as 'the value-range of $F = q$' if q is not given as a name of the form 'the value-range of . . . ', so that his system would turn out to be incomplete (although it *prima facie* already was, based on Goedel's further results). As a solution for the indeterminacy, his procedure however consists simply in stipulate truth-values to be value-ranges, in a way that it still leaves open whether an ordinary object (Caesar, the moon) is a value-range or not. Motivated to this fact many influential scholars claim that despite all his efforts Frege was unable to solve Caesar

Problem. As the main exponent of this thesis one might find Dummett, followed by Wright, Heck and Parsons. In this paper I claim that this view is mistaken. Primarily I suggest that the conviction that Frege's additional stipulation is inadequate to fully determine value-range names is based on a misleading view about Frege's domain of first-order variables. I also argue there is a crucial difference between the nature of Caesar Problem and the referential indeterminacy problem so that the former is essentially metaphysical whereas the latter is semantic. At last I claim that Caesar Problem is posed as an objection to Hume's principle disqualification for establishing cardinal numbers *(Anzahl)* as logical objects. As extensions are fundamental logical objects in Frege's view, Caesar objection could not possibly concern his explicit definition 'the number of *F*s is the extension of the concept "equinumerous with *F*"'.

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Artifacts of Unbelievability: If You Really *Believe that Modal Realism is False, Then it Actually is*

I propose a simple *reductio* argument against Lewisian modal realism (Lewis 1986), based on doxastic contents and actually held beliefs. There are some things one might (or does indeed) believe that are in fact 'unbelievable' on modal realism: They cannot possibly be believed.

The argument takes doubts about modal realism, understood as belief in the actual or merely possible falsity of its central ontological thesis (viz. that there exist concrete worlds besides our own) as its starting point. I will give an argument to the effect that within the theoretical setup of (Lewis 1986), proper doubt about modal realism is impossible to entertain, or equivalently, that if you truly believe that modal realism is (possibly) false, then it actually is. Since modal realism is in fact doubted, it eventually defeats itself. The argument generalizes within modal realism, as any statement that is false with *absolute* necessity can be shown not to be believable.

I will discuss which counterarguments are available, and conclude that most of them are either based on untenable premises or yield consequences that are unacceptable to the modal realist. I will ask whether the argument can also be generalized against other theories of doxastic content.

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Aspects of the theory of modal functions

We present some aspects of the theory of modal functions, which is the modal correlate of the theory of truth-functions. While the formulas of classical propositional logic express truth-functions, the formulas of modal propositional logic (S5) express modal functions. We generalize some theorems of the theory of truth-functions to the modal case; in particular, we show the functional completeness of some sets of modal functions and define a (new) notion of 'truth-functional reduct' of modal functions, as well as the composition of modal functions in terms of such reducts.

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Two Leibnizian Theses and the Plural Predication

In his A Critical Exposition of the Philosophy of Leibniz, Bertrand Russell, referring to the principle of the identity of indiscernibles (PII) states that the difficulty is to prevent its proving that there cannot be two substances at all on the principles of Leibniz' logic, the identity of indiscernibles does not go far enough. [Leibniz] should, like Spinoza, have admitted only one substance. (pp. 58-9) This is highly surprising given the fact that Leibniz embraces the indiscernibility of identicals (LL) as a complementary principle and maintains that differences in predicates provide a sufficient ground for the diversity between substances. Without discussing Russell's remark in the context of Leibniz' philosophy in particular, I argue that there is a tension between PII and LL and that once PII is endorsed Spinozistic monism follows.

Sketched in very rough terms, the tension is the following. According to PII, there can be no two distinct substances sharing all their predicates. That is: PII entails that there can be no brute facts of diversities between substances. But if, as PII states, differences in predicates are taken to be a necessary condition for diversity (and thus brute facts of diversities are denied), then such differences can no longer be taken to be a sufficient condition for diversityLL will be of no help. Russell explains this in the following cryptic way: "the numerical diversity of the substances is logically prior to their differences in predicates until predicates have been assigned, the two substances remain indiscernible; but they cannot have predicates by which they cease to be indiscernible unless they are first distinguished as numerically distinct." (ibid.) But if, as Russell argues, differences in predicates presuppose numerical diversities and therefore cannot account for diversities as such, then, once PII is endorsed, we are not better off in accounting the plurality at worlds that we pre-theoretically describe as containing two discernible substances than we are in accounting the plurality at worlds that we describe as containing two indiscernible substances. So, not only that there cannot be a numerical diversity between indiscernible substances but there cannot be a diversity between, say, a blue rectangular book and a red circular book. As Russell puts it, PII proves that there cannot be two substances at all. (ibid.)

In reconstructing Russell's argument, I will appeal to a standard version of the bundle of universals theory (BT), which is a metaphysics similar to Leibniz' in basic assumptions. BT allows nothing but universals in its metaphysical groundfloor and uses only two primitives: "bundled-up" and "being distant from". And "plural quantification", as presented by McKay (2006), will be used in formulating bundle-theoretic claims, illustrating Russell's point. The idea is to get the perspicuity of a first-order logic, quantifying over nothing but universals and then use plural quantification to eliminate commitments to bundles that can only be expressed by using second-order logic.

The argument will be given in two different stages. First, I argue that BT, expressed in the first-order logic with plural quantification, is compatible with PII and thus denies brute facts of diversities. To show this, Max Black's case where two indiscernible spheres are separated by distance will be used. In the bundle-theoretic language, Black's will be described as follows: F's [F's are bundled-up and G's (G's are bundle-up and F's bears the distance relation R to G's)], where F's=G's Now this description doesn't include the fact there is one single bundle. But nor does it exclude this fact. And only after excluding this fact, we can talk about the plurality in Black's world. That means: there can't be brute facts of diversities. In the second stage, I show that differences in predicates cannot ground numerical diversities. Consider the bundle-theoretic description of a world that we pre-theoretically describe as containing two discernible spheres. F's [F's are bundled-up and G's (G's are bundle-up and F's bears the distance relation R to G's)], where F'sG's This description does not include the fact that there is one single bundle, which is both F's and G's. But it doesn't exclude this fact either. I will consider a number of strategies that one may use in accounting the plurality of this world. But, as will be shown, none of these strategies will work unless one embraces brute facts of diversities.

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The Problem of Relations
The lecture will be about the question: What is the nature of the combination between a relation and its terms in a relational situation (state of affairs)? Various possible answers to this question will be described and discussed.

4.2.3 Many-Valued Logics

This workshop is organized by

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Many-valued logics are non-classical logics whose intended semantics has more than two truth-values. Their study started in the early 20th century as a rather marginal topic with some works of Łukasiewicz and Post on finitely-valued logics. However, in the last decades, many-valued logics have gained more and more prominence and have attracted an increasing number of researchers studying a growing family of logics arising from a heterogenous variety of motivations and yielding numerous applications.

Actually, nowadays many-valued logics occupy a central part in the landscape of non-classical logics, including well-known systems such as Kleene logics, Dunn-Belnap logic and other bilattice-valued logics, n-valued Łukasiewicz logics, fuzzy logics including Łukasiewicz infinitely-valued logic, Gödel-Dummett logic and many others), paraconsistent logics, relevance logics, monoidal logic, etc. Moreover, other systems like intuitionistic, modal, or linear logic whose intended semantics is of a different nature, can also be given algebraic semantics with more than two truth values and hence, can be fruitfully studied from the point of view of Algebraic Logic as many-valued systems.

Research on such variety of logics has benefited from connections with other mathematical disciplines like universal algebra, topology, and model, proof, game and category theory, and has resulted in many applications in other fields like philosophy and computer science.

This UNILOG workshop is organised in conjunction with the MarieCurie IRSES project MaToMUVI. It welcomes submissions devoted to the study of any aspect of any kind of many-valued logics. Especially welcomed are contributions providing general for studying classes of many-valued logics.

The invited keynote speakers of this workshop are Manuela Busaniche (page 61) and Vincenzo Marra (page 78).

Contributed Talks

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Free algebras in the varieties generated by Chang's MV-algebra and by Jenei's rotation of product t-norm

Chang's MV-algebra is the prototypical example of a linearly ordered MV-algebra having infinitesimals, and it is also the prototypical example of an MV-algebra that is not simple but is still local, that is, it has a unique maximal ideal. In this talk we discuss different characterisations of the free algebras in the variety $\mathbb{DLMV}$ generated by Chang's MV-algebra (the name reflects the fact that this variety is axiomatised by adding to MV-algebra equations the Di Nola–Lettieri axiom $2(x^2) = (2x)^2$, see [1,2]). In particular we consider the representation by means of weak Boolean products of local MV-algebras arising from the disconnected rotation of cancellative hoops (cfr. [1,3,5]), and a more concrete representation by means of a class of continuous functions (w.r.t. the usual Euclidean topology) from a power of $[0,1] \setminus \{1/2\}$ to $[0,1] \setminus \{1/2\}$. We also discuss the variety $\mathbb{J}\Pi$ of MTL-algebras generated by Jenei's rotation [4] of product t-norm. We introduce characterisations of the free algebras in this variety both as weak Boolean products and by a class of functions from a power of $[0,1]$ to $[0,1]$. We show that functions of free DLMV-algebras are obtained by restricting the domain of the functions of free JΠ-algebras to the set of points having no 1/2 components.

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State operators on non-associative generalizations of MV–algebras

Commutative basic algebras are a non-associative generalization of MV-algebras, that means of algebraic semantics of the infinite valued Łukasiewicz propositional logic. States on MV-algebras have been introduced by Mundici (1995) as averaging processes for formulas in the Łukasiewicz logic. Moreover, states constitute measures on MV-algebras, hence are related to probability, and therefore to reasoning under uncertainty.

Parallel to the investigation of states on MV-algebras, various probabilistic logics have been introduced. For example, Hájek (1998) introduced a fuzzy logic $FP(\text{Ł})$ with a modality P (interpreted as probably) which has the following semantics: The probability of an event a is presented as the truth value of $P(a)$. Along these lines Flaminio and Godo (20007) introduced another fuzzy logic, called $FP(\text{Ł}, \text{Ł})$, with modality in which one can study probability over many-valued events. Flaminio and Montagna (2009) brought a unified treatment of states and probabilistic many-valued logics in a logical and algebraic settings. From the logical point of view, they extended the system $FP(\text{Ł}, \text{Ł})$ by dropping the restrictions on its formulas. The logic obtained in this way (with the rules Modus Ponens and Necessitation) has been called $SFP(\text{Ł}, \text{Ł})$. The semantic counterpart of $SFP(\text{Ł}, \text{Ł})$ is constituted by MV-algebras with an internal state (or state MV-algebras). State MV-algebras are MV-algebras with a unary operation, called a state operator, satisfying some properties of states and form a variety of algebras of the corresponding type.

Commutative basic algebras have been introduced by Chajda, Halaš and Kühr (2009) and are non-associative generalizations of MV-algebras, more precisely, MV-algebras coincide with associative commutative basic algebras. Analogously as MV-algebras are an algebraic counterpart of the propositional infinite valued Łukasiewicz logic, commutative basic algebras constitute an algebraic semantics of the non-associative propositional logic $\mathcal{L}_{CBA}$ introduced by Botur and Halaš (2009), i.e., a non-associative generalization of the Łukasiewicz logic.

In the paper we enlarge the language of commutative basic algebras by adding a unary operation satisfying some algebraic properties of states. The resulting algebras are state commutative basic algebras which can be taken as an algebraic semantics of a non-associative generalization of Flaminio and Montagna's probabilistic logic. We present basic properties of such algebras and describe an interplay between states and state operators.

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Monadic MV-algebras are Equivalent to Monadic ℓ-groups with Strong Unit

In this paper we extend Mundici's functor Γ to the category of monadic MV-algebras. More precisely, we define monadic ℓ-groups and we establish a natural equivalence between the category of monadic MV-algebras and the category of monadic ℓ-groups with strong unit. Some applications are given thereof.

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Unification problems in finite MV-algebras with constants

In this work we deal with algebraic counterparts of expansions of Lukasiewicz logic enriched with finite number of truth constants. Denote these varieties by MVmSn. We show that these varieties contain non-trivial minimal subvariety generated by finite linearly ordered algebra which is functionally equivalent to Post algebra.

The analysis and characterizations of appropriate varieties and corresponding logical systems are given. Free and Projective algebras are studied in these varieties as well as projective formulas and unification problems.

We give two kinds of representations of MVmSn-functions. One is constructive algorithm for construction of conjunctive and disjunctive normal forms for these polynomials. Second is representation of these polynomials with McNaughton-style functions.

Our main considerations are unification problems in the corresponding varieties. We study it through projectivity. Connections between projective algebras and projective formulas have been established. Unification types of these varieties are unitary.

One of major parts of our work is the construction of unification algorithms for finding the most general unifiers for MVmSn-formulas.

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Approximations in algebras of the non-commutative Łukasiewicz logic

MV-algebras are an algebraic counterpart of the propositional Łukasiewicz infinite valued logic. As is known (Mundici, 1995), one can consider generalizations of classical measures to MV-algebras in the algebraic form of so-called states, hence MV-algebras are related to reasoning under uncertainty. Rough sets were introduced by Pawlak (1982) to give a new mathematical approach to vagueness. In the classical rough set theory, subsets are approximated by means of ordinary sets (lower and upper approximations) which are composed, e.g., by some classes of given equivalences. This leads to study of such kinds of rough sets which are closely related to the structures of MV-algebras from the algebraic point of view (Rasouli and Davvaz, 2010), and so combine, in such cases, two different approaches to uncertainty.

gMV-algebras are non-commutative generalizations of MV-algebras. Leuştean (2006) introduced the non-commutative Łukasiewicz infinite valued logic and proved that gMV-algebras can be taken as an algebraic semantics of this logic. Moreover, gMV-algebras, similarly as MV-algebras, are also related to reasoning under uncertainty (Dvurečenskij, 2001).

In the paper we study approximation spaces in gMV-algebras based on their normal ideals.

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On the relationship between tetravalent modal algebras, symmetric Boolean algebras and modal algebras for **S5**

In 1954, Gr. Moisil introduced the notion of *symmetric Boolean algebras*, which were studied in detail by A. Monteiro in the 1960's under the name of *involutive Boolean algebras*. A **symmetric Boolean algebra** is a structure $\langle A, \wedge, \vee, \sim, T, 0\rangle$ where the reduct $\langle A, \wedge, \vee, \sim, 0\rangle$ is a Boolean algebra and T is an involutive automorphism of A, that is, $T : A \to A$ is an automorphism such that $T(T(x)) = x$ for every $x \in A$. On the other hand, an **involutive Boolean algebra** (IBA) is a structure $\langle A, \wedge, \vee, \sim, \neg, 0\rangle$ where $\langle A, \wedge, \vee, \sim, 0\rangle$ is a Boolean algebra and $\neg$ is a De Morgan negation, that is, such that $\neg(x \vee y) = \neg x \wedge \neg y$ and $\neg\neg x = x$ (and so $\neg 0 = 1$). As observed by Monteiro, both kinds of structures coincide.

Tetravalent modal algebras were first considered by A. Monteiro. A **tetravalent modal algebra** (TMA) is an algebra $\langle A, \wedge, \vee, \neg, \Box, 0\rangle$ such that the reduct $\langle A, \wedge, \vee, \neg, 0\rangle$ is a De Morgan algebra and the unary operation $\Box$ satisfies the following two axioms: $x \vee \neg\Box x = 1$, and $\Box x \vee \neg x = x \vee \neg x$. The class of all tetravalent modal algebras constitutes a variety, which is generated by a four-element TMA (which justifies its name).

In this paper we prove, on the one hand, that TMAs plus a Boolean complement are the same as Boolean algebras plus a De Morgan negation, that is, IBAs. On the other hand, we prove that IBAs can be seen as modal algebras for modal logic **S5** satisfying additional equations such that the variety of IBAs is generated by the Henle algebra H_2. Thus, the logic that can be naturally associated to IBAs is a proper normal extension of **S5**. A Hilbert-style axiomatization for this logic is also obtained.

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On extensions of the Belnap-Dunn logic

The Belnap-Dunn logic, sometimes called *first degree entailment* [3, 1, 2], is a well-known four-valued logic having several applications in both logic and computer science. In this contribution I would like to address the following questions: how many extensions does the Belnap-Dunn logic have? And how does the lattice of its extensions look like?

At present I am not able to answer either of the above in full generality. However, I will present some partial results suggesting that the landscape of extensions of the Belnap-Dunn logic is much wider and more complicated that one would expect, the main one being that there are at least countably many of them.

Formally, by Belnap-Dunn logic ($\mathcal{BD}$) I mean the propositional logic in the language $\langle \wedge, \vee, \neg \rangle$ defined by the four-element Belnap lattice and axiomatized, for instance, in [4]. By *extension* I mean any logic *in the same language* that is a strengthening of $\mathcal{BD}$ (e.g., logics obtained by adding new axioms or rules to a syntactic presentation of the Belnap-Dunn logic).

It is easy to show that all extensions of $\mathcal{BD}$ are intermediate between $\mathcal{BD}$ itself and classical propositional logic (presented in the conjunction-disjunction-negation language). Another easy result is that the only proper axiomatic extension of $\mathcal{BD}$ (different from classical logic) is Priest's *logic of paradox* [9]. Hence, the main interest lies in those extensions that are obtained by adding new rules rather than axioms.

As far as I am aware of, the only other extensions of $\mathcal{BD}$ considered so far in the literature are the so-called *strong Kleene logic* [6], the Kleene logic of order [4] and the "Exactly True logic" introduced in [8] (and also independently in [7]). As shown in [4], the natural algebraic counterpart of the Belnap-Dunn logic is the variety of De Morgan algebras. This seems to indicate that, in order to understand the extensions of $\mathcal{BD}$, one should look at the structure of the lattice of sub-quasi-varieties of De Morgan algebras. However, $\mathcal{BD}$ is not protoalgebraic [4, Theorem 2.11], hence not algebraizable, which means that one cannot use the well-known correspondence result between finitary extensions of a logic and sub-quasi-varieties of the corresponding algebraic semantics (and even if we could, we know from [5] that the structure of the lattice of quasi-varieties of De Morgan algebras is rather complex and not yet fully understood).

Despite this, I will show that in certain cases the algebraic theory of De Morgan algebras does provide us with some insight about extensions of $\mathcal{BD}$. In particular, I will use a result of [5] to construct a countable chain of extensions of $\mathcal{BD}$ having as a limit a non-finitely axiomatizable logic.

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Adding Connectives to Intuitionistic Logic

Already in the 1960s Kuznetsov proposed to add to intuitionistic logic a connective called successor. In the 1970s Rauszer proposed adding co-conditional and co-negation. In 2001 Humberstone proposed adding another connective he called the strongest anticipator. Some properties usually considered are the disjunction property, univocity and conservatism.

We study the mentioned connectives in the context of intuitionistic logic or some of its fragments. However, we are also interested in the behaviour, in the mentioned context, of connectives such as Baaz-Monteiro's delta, that appears frequently in the context of fuzzy logic.

Axioms may not be enough for the axiomatization, i.e. in some cases it is necessary to add a rule. In some of these cases, when we are in the presence of a negation, we pay attention to the issue of paraconsistency, which is a property that arises naturally depending on the sort of rule given to the added connective. From a semantical point of view, the choice is between a truth-preserving or a truth-degree-preserving consequence.

We mostly work at the propositional level. However, in some cases we provide results for the first order extension.

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A contribution towards a cartography of fuzzy equivalence operators

Extending the basic boolean operators of conjunction, disjunction and negation into the fuzzy domain was an enterprise which was soon to reach some agreement upon imposing a few basic properties on such operators. Indeed, while the community working on fuzzy systems eventually settled around 't-norms as the permissible interpretations of conjunction', the mathematical fuzzy logic community also took their basic fuzzy propositional logic to be 'the logic of continuous t-norms'. The story concerning permissible interpretations of disjunction and negation did not differ much. As for the conditional, so important from the logical viewpoint in allowing for the internalization of entailment at the object-language level, there was already two competing approaches on the market: one of adhering to the logical motivation and defining implication as the residuum of a left-continuous t-norm, the other of wearing the algebraic hat and simply writing down the equations considered to be relevant in characterizing any such an operator.

Concerning the biconditional, the latter situation was replicated, and the schism between the two communities seems in fact to have been aggravated, given the usual role of such operator as establishing a particularly useful *equivalence* between logical sentences. On the one hand, using the narrow glasses of abstract algebraic logicians, the very study of equivalence as just another operator was equivocated, as the aim of any such operator could only be to serve the higher purpose of helping to algebraize the underlying deductive system, generalizing the age-old approach to logic based on finding suitable congruences that identify sentences with the same meaning. On the other hand, the typical broad fuzzy mind, semantically and pragmatically motivated, continued to see a lot of sense in looking at equivalences as operators that could serve various purposes, ranging from measuring proximity to generalizing the notion of similarity, as well as calculating degrees of indistinguishability between propositions, just like a fuzzy equality predicate could in principle mirror how (in)distinct two objects appear to be. In particular, a collection $\mathcal{B}_0$ of *fuzzy equivalence* operators has indeed been characterized in [1] by a number of more or less intuitive axioms.

In our present study we tread a third path, away from the above heated debate, and consider different *defining standards* for a fuzzy equivalence $\alpha \Leftrightarrow \beta$ based on patterns the classical logician would feel comfortable to use in showing this operator to be definable in terms of other operators she is more acquainted with. An initial immediate standard is indeed the one that trades the pattern $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$ for equivalence. A collection $\mathcal{B}_1$ of fuzzy equivalence operators based on such definition may be distinguished as one in which conjunction has the properties of a left-continuous t-norm, and implication is its residuum; a collection $\mathcal{B}_2$ may be formed in a similar way if we now consider arbitrary t-norms. In [2] we have shown that $\mathcal{B}_1$ is indeed properly contained in $\mathcal{B}_2$, and that by its turn $\mathcal{B}_2$ is properly contained in $\mathcal{B}_0$. Furthermore, in [3] we have shown that each of the above mentioned collections of fuzzy equivalence operators is closed under automorphisms, which means that the action of an automorphism can be used to define a natural quotient structure over such collections of operators. An appealing next step towards uncovering details of the relatively unexplored chart of fuzzy equivalence operators would consist in entertaining other defining standards for fuzzy equivalence, as inherited from classical logic. For instance, we may consider fuzzy equivalences that abbreviate the pattern $(\alpha \vee \beta) \Rightarrow (\alpha \wedge \beta)$, relativized to left-continuous t-norms and their dual s-norms, appropriately connected by fuzzy residual implications. This particular definition gives rise to a novel collection of operators $\mathcal{B}_3$, which may easily be checked to be more comprehensive than $\mathcal{B}_2$, and neither to contain nor to be contained in $\mathcal{B}_0$.

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Decidability of Bi-modal Gödel Logics

Sometimes it is needed in approximate reasoning to deal simultaneously with both fuzziness of propositions and modalities, for instance one may try to assign a degree of truth to propositions like "John is possibly tall" or "John is necessarily tall", where "John is tall" is presented as a fuzzy proposition. Fuzzy logic should be a suitable tool to model not only vagueness but also other kinds of information features like certainty, belief or similarity, which have a natural interpretation in terms of modalities. In this context, it is natural to interpret fuzzy modal operators by means of Kripke models over fuzzy frames.

If we take Gödel Kripke models GK then its underline fuzzy modal logic is the Gödel bi-modal logic $\mathcal{G}_{\Box\Diamond}$, as it was presented in [2]. There, a prominent question left unanswered is the decidability of $\mathcal{G}_{\Box\Diamond}$ and its extensions, since theses logics do not have the Finite Model Property (FMP) under the above mentioned semantics. In [3], the author proposes a new semantics $GM_{\Box}$ (for the $\Box$-fragment $\mathcal{G}_{\Box}$) and he shows that $GM_{\Box}$-validity coincides with $GK_{\Box}$-validity. In addition, he proves that under his new semantics $\mathcal{G}_{\Box}$ has the FMP. Finally, using the axiomatization given in [1], he obtains the decidability of the validity problem of $GK_{\Box}$.

In this presentation, we generalize this semantics for the bi-modal case, we prove that, under this semantics, $\mathcal{G}_{\Box\Diamond}$ has the FMP and, finally, we are going to conclude the decidability of $\mathcal{G}_{\Box\Diamond}$.

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The MV–structure of Intermediate Syllogisms

We study Peterson's intermediate syllogisms [1], i.e. inference systems (called Figures) that extend Aristotelian syllogisms on categorical propositions containing 'All' and 'Some' by accepting three intermediate categorical propositions containing 'Many', 'Most' and 'Almost all'. The 105 valid intermediate syllogisms, out of 4000 possible, are rather an empirical fact of correct reasoning than a theory of possible way of reasoning. Indeed, no further justification or validation is needed to recognize e.g. that 'All M are P' and 'Many M are not P' are two (Boolean) intermediate categorical propositions that are conflicting, i.e. are contradictory. This is a simple fact we accept. In this paper we do not define any new theory on Peterson's intermediate syllogisms; we only demonstrate that, by associating certain values V, W and U on standard Łukasiewicz algebra with the first and second premise and the conclusion, respectively, the validity of the corresponding intermediate syllogism is determined by a simple MV-algebra equation. Indeed, all valid syllogisms (and only them) in Figure 1 and Figure 2 are determined by equations of type $W \oplus U = 1$, $U^* \oplus W = 1$, or $W^* \oplus U = 1$. In Figure 3 the crucial conditions are $V \odot W \neq 0$ and $V^* \odot W \neq 0$. In Figure 4 validity of a syllogism depends only on the order of the values V, W and U. These observations justify the title of this paper. We also discuss possible extensions of Peterson's system. We claim that, due to the empirical nature of the 105 valid intermediate syllogisms including the original 24 Aristotelian, a proper extension or restriction must be conservative in a sense that validity or invalidity of any existing syllogism must remain in this extension or restriction. In this respect our approach differs from Peterson's system of fractional syllogisms. Of course, a proper extension must be based on a linguistic analysis of new quantifiers, their relation to the existing quantifiers, and pairs of contradictory intermediate categorical propositions. In practice, this implies that an extension of Peterson's syllogistic system must start by a conservative extension of Peterson's square, similarly than Peterson's square is obtained from Aristotle's square by a conservative extension. Finally, we show how Peterson's intermediate syllogisms can be viewed as fuzzy theories in the sense of Pavelka's fuzzy propositional logic [2]; after all, intermediate syllogisms deal with intermediate categorical *propositions*. This extends valid Boolean intermediate syllogisms to cases where premises are true to a degree, and how the unique degree of truth of the corresponding conclusion is determined. Thus, we introduce a fuzzy version of Peterson's intermediate syllogisms.

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On quantifiers on pocrims

Monadic MV-algebras (MMV-algebras, in short) were introduced and studied by J. Rutledge [7] as an algebraic model of the predicate calculus of the Łukasiewicz infinite valued logic in which only a single individual variable occurs. MMV-algebras were also studied as polyadic algebras by D. Schwarz [8,9]. Recently, the theory of MMV-algebras has been developed in [1,2,3]. The results have been recently extended in [6] for GMV-algebras (pseudo-MV-algebras), which form a non-commutative generalization of MV-algebras.

Recall that monadic, polyadic and cylindric algebras, as algebraic structures corresponding to classical predicate logic, have been investigated by Halmos in 60's and by Henkin, Monk and Tarski. Similar algebraic structures have been considered for various logics in [4,5].

The aim of our talk is to built up the theory monadic operators in a more general setting, namely for bounded pocrims. Bounded pocrims form a large class of algebras containing as proper subclasses the class of BL-algebras (an algebraic semantics of Hájek's BL-logic) as well as the class of Heyting algebras (algebras of intuitionistic logic).

We show that for so-called normal pocrims, i.e. those satisfying the identity $\neg\neg(x \odot y) = \neg\neg x \odot \neg\neg y$ (where $\neg x = x \to 0$), there is a mutual correspondence between existential and universal quantifiers. Further, the correspondence of existencial quantifiers with the m-relatively complete substructures will be discussed.

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Dynamic Algebras as an Axiomatization of Modal and Tense Logics

The aim of the paper is to introduce and describe tense operators in every propositional logic which is axiomatized by means of an algebra whose underlying structure is a bounded poset or even a lattice. We introduce the operators G, H, P and F without regard what propositional connectives the logic includes. For this we use the axiomatization of universal quantifiers as a starting point and we modify these axioms for our reasons. At first, we show that the operators can be recognized as modal operators and we study the pairs (P, G) as the so-called dynamic pairs. Further, we get constructions of these operators in the corresponding algebra provided a time frame is given. Moreover, we solve the problem of finding a time frame in the case when the tense operators are given. In particular, any tense algebra is representable in its Dedekind-MacNeille completion. Our approach is fully general, we do not relay on the logic under consideration and hence it is applicable in all the up to now known

cases.[1]

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Complexity of finitely valued Łukasiewicz possibilistic modal logics

Consider the language L_k of Łukasiewicz logic with *Var* being the countable set of its propositional variables, the binary connective $\to$, and truth constants $\overline{c}$ for every rational $c \in S_k = \{0, \frac{1}{k}, \ldots, \frac{k-1}{k}, 1\}$. The set $Form$ of formulas in L_k is defined as usual. Let us denote by $L(\Box)$ the language obtained by adding a unary modality $\Box$ to L. The resulting set of formulas will be denoted by $Form(\Box)$.

We will henceforth consider as models triples $M = (W, e, \pi)$ with W a set of possible worlds, $e : W \times Var \to S_k$ is a mapping that naturally extends to a mapping $W \times Form \to S_k$ by the truth functionality of $\to$ and by requiring that, for every constant $\overline{c}$, $e(w, \overline{c}) = c$, and that $\pi : W \to S_k$ is a possibility distribution. A model $M = (W, e, \pi)$ is called *normalized* if there exists a $w \in W$ such that $\pi(w) = 1$.

Given a model $M = (W, e, \pi)$ and a world $w \in W$, the truth value of a formula $\Phi \in Form(\Box)$ is defined inductively as follows. If $\Phi \in Form$, then $\|\Phi\|_{M,w} = e(w, \Phi)$ and if $\Phi = \Box\Psi$, then $\|\Box\Psi\|_{M,w} = \inf\{\pi(w') \to \|\Psi\|_{M,w'} : w' \in M\}$. The truth value of compound formulas is defined a usual.

For any formula $\Phi \in Form(\Box)$, we will denote by $\#\Phi$ its complexity which is defined inductively as follows: $\#\overline{c} = 1$, $\#p = 1$ for $p \in Var$, $\#(\Phi \to \Psi) = 1 + \#\Phi + \#\Psi$, and $\#(\Box\Phi) = 1 + \#\Phi$.

We can then prove the following lemma.

(LEMMA) For every formula $\Phi \in Form(\Box)$ and for every (not necessarily normalized) model $M = (W, e, \pi)$ and $w \in W$, there exists a model $M' = (W', e', \pi')$ and $w' \in W'$ such that $|W'| \leq \#\Phi$ and $\|\Phi\|_{M,w} = \|\Phi\|_{M',w'}$.

The following result fixes the complexity for both the problem $\mathbf{Sat}^{=1}$ of deciding for a formula $\Phi \in Form(\Box)$ whether there exists a model $M = (W, e, \pi)$ and $w \in W$ such that $\|\Phi\|_{M,w} = 1$, and for the problem $\mathbf{Sat}^{>0}$ of deciding whether there exists a model $M = (W, e, \pi)$ and $w \in W$ such that $\|\Phi\|_{M,w} > 0$. It is worth noticing that in (Bou et al, 2011) the authors fixed a similar problem, but with respect to generic models, to PSPACE-complete.

(THEOREM) The decision problems $\mathbf{Sat}^{=1}$ and $\mathbf{Sat}^{>0}$ are NP-complete, even if we only consider normalized models.

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Many-Valued Degrees of Memberships in Coalitions: Logical and Strategical Consequences

Games in coalitional form were introduced by von Neumann and Morgenstern. They derived the characteristic function of coalitions from a strategic n-player game in normal form by using the minimax of the total sum of payments imputed to each coalition. A strategy space in such a game is thus identified with the filter generated by a considered coalition, the subset of the player set. Vice versa, every (superadditive) coalition game gives rise to some strategic normal form game. In particular, this approach makes possible to decompose a simple game, which is represented by a monotone boolean formula, into a min-max form other that the usual CNF of the formula.

In this contribution we make an effort to recover the same kind of strategic min-max decomposition for games with fuzzy coalitions. We employ the framework of simple coalition games in Łukasiewicz logic.

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Game Semantics for Deterministic and Nondeterministic Many-Valued Logics

We connect two general paradigms of logic that seem to be hardly related at all at a first glance: semantic games, as introduced by Hintikka, on the one hand side, and deterministic as well as nondeterministic matrix based semantics on the other hand side. Hintikka's characterization of classical logic in terms of a competitive game between a Proponent, who seeks to verify that a given formula is true in a given model, and an Opponent, who challenges the Proponent's claim. While various extensions and variants of Hintikka's original game, which cover a broad range of nonclassical logics, have been described in the literature, it has so far seemingly never been attempted to uniformly define semantic games for all finitely valued logics, i.e., all logics characterized by finite truth tables. We generalize the classical semantic game to $\mathcal{M}$-games and show that winning strategies for the Proponent correspond to valuations with respect to a collection of finite truth tables (matrices) $\mathcal{M}$. However $\mathcal{M}$-games will only serve as intermediate station towards a more ambitious goal. Arnon and Lev have introduced the concept of non-deterministic matrices, which, following Avron and his collaborators, we will call Nmatrices. We will show that for every Nmatrix semantics $\mathcal{N}$ there is a corresponding $\mathcal{N}$-game that looks exactly like an $\mathcal{M}$-game: each rule refers to a connective and a truth value and specifies choices by Proponent and Opponent in a format that can be directly extracted from the corresponding truth table. However, it turns out that the Proponent's winning strategies in an unrestricted $\mathcal{N}$-game neither match the static nor the dynamic valuations, that are commonly used when referring to Nmatrices. These winning strategies rather give rise to a further concept of nondeterministic valuation, introduced as 'liberal valuation' here. We argue that liberal valuations are interesting and useful even independently of semantic games. But we provide characterizations of dynamic and static valuations in terms of $\mathcal{N}$-games as well. We show that certain pruning processes, to be applied to the unrestricted game viewed as a tree, lead to restricted $\mathcal{N}$-games that are adequate for dynamic or static valuations, depending on the specific version of pruning. The pruning process can be described as a series of interactions between the two players, thus sticking with the spirit of game semantics.

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Semi-canonical Systems and Their Semantics
A canonical (propositional) Gentzen-type system is a system in which every rule has the following properties:

1. It introduces exactly one occurrence of formula in its conclusion;
2. The formula being introduced is of the form $\diamond(\psi_1, \ldots, \psi_n)$ where $\diamond$ is an n-ary connective;
3. Let $\diamond(\psi_1, \ldots, \psi_n)$ be the formula mentioned in Item 2. Then all the principal formulas in the premises belong to the set $\{\psi_1, \ldots, \psi_n\}$;
4. There are no restrictions on the side formulas.

In [1] a coherence criterion for canonical systems was introduced, and it was shown that a canonical system is non-trivial iff it is coherent iff it admits cut-elimination iff it has two-valued (non-deterministic) semantics.

A larger class of Gentzen-type systems which is also extensively in use is that of semi-canonical systems. A semi-canonical Gentzen-type system is a system in which every rule has the following properties:

1. It introduces exactly one occurrence of formula in its conclusion;
2. The formula being introduced is either of the form $\diamond(\psi_1, \ldots, \psi_n)$ or of the form $\neg\diamond(\psi_1, \ldots, \psi_n)$, where $\diamond$ is an n-ary connective and $\neg$ is a distinguished unary connective of the language;

3. Let $\diamond(\psi_1, \ldots, \psi_n)$ be the formula mentioned in Item 2. Then all the principal formulas in the premises belong to the set $\{\psi_1, \ldots, \psi_n, \neg\psi_1, \ldots, \neg\psi_n\}$;

4. There are no restrictions on the side formulas.

In this paper we show that each non-trivial semi-canonical system has a characteristic non-deterministic matrix having at most four truth-values. We also provide constructive criteria for the non-triviality of such a system and for the admissibility of the cut rule in it.

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Nmatrices for Modal Logic

In 1914, Lewis proposed the **S1-S5** hierarchy., what is generally stipulated the first work on formal modal logic. Although Lewis had proposed some many-valued matrices to show the independence of her axioms, in 1940 Dugundji proved that no system between **S1-S5** can be characterized by finite matrices.

Dugundji's result forced by one hand the algebraic approach of McKinsey and Tarski and by the other hand the develop of Kripke's relation semantic. The success of the last one overshadowed the algebraic semantic and even more heterodox approaches, as 4-valued non-deterministic matrices for **T**, **S4** and **S5** systems.

Before the first Kripke's works, Lemmon had showed that the whole hierarchy of Lewis could be formalized in terms of $\Box$. Changing the primitive operators, Lemmon has proposed others hierarchies: **D1-D5** and **E1-E5**. He also presented a systems weaker than **S1** called **S0.5**.

If we replace the axioms (T) by (D) in **T** and **E2** we obtain the systems **D** and **D2**, respectively. Let **D0.5** be the system obtained replacing also (T) by (D) in **S0.5**. We will see that those tree systems are correct and complete by 6-valued non-deterministic matrices.

Notwithstanding those modal system can be characterized by 4 and 6-valued Nmatrices semantic, the notion of level-valuation is also necessary. Ivlev was explored the axiomatic of 4-valued Nmatrices semantic without this machinery. For instance, S_a+ is the Kearns 4-values Nmatrices without level-valuations. Let **Dm** be be the system obtained replacing (T) by (D) in S_a+. We will see that **Dm** is the axiomatic versions of 6-valued Nmatrices semantic without level-valuations.

4.2.4 Between First and Second-Order Logic

This workshop is organized by

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Second-order logic raises a number of philosophical issues, particularly when it is contrasted with its first-order counterpart. Is second-order logic really logic? What are the requirements (if any) for something to be considered logic rather than, say, mathematics? How should we compare the ontological commitments of first-order logic with those of second-order logic? How should the incompleteness, consistency, and the Skolem-Löwenheim Theorem in first- and second-order logics be assessed? What are the implications of the "first-order thesis" and its criticisms? What are the connections between second-order logic and set theory? Do plural quantifiers provide a suitable understanding of second-order logic quantifiers? How should the model theory for second-order logic be developed? How should the historical shift from higher-order logic to first-order logic be understood? How should first- and second-order logics be compared and contrasted? How do all of these issues change when one considers systems that are intermediate between standard first-order logic and full second-order logic, such as, first-order logic with generalized quantifiers, infinitistic first-order logic, first-order logic with branching quantifiers, or monadic second-order logic? These and related issues will be examined in this session with the goal of assessing current debates as well as moving them forward.

The invited keynote speakers of this workshop are Arnold Koslow ((page 77)) and Edward N. Zalta ((page 77)).

Contributed Talks

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Ancestral Logic

Many efforts have been made in recent years to construct computerized systems for mechanizing general mathematical reasoning. Most of the systems are based on logics stronger than first-order logic (FOL). However, there are good reasons to avoid using full second-order logic (SOL) for this task. We investigate a logic which is intermediate between FOL and SOL, and seems to be a particularly attractive alternative to both: *Ancestral Logic*. This is the logic which is obtained from FOL by augmenting it with the transitive closure operator, TC. The expressive power of ancestral logic is equivalent to that of some of the other known intermediate logics (such as weak SOL, ω-logic, etc), yet there are several reasons to prefer it over the others. One of them is that it seems like the easiest choice from a *proof-theoretic* point of view. Another important reason is simply the simplicity of the notion of transitive closure.

We argue that the concept of transitive closure is the key for understanding finitary inductive definitions and reasoning, and we provide evidence for the thesis that logics which are based on it (in which induction is a logical rule) are the right *logical* framework for the formalization and mechanization of Mathematics. We show that with TC one can define all recursive predicates and functions from 0, the successor function and addition, yet with TC alone addition is not definable from 0 and the successor function. However, in the presence of a pairing function, TC does suffice for having all types of finitary inductive definitions of relations and functions.

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Second-order Logic and Unrestricted Quantification
The possibility of unrestricted quantification is often taken to be obvious, since even those who deny it seem to presuppose its existence in order they express the claim that no quantification is unrestricted (Lewis [1991], p. 68, and Williamson [2003]). Presumably, the argument goes, the quantification in the previous sentence is unrestricted. In this paper, I assess critically and reject various arguments for the possibility of unrestricted quantification, and I develop an alternative that acknowledges the restrictions in each kind of quantification. As an example, I examine second-order quantification, and indicate the nature of the restrictions in this case, contrasting the resulting proposal with the conception advanced in Hellman [2006].

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Possible Predicates, Actual Properties, and Hale's Principle

Bob Hale has recently defended the legitimacy of second-order logic, and the existence of the properties that fall in the range of second-order quantifiers, via an argument for what I shall call Hale's Principle:

> HP: A property P exists if and only if it is possible that there is a predicate "$\Phi(x)$" such that "$\Phi(x)$" holds of the Ps.

I will not examine the arguments for and against Hale's Principle, but will instead investigate exactly how much second-order logic one obtains via mobilization of the principle. Obviously, the strength of the second-order logic obtained depends crucially on what predicates one takes to be possible in the relevant sense. It turns out that the assumptions that are required in order to obtain a certain philosophically important sort of categoricity result within second-order logics based on Hale's Principle are both relatively weak and (so I will argue) philosophically plausible – in particular, we need only assume that it is at least (logically) possible to construct countably infinite conjunctions and disjunctions via supertasks.

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Axiomatizations of arithmetic and the first-order/second-order divide

Hintikka (1989) distinguishes two functions of logic and logical apparatuses for the foundations of mathematics: the descriptive use and the deductive use. The descriptive use is what underpinned the pioneer work on the foundations of mathematics of Dedekind, Peano, Hilbert, etc. With the exception of Frege, the deductive perspective only became widely adopted after Principia Mathematica, i.e. after a formalized approach not only to axioms but also to rules of inference became available. Now, it is a well-known fact that any first-order axiomatization of arithmetic (such as first-order Peano Arithmetic) cannot be categorical in that it will allow for non-standard models. Second-order axiomatizations can exclude non-standard models (as noticed by Dedekind himself in his famous 1890 letter to Keferstein), and thus be categorical. However, the move to the second-order framework entails that there is no longer an effective notion of logical consequence underlying the axiomatization, as second-order logic is something of a deductive disaster. This is just one particular instance of a phenomenon widely discussed by computer scientists, namely the tension between the orthogonal desiderata of expressiveness and tractability for a given formal system (Levesque and Brachman 1987). Generally, the rule of thumb is that expressiveness and tractability are inversely proportional; in other words, you can't have your cake and eat it. First-order axiomatizations of arithmetic are non-categorical because they do not have sufficient expressive power to exclude non-standard models; but moving from first- to second-order theories entails a significant loss of deductive power. So in the case of axiomatizations of arithmetic (as elsewhere), both desiderata cannot be simultaneously satisfied: one goes at the expenses of the other. Thus, one conclusion to be drawn is that the choice of the underlying logic will also depend on the goal of the axiomatization: if descriptive, then it is advantageous to choose a formalism with a high level of expressive power; if deductive, then one is better off with a less expressive but more tractable formalism. From this point of view, the 'dispute' between first- and second-order logic and the question of which one is to count as logic properly speaking may be misguided: different tools are needed for different applications.

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The Concept of Subset in Second-Order Logic

Second order language seems to be designed to talk about very particular structures, having a non-empty set as universe of individuals and for every natural number n, a universe of $n-$ary relations on that set. So, to determine a structure, we have to fix the *notion of subset* and there are basically two ways: (1) The notion of subset is taken from the background set theory and it is fixed. (So it is treated as a "logical" concept.), (2) The notion of subset is explicitly given within each structure. (So it is treated as a concept "defined in the structure".) These two criteria for the concept of subset are directly related to the Zermelo hierarchy and Gödel's constructible universe. When choosing general structures Gödel's universe plays a crucial role.

We have $\mathcal{S}.\mathcal{S}$, the class of standard structures, and accordingly, we find $\models_{\mathcal{S}.\mathcal{S}}$, the set of validities in the class. We know that there is no complete calculus for $\models_{\mathcal{S}.\mathcal{S}}$, since this set is not recursively enumerable.

But even knowing that there is no calculus in the standard sense, we have certain deductive rules which are sound and so we define a second-order calculus SOL. A weaker calculus is also defined, in fact, a many-sorted calculus MSL. Since $\vdash_{SOL}$ is a proper subset of $\models_{\mathcal{S}.\mathcal{S}}$, in order to get the right semantics for it, we need to widen the class of structures to reduce the set of validities. So Henkin defined structures in a wider sense (which he called frames) and general structures. They produce $\models_{\mathcal{F}}$ and $\models_{\mathcal{G}.\mathcal{S}}$ and it happens (not by chance) that they are exactly the sets $\vdash_{MSL}$ and $\vdash_{SOL}$

Therefore, when we interpret general validity as being true in all general models, and redefine all the semantic notions referring to this larger class of general structures, general completeness (in both weak and strong senses), Löwenheim-Skolem and all those theorems can be proven as in first order logic.

It is clear that you cannot have both: expressive power plus good logical properties.

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Logic with a Single Type: Motivations and Metatheory

Practical problems of representing information and sharing it among distributed knowledge bases have led to interesting theoretical developments in logic. Among these is the development of a very flexible framework for knowledge representation called Common Logic (CL). Particularly relevant to the theme of the workshop is that the syntax of CL is very free — there is only a category of names in the non-logical lexicon of a language and any finite string of names is an atomic formula. Thus, standard syntactically second-order constructs like '$\exists FFx$' are legitimate, as are highly untyped constructs like (the logical falsehood) '$\exists F\forall x(Fx \leftrightarrow \neg xx)$'. Predication is also variably polyadic: 'Fx', 'Fxy', '$Fxyz$', etc are all legitimate. However, this fragment of CL can be given a complete proof theory so, these constructs notwithstanding, CL isn't really higher-order. In this talk, I will motivate these and other syntactic features of CL. I will also discuss its basic proof theory and will suggest two non-traditional axiom schemas that fall naturally out of the framework and will discuss their logicality. I will then discuss two different completeness proofs, one that proves it directly for CL as given and one that proceeds by way of a scheme for translating a given CL language into a "traditional" first-order language. Finally, I will discuss a natural extension of CL suggested by variable polyadicity that introduces a class of "sequence variables" that range over finite sequences of elements of the domain. The addition of sequence variables increases the expressive power of CL dramatically, which I will demonstrate with some examples. Unsurprisingly, however, this leads to a loss of compactness and, hence, the extension is not first-order. I will close with a brief discussion of where CL with sequence variables lies on the spectrum of expressibility.

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Logical consequence and the first-/second-order logic distinction

My talk investigates the idea that the aim of formalisation is to capture natural language inferential relations. I consider some ways of making this idea precise and evaluate them with reference to both first- and second-order logic.

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Three Questions About Genuine Logic

What logic is "genuine" logic? I approach this problem by considering three questions about genuine logic.

Fix a formal language L containing $\aleph_0$-many sentences. Since an argument in L is a pair of a set of sentences (the premises) and a sentence (the conclusion), there are $2^{\aleph_0}$-many arguments (valid or not) in L. Therefore, there are $2^{2^{\aleph_0}}$-many sets of arguments, one of which is the set of all valid arguments of genuine logic.

How can we find the set of all valid arguments of genuine logic among the $2^{2^{\aleph_0}}$-many candidates? Suppose that there is a necessary condition for genuine logic. This condition divides all the candidates into the ones satisfying it and the others. We can reject the latter candidates as inadequate for genuine logic. If there is another necessary condition, then we can do the same filtering, i.e., discarding the remaining candidates that do not meet the second necessary condition. We can narrow down the candidates by imposing necessary conditions on them. After screening with sufficiently many necessary conditions, if there is a set left in the candidate list, then the set would be the set of all valid arguments of genuine logic.

In order to find the necessary conditions used for this narrowing-down strategy, I think that the following three questions are important.

(I) *Should genuine logic be truth-preserving?* Since the Tarskian characterization of logical consequence was proposed, logical validity has often been identified with semantic validity, which is defined as: if all premises are true, then the conclusion must also be true. Under this identification, logically valid arguments are all and only semantically valid arguments. Is it necessary for genuine logic to be semantically characterized? If so, what kind of semantics should be associated with genuine logic?

(II) *What logical constants should genuine logic contain?* Logical constants play a crucial role in logic. Different logics include different logical constants. Second-order logic is characterized by second-order quantifiers, which are not used in first-order logic. Genuine logic would also have logical constants and be demarcated by them. The question then is: What constants are they?

(III) *Should genuine logic be compact?* I think that satisfying compactness is a necessary condition for logic to be general. By "a discipline A is more general than a discipline B," I mean that all valid arguments in A are also valid in B. The connection between compactness and generality can be exemplified, for example, by the following facts: (1) every valid argument in first-order logic (compact) is also valid in first-order logic on the class of finite models (not compact) but not vice versa; (2) every argument that is valid in the Henkin second-order logic (compact) is also valid in the full second-order logic (not compact) but not vice versa. Requiring compactness thus means requiring "more general" generality. But, is this requirement necessary for genuine logic?

In the presentation, I will discuss these three questions and explain how they help us to filter the $2^{2^{\aleph_0}}$-many candidates.

4.2.5 Generalizing Truth-Functionality – GeTFun 1.0

This workshop is organized by

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The Fregean-inspired Principle of Compositionality of Meaning (PoC), for formal languages, may be construed as asserting that the meaning of a compound expression is deterministically (and often recursively) analysable in terms of the meaning of its constituents, taking into account the mode in which these constituents are combined so as to form the compound expression. From a logical point of view, this amounts to prescribing a constraint ?that may or may not be respected? on the internal mechanisms that build and give meaning to a given formal system. Within the domain of formal semantics and of the structure of logical derivations, PoC is often directly reflected by metaproperties such as truth-functionality and analyticity, characteristic of computationally well-behaved logical systems.

The workshop GeTFun is dedicated to the study of various well-motivated ways in which the attractive properties and metaproperties of truth-functional logics may be stretched so as to cover more extensive logical grounds. The ubiquity of non-classical logics in the formalization of practical reasoning demands the formulation of more flexible theories of meaning and compositionality that allow for the establishment of coherent and inclusive bases for their understanding. Such investigations presuppose not only the development of adequate frameworks from the perspectives of Model Theory, Proof Theory and Universal Logic, but also the construction of solid bridges between the related approaches based on various generalizations of truth-functionality. Applications of broadly truth-functional logics, in their various guises, are envisaged in several areas of computer science, mathematics, philosophy and linguistics, where the ever increasing complexity of systems continuously raise new and difficult challenges to compositionality. Particular topics of interest include (but are not limited to) the following:

a) Algebraic valuation semantics;

b) Curry-Howard proof systems and Focusing;

c) Controlled non-determinism: Metalogical properties;

d) Distance-based reasoning;

e) Information sources;

f) Labeled deductive systems;

g) Many-valuedness meets bivalence;

h) Non-determinism and premaximal paraconsistency logics, etc;

i) Semantics-informed proof theory of modal and fuzzy logics;

j) Truth-values.

The invited keynote speakers of this workshop are Agata Ciabattoni (page 65), Edward Hermann Haeusler (page 73), Beata Konikowska (page 75) and Heinrich Wansing (page 85).

Contributed Talks

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A Dissimilarity-Based Approach to Handling Inconsistency in Non-Truth-Functional Logics

Many commonly used logics, including classical logic and intuitionistic logic, are trivialized in the presence of inconsistency, in the sense that inconsistent premises lead to the derivation of any formula. For such logic $\vdash$, it is often useful to define an *inconsistency-tolerant* variant $\mid\!\sim$ of $\vdash$ with the following properties:

- Faithfulness: $\mid\!\sim$ and $\vdash$ coincide with respect to consistent premises (i.e., for every satisfiable set of formulas Γ and every formula ψ, it holds that $\Gamma \mid\!\sim \psi$ iff $\Gamma \vdash \psi$).

- Non-Explosiveness: $\mid\!\sim$ is not trivialized when the set of premises is not consistent (i.e., if Γ is not satisfiable then there is a formula ψ such that $\Gamma \not\mid\!\sim \psi$).

A common way of defining an inconsistency-tolerant variant of a given logic is by incorporating distance-based considerations for that logic. This method is very standard in the context of propositional classical logics, e.g. for belief revision, database integration and consistent query answering. It involves distance functions that supply numeric estimations on how 'close' a given interpretation is to satisfying the premise formulas. However, this approach is tailored for classical logic, and cannot be easily imported to other, non-classical formalisms, in particular to those that may not have truth-functional semantics.

For adjusting the distance-based approach to semantics that are not necessarily truth-functional, we introduce the weaker notion of *dissimilarities*. Dissimilarity functions provide quantitative indications on the distinction between their arguments, without preserving identities. Thus, two objects with zeroed dissimilarity need not be the same. This weakening of the notions of (pseudo) distances allows us to define, in a uniform and general way, a variety of inconsistency-tolerant logics, including those whose semantics need not be truth-functional (such as those that are defined by non-deterministic matrices, see [1]).

Generally, a dissimilarity-based entailment relation $\mid\!\sim$ induces a preferential semantics (in the sense of Shoham, [2]), defined by: $\Gamma \mid\!\sim \psi$ iff ψ is satisfied by all the interpretations that are 'as similar as possible' to Γ. We examine the basic properties of the entailment relations that are obtained by this way, provide general methods of generating such entailments, and exemplify dissimilarity-based reasoning in various semantic structures, including multi-valued semantics, non-deterministic semantics, and possible-worlds (Kripke-style) semantics. This general approach may be used for extending traditional distance-related methodologies to handle real-world phenomena (such as incompleteness, uncertainty, vagueness, and inconsistency) that are frequently in conflict with the principle of truth-functionality.

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A Theorem on Compositionality and Recursion

Pagin and Westerståhl in their paper "Compositionality I-II" (2010) write: "Standard semantic theories are typically both recursive and compositional, but the two notions are mutually independent". The reason why the property of recursivity (PR) does not imply the principle of compositionality (PC) is obvious: recursive functions may contain syntactic terms as arguments, as (roughly) in $\mu(\alpha(e)) = r_\alpha(\mu(e), e)$, where $\mu(e)$ is a semantic argument, and e is a syntactic argument of the recursive funcion r_α; whereas the form of compositional functions brings to the picture only arguments of a semantic nature, as in $\mu(\alpha(e)) = r_\alpha(\mu(e))$. Conversely, the authors point out that, in order for PC to imply PR, we should require that the compositional function r_α be as well recursive. Alas, there is no such requirement in any standard formulation of PC.

Notice that, unlike the direction $PR \Rightarrow PC$, whose negative proof relies directly on the fact that syntactic arguments would make possible the failure of substitutivity —which is shown by Hodges in his "Formal features of compositionality" (2001) to be equivalent to PC, under an additional weak hypothesis—, there is no actual proof of the failure of the converse. All that is said is that the standard formulations of PC do not require the compositional function to be recursive.

The aim of this talk is to prove a theorem whose content is $PC \Rightarrow PR$ (a few more reasonable hypotheses are assumed which will be made explicit), through showing that given PC, $\neg PR$ entails a contradiction. We call *semantic operator* any function taking semantic arguments into semantic values (these and other related notions will be more precisely defined in the talk following Hodges' algebraic setup).

(THEOREM) Suppose PC. Then, any semantic operator r_α is recursive.

The formulation of PC does not require PR, but implies it. And if the former implies the latter, it is natural that the formulation of the former saves the explicit formulation of the latter.

I will conclude the talk by proposing as a consequence of the Theorem and of a result in Pagin's "Communication and the complexity of semantics" (2012) a general argument for the suitability of compositionality derived from the standard arguments variously known as productivity, learnability, systematicity and so on. Roughly, it says that compositionality is a particularly high-performative kind of recursion operation for meaning functions. If recursion does not imply compositionality, on the other hand compositionality implies a very powerful kind of recursion. I will call it the "Frege-Pagin argument", since it puts the famous Fregean astonishment about the power of languages, together with the astonishment contained in Pagin's result about the efficiency with which languages realize their power.

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Toward a Non-deterministic Fuzzy Logic

Fuzzy set theory was introduced as a mathematical framework to deal with approximate reasoning. The main idea is to consider truth degrees (values in the unit interval) instead of the classical binary values in order to capture vague concepts and reasoning. Since the Zadeh's seminal work [11] many extensions of fuzzy set theory have been proposed among them many-valued fuzzy sets. This work proposes an investigation on the field of *hesitant fuzzy sets* (shortly HFSs) which was proposed by Torra in [5] and [6] in order to deal with multi-criteria decision making. The idea rests on the intuition that a set of possible values grouped together based on certain criteria, built to define the membership degree of an element, provides the "best" given alternative.

Our investigation focuses on the study of *aggregation functions for HFSs* which have an important role in fuzzy reasoning based (see [4]). The action of aggregations allow the reduction of a set of hesitant fuzzy degrees (estimating membership degrees of distinct elements) to a unique representative and "meaningful" HFS. So, it is possible to can make use of different information expressed by means of hesitant fuzzy degrees, e.g provided by several sources or expertises, that lead to a conclusion or a decision, which can be aggregated in a single hesitant fuzzy degree.

In [8], aggregation operators for hesitant fuzzy information were introduced and the relationship between intuitionistic fuzzy set and hesitant fuzzy set were also presented, including some operations and aggregation operators for hesitant fuzzy elements. However, this work did not provide any kind of theoretical approach for a general notion of aggregation operators for hesitant fuzzy degrees.

Usually, applications based on hesitant fuzzy sets requires only finite non empty hesitant fuzzy degrees (typical hesitant fuzzy sets - THFS). We consider only this context to study aggregation operators which are defined over the set $\mathbb{H} \subseteq \wp([0,1])$ of all finite non-empty subsets of the unitary interval $[0,1]$.

In [2] it was introduced the notion of finite hesitant triangular norms (FHTNs) and investigated the action of $\mathbb{H}$-automorphisms over such operators. The concept of overlap functions was introduced by Bustince, Fernandez, Mesiar, Montero and Orduna [3] in 2010 with the aim to deal with problems of classification, like in the field of image processing, which naturally encompasses the overlapping problem. In this case, the degree of overlap between two functions, which represent both the object and its background (in a scale of L levels of gray), can be interpreted as the representation of a lack of an expert's knowledge aiming to determine if a certain pixel belongs to the object or to its background. Then, the overlap functions are defined as a measurement of such overlapping degree. Overlap functions can also be applied in decision making which is based on fuzzy preference relations. In both applications, the property of associativity, which is in T-norms axiomatic, is not a required property for combination/separation operators.

According to Arnon Avron [1]:

> "The principle of truth-functionality (or compositionality) is a basic principle in many-valued logic in general, and in classical logic in particular. According to this principle, the truth-value of a complex formula is uniquely determined by the truth-values of its subformulas. However, real-world information is inescapably incomplete, uncertain, vague, imprecise or inconsistent, and these phenomena are in an obvious conflict with the principle of truth-functionality. One possible solution to this problem is to relax this principle by borrowing from automata and computability theory the idea of non-deterministic computations and apply it in evaluations of truth-values of formulas. This leads to the introduction of non-deterministic matrices (Nmatrices): A natural generalization of ordinary multi-valued matrices, in which the truth-value of a complex formula can be chosen non-deterministically out of some non-empty set of options."

Definition: A non-deterministic matrix, Nmatrix or NM for short, for a propositional language $\mathcal{L}$ is a tuple $\mathcal{M} = \langle \mathcal{V}, \mathcal{D}, \mathcal{O} \rangle$ such that:

1. $\mathcal{V}$ is a non-empty set of truth values;
2. $\mathcal{D}$ is non-empty proper subset of $\mathcal{V}$; and
3. $\mathcal{O}$ is the set of functions $\tilde{\diamond}$ from $\mathcal{V}^n$ to $\mathcal{P}(\mathcal{V}) - \{\emptyset\}$ and
   $\diamond$ is a n-ary connective in $\mathcal{L}$.

We propose to introduce the notion of finite hesitant overlaps (FHOs, i.e. overlap functions for hesitant degrees, $\mathbb{H}$, and a method to obtain a FHO $\Theta(S)$ from a finite set of overlaps S. We also propose the introduction of **non-deterministic overlaps for fuzzy FNM**, i.e. we generalize the notion of overlaps for functions of the form $O : [0,1] \times [0,1] \to \mathbb{H}$. Finally, we will prove that every FHO $\Theta(S)$ obtained from a finite set of overlaps S determines a non-deterministic overlap. We will will also demonstrate that: if the overlaps in S preserve elements in $\mathcal{D}$ — i.e. for each $O \in S$, $O(x,y) \in \mathcal{D}$ iff $x, y \in \mathcal{D}$ — then the resulting non-deterministic overlap obtained via the previous construction will satisfy the condition (CPD).

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On the correctness of the Thomason embedding

The aim of this work is to argue that of the myriad embeddings of propositional languages into modal ones (*e.g.*, those of Gödel [3], Łukowski [4], and Thomason [6]) that of Thomason most correctly captures real semantic compositionality. In turn, logics resembling Nelson's logic of constructible falsity [5] suggest themselves as extremely well-motivated.

To begin, I argue that every proposition has as a part some "preamble," a collection of conditions by the lights of which that proposition is to be understood. The most famous example of a preamble is that of intuitionistic logic, in which a proposition A bears the preamble "there is a constructive proof that..." [1]. Even in the case of classical logic, in which the preamble is less explicit, the Gödel embedding into the modal logic Triv (and that logic's soundness and completeness with respect to one-point, reflexive frames) provides a clear and salient picture for the preamble of "it is actually true that...." The notion of a preamble serves to underscore that when an intuitionist utters "B" and a classical mathematician utters the same, there are in a sense two distinct propositions being uttered while there is still something common between them.

Let a preamble be denoted by $\bigcirc$ and let $\circ$ denote an arbitrary connective. Modulo the above observation, the Principle of Compositionality described by Frege [2] or Wittgenstein [7] entails then that the semantic interpretation of the complex proposition denoted by $A \circ B$ is not a function of A and B but, rather, ought to be regarded as a function of $\bigcirc A$ and $\bigcirc B$. This observation supports that, as general schemata, such embeddings have latched onto the right intuition.

In essence, when dealing with this approach to logic, there are two dimensions to be analyzed: the question of the correct embedding and the question of the correct preamble. The latter question won't be taken up here but an argument is given that there is a correct approach with respect to the former.

For one, I argue that the Principle of Compositionality *does not* entail that semantic complexity must track syntactic complexity; this is supported by many adherents of this principle, *e.g.*, Wittgenstein's claim that tautologies all mean the same thing (that is, nothing) irrespective of their syntactic complexity [7]. Granted this, I argue that negated atomic propositions, despite their greater *syntactic* complexity, are not complex but are themselves *de facto* atoms. This follows from a host of quite innocuous principles. Appealing to the antisymmetry of the meronomic relation, a squeezing argument can be made from the claim that negation is an involution. Of course, this is anathema to a hardened intuitionist but the argument may be rehearsed on the basis of simpler cases, such as the existence of contradictory predicates (*e.g.*, $\geq$ and $<$ on the natural numbers) neither of which is obviously more semantically fundamental than the other. A general picture, then, is that a negated atom with preambles made explicit ought not to be depicted as $\bigcirc\neg\bigcirc A$ but, rather, as $\bigcirc\neg A$.

This entails that the Thomason embedding, in its symmetric treatment of both positive and negated atoms, better represents how complex propositions ought to be considered compositionally. To the classical logician, inasmuch as the Thomason embedding into $\mathsf{S5}$ yields classical logic, this bears the consequence that the Thomason embedding better elucidates the semantic composition of propositions. With respect to constructive logics, however, this suggests a deficiency in the sort of semantic compositionality to which the intuitionist is beholden and reinforces the strength of Nelson's approach.

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Alternative Supervaluation for Kripke's Theory of Truth

In 1982 Gupta presented the following scenario in order to criticize Kripke's theory of truth: Assume that the sentences

A1: Two plus two is three
A2: Snow is always black
A3: Everything B says is true
A4: Ten is a prime number
A5: Something B says is not true

are all that is said by a person A, and the sentences

B1: One plus one is two
B2: My name is B
B3: Snow is sometimes white
B4: At most one thing A says is true

are all that is said by a person B. The sentences A1, A2, and A4 are clearly false and B1, B2, and B3 are clearly true. So it seems unobjectionable to reason as follows: A3 and A5 contradict each other, so at most one of them can be true. Hence at most one thing A says is true. But that is what B says with his last sentence, so everything B says is true. This is again what A says with A3 and rejects with A5, so A3 is true and A5 false.

But contraintuitively Kripke's theory, in its strong Kleene schema, minimal fixed point version, tells us that A3, A5, and B4 are all undefined. The reason is that the evaluation of A3 and A5 awaits the determination of B4, which in turn cannot get a truth value before A3 or A5 do.

By adding extra truth predicates to some of the sentences, other versions of the problem can be formulated for which virtually the same intuitive argument for all of the sentences having proper truth values, can be given. Gupta showed that none of the supervaluation versions of Kripke's theory can handle all of those versions. I will show that neither can Gupta's Revision Rule theory.

In addition, when turning to Kripke's supervaluation version or Gupta's theory, in an attempt to make some progress on the problem, compositionality is given up. For instance, there will be true disjunctions without a true disjunct.

I will present an alternative method of supervaluation for Kripke's theory, employing trees, that not only gives the intuitively correct verdict in all versions of the problem, but does so in a way that results in a fully compositional fixed point.

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Finite-valued Semantics for Canonical Labelled Calculi

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A useful semantics is an important property of formal calculi. In addition to providing real insights into their underlying logic, such semantics should also be *effective* in the sense of naturally inducing a decision procedure for its calculus. Another desirable property of such semantics is the possibility to apply it for characterizing important syntactic properties of the calculi, which are hard to establish by other means.

Recently some systematic methods for constructing such semantics for various calculi have been formulated. In particular, *labelled sequent calculi* with generalized forms of cuts and identity axioms and natural forms of logical rules were studied in this context. Such calculi, satisfying a certain coherence condition, have a semantic characterization using a natural generalization of the usual finite-valued matrix called *non-deterministic matrices*, which is effective in the above sense.

In this talk, we show that the class of labelled calculi that have a finite-valued effective semantics is substantially larger than all the families of calculi considered in the literature in this context. We start by defining a general class of fully-structural and propositional labelled calculi, called *canonical labelled calculi*, of which the previosuly considered labelled calculi are particular examples. In addition to the weakening rule, canonical labelled calculi have rules of two forms: primitive rules and introduction rules. The former operate on labels and do not mention any connectives, while the latter introduce exactly one logical connective of the language. To provide semantics for all of these calculi in a systematic and modular way, we generalize the notion of non-deterministic matrices to *partial non-deterministic matrices* (PNmatrices), in which empty sets of options are allowed in the truth tables of logical connectives. Although applicable to a much wider range of calculi, the semantic framework of finite PNmatrices shares the following attractive property with both usual and non-deterministic matrices: any calculus that has a characteristic PNmatrix is decidable. We then apply PNmatrices to provide simple *decidable* characterizations of the crucial syntactic properties of strong analyticity and strong cut-admissibility in canonical labelled calculi. Finally, we demonstrate how the theory of labelled canonical calculi developed here can be exploited to provide effective semantics also for a variety of logics induced by calculi which are not canonical. One such example is calculi for paraconsistent logics known as C-systems.

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Malinowski Modalization and the Leibniz Hierarchy

We use the framework of *fibring of logical systems*, as presented by Fernández and Coniglio in [2], as a way to combine logical systems, to show that the *modal systems* of Malinowski [4], which were originally defined as structural deductive systems with a given set of theorems (including all classical tautologies) and closed under modus ponens, can be obtained as fibrings of classical propositional logic with appropriately defined modal implicative logics. The goal of these results, besides providing additional examples of the usefulness and broad applicability of the fibring process, is to study this process with respect to the Leibniz (algebraic) hierarchy of logics (see [3]).

In another direction we extend the constructions of Malinowski to apply not only to extensions of classical propositional calculus, but, also, of any arbitrary equivalential logic $\mathcal{S}$, in the sense of abstract algebraic logic, that has the deduction detachment theorem with respect to an implication system forming part of its equivalence system. For instance, classical propositional calculus falls under this framework, since it has the deduction detachment theorem with respect to$\{x \to y\}$ as well as being equivalential with respect to the equivalence system $\{x \to y, y \to x\}$. We extend Malinowski's result (Theorem II.4 of [4]) asserting that $\vec{\mathrm{K}}$ is equivalential to show that a similarly defined logic over an arbitrary equivalential deductive system with the deduction detachment theorem (and not just over classical propositional calculus) is equivalential.

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Do not be afraid of the Unknown

We propose a useful model for representing and reasoning about information collected by single agents or combinations thereof. The model is based on a generalized notion of entailment that accommodates both for distinct doxastic attitudes of the involved agents and for distinct aggregating strategies used by their societies. One of the advantages of this model is that it provides a natural interpretation for non-classical logical values characterizing 'the unknown': some sentence unbeknownst to a given agent might be a sentence which the agent has reasons to accept and also reasons to reject; alternatively, the sentence might be unbeknown to the agent if she simultaneously has reasons not to accept it and reasons not to reject it. Semantically, while the former agent might accept truth-value gluts, the latter agent might accept truth-value gaps. In an analogous fashion, for societies of agents, some sentence may be qualified as unknown if none of the involved agents accept it and none of the agents reject it, either; alternatively, the sentence may be called unknown if not every agent from that society rejects it and not every agent accepts it. In this case, the former society is gappy, the latter glutty. As we will show, yet another advantage of our generalized notion of entailment and of its underlying high-dimensional structure of truth-values is that they provide a simple and appealing framework for the uniform representation of many known logics.

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The mistery of duality unraveled: dualizing rules, operators, and logics

Though one often finds in the literature the concept of 'dualization' employed to describe a certain form of opposition that holds between statement-forms, between logical operators, and even between logics themselves, only rarely a formal definition of duality is actually offered by the very authors that embrace the terminology. Basic as it might seem, proper definitions of duality appear also not to have been incorporated in logic textbooks and courses beyond the level of side comments, exercises, or cheap talk. The current scarcity of basic material available addressing themes of Universal Logic has also not helped the development of firm grounds for setting up a theory of logical duality generous enough to cover non-classical territory.

In the presence of a classical negation, a common syntactic approach used to produce the dual of a given logical connective is the one we might call De Morgan Method: Exchange each atomic argument by its negated counterpart, and add also a negation over the whole expression. Semantically, still on a classical context, this gives rise to an approach we might call Inversion Method: Draw a truth-table for the formula and exchange 0s and 1s both for the atoms in the input and for the complex formula in the output. A third approach we might call Symmetry Method proposes to read systematically from right to left any semantical clause over the set of valuations and any proof-theoretical statement that is naturally read from left to right, and vice-versa. The latter approach gets complicated when the underlying formalism is asymmetric (such as the case of Gentzen systems for intuitionistic logic), despite the lasting potential informativeness of the dualization procedure (the notion of constructive truth of intuitionistic logic, for instance, may dualize into a notion of constructive falsity, and the verification methodology is dualized into falsification, as in [3]).

A straightforward abstract formal definition of duality may be found in [2]. In the present contribution we show that this definition encompasses all the above mentioned approaches, and applies equally well to logics that either extend or diverge from classical logic. The particular methods applicable to classical logic generalize in a natural way to modal logics and to many-valued logics, for instance. We shall illustrate in particular how finite-valued logics are dualized by taking advantage of their bivalent representations, following the algorithms surveyed in [1].

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(Non-)Compositionality in many-valued and two-dimensional logics

Usually researchers try to make the concept "compositionality" as precise as possible. Then they discuss whether systems of formal or natural languages are compositional or not. Against this mainstream it will be shown that the concept "non-compositionality" can be made precise within non-classical logics like many-valued logics as well as in two-dimensional classical logics. It will be discussed how we can get compositionality back. The steps we can make from non-compositionality to compositionality can be formalized as *degrees* of compositionality.

In the first part of the presentation the situation will be characterized mainly semantically with respect to 3- and 4-valued logics. In the second part we reconstruct the situation equivalently within a syntactically extended – two-dimensional – but nevertheless classical framework. The propositional variables become ordered pairs of classical formulas. 3- and 4-valued operators will be reconstructed as reduction operators with two-dimensional arguments. The reduction of each complex formula yields finally another ordered pair of classical formulas. It is possible to define several notions of validity for ordered pairs of classical formulas using only the classical vocabulary. In this environment the semantic steps back to compositionality within many-valued logics have interesting syntactical counterparts within our two-dimensional framework.

The general aim of my argumentation is that non-compositionality is not our enemy. There is an interesting analogy between extensionality and non-extensionality/intensionality. If we have a fruitful notion of non-compositionality we can precisely formulate cases of non-compositionality and the conditions / strategies / stages under which we get compositionality back. These cases are very important with respect to applications of formal methods to represent natural language expressions, e.g., sentence connectives.

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Developing a Hierarchy of Composition-Nominative Logics

Mathematical logic is widely used for investigation of programs; still, some program features such as *partiality of functions*, *elaborated system of data types*, *behavioral non-determinism* etc. are difficult to investigate by traditional logic. To cope with such difficulties we propose to construct *logics based directly on program models*.

To realize this idea we first construct models of programs using *composition-nominative approach* [1]. Principles of the approach (*development* of program notions *from abstract to concrete*, *priority of semantics*, *compositionality* of programs, and *nominativity* of program data) form a methodological base of program model construction. These principles specify program models as *composition-nominative systems* (CNS) consisting of *composition*, *description*, and *denotation* systems.

A composition system can be specified by two algebras: *data algebra* and *function algebra* (with compositions as operations). Function algebra is the main semantic notion in program formalization. Terms of this algebra define *syntax* of programs (descriptive system), and *ordinary procedure of term interpretation* gives a denotation system.

CNS are classified in accordance with levels of data consideration: *abstract*, *Boolean*, and *nominative*. The last level is the most interesting for programming. Data of this level are constructed over sets of names V and basic values A and are called *nominative data*. Such data can be considered as states of program variables. We identify *flat and hierarchic nominative data*. Partial mappings over flat nominative data are called *quasiary*. Such mappings do not have fixed arity.

Having described program models of various abstraction levels we develop a hierarchy of semantics-based logics which correspond to such models. Obtained logics are called *composition-nominative logics* (CNL).

At the abstract level such predicate compositions as disjunction and negation can be defined. Logics obtained are called *propositional CNL of partial predicates*.

At the nominative level we have two sublevels determined respectively by flat and hierarchic nominative data.

Three kinds of logics can be constructed from program models at the flat nominative data level:

- pure quasiary predicate CNL based on classes of quasiary predicates;
- quasiary predicate-function CNL based on classes of quasiary predicates and ordinary functions;
- quasiary program CNL based on classes of quasiary predicates, ordinary functions, and program functions.

For logics of pure quasiary predicates we identify *renominative*, *quantifier*, and *quantifier-equational* levels. Renomination, quantification, and equality are new compositions respectively specific for these levels [2].

For quasiary predicate-function logics we identify *function* and *function-equational* levels. Superposition and equality are respectively additional compositions for these levels.

To preserve properties of classical first-order predicate logic in first-order CNL we restrict the class of quasiary predicates. Namely, we introduce a class of equitone predicates and its different variations. These predicates preserve their values under data extension. Logics based on such predicate classes are the closest generalizations of the classical first-order logic that preserve its main properties. These logics are called *neoclassical logics* [2]. We develop algorithms for reduction of the satisfiability and validity problems in CNL to the same problems in classical logic [3].

At the level of *program logics* we introduce monotone logics of *Floyd-Hoare* type.

In a similar way we define several logics over hierarchic nominative data.

Let us admit that CNL differ from *nominal* logic [4] which shares some similarities with the logics constructed. But the predicates considered in the nominal logic have to be equivariant, that is, their validity should be invariant under name swapping. In our approach we consider general classes of partial quasiary predicates.

The main results for the constructed hierarchy of logics are the following:

- properties of constructed many-sorted algebras which form a semantic base of corresponding CNL are studied;
- for each predicate CNL a calculus of sequent type is constructed, its soundness and completeness is proved;

- a comparison of constructed predicate CNL with classical logic is made; subclasses of first-order CNL having properties of classical logic are identified.

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On logical harmony, sequent systems and modality

Although there is no precise notion of logical harmony, intuition given by Gentzen himself suggests that there should be a proper *balance* between rules for each connective in a deductive system in order to guarantee that the system itself has "good properties".

In the natural deduction setting, Francez and Dyckhoff [1] proposed a construction of general elimination rules from given introduction rules, and proved that the constructed rules satisfy the so called *local intrinsic harmony.* The main idea is, of course, to use the *inversion method* in order to go from I-rules to E-rules.

In the present work, we will define the notions of general and local intrinsic harmony in the sequent calculus setting and use the logical framework SELLF in order to construct left rules from right ones for a given connective. The idea is to build *coherent rules* [2] so that the resulting systems will have *the* good properties of cut-elimination, subformula property and analicity. Moreover, we go one step further and show how to build harmonical systems to a number of modal connectives and mathematical theories [3].

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A framework for specifying and reasoning in sequent calculus systems

It has been shown that linear logic with subexponentials can be successfully used as a framework for specifying proof systems [1] as well as proving fundamental properties about some of them [2], such as whether the systems admits cut-elimination. This provides a straightforward and uniform way of implementing sequent calculus proof systems, thus allowing one to search for proofs, and prove properties that would otherwise require an error-prone analysis of a number of cases. We implemented this framework and made available on the web a tool called TATU [5] for proving properties of systems. In particular, by using linear logic with subexponentials as a programming language we implemented several sequent calculus proof systems. Among the most interesting ones were (1) the system G3K for modal logics [3], whose cut-elimination property was proved, and (2) the sequent calculus for paraconsistent logics [4], in which we can do proof search.

In this talk I will present the theory underlying TATU [5], a system demonstration and the expected improvements for the future.

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Toward a notion of i-distance-based reasoning
Distance-based reasoning is a common technique for reflecting the principle of minimal change in different scenarios where information is dynamically evolving, such as belief revision, data-source mediators, knowledge recovery from knowledge-bases, pattern recognition, machine learning and decision making in the context of social choice theory. In distance-based semantics, a metric (distance) d is defined on the space of valuations, and can be used to measure the relevance of a valuation to a set of premises. Thus, given a theory of assumptions, instead of considering only the set of models of this theory (which is empty in case the theory contains some inconsistent information), we consider only those valuations that are d-closest (or most plausible) to the theory. The advantage of this approach is that the set of the most plausible valuations of a theory is, unlike its set of models, never empty, and so reasoning with inconsistent set of premises is never trivialized.

The method of distance-based reasoning has so far been mainly applied in the context of standard two-valued semantics. Recently Arieli and Zamansky have extended this approach to the context of two-valued non-deterministic semantics. This combination leads to a variety of entailment relations that can be used for reasoning about non-deterministic phenomena and are inconsistency-tolerant (e.g., in the context of model-based diagnostic systems).

The concept of distance can be generalized by using two methods: (1) Changing its axiomatic or (2) changing the nature of their values. We propose to investigate distance-based reasoning from the perspective of i-distances. An i-distance is a generalization of metrics which falls in the second approach, it is built as an ordered structure in which the standard metrics are instances. The idea is to investigate how enriched values of distances can be able to provide more accurate answers for distance-based reasoning methods. The idea of i-distance was introduced by the authors in [1,2] and recentely refined in [3].

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The Tractarian Collapse: on contradictions, nonsenses, mutilated truth tables and limits of truth-functionality

The meaningfulness of elementary propositions does not guarantee in some contexts the meaningfulness of the complex propositions built through their articulations. Some articulations have to be ad hoc forbidden in the construction of some complex propositions (eg. By ascriptions of colors and by measurements). A complex proposition could not thus be built exclusively over its elementary base and logical operators could not freely vary over propositions without being sensitive about which propositions they are connecting. This represents a challenge, in general, for any theory which defends an all-encompassing truth-functionality and, in particular, for the influential Philosophy of Tractatus. We know that propositions in the Tractatus, and also in Some Remarks of Logical Form are either elementary or molecular. These must be possible so we can reduce truth-functionally molecular propositions to elementary, bipolar and logically independent propositions. Thus, the truth value of these elementary propositions would always be compatible with the distribution of truth values of others. We would have, then, tautologies and contradictions as extreme cases of this combinatorial and neutral game. Nevertheless, the ascription of colors could not be trivially a case for displaying atomic logical independence, i.e., we would still have implications and exclusions in this context. In any form, the output of the tractarian passage 6.3751 is unsatisfactory in at least two lines of argumentation: with numbers and with velocities of particles, because they imply the mutilation or restriction of the articulatory horizon of the truth tables, blocking ad hoc the free distribution of values of truth to propositions. Here there must be a restriction in the distributions of truth-values in molecular propositions. It must be clear that the problem here is less with the falsehood than with the absurdity. Contradictions belong to symbolism, while nonsense or absurdities should not. This necessary mutilation of truth tables shows interesting cases of logical dependency between some propositions and their internal components. What is evident here is the ineptitude of the truth table or of any scheme of truth-functionality to explain the exclusion of color in particular and of degrees in general. The truth table notation does not capture the logical multiplicity of the system in which it is used, i.e., it allows the articulation of symbols of things that cannot be articulated in reality. It is a nonsense that the truth tables thought as a notation does not prevent, even though being created to prevent it. Here we clearly see how the limitation of the truth table shows a limitation in the conceptual framework of the Tractatus, and vice versa.

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Suszko-Reduced Logics and Truth-Functionality

In the 70s of the last century many-valued logics were already established. They had a well-founded theoretical basis as well as their eligibility in practice. Then Roman Suszko entered the stage. He claimed that many-valued logics does not really exist. The various truth-values are nothing but algebraic values and by no means "degrees of truth". Suszko's thesis did not attack the application of many-valued logics in industry and technology but the understanding of how to interpret the various degrees of truth. Meanwhile this challenging statement would have been forgotten if its representative were not Roman Suszko, one of the famous Polish logicians of that time, so that one dealt seriously with the problem how to handle that thesis. However, it took more than 10 years until Suszko's thesis recieved a formal framework by Malinowski. Further questions remained. How to make a two-valued logic from a many-valued one? What kind of structure has the two-valued logic and how to deal with the biggest deficit of the Suszko-Reduced-Logic – the loss of truth-functionality. In 2005 the paper *Two's company: "The humbug of many logical values"* was published by Carlos Caleiro et.al. in which a procedure for constructing a two-valued semantic was specified. This procedure is based on a suitable separation of truth-degrees through various formulas of many-valued logic. The aim of that characterisation is the limitation of homomorphisms which can be used for the valuation of the reduced many-valued logic. For it is *clear* that a reduced many-valued logic usually is no longer truth-functional and in order to guaranty the adequacy of such logics the homomorphisms need limitation.

In my master thesis "Logik und Relationale Strukturen" I intensively dealt with Suszko's and Caleiro's works. The target of my work was to specify precisely the limitations of the homomorphisms of the Suszko reduced logics. Already in *Two's company* the formal means had been provided. However, there was no specification of general procedures for definite logics. I was able to develop a procedure for any many-valued Łukasiewicz logic $Ł_n$ which generates a sequence of formulas which separates the truth values depending on the number of truth degrees, so that a two-valued pendant arises. Furthermore, it was shown that this two-valued logic is nevertheless a truth-functional one. Using suitable interpreting it can be shown that the new two-valued logic is isomorphic to its pendant. Simultaniously I showed by means of a counter-example that there actually are many-valued logics which are not trivial but cannot be made two-valued. At this point I stress the fact that my approach does not basically differ from Caleiro's. On the contrary I have completely taken over the ideas of separating the truth-degrees.

However I have developed the idea further. I will provide an effective procedure for gaining a two valued semantic for $Ł_n$ the n-valued Łukasiewicz' logic. And I will show that Suszko reduced logics actually remain truth functionality.

4.2.6 Non-Classical Mathematics

This workshop is organized by

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The 20th century has witnessed several attempts to build (parts of) mathematics on grounds other than those provided by classical logic. The original intuitionist and constructivist renderings of set theory, arithmetic, analysis, etc. were later accompanied by those based on relevant, paraconsistent, contraction-free, modal, and other non-classical logical frameworks. The bunch of such theories can be called non-classical mathematics and formally understood as a study of (any part of) mathematics that is, or can in principle be, formalized in some logic other than classical logic. The scope of non-classical mathematics includes any mathematical discipline that can be formalized in a non-classical logic or in an alternative foundational theory over classical logic, and topics closely related to such non-classical or alternative theories. (For more information about Non-Classical Mathematics have a look here).

Particular topics of interest include (but are not limited to) the following:

a) Intuitionistic, constructive, and predicative mathematics: Heyting arithmetic, intuitionistic set theory, topos-theoretical foundations of mathematics, constructive or predicative set and type theories, pointfree topology, etc.

b) Substructural mathematics: relevant arithmetic, contraction-free naive set theories, axiomatic fuzzy set theories, fuzzy arithmetic, etc.

c) Inconsistent mathematics: calculi of infinitesimals, inconsistent set theories, etc.

d) Modal mathematics: arithmetic or set theory with epistemic, alethic, or other modalities, modal comprehension principles, modal treatments of vague objects, modal structuralism, etc.

e) Non-monotonic mathematics: non-monotonic solutions to set-theoretical paradoxes, adaptive set theory, etc.

f) Alternative classical mathematics: alternative set theories over classical logic, categorial foundations of mathematics, non-standard analysis, etc.

g) Topics related to non-classical mathematics: metamathematics of non-classical or alternative mathematical theories, their relative interpretability, first- or higher-order non-classical logics, etc.

The invited keynote speaker is Arnon Avron (page 57).

Contributed Talks

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A Sober Librationist Interpretation of ZF

The librationist system now named £ (libra) is detailed some in [1]. £ reminds of paraconsistent systems but the author thinks of it as *parasistent*, *bialethic* and consistent and that some paradoxical sentences are *complementary* and not contradictory with their own *negjunctions* ("negations"); this has been made more precise in [3]. £ is semi-formal (*contentual*) and negjunction ("negation") complete and not recursively axiomatizable; we count it as a *theory of sorts*, to distinguish from sets. Let us agree that a theory T extends S soberly iff (i) the set of theorems of S is a proper subset of the set of theorems of T and (ii) no theorem A of T is such that the negjunction ("negation") of A is a theorem of S; T soberly interprets S iff T interprets S and for no *intrepretans* A^i in T of the *interpretandum* A of S does T both have A^i and its negjunction as theorems. [1] shows that £ soberly interprets $\mathrm{ID}_{\prec\omega}$ plus Bar Induction so that by established results of proof theory £ is stronger than the *Big Five* of Reverse Mathematics. £ is *unordered* and *untypical* and it accommodates a fixed-point construction I call *manifestation point* which originates with [9] and [4]: If $A(x, y)$ is a formula with x and y free then there is a sort f^A so that it is a maxim in £ that $\forall x(x \in f^A \equiv \mathbf{TT}A(x, f^A))$; a maxim in £ is a theorem whose negjunction is not also a theorem of £, and minors are theorems which are not maxims. In combination with isolated partial axiomatic and inferential principles and alethic comprehension AC_M which connects sort abstraction with the truth operator $\mathbf{T}$, see [1, §3], we now use manifestation points in order to define a rather complex manifestation point E^u which captures the notion of being a sort definable relative to u (Cfr. [7, Ch. 5] for a related construction). We next define a manifestation point $\dot{Z}$ and thereby bring into existence also $E^{\dot{z}}$ which $\dot{Z}$ calls upon in order to isolate the notions of $\dot{Z}$-definable power and *$\dot{Z}$-carptum* (that which is picked) in order to get replacement relative to $\dot{Z}$; the sort $\dot{Z}$ is the manifestation point of the least sort containing an empty sort (here defined by a Leibnizian identity relation $\stackrel{\dot{z}}{=}$ relative to $\dot{Z}$) and closed under *omegation of* (the least sort containing the operand and all its $\stackrel{\dot{z}}{=}$ successors), $\{\}_{\dot{z}}$-pairing, definable power relative to $\dot{Z}$, the $\dot{Z}$-carptum, union and next inaccessible$_{\dot{z}}$. By [8] ZF with replacement minus extensionality does not suffice to interpret ZF. [6] shows that it interprets Z. However, [5] gives an interpretation of ZF with collection in a system S which is ZF with collection and a weak power set notion minus extensionality. I will show that £ soberly interprets S via $\dot{Z}$ if ZF plus "*there are omega inaccessibles*" has a standard model; so by the result of [5], £ then soberly interprets ZF.

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Fuzzy Sets and Fuzzy Classes in Universes of Sets

Traditionally, the concept of fuzzy sets is closely related to a fix set of interesting objects called the "universe of discourse" and a fuzzy set theory is developed over the set of all fuzzy subsets of this universe. However, this restriction to one universe seems to have several disadvantages. One can simply recognize that from the practical reasons it is more natural to deal with fuzzy sets over different universes as in the case of fuzzy sets of "fresh apples" and "fresh pears" than to suppose one common universe for all fuzzy sets, it means to consider a common universe (basket) of all apples and pears. Further, the presumption of one fix set as a common universe for all fuzzy sets brings some limitation on fuzzy sets constructions. For example, the concept of power fuzzy set cannot be introduced if one deals with the set of all fuzzy subsets of a fix universe.

Practically, it means that an adequate fuzzy set theory cannot be developed on a set of all fuzzy subsets of a fix universe. It should be noted that an analogical disadvantage has been also recognized by S. Gottwald and, therefore, he proposed in [1] a cumulative system of fuzzy sets.

In the presentation, we will introduce a universe of sets over which fuzzy sets are built. The definition of the universe of sets is primarily based on the axioms of Grothendieck universe and we only add an axiom ensuring that fuzzy sets with membership degrees interpreted in a complete residuated lattice are objects of this universe of sets. Recall that a Grothendieck universe is a set in which the whole set theory may be developed (see e.g. [2]). Some of the examples and properties of the universe of sets will be demonstrated. Further, we will establish the concept of fuzzy set in a universe of sets and show several constructions of fuzzy objects and fuzzy relations that are well known in the fuzzy set theory. Finally, we will define the informal but very useful notion of fuzzy class in a universe of sets which generalizes the concept of fuzzy set. Some properties of fuzzy classes will be also presented.

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Constructive Lessons for Paraconsistency

We discuss several cornerstone theorems of classical mathematics which come apart at the seams when viewed under the paraconsistent microscope. In particular, we investigate results concerning order and locatedness—a constructive concept—within a framework of analysis founded on a variety of paraconsistent logic. Practitioners of constructive mathematics have shown that there are crucial assumptions, implicitly made on the classical view (a result of the validity of omniscience principles), which separate out different versions of the same theorem. Here we shed light on what happens from the paraconsistent perspective. Again, we find (perhaps unsurprisingly) that one classical theorem has many paraconsistently distinguishable versions. But we find (perhaps surprisingly) that constructive techniques that play a central role in highlighting these differences can often be adapted to paraconsistency.

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Escher's Impossible Images
M.C. Escher is justly famous for his exploration of the conceptual limits of geometrical content. He worked through several themes, including ambiguity, points of view, the "conflict" between the flat and the spatial, and, of course, impossibility. This talk will concentrate on the theme of impossibility. To focus the discussion, the question to be asked is: how many impossible images did Escher produce? Opinion differs, which is hardly surprising. First, various appropriate distinctions need to be made, particularly the sense of "impossible" in play. We then offer a set of five templates of impossible images to be found in the literature, against which impossibility can be judged. One such template is Escher's principal novelty, which we call "Escher's Cube". A brief illustration of the techniques of inconsistent geometry is given, by focusing on how to describe Escher's Cube. In particular, a notion of degree of inconsistency can be developed. Finally, an answer is proposed to the question of how many impossible images there are to be found in Escher.

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Remarks on naive set theory based on da Costa's idea
There are several naive set theories based on paraconsistent logic. However, not every paraconsistent logic is thought to be suitable for developing naive set theory. Indeed, systems following the approach suggested by Newton C. A. da Costa are thought to be inappropriate due to the presence of classical negation. The aim of the paper is to challenge this view, and claim that it is in fact possible to deal with naive set theory based on systems that reflect da Costa's idea.

Two antecedent bodies of work play an important role in the present paper. The first comes from the work of Greg Restall and is based on the Logic of Paradox (**LP**) of Graham Priest. The main result of Restall is that naive set theory based on **LP** is actually non-trivial, i.e. there is a statement that is not provable in the developed theory. There are also some problems related to the theory, noted by Restall as well, but the point is that naive set theory need not be trivial or logically uninteresting when the underlying logic is **LP**.

The second body of work concerns Logics of Formal Inconsistencies (LFIs), developed by Walter Carnielli, Marcelo Coniglio, and João Marcos. The characteristic feature of LFIs is that they have an additional connective beside classical connectives called consistency (or inconsistency) that controls the behavior of contradiction. We may say that consistency shows explicitly when we can apply inferences of classical logic. There are many systems of LFIs developed in the literature, but here we will focus on the result that if we add a consistency (or inconsistency) connective to **LP**, then we obtain a system known as **LFI1** in which classical negation can be defined.

Based on these works, we show that naive set theory based on **LFI1** is also not trivial by following the proof of Restall. The point to be emphasized is the following. In formulating the axioms of naive set theory, we make use of biconditional. In the case of naive set theory based on **LP**, material biconditional is defined by paraconsistent negation. On the other hand, in the case of **LFI1**, there are two possibilities which have not been considered on a par. Indeed, all authors have, at least to the best of my knowledge, taken material biconditional to be definable in terms of classical negation. There is, however, another possibility—viz. to define material biconditional by paraconsistent negation. And it is this latter strategy that leads us to the desired result.

The general lesson that we learn from the result is as follows. The presence of classical negation, which is often the case for systems based on da Costa's idea, is not necessarily harmful if we use paraconsistent negation in formulating what are otherwise problematic principles such as naive comprehension. All is not without problems, however. Indeed, we need to examine the problem raised by Restall in the case of the **LP**-based theory, and we also need to give a separate justification for which negation is appropriate in formulating certain principles. For the former, the presence of consistency connective might help us, and for the latter, more discussion is required which I take up in the present paper.

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A semantical approach to conservativity of classical first-order theories over intuitionistic ones
The well-known conservativity result is that of Π_2-conservativity of Peano Arithmetic over its intuitionistic counterpart, Heyting Arithmetic and states that for every Π_2-formula A, if A is (classically) provable in the former then it is also (intuitionistically) provable in latter. In the typical proof of this fact one uses the so-called Gödel-Gentzen negative translation and the Friedman translation. For more details see [3].

We show that some conservativity results can be proven by means of semantical methods using Kripke models. We will consider arbitrary intuitionistic first-order theories, and say that a formula A is a $\forall\exists$-formula if it is of the form $\forall x \exists y B$ where B is either a quantifier-free or a semi-positive formula. We will also consider the so-called T-normal Kripke models, introduced by S. Buss in [1], i.e., models whose all the worlds are classical first-order structures that validate the theory T.

Using the pruning technique introduced in [2] we show that in some cases of conservativity results we can skip the assumption that the theory in question is invariant with respect to negative translation. More specifically, we show the conservativity result for all intuitionistic first-order theories that are invariant with respect to the Friedman translation and are complete with respect to conversely well-founded Kripke models with constant domains.

Moreover, we prove that if T is an intuitionistic theory which is complete with respect to a class of T-normal Kripke models than the classical counterpart of T is conservative over T with respect to all the formulas of the form $A \to B$ where A is semipositive and B is a $\forall\exists$-formula. Note also that one can show that each theory T which is invariant with respect to negative translation and Friedman translation is complete with respect to the class of Kripke models in question.

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Adding the ω-rule to Peano arithmetic by means of adaptive logic

In this talk I present a special extension of Peano arithmetic (PA). This extension validates the ω-rule and is therefore negation complete. Hence, the theorems of the presented theory are exactly those arithmetical formulas that are true in the standard model of arithmetic.

The solution presented here makes use of so called *adaptive logic (AL)*. This is a class of logics for various kinds of defeasible reasoning forms. They have a unifying format, with a dynamic proof theory, an intuitive semantics and a general metatheory. The advantage of AL's is that they are close to standard *Tarski logics* (reflexive, monotonic and transitive logics) but are able to define a much larger consequence set. Every AL is built up from a (relatively weak) Tarski logic, called the *Lower Limit Logic (LLL)*. The AL allows for the application of all the LLL-rules PLUS certain instances of the rules of a richer Tarski logic (the *Upper Limit Logic (ULL)*), i.e. those rules that are valid in the LLL on the condition that formulas of a specific logical form (called the *abnormalities*) are false. AL's adapt themselves to a premise set, i.e. the premises determine which ULL-consequences are derivable. In adaptive proofs, the application of ULL-rules results in a line that is derived on a condition (that some abnormalities are false). If this condition is falsified later, the line is marked and the formula of that line is no longer considered as derived.

The non-logical axioms of the presented theory are the traditional axioms of PA. The underlying logic is a new adaptive logic inspired by the ideas behind the adaptive logics for (empirical) induction devised by Diderik Batens (cf. [1] and [2,Chapter 3]). The logic is formulated in the format of Lexicographic

Adaptive Logics, developed by Christian Straßer and Frederik Van De Putte to formalize reasoning with abnormalities that have different priorities. The LLL is Classical Logic and the abnormalities are Σ_n-formulas of arithmetic in prenex normal form. The complexity of the abnormalities determines their priority order. The strategy is Reliability.

To illustrate why one needs a prioritized adaptive logic: here is a typical issue[4]. The PA-theorem $\neg\forall x\exists y\mathsf{Prf}(y,x) \vee \neg\forall x\neg\mathsf{Prf}(x,\ulcorner G\urcorner)$ is a minimal disjunction of abnormalities of our theory, where $\ulcorner G\urcorner$ is the coding of the Gödel sentence and $\mathsf{Prf}(\alpha,\beta)$ is a formula that is true iff α is the coding of a PA-proof of the formula of which β is the coding. This follows immediately from the conjunction of Gödel's two imcompleteness theorems. This means that one cannot conclude both generalizations $\forall x\exists y\mathsf{Prf}(y,x)$ and $\forall x\neg\mathsf{Prf}(x,\ulcorner G\urcorner)$, but a flat adaptive logic would give no clue as to which one of those generalizations can be concluded and which one cannot. Of course, the ω-rule allows us to derive the second generalization (in view of the first Gödel incompleteness theorem) and hence also the negation of the first generalization. This is achieved in the prioritized adaptive logic by giving priority to generalizations that are less complex.

It is expected that one can also devise a non-prioritized adaptive logic with the Minimal Abnormality strategy which results in exactly the same arithmetical theory, provided that the non-logical axioms of the theory are also the PA-axioms. The provisional results concerning this expectation, however, suggest that this approach is less elegant than the prioritized approach.

The philosophical motivation behind this approach to arithmetic is twofold. First, the logic is an arguably formal first order logic which has a complete proof theory with finite and recursive proofs with finitary rules. This is a remarkable result. As far as I know, until now, there was no logic with such a proof theory capable of structuring the set of true arithmetical sentences in the intended model of arithmetic. Of course, the obtained result would be impossible, in view of Gödel's first incompleteness theorem, if finite adaptive proofs could always serve as final demonstrations of their conclusions. The dynamic adaptive logic proofs are conceived in such a way that the final conclusions of the proofs are those that are in some particular sense *stably* derived in the proof (given all possible extensions of the proof). This dynamic characterization of the notion 'proof', makes it possible that adaptive logic consequence relations can be sufficiently complex to structure highly complex theories like the one which we consider here (see [3]).

Second, there are remarkable similarities between the logic and the existing formalization of inductive reasoning for empirical contexts by means of inductive adaptive logics. Also in actual mathematical practice, more particularly in the creative process that leads to the formulation of mathematical theorems and their proofs, one can observe reasoning methods similar to inductive generalization. The here presented logic may serve as an appropriate formalization of this type of reasoning.

[4]I assume here that PA is consistent

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Fixed Point Theorems in Non-Classical Mathematics
In many non-classical set theories, there is a well-known theorem to the effect that every function whatsoever has a fixed point [1, 3, 4, 7, 8, 9]. This result is a descendent of the fixed point theorem for untyped lambda calculus, itself related both to Kleene's second recursion theorem, as well as Curry's paradox [2, 5, 6]. Thus the history shows that having fixed points involves two dueling properties – a positive aspect, in providing the existence of solutions to equations, and a negative aspect, in generating contradiction and the possibility of incoherence.

In this talk I consider whether a paraconsistent set theory has a similar fixed point property, and what such a property means in an inconsistent (but non-trivial) setting. More broadly, I aim to characterize a large fragment of paraconsistent mathematics as being about structures that are (isomorphic to) complete lattices carrying automorphisms – that is, like the original untyped lambda calculus, settings that would be expected to have fixed points. The aim of the talk is to understand why fixed points arise so readily in much non-classical mathematics, and to point to good work such theorems can do.

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4.2.7 Intuitionistic Modal Logic and Applications (IMLA 2013)

This workshop is organized by

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Constructive modal logics and type theories are of increasing foundational and practical relevance in computer science. Applications of constructive modal logics are in type disciplines for programming languages, meta-logics for reasoning about a variety of computational phenomena and explanatory frameworks in philosophical logic. The workshop aims at developing and explaining theoretical and methodological issues centered around the question of how the proof-theoretic strengths of constructive logics can best be combined with the model-theoretic strengths of modal logics. Practical issues center around the question of which modal connectives with associated laws or proof rules capture computational phenomena accurately and at the right level of abstraction.

Topics of interest of this workshop include but are not limited to:

a) applications of intuitionistic necessity and possibility;

b) (co)monads and strong (co)monads;

c) constructive belief logics and type theories;

d) applications of constructive modal logic and modal type theory to formal verification, abstract interpretation, and program analysis and optimization;

e) modal types for integration of inductive and co-inductive types;

f) higher-order abstract syntax;

g) strong functional programming;

h) models of constructive modal logics such as algebraic, categorical, kripkean, topological, and realizability interpretations;

i) notions of proof for constructive modal logics;

j) extraction of constraints or programs from modal proofs;

k) proof search methods for constructive modal logics and their implementations.

The invited keynote speakers are Yuri Gurevich (page 74), Luca Viganò (page 84) and Gianluigi Bellin (page 58).

Contributed Talks

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An algebraic axiomatization of IKt system
Ewald in [1] considered tense operators G (*it will always be the case*), H (*it has always been the case*), F (*it will be the case*) and P (*it was the case*) on intuitionistic propositional calculus and constructed an intuitionistic tense logic system called IKt. The aim of this paper is to show an algebraic version of the IKt system. It is worth mentioning that our axiomatization is different from Chajda's in [2], for tense intuitionistic logic.

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Logic of Negation-Complete Interactive Proofs
We produce a decidable classical normal modal logic of internalized *negation-complete* or *disjunctive* non-monotonic interactive proofs (LDiiP) from an existing logical counterpart of non-monotonic or instant interactive proofs (LiiP). LDiiP internalizes agent-centric proof theories that are negation-complete (maximal) and consistent (and hence strictly weaker than, for example, Peano Arithmetic) and enjoy the *disjunction property* (like Intuitionistic Logic). In other words, internalized proof theories are *ultrafilters* and all internalized proof goals are *definite* in the sense of being either provable or disprovable to an agent by means of disjunctive internalized proofs (thus also called *epistemic deciders*). Still, LDiiP itself is classical (monotonic, non-constructive), negation-incomplete, and does not have the disjunction property. The price to pay for the negation completeness of our interactive proofs is their non-monotonicity and non-communality (for singleton agent communities only). As a normal modal logic, LDiiP enjoys a standard Kripke-semantics, which we justify by invoking the Axiom of Choice on LiiP's and then construct in terms of a concrete oracle-computable function. Our agent-centric notion of proof is also a *negation-complete disjunctive explicit* refinement of standard KD45-belief, and yields a *disjunctive* but negation-*in*complete *explicit* refinement of standard S5-knowledge.

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J-Calc: A typed lambda calculus for Intuitionistic Justification Logic
In this paper we offer a system J-Calc that can be regarded as a typed λ-calculus for the $\{\rightarrow, \bot\}$ fragment of Intuitionistic Justification Logic. We offer different interpretations of J-Calc, in particular, as a two phase proof system in which we proof check the validity of deductions of a theory T based on deductions from a stronger theory T'. We establish some first metatheoretic results.

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Hypothetical Logic of Proofs
We study a term assignment for an intuitionistic fragment of the Logic of Proofs (LP). LP is a refinement of modal logic S4 in which the assertion $\Box A$ is replaced by $[[s]]A$ whose intended reading is "*s is a proof of A*". We first introduce a natural deduction presentation based on hypothetical judgements and then its term assignment, which yields a confluent and strongly normalising typed lambda calculus λ^{IHLP}. This work is part of an ongoing effort towards reformulating LP in terms of hypothetical reasoning in order to explore its applications in

programming languages.

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A graph calculus for proving intuitionistic relation algebraic equations
We introduce a diagrammatic system in which diagrams based on graphs represent binary relations and reasoning on binary relations is performed by transformations on diagrams. We prove that if a diagram D_1 is transformed into a diagram D_2 using the rules of our system, under a set Γ of hypotheses, then "it is intuitionistically true that the relation defined by diagram D_1 is a subrelation of the one defined by diagram D_2, under the hypotheses in Γ". We present the system formally, prove its soundness, and left the question of its completeness for further investigation.

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If, not when
We present a logic of verified and unverified assertions and prove it sound and complete with respect to its possible-worlds semantics. The logic, a constructive modal logic, is motivated by considerations of the interpretation of conditionals in natural language semantics, but, we claim, is of independent interest.

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A rich language for negative modalities
The paper studies a modal language for negative operators– an intuitionistic-like negation and its independent dual– added to (bounded) distributive lattices. For each negation an extra operator is added to describe the models in which it behaves classically. The minimal normal logic with these operators is characterized. Statements of all important results are provided and proofs of main propositions are sketched.

4.2.8 SHAPES 2.0 – The Shape of Things

This workshop is organized by

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Shape, Form, and Structure are some of the most elusive notions within diverse disciplines ranging from humanities (literature, arts) to sciences (chemistry, biology, physics etc.) and within these from the formal (like mathematics) to the empirical disciplines (such as engineering and cognitive science). Even within domains such as computer science and artificial intelligence, these notions are replete with commonsense meanings (think of everyday perception and communication), and formalisations of the semantics and reasoning about shape, form, and structure are often ad hoc. Whereas several approaches have been proposed within the aforementioned disciplines to study the notions of shape, form and structure from different viewpoints, a comprehensive formal treatment of these notions is currently lacking and no real interdisciplinary perspective has been put forward.

This workshop will provide an interdisciplinary platform for the discussion of all topics connected to shape (broadly understood): perspectives from psycholinguistics, ontology, computer science, mathematics, aesthetics, cognitive science and beyond are welcome to contribute and participate in the workshop. We seek to facilitate a discussion between researchers from all disciplines interested in representing shape and reasoning about it. This includes formal, cognitive, linguistic, engineering and/or philosophical aspects of space, as well as their application in the sciences and in the arts.

We also welcome contributions on the relationship among representations of shape at different levels of detail (e.g. 2D, 3D) and in different logics, and with respect to different qualitative and quantitative dimensions, such as topology, distance, symmetry, orientation, etc.

Form and Function in Natural and Artificial Systems

Within the philosophy and practice of design, the ontological notions of shape, form and structure have a further role of constraining function, malfunction, and behaviour of things. In this perspective, the decision-making process in design is a trade-off between physical, logical and cognitive laws and constraints that intertwine shapes and functionalities. Here, the spatio-linguistic, conceptual, formal, and computational modeling of shape serves as a crucial step toward the realization of functional affordances. This line of thought extends to several other disciplines concerned not only with the design of technical systems, but also with the understanding of biological as well as socio-technical systems. For instance, in biochemistry the shape of molecular entities (proteins, small molecules) has a direct effect on their interactions which give rise to the capacities they can manifest and, in turn, to the processes of life and death. Representing and reasoning with the shapes and realizable functionalities of these entities is essential to understand basic biological processes. Of special importance, in this as well as other contexts, is the understanding of shape complementarity, that is, categorising the shapes of holes as well as the shapes of the entities that can fit into those holes, which can either facilitate or block the functionality of the overall system.

The invited keynote speakers of this workshop are Anthony Galton (page 71), Simon Colton (page 66), Roberto Marcondes Cesar Jr. (page 64), Barbara Tversky (page 83) and Roberto Casati (page 64).

Contributed Talks

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Sketch Learning by Analogy
Sketches are shapes that represent objects, scenes, or ideas by depicting relevant parts and their spatial arrangements. While humans are quite efficient in understanding and using sketch drawings, those are largely inaccessible to computers. We argue that this is due to a specific shape based representation by humans and hence the use of cognitively inspired representation and reasoning techniques could lead to more proficient sketch processing. We also propose a three-level system for sketch learning and recognition that builds on concepts from cognitive science, especially from analogy research, to map and generalize sketches.

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The Shape of Empty Space
We propose a human-centred model for abstraction, modelling and computing in function-driven spatial design for architecture. The primitive entities of our design conception ontology and computing framework are driven by classic notions of 'structure, function, and affordance' in design, and are directly based on the fundamental human perceptual and analytical modalities of visual and locomotive exploration of space. With an emphasis on design semantics, our model for spatial design marks a fundamental shift from contemporary modelling and computational foundations underlying engineering-centred computer aided design systems. We demonstrate the application of our model within a system for human-centred computational design analysis and simulation. We also illustrate the manner in which our design modelling and computing framework seamlessly builds on contemporary industry data modelling standards within the architecture and construction informatics communities.

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A new Molyneux's problem: Sounds, shapes and arbitrary crossmodal correspondences
Several studies in cognitive sciences have highlighted the existence of privileged and universal psychological associations between shape attributes, such as angularity, and auditory dimensions, such as pitch. These results add a new puzzle to the list of arbitrary-looking crossmodal matching tendencies whose origin is hard to explain. The puzzle is all the more general in the case of shape that the shapes-sounds correspondences have a wide set of documented effects on perception and behaviour: Sounds can for instance influence the way a certain shape is perceived (Sweeny et al., 2012). In this talk, we suggest that the study of these crossmodal correspondences can be related to the classical cases of crossmodal transfer of shape between vision and touch documented as part of Molyneux's question. In addition, these studies reveal the role that movement plays as an amodal invariant in explaining the variety of multimodal associations around shape.

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Structure, Similarity and Spaces

Much of the discussion about shape representation during the last two decades is fundamentally related to questions about the representation of parts. Inspired by the cognitive processes governing how people represent and think about parts, we provide a brief summary of our framework for representing part structures. It extends the Theory of Conceptual Spaces, where concepts are represented by regions in a mathematical space. We propose a special kind of conceptual space that can represent the part structure of a concept. The structure space of a whole is formed by the product of its parts. In this space, structural similarity judgements between concepts and between objects is reduced to distance measurements; i.e. objects that share a similar part structure are more close together in the space. We are still developing a more formal theory around these notions and we expect to be able to apply it in some real-world problems, particularly in object recognition.

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Local Qualities, Quality Fields, and Quality Patterns

When we describe the shape of certain entities, like a vase or a river, we refer to their qualities in different ways. A river has (more or less) a definite length, but its width varies with the distance from the source, typically getting higher towards the end. Similarly, a vase has a definite height, but its width may vary, reflecting a certain pattern that often marks a particular style. So, at least for certain entities, quality kinds such as length, height and width dont behave in the same way: length or height just inhere to these objects with no need of further qualification, while width requires a spatial localisation in order to be determined. We shall say that length and height, in these examples, are global qualities, while width is a local quality. Note that a local quality of a certain object does actually inhere to a part of that object, but, despite this fact, we tend to consider it, from the cognitive point of view, as a quality of the whole object: so, we rarely say the width of this river stretch is 100 meters, but we prefer to say the rivers width is 100 meters here. Analogously, we say the depth of the Adriatic Sea is much higher along the Croatian coast than along the Italian coast, referring to the rivers width or the seas depth as one single entity, although, so to speak, spread out in space. In many simple cases, this the way we describe the shape of a certain object in terms of the behaviour of a local quality along a spatial dimension. In this paper I would like to explore the way qualities of things behave with respect to the parts of such things. Building on the notion of individual quality introduced in the DOLCE ontology, I will introduce the new notions of local quality, quality field and quality pattern, stressing their cognitive role in many practical situations. I will first discuss Johanssons distinction between inclusive and exclusive properties, which I will take as a basis for my distinction between global and local individual qualities. Basically, the idea is that, given a certain individual quality q of kind Q with a value v inhering to a thing x, q is a global individual quality of x iff, necessarily, there exists a proper part y of x such that Q of y has a value w different from v. q will be a local individual qualityof x otherwise. I will then introduce the notion of a quality field as the mereological sum of all local qualities of a certain kind inhering to some thing (endurant or perdurant). I will argue that an expression like the rivers width or the depth of the sea actually refers to a quality field, and not to an individual quality. Quality fields will be used to introduce the further notion of quality pattern, and to analyse the distinction between variation and change. Consider for instance the Adriatic Sea, whose depth changed in the last 2000 years. At the Roman age, this field exhibited local variations corresponding a certain pattern, which is different from the pattern we observe today. The whole quality field did genuinely change in time, keeping its identity, while some of its individual qualities changed their value. So a quality pattern is different from a quality field, since its actual value distribution is essential to it, and not essential for the quality field.

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Shape Perception in Chemistry
Organic chemists make extensive use of a diagrammatic language for designing, exchanging and analysing the features of chemicals. In this language, chemicals are represented on a flat (2D) plane following standard stylistic conventions. In the search for novel drugs and therapeutic agents, vast quantities of chemical data are generated and subjected to virtual screening procedures that harness algorithmic features and complex statistical models. However, in silico approaches do not yet compare to the abilities of experienced chemists in detecting more subtle features relevant for evaluating how likely a molecule is to be suitable to

a given purpose. Our hypothesis is that one reason for this discrepancy is that human perceptual capabilities, particularly that of 'gestalt' shape perception, make additional information available to our reasoning processes that are not available to in silico processes. This contribution investigates this hypothesis.

Algorithmic and logic-based approaches to representation and automated reasoning with chemical structures are able to efficiently compute certain features, such as detecting presence of specific functional groups. To investigate the specific differences between human and machine capabilities, we focus here on those tasks and chemicals for which humans reliably outperform computers: the detection of the overall shape and parts with specific diagrammatic features, in molecules that are large and composed of relatively homogeneous part types with many cycles. We conduct a study in which we vary the diagrammatic representation from the canonical diagrammatic standard of the chemicals, and evaluate speed of human determination of chemical class. We find that human performance varies with the quality of the pictorial representation, rather than the size of the molecule. This can be contrasted with the fact that machine performance varies with the size of the molecule, and is absolutely impervious to the quality of diagrammatic representation.

This result has implications for the design of hybrid algorithms that take features of the overall diagrammatic aspects of the molecule as input into the feature detection and automated reasoning over chemical structure. It also has the potential to inform the design of interactive systems at the interface between human experts and machines.

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Shapes as property restrictions and property-based similarity
Varied approaches to the categorization of shapes and forms, as well as their mutual similarity and connectedness, are of great importance for the development of many scientific fields. In different contexts, domain knowledge can be represented semantically using ontologies expressed in OWL, where domain concepts are organized hierarchically and have their features defined as properties. Properties are used in OWL for defining classes with property restrictions, using *value constraints* and *cardinality constraints*.

In the domain of shape, form and structure representation, there were some attempts at modeling shapes ontologically, as an exhaustive class hierarchy. But instead of forcing this somehow artificial categorization upon the shape world, we would do the shapes more justice by defining them as property restrictions on classes. We can start by defining many different properties which would help us precisely describe the shapes we need. In this way, there is no need to a-priory decide which categorization should happen higher up in the hierarchy, they can peacefully co-exist together. The process is versatile and applicable in many different contexts. It also enables very natural comparison of shapes and establishes their similarity based on properties.

If we define shapes as property restrictions (on values and cardinality), we can find similar shapes by comparing their properties, starting from Tversky's feature-based model of similarity. Given two shapes S_1 and S_2, for each property p, we calculate how much the property p contributes to common features of S_1 and S_2, distinctive features of S_1 and distinctive features of S_2, respectively. How these values are calculated depends on how the property p is defined in each of S_1 and S_2. The property-based similarity of equivalent classes is equal to 1. For instances we simply compare the values-property pairs declared for each instance. The subclass relation is taken into account, providing each class with the property definitions inherited from parent classes.

Apart from modeling shapes as property restrictions on classes, this approach would bring new insights into modeling forms and patterns as well, as it avoids strict categorizations, providing a flexible environment for expressing various features of complex forms.

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The Shape of Absolute Coincidences. Salmon's Interactive Fork Model as shape of coincidental processes.
According to a particular view, chance events are not uncaused but they are simply the result of intersecting causal lines. More precisely, the intersections between different processes that belong to independent causal chains are the origin of accidental events, called *absolute coincidences.*

In the talk I will provide a new account devoted to showing the strong relation between absolute coincidences and Salmon's interactive fork criterion, in an attempt to endorse the idea that coincidences can be shaped in terms of a causal model.

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Representing numbers and figures in problem-solving activities in mathematics
In our presentation we rely on research by Grosholz (2007) and consider her thesis of the irreducibility of shape in the sense of iconic representation in mathematics. Against this background, we aim to discuss the epistemic value of iconicity both in the representation of numbers in arithmetic and figures in the case of geometry.

We bring in two case-studies selected from Leibniz's work with notations and diagrams in problem-solving contexts of work. In our first case-study that concerns the representation of number systems Leibniz argues for the view that the iconic aspects present in binary notation reveal structural relations of natural numbers that remain concealed in other numerical modes of representation such as the system of Arabic numerals. In our second case-study, we show how Leibniz designs a method which allows him to re-conceive a given shape – triangles – by transmuting it into another kind of shape – rectangles – as part of his strategy to solve the thus far unsolved problem of the squaring of the circle.

In the case of arithmetic, we focus on the idea that representations "articulate likeness by visual or spatial means". Grosholz suggests that even highly abstract symbolic reasoning goes hand in hand with certain forms of visualizations. To many this may sound polemical at best. Granted to the critic that "shape is irreducible" in geometry as it is the case with geometrical figures, but what is the role of iconicity in the representation of numbers, and more generally, what is involved in visualizing in arithmetic?

In visualizing we need to understand articulated information which is embedded in a representation, such articulation is a specific kind of spatial organization that lends unicity to a representation turning it intelligible. In other words, spatial organization is not just a matter of physical display on the surface (paper or table) but "intelligible spatiality" which may require substantial background knowledge so that the ability to read off what is referred to in a representation will depend on relevant training and expertise of the reader. Such cognitive act is successful only if the user is able to decode the encrypted information of a representation while establishing a meaningful relationship between the representation and the relevant background knowledge which often remains implicit.

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Shaping up: The Phenotypic Quality Ontology and Cross Sections

The Phenotypic Quality Ontology (PATO) uses the notion of a cross section to relate two- and three-dimensional shapes and to describe the shape of biological entities. What is a cross-section? What is a truthful ontological account of cross sections? In this communication I explore potential answers to these questions, approaching the task from philosophical and ontological perspectives, and provide a preliminary examination of the PATO shape hierarchy. Some critical observations, questions, and suggestions for the shape portion of PATO are presented.

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Dynamic Assembly of Figures in Visuospatial Reasoning

An exploratory, qualitative experiment sheds light on the depictive theory of mental imagery. The study analyzes the very operations subjects undertake

when solving visuospatial tasks. Preliminary results indicate that subjects do not make use of stable mental images: instead, they continuously assemble and re-assemble different perspectives through the guidance of heuristics and prototypes. These observations allow a reinterpretation of mental imagery. We want to forward the hypotheses that a) the assembly process itself is of much higher importance than usually acknowledged; b) that an assembled perspective (or figure) is defined by one's orientation towards certain operations; and c), that heuristics and prototypes are instantiated by a heterarchical organization of mental operations.

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Mental "structures" in the Berlin school of Gestalt Psychology: can sensation be described as "structural"?

It is not exaggerated to affirm that the modern notion of structure arises in Koffkas Growth of the Mind and in his following article, "Perception : An introduction to the Gestalt-theorie" (1922). The importance of the notion of structure as Koffka uses it lies in the fact that it is designed to replace the old empiricist notion of "sensation" as a real and separable element of the phenomenal field, corresponding to a definite stimulus. But, yielding to many suggestions by Köhler, Koffka does not only understand the interdependency of sensations in a structure as a causal one: in fact, he decidedly understands it as a logical one. Thus he defines structures as "very elementary reactions, which phenomenally are not composed of constituent elements, their members being what they are by virtue of their 'member-character,' their place in the whole; their essential nature being derived from the whole whose members they are" ("Perception", p. 543).

I mean to show that the parts in such structures can only be what it is classical to name "relational attributes" or "relational predicates". In other words, structures are now internal relations between their terms, and more precisely still "directly constitutive internal relations", not internal relations reducing to the existence of their terms as were the internal relations against which Russell struggled, but relations to which their terms reduce (I shall develop this point further on in my talk). But the real importance of this notion of structure is that it rests and is built upon a truly impressive amount of empirical data. Nevertheless, I want to show that Koffka's conception of sensation is fundamentally impossible to conceive, and that the belief that it is empirically grounded rests mainly on a confusion between abstraction of a sense-datum and real separation of the stimuli underlying such a datum. As a consequence, phenomenal structures, if they exist, can only be external to their terms, as they are in Köhler's view, in spite of many ambiguities in his formulations. However, I will end by showing that, correctly understood, the notion of structure can still be of great help in phenomenology and psychology since it provides a naturalistic means to understand how a non-intentional "meaning" can be passively present at a sensory level.

Keywords: Structure; Internal relations; Sensation; Gestalt psychology; Phenomenology.

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Declarative Computing with Shapes, and their Shadows
We present a preliminary concept and a prototypical implementation of a declarative computing framework that is capable of reasoning about 3D physical entities, and the shadows that they cast in open or uniformly lit environments. For this paper, we restrict our scope of ‘uniform lighting’ to sunlight, and its incidence on a given geospatially and temporally referenced location.
The model extends traditional techniques from computational geometry and computer graphics that are primarily motivated by simulation or visualisation. In particular, our declarative framework is capable of deriving and reasoning about the objects and their cast shadows in a knowledge processing sense, e.g., involving qualitative abstraction and semantic specification of requirements, query capability, ensuring conceptual consistency of design requirements. Our ontology of objects and shadows, and the resulting computational framework serves as a foundational engine for high-level conceptual (spatial) design assistance technology.
The capabilities demonstrated in this paper are aimed at applications in spatial design, chiefly encompassing Computer-Aided Architecture Design (CAAD), Urban Planning, and Interior Design

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Statistical Invariants of Spatial Form: From Local AND to Numerosity
Theories of the processing and representation of spatial form have to take into account recent results on the importance of holistic properties. Numerous experiments showed the importance of “set properties”, “ensemble representations” and “summary statistics”, ranging from the “gist of a scene” to something like “numerosity”. These results are sometimes difficult to interpret, since we do not exactly know how and on which level they can be computed by the neural machinery of the cortex. According to the standard model of a local-to-global neural hierarchy with a gradual increase of scale and complexity, the ensemble properties have to be regarded as high-level features. But empirical results indicate that many of them are primary perceptual properties and may thus be attributed to earlier processing stages. Here we investigate the prerequisites and the neurobiological plausibility for the computation of ensemble properties. We show that the cortex can easily compute common statistical functions, like a probability distribution function or an autocorrelation function, and that it can also compute abstract invariants, like the number of items in a set. These computations can be performed on fairly early levels and require only two well-

accepted properties of cortical neurons, linear summation of afferent inputs and variants of nonlinear cortical gain control.

4.2.9 Abstract Proof Theory

This workshop is organized by

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At present we do not have *Abstract Proof Theory* as a branch of Mathematical Logic, in the same sense in which we do have Abstract Model Theory – there is nothing so far comparable e.g. to Lindström's theorem in proof theory. But, to borrow an expression used by Kreisel, although we do not have a science (a discipline), we do have a natural history (scattered results).

The changing scope of logic through its history also has important philosophical implications: is there such a thing as the essence of logic, permeating all these different developments? Or is the unity of logic as a discipline an illusion? What can the study of the changing scope of logic through its history tell us about the nature of logic as such? What do the different languages used for logical inquiry – regimented natural languages, diagrams, logical formalisms – mean for the practices and results obtained?

We invite submissions that belong to this intended field.

The following is a non-exclusive list of possible topics:

a) Abstract deductive systems (generalized rules in natural deduction and in sequent calculi);

b) The nature and role of ordinal analysis;

c) The identity problem (When are two proofs identical?);

d) Abstract normalization, strong normalization and confluence;

e) Inversion principles;

f) Reducibility predicates;

g) Reducibility and definability;

h) Translations and interpretations;

i) The role of categorical logic.

Contributed Talks

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Towards Powerful and Decidable Formalizations Through Schematic Representation

Work done by Aravantinos et al.[1] on propositional schemata provides a way to give a finite formalization of an infinite set of propositional sentences by iterating a conjunction (disjunction) of the formula indexed by a free numeric variable. They also provide a decision procedure for a particular type of schemata, namely regular schemata. Regular schemata allow formalization of induction on propositional formulas up to the ordinal ω.

While regular propositional schemata are quite powerful tools, they are not expressive enough to formalize theorems such as the pigeonhole principle (PHP). While it is well known that an instance n of PHP (PHP_n) can be formalized in propositional logic [2], see Equation 1, a formalization of the set $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ requires induction over the ordinal $\omega\cdot\omega$. There are classes of schemata powerful enough to formalize $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$, however, all known classes of schemata which are powerful enough are undeciable for satisfiability [1].

$$\bigwedge_{i=1}^{n+1}\bigvee_{j=1}^{n} P_{i,j} \rightarrow \bigvee_{i=0}^{n+1}\bigvee_{m=i+1}^{n+1}\bigvee_{j=1}^{n}(P_{i,j}\wedge P_{m,j}) \tag{7}$$

Our goal is to study the relationship between induction and schemata, namely, finding classes of schemata powerful enough to represent induction up to a given countable ordinal α while remaining decidable for satisfiability. The set $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ is a perfect choice for studying this schemata to induction relationship, being that it is an elementary theorem which requires a larger than expected ordinal to formalize. Thus, we ask two main questions: what is the weakest class of schemata that can formalize $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$, and is this class decidable.

Many of the simple extensions of regular schemata have been proved undeciable. However, our prior work Cerna [3] lead to the discovery of a class of schemata (linkable schemata), which can express induction up to the ordinal $\omega\cdot m$, where m is a finite ordinal. Linkable schemata are created by changing the indexing terms used in regular schemata. Aravantinos et al. [1] used what they call linear expressions, essentially a form of Pressburger arithmetic. In Cerna [3] we instead use the following terms:

Definition 1. *Given the alphabet,* $\Sigma = \left\{0, S, \dot{0}, \mathbb{S}, \langle\cdot,\cdot\rangle\right\}$ *we construct the set of* $\mathbb{L}$*-terms,* $\mathbb{L} = \left\langle S^n(0), \mathbb{S}^m(\dot{0})\right\rangle$ *where* S *and* $\mathbb{S}$ *are successor functions,* 0 *and* $\dot{0}$ *are constants, and* n,m *are the number of nested successor functions. We also have two countably infinite distinct sets of variables* $\mathcal{V}_f$ *and* $\mathcal{V}_b$*. The set* $\mathcal{V}_f$ *ranges over the inductively constructed terms* $\mathbb{S}^n(\dot{0})$*, while the set* $\mathcal{V}_b$ *ranges over all the* $\mathbb{L}$*-terms. We will refer to the first part of a* $\mathbb{L}$*-term as the index numeral (*α *is the index numeral in* $\langle\alpha,\beta\rangle$*) and the second part of a* $\mathbb{L}$*-term as* *the numeric value (β is the numeric value in $\langle \alpha, \beta \rangle$).*

Definition 2. *Given a term $\langle \alpha, \beta \rangle \in \mathbb{L}$, we define its cardinality within $\mathbb{N}$ as $|\langle \alpha, \beta \rangle| = \{\ \#\ of\ \mathbb{S}\ in\ \beta\ \}$.*

Representing the $\mathbb{L}$-term $\langle \alpha, \beta \rangle$ as an ordinal is simply $\omega \cdot \alpha + \beta$. Thus, intuitively, it seems as if $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ can be represented if we create a new set of variables ($\mathcal{V}_n$) that ranges over the index numerals. This seems to allow a representation of the ordinal $\omega^2 \cdot m$. However, the inner schema in the nesting is repeated, thus these new schemata are no more powerful than linkable schemata. To get around this repetition problem we extend the term language of linkable schemata. We add functions from the set $\mathbb{F} = \{f | f : \mathbb{L} \rightarrow \mathbb{L}\ \&\ \exists \lambda \forall \alpha | f(\alpha) | \leq |\lambda|\}$, e.g $f(\langle 3, 10 \rangle) = \langle 1, 5 \rangle$. We use the theory $(\mathbb{L}, <)$ extended by functions from $\mathbb{F}$ in a schematic Π_0^0-predicate logic. We only allow the predicate $<$ and construct $\leq$ and $=$ from $<$. Aravantinos et al.[1] considered atoms of the form $a < b$ as iterations, e.g $\vee_{i=a}^{b} \top$. We instead consider them as binary predicate symbols.

We can formalize a weaker form of $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ in this logic, where we check the numbers assigned to the pigeons in canonical order and see if any pigeons are neighbours (Equation 2), i.e. assigned the same number. This formalization requires induction over the ordinal $\omega + r$, where r is a finite ordinal.

$$\left(f(\langle 1, m + \bar{1} \rangle) \leq (\langle 0, m \rangle) \wedge \langle 0, \bar{1} \rangle \leq f(\langle 1, \bar{1} \rangle) \wedge \bigwedge_{i=\langle 1, \bar{1} \rangle}^{\langle 1, m \rangle} f(i) \leq f(i + \langle 1, \bar{1} \rangle) \right) \rightarrow \bigvee_{i=\langle 1, \bar{1} \rangle}^{\langle 1, m \rangle} f(\bar{i}) = f(i + \langle 1, \bar{1} \rangle) \tag{8}$$

It remains open whether or not this method can be extended to formalize $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$. Also, it is not yet known if this method can formalize induction over the ordinal ω^2. Foreseeable extensions of this work include introduction of variables over index numerals and construction of a decision procedure for the new schematic logic, if it is found to be decidable or semi-decidable.

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Quantified Modal Logic: a proof-theoretic approach
In first-order modal logic, interesting questions arise when the modal operators $\Box$ (necessity) and $\Diamond$ (possibility) logically interacts with the first-order quantifiers $\forall$ (universal) and $\exists$ (existential). From a semantical point of view, if we take into account possible-worlds semantics, we have the interaction of two types of quantifiers that range over the objects and over possible worlds. This interaction leads to many complications depending also on other decisions, e.g. whether the domain of discourse should be fixed or allowed to vary from world to world, within the model [3].

The interaction between the modal operators combined with the first-order quantifiers $\forall$ (universal) and $\exists$ (existential) in the Gentzen-Prawitz style has not been analytically studied yet. In our talk we will first present a system in natural deduction in the Gentzen-Prawitz style for the modal logic $S4$-quantified, called $QS4$, that is an extension of the propositional modal system presented in [2]. Then, we will show that $QS4$ is sound and complete with respect to models in which different domains can be associated with different worlds. Besides, we will show that the system QS4 satisfies the substitution property and the Normalization Theorem. In particular, as a case study, we would like to call attention to [1] where another approach—different for the one we propose—for formalizing proof systems for quantified modal logics based on labelled natural deduction is presented. We observe that in this labelled natural deduction systems class, some modal axioms can not be axiomatized, e.g. the Löb axiom, $(\Box(\Box A \to A) \to \Box A)$ of Provability Logic.

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Reducibility method and resource control

The basic relationship between logic and computation is given by the Curry-Howard correspondence [4] between simply typed λ-calculus and intuitionistic natural deduction. This connection can be extended to other calculi and logical systems. The *resource control* lambda calculus, $\lambda_{®}$ [2], is an extension of the λ-calculus with explicit operators for erasure and duplication, which brings the same correspondence to intuitionistic natural deduction with explicit structural rules of weakening and contraction on the logical side [1]. It corresponds to the $\lambda_{\mathsf{cw}}lcw$-calculus of Kesner and Renaud [5]. In $\lambda_{®}$ in every subterm every free variable occurs exactly once, and every binder binds (exactly one occurrence of) a free variable.

The main computational step is β reduction. But there are also reductions which perform propagation of contraction into the expression and reductions which extract weakening out of expressions. This discipline allows to optimize the computation by delaying duplication of terms and by performing erasure of terms as soon as possible.

Our intersection type assignment system $\lambda_{®}\cap$ integrates intersection into logical rules, thus preserving syntax-directedness of the system. We assign restricted form of intersection types to terms, namely strict types, therefore minimizing the need for pre-order on types. By using this intersection type assignment system we prove that terms in the calculus enjoy strong normalisation if and only if they are typeable. We prove that terms typeable in $\lambda_{®}$-calculus are strongly normalising by adapting the reducibility method for explicit resource control operators.

The reducibility method is a well known framework for proving reduction properties of λ-terms typeable in different type systems [3]. It was introduced by Tait [6] for proving the strong normalization property for the simply typed lambda calculus. Its main idea is to relate terms typeable in a certain type assignment system and terms satisfying certain realizability properties (e.g., strong normalisation, confluence). To this aim we interpret types by suitable sets of λ-terms called saturated sets, based on the sets of strongly normalizing terms. Then we obtain the soundness of type assignment with respect to these interpretations. As a consequence of soundness we have that every term typeable in the type system belongs to the interpretation of its type. This is an intermediate step between the terms typeable in a type system and strongly normalizing terms. Hence, the necessary notions for the reducibility method are: type interpretation, variable, reduction, expansion, weakening and contraction properties (which lead to the definition of saturated sets), term valuation, and soundness of the type assignment. Suitable modified reducibility method leads to uniform proofs of other reduction properties of $\lambda_{®}$-terms, such as confluence or standardization.

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Semantic Investigation of Basic Sequent Systems

Our research aims at a unified semantic theory for Gentzen-type systems and their proof-theoretic properties. We put our focus on the family of basic systems a large family of fully-structural propositional sequent systems including arbitrary derivation rules of a certain general structure. Various sequent calculi that seem to have completely different natures belong to this family. This includes, for example, standard sequent calculi for modal logics, as well as multiple-conclusion systems for intuitionistic logic, its dual, and bi-intuitionistic logic. We present a general uniform method, applicable for every system of this family, for providing (potentially, non-deterministic) strongly sound and complete Kripke-style semantics. Many known soundness and completeness theorems for sequent systems easily follow using this general method. The method is then extended to the cases when: (i) some formulas are not allowed to appear in derivations, (ii) some formulas are not allowed to serve as cut-formulas, and (iii) some instances of the identity axiom are not allowed to be used. This

naturally leads to semantic characterizations of analyticity (in a general sense), cut-admissibility and axiom-expansion in basic systems. In turn, the obtained semantic characterizations make it possible to provide semantic proofs (or refutations) of these proof-theoretic properties. In many cases such proofs are simpler and easier to verify than their proof-theoretic counterparts. We believe that these results provide useful tools, intended to complement the usual proof-theoretic methods.

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Glue Semantics for Proof Theorists

If abstract proof theory is to earn its stripes as really *abstract*, it needs to show that it has extensive applications, meaning that its applications range over several different domains. In this note we discuss a class of applications of abstract proof theory to logic systems concerned with the meaning of natural language sentences. The class of applications we have in mind is a generalization of "Glue Semantics" but also goes by the name of "(Natural language) Proof-theoretic semantics".

Quoting Wikipedia "Glue semantics (Dalrymple et al. 1993) is a linguistic theory of semantic composition and the syntax-semantics interface which assumes that meaning composition is constrained by a set of instructions stated within a formal logic, Linear logic". Glue semantics embodies a notion of 'interpretation as deduction' closely related to categorial grammar's 'parsing as deduction'.

Syntactic analysis of a sentence yields a set of glue premises, which essentially state how bits of lexical meaning attach to words and phrases. Deduction in (linear) logic then combines the premises to derive a conclusion that attaches a meaning to the sentence as a whole.

Glue semantics has been used with many different syntactic frameworks (LFG, HPSG, Categorial Grammar and Tree-adjoining grammar) and with many different theories of semantics such as Discourse Representation Theory, Intensional logic and traditional First-order logic. Our goal in this paper is to present it as a logic system whose proof theory works in ways very different from most traditional proof theory and discuss this contrast.

The main point we want to make is that unlike traditional proof theory where cuts with axioms $A \vdash A$ are considered trivial, so that our main aim is to eliminate important cuts so that they become non-important i.e. so that they become cuts with axioms of the form $A \vdash A$, for the proof theory associated with Glue semantics, our main task is to discover where do we need to have such trivial cuts with axioms of the form $A \vdash A$. This is because instead of trivial, cuts with axioms are actually what makes true semantic ambiguities show up.

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The Constructivist Semantic of Problems
Intuitionistic logical constants are characterized by means of BHK clauses. The "K" is a reference to Kolmogorov who in 1925 had made a proposal for defining intuitionistic logic and which was later extended with the inclusion of *ex falso quodlibet* principle. In 1932 [Kol32] this principle is present and the author gives a semantics alternative to the one presented by Heyting in 1930. There, Kolmogorov uses the concept of problem.

Here we are proposing a partial analysis of this semantics of problems. We want to show that this semantics involves a logic of actions. A semantic of problems provide a way to examine the characterization of the implication logical constant. We intend to show that the intuitionistic proposal over implication, –Kolmogorov proposal here comprised– is not the first option that could come to mind in regard of the notion of problem. We finish by pointing what seems to us to be open questions and paths for investigation in this semantics.
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Relating Focused Proofs with Different Polarity Assignments
Focused proof systems have been successfully used as meta-level logic to faithfully encode different logics in different proof systems, such as Natural Deduction and Sequent Calculus [1,2].

More interestingly, a single theory can be used in order to encode these systems and just rely on the different annotation of polarity to atoms to obtain, from the same theory, one system or another [3].

In this work, we investigate how a given focused proof where atoms are assigned with some polarity can be transformed into another focused proof where the polarity assignment to atoms is changed. This will allow, in principle, transforming a proof obtained using one proof system into a proof using another proof system.

More specifically, using the intuitionistic focused system LJF [4] restricted to Harrop formulas, we define a procedure, introducing cuts, for transforming a focused proof where an atom is assigned with positive polarity into another focused proof where the same atom is assigned negative polarity and vice-versa. Then we show how to eliminate these cuts, obtaining a very interesting result: while the process of eliminating a cut on a positive atom gives rise to a proof with one smaller cut, in the negative case the number of introduced cuts grows exponentially.

This difference in the cut-elimination algorithm seems to be related to the different evaluation strategies according to the Curry-Howard isomorphism, where cut-elimination corresponds to computation in a functional programming setting. In particular, we will show that how the polarities of atoms is assigned is related to call-by-value and call-by-name reduction strategies.

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4.2.10 Relevant Logics

This workshop is organized by

Ross T. Brady
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Edwin Mares
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Relevant Logic is an interesting family of logical systems taking in account meaningful correlations between hypotheses and conclusions.

We are looking for papers on topics such as:

a) the notion of relevance;

b) the use of relevant logic in formalizing theories, such as arithmetic, set theory, and scientific theories;

c) the use of relevant logic in formalizing philosophical theories, such as epistemological theories, and theories of belief revision;

d) the relationship between relevant logic and other logical systems.

The invited keynote speaker of this workshop is J. Michael Dunn (page 69).

Contributed Talks

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The Classical Constraint on Relevance Logics

Definition 1

- By a *relevant language* we mean any set of connectives which includes $\to$ and all its other connectives are taken from $\{\sim, \otimes, +, \wedge, \vee, \mathbf{T}, \mathbf{F}\}$.
- By the classical connectives we mean the following connectives: $\neg$, $\supset$, $\wedge$, $\vee$, $\mathbf{T}$, and $\mathbf{F}$.

Definition 2 R^- is the system in the language $\{\to, \sim, \wedge, \mathbf{T}\}$ which is obtained from $R_{\underset{\to}{\sim}}$ by adding the following axioms and inference rule:

Axioms: (i) $\varphi \wedge \psi \to \varphi$ (ii) $\varphi \wedge \psi \to \psi$ (iii) $\varphi \to \mathbf{T}$

Relevant Adjunction Rule: From $\varphi \to \psi$ and $\varphi \to \theta$ infer $\varphi \to \psi \wedge \theta$.

Note The other connectives that may be used in a relevant language (as defined above) are introduced in R^- in the standard way ($\varphi \otimes \psi = \sim(\varphi \to \sim\psi)$,$\varphi + \psi = \sim \varphi \to \psi$, $\varphi \vee \psi = \sim(\sim\varphi \wedge \sim\psi)$, $\mathbf{F} = \sim\mathbf{T}$). Hence we take them below as belonging to the language of R^-.

Definition 3 By an *extension of a fragment of* R^- we mean a set $\mathcal{L}$ of sentences in some relevant language $\mathbf{L}$ which has the following properties:

1. $\mathcal{L}$ is closed under substitutions of sentences of $\mathbf{L}$ for atomic formulas.
2. $\mathcal{L}$ includes every theorem of R^- in its language.
3. $\mathcal{L}$ is closed under MP for $\to$.
4. $\mathcal{L}$ is non-trivial: there is some sentence ψ of $\mathbf{L}$ such that $\psi \notin \mathcal{L}$.

Definition 4 Let φ be a sentence in some relevant language. Its *classical translation* φ^c is the sentence in the classical language obtained from φ by replacing every occurrence of $\sim$ by $\neg$, every occurrence of $\to$ by $\supset$, every occurrence of $\otimes$ by $\wedge$, and every occurrence of $+$ by $\vee$.

Definition 5 A sentence φ in the language of R is a *classical tautology* if its classical translation φ^c is a classical tautology.

Main Theorem: *Any sentence in some extension of a fragment of* R^- *is necessarily a classical tautology.*

In other words: the only axioms schemas that can be added to R^- or any of its relevant fragments to produce a non-trivial logic are classical tautologies. Hence any such extension is necessarily contained in classical logic.

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Metavaluations
The presentation will be a general account of metavaluations and their application, as an alternative to standard model-theoretic approaches. They work best for metacomplete logics which include the contraction-less relevant logics, with possible additions of Conjunctive Syllogism, $((A \to B) \wedge (B \to C)) \to (A \to C)$, and the irrelevant, $A \to (B \to A)$, and which include, I believe, the main entailment logics. Indeed, metavaluations focus on the properties of theorems of the form $A \to B$, splintering into two types according to key properties of negated entailment theorems (see below). Metavaluations have an inductive presentation and thus have some of the advantages that model theory does, but, in essence, they represent proof rather than truth and thus more closely represent proof-theoretic properties, such as the Priming Property, if $A \vee B$ is a theorem then A is a theorem or B is a theorem, and the negated-entailment properties, $\sim(A \to B)$ is a non-theorem (for M1-logics) and $\sim(A \to B)$ is a theorem iff A is a theorem and $\sim B$ is a theorem (for M2-logics). Topics to be covered are their

impact on naive set theory and paradox solution, and also Peano arithmetic and Gödel's First and Second Theorems.

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What is Entailment?
There have been various attempts to characterize entailment, most of which attempt to represent the rules that generate the logic within the logic itself. In a recent paper, Francesco Paoli and I argue that many of the logical paradoxes arise from attempting to represent these "external rules" internally within a logical system. But if we reject the idea the idea that a theory of entailment ought to represent the external rules of the logic in it, what is the aim of a theory of entailment? This paper outlines an approach to entailment that is semantically based. It is based on some ideas found in the writings of C.I. Lewis. I apply these ideas both to modal logic and to relevant logic.

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Dialetheic Paths to Triviality
Slaney showed in his paper ***RWX*** *is not Curry Paraconsistent* that **DWX** extended by the permutation axiom $(A \to (B \to C)) \to (B \to (A \to C))$ is too strong for naïve theories. This paper extends his, and others, results by showing that various paraconsistent relevant logics are too strong for naïve theories. The main focus is on permutation principles and the logic **DWX**.

It is shown that **DWX** extended by either $A \to (B \to C), B \vdash A \to C$ (Ackermann's δ rule), or $(A \to \neg A) \to \neg A$ (*Consequentia mirabilis*) trivialize any naïve theory if the permutation rule $A \to (\top \to B) \vdash \top \to (A \to B)$ is a rule of the logic. The presence of this last rule is sufficient for proving at least one implication sentence "trivially true", such as the sentence $\top \to (\bot \to \bot)$. It is then shown that **TWX** extended by either Consequentia mirabilis or Ackermann's δ rule is too strong for naïve theories if one trivially true implication sentence $\top \to (A \to B)$ is a theorem. It is also shown that if the meta-rule of reasoning by cases is added to **DWX** extended by Ackermann's δ rule, then the sentence $\top \to (\bot \to \bot)$ can't be added, on pain of triviality, to any naïve theory.

Ackermann's δ rule is a weak permutation rule which holds in the logic **EW**. It is shown that **EW** extended by the axiom $((A \to B) \wedge (B \to C)) \to (A \to C)$ (*Conjunctive syllogism*) proves any naïve theory trivial.

The rules governing the fusion connective suffice, in the presence of the negation-principles of **DWX**, for deriving the sentence $\top \to (\bot \to \bot)$. Logics such as **DWX** extended by reasoning by cases and Ackermann's δ rule can't therefore treat a naïve theory non-trivially if the fusion connective is part of the logic, even if the language of the naïve theory does not include the fusion connective. It is shown that if the language of the theory does include the fusion connective, then the logic **DW** extended by conjunctive syllogism, and **DWX** extended by either consequentia mirabilis or Ackermann's δ rule are all too strong for naïve theories.

The last proof shows that the logic **TWX** extended by reasoning by cases is too strong for naïe theories if both the fusion connective and the Ackermann constant is part of the language of the theory.

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Curry's Paradox and Sub-structural Proof Theory for Relevant Logic

Curry's paradox is well known. It comes in both set theoretic and semantic versions. Here we will concentrate on the semantic versions. Historically, these have deployed the notion of truth. Those who wish to endorse an unrestricted T-schema have mainly endorsed a logic which rejects the principle of Absorption, $A \to (A \to B) \models A \to B$. High profile logics of this kind are certain relevant logics; these have semantics which show how and why this principle is not valid. Of more recent times, paradoxes which are clearly in the same family have been appearing; but these concern the notion of validity itself. The standard semantics of relevant logics seem powerless to address these. But they can. This note shows how.

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Thither relevant arithmetic!

In their 1992 paper "Whither Relevant Arithmetic?", Friedman and Meyer reported some Bad News. They showed that first-order relevant Peano arithmetic $R^{\#}$ is weaker than classical Peano arithmetic, by showing that the ring of complex numbers is a model of $R^{\#}$, and that there is a classical theorem that fails in this model. Meyer had hoped that simply grafting the Peano axioms into R would, on the one hand, produce a theory at least as good as the classical counterpart, in proving all its theorems, but on the other hand, a theory better than the classical counterpart, in that $R^{\#}$ supports a (finitary) proof of its own absolute consistency. Since the Friedman/Meyer result shows that $R^{\#}$ is incomplete with respect to classical PA, thereby hopes for relevant arithmetic were dashed.

In this talk I want to look at the facts of the situation again, from a more general setting. In stepping back from the details (say, about the admissibility of γ), two related themes emerge: the size of models, and the role of logic in metatheory. Friedman and Meyer conclude their paper by observing that $R^{\#\#}$, which replaces an induction axiom with Hilbert's ω-rule, is complete, but now at the cost of being infinitary. Meyer asks readers to look for a magical midpoint, named $R^{\#1/2}$ – strong enough to capture arithmetic, weak enough to be finite. I will consider whether there is indeed a way to reconcile this, via a relevant metatheory, and its corresponding Löwenheim/Skolem property: *every* theory has a *finite* model. I will canvass some recent work by Restall and also Mortensen, and gesture at possibilities for relevant arithmetic in the style of Routley's DKA.

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The one variable pure implication fragment of T is infinite

In 1970 R. K. Meyer [1] posed the problem of determining the structure of the pure implication fragments of substructural logics restricted to formulae in one propositional variable. In particular, he asked how many equivalence classes of such formulae there are, where equivalence is defined as mutual derivability by the canons of one logic or another. The one-variable fragment is an interesting abstraction of a propositional logic, as it consists in waiving all distinctions between the "content" of formulae, as represented by the different atoms, leaving only the "shape" given by the connectives. The most basic question is whether the number of equivalence classes is finite or infinite. In the years since 1970, Meyer's question has been answered for most logics in the class, but two logics have resisted: these are the Anderson-Belnap systems [2,3] **T** of "ticket entailment" and **E** of entailment.

In this paper, the question is answered for $\mathbf{T}_{\to}$. The argument is algebraic, giving a recipe for constructing, for any finite n, an algebraic model of $\mathbf{T}_{\to}$ of order at least n with a single generator. The construction proceeds by defining a certain kind of totally ordered model of the better-known relevant logic $\mathbf{R}_{\to}$ and then embedding it in a model of $\mathbf{T}_{\to}$ by adding two further elements. The way this is done is not without interest, particularly as it shows how very different is the concept of the modality $\Box$ in **T** from that associated with stronger logics like **R** and **E**.

The result leaves $\mathbf{E}_{\to}$ as the only well-known logic for which Meyer's question is still open. It does not seem likely that the technique used in this paper will extend in any natural way from $\mathbf{T}_{\to}$ to $\mathbf{E}_{\to}$, so the latter must await another investigation.

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4.2.11 Thinking and Rationality

This workshop is organized by

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Juliana Bueno-Soler
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Contemporary logic with its plethora of systems for approaching topics such as reasoning about truth, about knowledge, about belief, about preference, and for reasoning under more liberal attitudes (by dropping certain classical laws) such as in reasoning under constructive paradigms (intuitionism), reasoning under contradictions (paraconsistency), multi-alethic reasoning (many-valuedness), reasoning under uncertainty (fuzzyness), and so on, can be confronted and complemented with other tools for modeling decision making and intelligent interaction that take into account the information states of agents and the information flow such as belief revision, preference revision, multimodal and dynamics logics, reasoning in societies against individual reasoning, etc.

But, on the other hand, abandoning some classical laws and adding a dynamic side to logics has the price of imposing more severe constraints for the mathematical and formal rendering of logics. How this affects rationality and the foundational aspects of the idea of critical thinking ? if we accept not to have a unique theory of rationality, how could we then expect to reason collectively as in science and, collective decisions and in dialogical argumentation? This workshop aims at discussing the philosophical and logical roots of thinking and rationality broadly conceived, as well as their connections to contemporary logic and information.

Topics include (but are not restricted to):

a) Game theory and logic;

b) Social choice theory and logic;

c) Logic and information;

d) Logics between statics and dynamics;

e) Probabilistic reasoning;

f) Decision making;

g) Causal reasoning and counterfactual thinking;

h) Economic roots on logic;

i) Logical roots on economics;

j) Cultural influences on thinking and logic;

k) Different rationalities.

The invited keynote speaker of this workshop is Otávio Bueno (page 59).

Contributed Talks

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Logics of Formal Inconsistency and Non-Monotonicity

Nonmonotonic Logics are those which monotonicity does not hold, i.e., when new information appears we no longer can have the same conclusion that we had before. We have to make belief revisions and work with this new set of information in a other way, so we do not have contradictions that lead to inconsistency.

Otherwise, contradictions can be quite informative, they should not be eliminated as soon as they appear. The Logics of Formal Inconsistency (LFI's) provides a reasonable framework for approaching contradictions, internalizing the notions of consistency and inconsistency at the language-object level. This work intends to research interactions between LFI's and Nonmonotonicity in order to find a plausible form for simultaneously approaching nonmonotonicity and contradictions.

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A simple solution to Ulam's game with n lies

In 1976, Stanislaw Ulam published an autobiographical work called "Adventures of a Mathematician" that contained the following game: "Someone thinks of a number between one and one million (which is just less than 2^{20}). Another person is allowed to ask up to twenty questions, to each of which the first person is supposed to answer only yes or no. Obviously the number can be guessed by asking first: Is the number in the first half million? then again reduce the reservoir of numbers in the next question by one-half, and so on. Finally the number is obtained in less than $\log_2$ (1000000). Now suppose one were allowed to lie once or twice, then how many questions would one need to get the right answer?" [11], p. 281. However, since in 1964 there was a description of this same problem in the MIT doctoral thesis of Elwyn Berlekamp [1]. Several researchers from diverse fields of knowledge have developed studies on Ulam's games (cf. [9], [10], [7], [8], [6], [2] and [4]). Several articles have been published containing solutions to the problem presented by Berlekamp and Ulam and allowing one, two, three or more false answers. In *The logic of Ulam's game with lies* [5], Mundici shows that the semantics of Lukasiewicz's logic can be naturally attached to the solution of Ulam's Games with lies. The paper [3] of Claudio Marini and Franco Montagna discusses some generalizations of Ulam's Games with lies. Some of these generalizations are probabilistic variations of the game while others differ from the original game by allow more than one number to be guessed. The present work shows a relatively simple (and systematic) solution to a variant of the Ulam's Game with n lies. It also proposes a classification of variations of Ulam's Games based on the algorithms used to calculate the answers (by *Responder*) and on the interpretation of the terms of the problem.

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Conditionalism about the normativity of rationality
According to the “wide-scope” conception of rationality (WSR), rationality requires various things of you. For instance, it requires of you that you to intend to F if you believe you ought to F and that you believe q if you believe p and you believe if p then q. On this conception of rationality, its requirements are consistency requirements, and nothing more. Notoriously, however, one can be consistent in ridiculous and morally abhorrent ways. Given this, it is far from clear if, or to what extent, we ought to satisfy these requirements. John Broome has recently declared that he is officially agnostic about the normative status of wide-scope rationality, and he has provided arguments for his agnosticism. I think Broome's arguments fail to show that we should be agnostic about the normativity of wide-scope rationality, and the aim of this paper is to show why he is mistaken. Briefly put, I will argue that normative requirements are normative in their own right, but only under certain conditions. In section 1 I will explain in more detail the wide-scope conception of rationality. In section 2 I will outline Broomes reasons for being agnostic about the normative status of wide-scope rationality. Section 3 will finish by showing why Broome is mistaken and why we need not, and should not, be sceptical about the normativity of rationality.

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Rational and reasonable inferences
The paper aims to recovering the traditional distinction between rationality and reasonabless in order to see if it makes sense to apply it to the analysis of inference or to use it in the construction of a general theory of inference. It has been sometimes held that there are behaviours which are rational but not reasonable, and one could also maintain that some behaviours are reasonable but not rational. In the philosophy of inferential behaviour, an old point of view consists in identifying deductive logic with rational inference and inductive logic with reasonable inference. This taxonomy is, however, too rough to be defendable. There are various kind of non-deductive logics which are not easily reducible either to deductive or inductive logic: suffice it to mention conditional logic and the logic of abductive inference. The second part of the paper takes into consideration theories of rational and/or reasonable inference which have been proposed by XX Century logicians, with special attention to the different intuitions developed on this topic by Hans Reichenbach and Robert Stalnaker. The third part of the paper aims to discussing the basic idea that rationality in inference consists essentially in the selection of the best conclusion among a set of admissible alternative conclusions. It is stressed that this is the common feature of various kinds of non-deductive inference (inductive, contrary-to-fact, abductive). The distinction between rational and reasonable conclusions de-

pends on the inferential tools which are used in the derivation of the conclusion. A paradigmatic example of unreasonable but rational inference is the one which is allowed by the impossibility (in any sense) of the antecedent or by the necessity (in any sense) of the consequent. It is held that reasonableness depends on some kinds of relevance of the antecedent for the consequent, so that every reasonable inference is rational, but the converse is not true. The sense of relevance which is intended in the proposed theory is however non coincident with the one which is intended in the framework of so-called relevant logic. The last part of the paper is devoted to clear the sense in which relevance is to be understood to match the intended idea of reasonableness.

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Polynomial Proof Systems: from the classical to the non-classical logics

During the last years a great number of logical non-classical been developed and studied, and consequently various methods for automated theorem proving tests have been proposed, such as sequential calculation, Analytical Tableaux, among others. Most of these methods are strongly related to the inherent characteristics of these new private and logical systems. To present efficient implementations of these calculations is not a specialized trivial task and this leads to difficulties in the maintenance and modification of these systems so that they can develop with the same efficiency as tasters of classical logic. Carnielli, in [CAR05] introduces a new algebraic proof method for general sentential logics which is particularly apt for finitely-many-valued logics and its particularization for PC, based on reducing polynomials over finite fields. The method can also be extended to cover certain non-finitely valued logics as well, provided they can be characterized by two-valued dyadic semantics. In this work, I present the polynomials developed to the finite-valued logic after having been translated by Suszko thesis, in a system bi-valuedness, for non-deterministic logic of Arnon Avron and some preliminary ideas for the predicate calculus.

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On a philosophical justification of logics of formal inconsistency

The distinctive feature of paraconsistent logics is not so much to accept contradictions but rather do not become trivial in the presence of them. It is because the principle of explosion $A, \sim A \vdash B$ does not hold without restrictions. Logics of formal inconsistency are a family of paraconsistent logics whose main feature is to express the notion of inconsistency inside the object language. There is a connective called ball: "A^o" means that A is consistent. Thus, we can distinguish consistent from inconsistent formulas and restrict the application of the principle of explosion, allowing contradictions but avoiding triviality. The aim of this talk is to present a philosophical justification for logics of formal inconsistency that is not committed with the so called dialetheism, the view according to which there are true contradictions, what ends up being tantamount to the claim that reality is contradictory. However, as I see the problem, this thesis cannot be accepted for the simple reason that there is no conclusive evidence of the occurrence of real contradictions, that is, contradictions in space-time phenomena or in mathematical objects. And what sometimes makes it difficult to recognize the philosophical significance of paraconsistent logics in general is precisely the (wrong) view that they are necessarily committed to accepting inconsistencies in the ontological level. I will defend a view according to which contradictions have an epistemic character. They are either yielded by thought, in the case of semantic and set theoretical paradoxes, or due to the way we perceive empirical phenomena and elaborate theories about them.

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Evading Gödel's theorems?

Although Gödel thought about connecting modal logic with the theory of provability, apparently he never developed such ideas significantly. It was George Boolos who used modal logic more explicitly to offer proofs of Gödel's Theorems in *The Unprovability of Consistency* (1979), which leaded to his *The Logic of Provability* of 1993. Such modal treatment made clear how Gödel's Second Incompleteness Theorem depends on a precise formulation of the provability predicate, as axiomatized by David Hilbert and Paul Bernays in *Grundlagen der Arithmetik*.

Discussions on how to avoid Gödel's objections to logic and arithmetic are not new. For instance, in a three-valued logic (dealing with truth-values T, F and undecidable) the scenario posed by the First Incompleteness Theorem would of course change: Instead of having true unprovable statements, we would just have undecidable statements. On another view, Dan Willard discusses a new version of the Second Incompleteness Theorem and shows how certain axiom systems with arithmetic power can (at least partially) evade the Second Incompleteness Theorem. However, I intend to discuss possible ways of evading Gödel's objections by means of the analysis of the notion of consistency, and to its connections to paraconsistent modal logics.

The proof of the Second Incompleteness Theorem essentially depends, not only on assumptions about provability and their formalization within the system, but also on a very particular notion of consistency. If an axiomatic system $\mathcal{S}$ is consistent in such sense, the First Incompleteness Theorem shows that there exists a sentence $\mathcal{A}$ (which claims not to be provable in $\mathcal{S}$) such that $\mathcal{A}$ and its classical negation $\neg\mathcal{A}$ cannot be proven in the system. Therefore $\mathcal{A}$ is true, but cannot be formally proved in $\mathcal{S}$.

If $\mathcal{A}$ is a case of the sentence constructed in the First Incompleteness Theorem, assuming that the consistency of the system can be proven from within the system itself leads to a contradiction. Indeed, the standard argument shows that if the system is consistent, then $\mathcal{A}$ is not provable. As the proof of this implication can be formalized within the system, the statement '$\mathcal{A}$ is not provable' is provable, but this is equivalent to $\mathcal{A}$. Therefore $\mathcal{A}$ can be proven in the system. But this is a contradiction, so the classical argument says, and this shows that the system must be inconsistent. I will discuss how this depends on the derivability conditions holding in a system $\mathcal{S}$ and on a narrow notion of consistency, and the effects of a paraconsistent negation in the formalized versions of the First and Second Incompleteness Theorems and the reach of such evasions to Gödel's results.

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Paraconsistent Description Logics from the formal-consistency viewpoint

Description Logics (DLs) are an extensively used formalism for class-based modeling and knowledge representation that intend to express properties of structured inheritance network. These systems constitute variants of multimodal versions of the familiar normal modal logic **K** and can be also interpreted as fragments of first-order logic with interesting computational properties. DLs convey relevant logical formalism for ontologies and for the semantic web, much used in artificial intelligence.

However, precisely because of its wide applicability, DLs may face serious difficulties in expressing knowledge bases (or ontologies) that contain contradictions.

Considering that the capacity of reasoning under contradictions is a much needed feature in enhanced versions of DLs, we introduce the description logic $\mathbf{Ci}\mathcal{ALCQ}$ based upon the Logics of Formal Inconsistency (LFIs), a class of powerful paraconsistent logics, in [3].

$\mathbf{Ci}\mathcal{ALCQ}$ is strong enough to encompass the "classical" description logic $\mathcal{ALCQ}$ [1,2], so our proposal not simply repairs contradictory (or inconsistent) ontologies, but genuinely generalizes the notion of description logic by enhancing the underlying logic with a weaker negation and with a primitive notion of consistency independent from negation (that is, independent from any notion of contradiction).

The new description logic $\mathbf{Ci}\mathcal{ALCQ}$ based upon the logics of formal inconsistency is semantically characterized by quite philosophically acceptable semantics, thus representing a natural improvement in the notion of description logics.

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New results on **mbC** *and* **mCi**

The *Logics of Formal Inconsistency* (LFIs), proposed by W. Carnielli and J. Marcos, play an important role in the universe of paraconsistency, since they internalize in the object language the very notions of *consistency* and *inconsistency* by means of specific connectives (primitives or not). This generalizes the pioneering work of N.C.A. da Costa on paraconsistency, which introduced the well-known hierarchy of systems C_n, for $n \geq 1$.

Carnielli and Marcos proposed a hierarchy of propositional LFIs starting from a logic called **mbC**, the weakest in that hierarchy, but enjoying interesting features. The language of **mbC** consists of a paraconsistent negation $\neg$, a conjunction $\wedge$, a disjunction $\vee$, an implication $\rightarrow$ and an unary connective $\circ$ for *consistency*. For each formula β, the formula $\bot_\beta =_{def} \beta \wedge \neg\beta \wedge \circ\beta$ is a bottom and so $\sim_\beta \alpha =_{def} \alpha \rightarrow \bot_\beta$ defines an explosive, classical negation. The logic **mbC** is axiomatized by considering the positive classical logic extended by the law of excluded middle $\alpha \vee \neg\alpha$ and the *gentle explosion law* $\circ\alpha \rightarrow (\alpha \rightarrow (\neg\alpha \rightarrow \beta))$. All the other systems studied by Carnielli and Marcos are extensions of **mbC** obtained by adding appropriate axioms.

In this paper the logic **mbC** is formulated in the signature $\{\bot, \neg, \circ, \rightarrow\}$ and proved to be equivalent to the usual formulation in the signature mentioned above. Since the replacement property does not hold in **mbC**, the equivalence between both formulations is stated by means of conservative translations. The advantage of the new signature is that there is just one bottom $\bot$ and so a single classical negation can be given by $\sim\alpha =_{def} \alpha \rightarrow \bot$ (from this, the other classical connectives $\vee$ and $\wedge$ are defined from the previous ones as usual in classical logic). Additionally, it allows to see in a clear way that **mbC** is an extension of propositional classical logic obtained by adding a paraconsistent negation $\neg$ and a consistency operator $\circ$.

Sequent calculi for **mbC** and its extension **mCi** are also presented in the new signature. The corresponding sequent calculi are shown to admit cut elimination and, as a consequence of this, two new results are proved: just like in classical logic, a negated formula $\neg\alpha$ is a theorem of **mbC** (resp., of **mCi**) iff α has no models. The other result gives an answer to an open problem in the literature: the logic **mbC** is shown to be not controllably explosive.

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Plausibility and Justification

In this work, we combine the frameworks of Justification Logics and Logics of Plausibility-Based Beliefs to build a logic for Multi-Agent Systems where each agent can explicitly state his justification for believing in a given sentence. Our logic is a normal modal logic based on the standard Kripke semantics, where we provide a semantic definition for the evidence terms and define the notion of plausible evidence for an agent, based on plausibility relations in the model. This way, unlike traditional Justification Logics, justifications can be actually faulty and unreliable. In our logic, agents can disagree not only over whether a sentence is true or false, but also on whether some evidence is a valid justification for a sentence or not. After defining our logic and its semantics, we provide a strongly complete axiomatic system for it and show that it has the finite model property and is decidable. Thus, this logic seems to be a good first step for the development of a dynamic logic that can model the processes of argumentation and debate in multi-agent systems.

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Logical knowledge and ordinary reasoning: more than considering, less than believing

Besson (2012) argues that the dispositional account for logical knowledge is doomed to fail. As a reply, Steinberger and Murzi (forthcoming) suggested a new criterion for logical knowledge that seems to avoid Besson's objections. In this paper I show that this new account is ?awed, because the demands for the agents are too weak. I offer a modi?ed criterion which is not affected by most of the objections.

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4.2.12 Medieval Logic

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As scholars are becoming more and more aware, one of logic's most fruitful periods filed the five centuries between, roughly speaking, 1100 and 1600. This medieval tradition on logic exhibits an extraordinary richness extending its reach from creative reinterpretations of the syllogism, researches on logical consequence, quantification, paradoxes, to treatments of the relation between logic and natural languages.

Since a couple of decades the material medieval logicians produced are being object of critical editions, on the basis of which new researches are on their way. Has little chance of losing who bet that there are quite a number of interesting logical reasonings waiting for us to be discussed in those texts.

This UNILOG Medieval Logic workshop will focus on the various and diversified contributions on logical questions raised in the Middle Ages.

Topics may include:

a) Consequences;

b) Epistemic paradoxes;

c) Modal logic;

d) Natural languages and formal logics;

e) Obligations;

f) Semantic paradoxes;

g) Sophismata literature;

h) Square of oppositions;

i) Theory of consequences;

j) Theory of supposition.

The invited keynote speaker of this workshop is Stephen Read (page 80).

Contributed Talks

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The Lullian Methods of Inconsistency Resolution

In this talk, I shall discuss some logical aspects which characterize two inconsistency resolution methods developed by Ramon Llull (1232-1316): The Contradictory Syllogisms and the Fallacy of Contradiction. The first evaluates pairs of inconsistent theses, in order to select one of them based on two groups of arguments and counter-arguments. This procedure requires the following adaptive strategy: after detecting an inconsistency, the underlying logic of Contradictory Syllogisms method behaves in a paraconsistent manner under inconsistent contexts, nullifying the application of some classic inference rules, although may also allow, under consistent contexts, an unrestricted application of such full-fledged rules. The other method is based on the Fallacy of Contradiction that affects those arguments that contain some kind of ambiguity and share the form 'No S is P and some S is P; therefore some S is P and not is P'. According to Llull, such arguments would appear to be invalid because they just simulate the derivation of a contradiction from an inconsistent pair of premises, although shown to be valid in light of the identification and removal of the ambiguity responsible for the apparent contradiction.

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How Many Distributions? The Late Medieval Semantics of Natural Language and Its Shifting Borders

Among the most celebrated achievements of late medieval logicians is the sort of formal semantics they developed with the so-called supposition theory. There is, however, more in late medieval semantics than supposition theory; or, rather, supposition theory belongs to a larger theoretical structure, which includes as well a semantics of syncategoremes, largely developed within the literature of *sophismata*. There, medieval logicians devoted highly sophisticated discussions to a wide range of linguistic phenomena. *Sophismata* not only provided the space in the 13$^{\text{th}}$ century at least where natural language was dealt with, they epitomised a certain way of doing logic, of analysing language by dialectically devising distinctions between a score of logical properties of a given syncategoreme (such as omnis, the universal quantifier). The resulting theoretical landscape is extremely rich, indeed able to handle many aspects of language; it provides logical analyses over an extended territory, setting (porous) borders with 'pragmatic' approaches only when improper uses of language, such as figures of speech, are involved. It also displays some consistency issues with, *inter alia*, supposition theory, as well as problematic consequences in terms of propositional ambiguity or, even, ontological commitment. In the 14$^{\text{th}}$ century, with authors like William of Ockham and most of those who follow, this situation

changes. The descriptive ambitions characterising the 13th century semantics and the sophismata literature give way and a greater concern for consistency emerges. *Sophismata*, while still around, lose the privilege of being the place where complexities of language are dealt with, Sums of logic gain considerably in sophistication, and most of the distinctions-properties of syncategoremes isolated by thirteenth century logicians are expelled from logic. Several of the linguistic phenomena which were investigated at length in the *sophismata* literature are relocated in a domain closely resembling contemporary classical' pragmatics outside logic. The borders shift dramatically.
I will look at the way 13th century logicians used *sophismata* to build a somewhat formal semantics, through the example of the quantifier omnis and the main properties ascribed to it. The picture obtained will be contrasted with the sort of analysis developed in the 14th century, and the reasons given for not accepting the properties identified by their predecessors will be examined. If my reconstruction is correct, it should allow to better understand the nature of the borders of formal' semantics in late middle ages, their porosity, and their mobility.

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Ockham's Calculus of Strict Implication
The aim of this contribution is to show that already in the 14th century (in his main work Summa Logicae and in the smaller writings Elementarium Logicae and Tractatus Minor) William Ockham developed a system of propositional modal logic which contains almost all theorems of a modern calculus of strict implication. This claim is somehow at odds with the "traditional" view put forward, in particular, by J. Salamucha and by E.J. Moody. These authors maintain that, although other medieval logicians like Albert of Saxony, Walter Burleigh, Johannes Buridanus, or Duns Scotus had a fair knowledge of principles which together determine a set of theorems formally similar to those of the system of strict implication (Moody 1953: 78), Ockham's calculus should be regarded as a system of material, rather than strict, implication. Thus, according to Salamucha's pioneering study (1935/1950), Ockham's conditional "si, tunc" should be interpreted as a truth-functional, rather than a modal, operator. Moreover, Moody (1953) raised some serious doubts whether in general the medieval 'if, then'-relation may be understood as a strict implication at all. In order to clarify this issue which has been discussed, e.g., by Boehner (1958) and by Adams (1973), one not only has to take into account Ockham's theory of the alethic modal operators "in sensu compositionis", but one also must take a closer look at his distinction between "simple" consequences and consequences ut nunc. It will be argued that if one were to follow Salamucha in interpreting Ockham's conditional "si, tunc" as kind of a material implication, then one would also have to interpret the related operators "stat cum" and "repugnat" in a non-modal, "material" way. This, however, turns out to be untenable both from a systematic and from a historical point of view. After this clarification of

the proper understanding of Ockham's conditionals, a summary of his "calculus" of strict implication will be presented which contains laws for the following propositional operators: negation p, conjunction p?q, disjunction p?q, strict implication p?q, possibility Mp, necessity Np, impossibility Ip, contingency Cp, compatibility S(p,q), incompatibility R(p,q). Literature Adams, M. McCord: "Did Ockham Know of Material and Strict Implication? A Reconsideration", Franciscan Studies 33 (1973), 5-37. Boehner, Ph.: "Does Ockham Know of Material Implication?", reprinted in E.M. Buytaert (ed.): Collected Articles on Ockham, St. Bonaventure 1958, 319-351. Moody, E.J.: Truth and Consequence in Mediaeval Logic, Amsterdam (North Holland Publishing Company) 1953. Salamucha, J.: "Die Aussagenlogik bei Wilhelm Ockham", (translated from the Polish by J. Bendiek O.F.M.) Franziskanische Studien 32 (1950), 97-134.

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Muslims' Logical Research: A Necessity for Unifying Controversial Drives
As an exquisite instrument, science of logic has grounded the way of mankind to achieve truth in the realm of thought. Throughout history, in the mainstream of both religious and philosophical traditions, logicians had been trying to discover the rules of true thought and of argumentation solidity for the sake of religious, philosophical, political purposes. In the meantime, by applying to the original religious resources like Quran and tradition and mostly with the intention of adjusting the pure principles of human thinking (i.e. philosophy and logic) to the religious creeds, great Muslim thinkers managed to make progress in logic and its elements significantly. To date, what has made logic an important science in all Islamic sects' seminaries and even universities, is the nature of logic; because it has both formulated and articulated the most basic rules of human thought, and through human history, maintains to be the purest kind of disinterestedly reasoning and of rationally profound thinking. Since the true sympathetic discourse among different sects towards the realization of Islamic unity is based upon valid argument and good dialog, considering logical rules and principles sounds necessary. In this paper, adopting both analytical and comparative approaches, the author, in addition to setting out a basic description of logic, strives to sort out a sketch of its impartial and unifying nature during the Islamic thought.

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Syncategoremata in Arabic Logic
In this talk, I examine the terms usually called 'syncategoremata' in some Arabic logicians' texts. My problem is the following: What terms are considered as syncategoremata in the Arabic logical texts? What role do they play in the proposition and / or the syllogism? How do they help determine the form of a proposition or of a syllogism? What about the evolution of these terms through

time? To answer these questions, I will focuse on three main logicians, namely Al Farabi, Avicenna and Averroes, and I will study the way they characterize this kind of terms by taking into account their definitions of the quantifiers and of the logical constants as well as their analyses of the propositions and of the categorical syllogisms. I will show in this comparative study that there is some kind of evolution in the way these terms are characterized, for the definitions provided in the texts of the authors considered are different. It appears that the characterization of these terms tends towards more preciseness and clarity through history. For instance, the quantifiers are clearly separated from negation in Averroes' texts, while al Farabi and Avicenna provide definitions where the quantifiers are mixed with negation. This notion appears as much clearer in Averroes' texts, which indicates to some extent the evolution of the notion of form in Arabic Logic. As to the logical constants, such as implication and disjunction, to cite the most used ones, they appear also to be differently defined from author to author, for Avicenna, for instance, considers several definitions of implication which are not all equivalent to the classical philonian meaning of this constant, while disjunction is given some distinct and various meanings in al Farabi's text. We will then examine those various definitions, and try to relate them with the general view about logic endorsed by our three authors. This will determine to what extent logic is considered as formal in these different systems and what are the role and function of the distinction matter vs form of the propositions and the arguments.

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The Medieval Octagons for Modal and Oblique Sentences

Jean Buridan, a Fourteenth-Century logician, presented three Octagons of opposition and equivalence. One for quantified sentences where the predicate admits modal operator, for example: "Every man possibly dispute", "No man necessarily dispute". Another Octagon for sentences where the predicate is explicitly quantified, as "Every man is some animal", "Some man is not all animal". A third Octagon is dedicated to oblique sentences, directly related to relations, such as "of every man some ass is running", "some ass of any man is not running". Walter Redmond has designed a special language for the treatment of these sentences. There are some logical operations related to the extensional interpretation of predicates, such as the conjunctive "descent", that is to interpret universally quantified sentences in terms of chains of sentences without quantifier and joined by conjunctions; disjunctive descent is to interpret particularly quantified sentences in terms of chains of singular sentences linked by disjunctions. These techniques also apply to Octagons of Opposition, especially the Octagon where subject and predicate are quantified, but also to Modal and Oblique Octagons. Using Redmond's Language we will show how to perform conjunctive and disjunctive descent for these sentences. Descents for these sentences lead us to a very strong analogy between quantification and modality on the one hand, and on the other lead us to multiple quantification for quantified relations. A novel aspect is the extensional treatment applied to relations. The Quantified Modal Octagon corresponds to the so called *de re* modal sentences, where the modal operator is placed inside the whole sentence. These kind of sentences admit descent since they are quantified, but descent this time is to be extended, in some way, to cover modal sentences. Now, each Octagon has a Converse Octagon when the subject and the predicate. are changed by each other which leads to new relationships among their sentences. In this paper we present the language L for Quantified Modal and Oblique sentences. We also show some aspects of the Octagons, such as the relationships between *disparatae* sentences, and their relations to the normal and converse Octagons.

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Al Farabi's modal of Logic
Abu Nasr Muhammad al Farabi (870-950), the second outstanding representative of the Muslim peripatetic (mashai) after al Kindi (801-873), was born in Turkestan around the year 870.

Al Farabi's studies first began in Farab, but then he traveled to Baghdad, where he studied logic with a Christian scholar named Yuhanna b. Hailan.

Al Farabi wrote numerous works dealing with almost every branch of science in the medieval world. Traditional biographers attribute to him more than one hundred works of varying length, most of which have survived.

In addition to a large number of books , he came to be known as the 'Second Teacher' (al-Mou'allim al-Thani) Aristotle being the First. One of the important contributions of Farabi was to make the study of logic more easy by dividing it into two categories, Takhayyul (idea) and Thubut (proof). In addition that Alfarabi (Ihsa al Ulum, Amin, p. 70) believe that, the objective of logic is to correct mistakes we may find in ourselves , others and mistakes that others find in us. Then, if we want to correct other people errors, we should correct them the same way we correct ours. However, if we want other people to correct our errors, we must accept this knowledgably.

If we do not know logic, then we have either to have faith in all people or accuse all people or even to differentiate between them. Such action will be without evidence or experiment.

His famous books in Logic include the Art of Logic, book of Kitab al-lhsa al Ulum and Fusus al-Hikam. In his book Kitab al-lhsa al 'Ulum (Enumeration of the Sciences) defines logic and compare it with the grammar and discusses classification and fundamental principles of science in a unique and useful manner. In my paper, I will be analyzed these are logical books and others such as Kitab al-qiyas (Book on the Syllogism), Kitab al-jadal (Book on Dialectic) and Kitab al-burhan (Book on Demonstration) to present his modal in logic.

4.2.13 Logical Quantum Structures

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This workshop intends to lighten from several points of view some foundational aspects of physical theories, mainly quantum theories, such as mathematical, philosophical, logical, metaphysical and ontological aspects.

The aim is to indicate some possible lines of philosophical and logical investigation which seem to be in need of deeper research in this field, so as to join people with common interest and to motivate the formation of research groups.

Some of the topics to be discussed are:

a) Quantum logic: approaches and perspectives;

b) Inconsistencies and paradoxes in quantum mechanics: how to consider them;

c) Identity and individuality: can quantum objects be treated as individuals?;

d) Modality in quantum mechanics: formal and interpretational issues;

e) Quantum computation: logical frameworks.quantum structures;

f) Formal aspects of quantum theories;

g) Non-reflexive quantum mechanics;

h) Paraconsistent quantum mechanics.

The invited keynote speakers of this workshop are Dennis Dieks (page 68), Bob Coecke (page 65) and Hector Freytes (page 70).

Contributed Talks

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Identity, Non Reflexive Logics and the Kochen-Specker Theorem in Quantum Mechanics

Non-Reflexive Logics are logics in which the principle of identity does not hold in general. Quantum mechanics has difficulties regarding the interpretation of particles and their identity, in general these problems are known in the literature as the problems of indistinguishable particles. Non-reflexive logics can be a useful tool to account for such quantum indistinguishable particles. From a more general physical perspective we will analyze the limits of considering such indistinguishable particles. We will argue that the problem of identity regarding quantum mechanics is much profound than commonly acknowledged and that already the Kochen-Specker theorem places limits to discuss about a single particle.

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Quantum Ontology in the Light of Gauge Theories
By using the conceptual framework provided by the symplectic reduction procedure in gauge theories, we propose a quantum ontology based on two independent postulates, namely the phase postulate and the quantum postulate. The phase postulate generalizes the gauge correspondence between first-class constraints and gauge transformations to the observables of unconstrained Hamiltonian systems. The quantum postulate establishes a faithful correspondence between the observables that allow us to identify the states and the operators that act on these states. According to this quantum ontology, quantum states provide a complete description of all the objective properties of quantum systems.

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Remarks on the semantics for non-reflexive logics
Non-reflexive logics are systems of logic developed to deal with non-individuals, i.e. particular entities whose status as individuals is denied. Of course, quantum particles are the main alleged instances of such items. Since there are many distinct characterizations of what an individual is, there should also be many distinct corresponding definitions of non-individuals. The standard metaphysical characterization of a non-individual states roughly that non-individuals are items having no identity conditions, so that if a and b are non-individuals, statements to the effect that they are equal or different should at least not have a truth value, and in some cases, not even make sense. Quantum entities are said to be non-individuals in precisely this last sense, so that some kind of non-reflexive logic should be employed for a philosophically satisfactory treatment of those entities; in this case, the logic joins the metaphysics. In most systems of non-reflexive logics, the task of dealing with non-individuals is performed by putting some restrictions on the relation of identity at the syntactical level. We demand that terms denoting non-individuals in the intended interpretation do not figure in the identity relation. That should be good enough to formalize the idea that statements of identity do not make sense for non-individuals. However, trouble appears when it comes to provide the semantics. If the semantic

for such logics is provided inside a classical metalanguage, unrestricted identity is re-introduced by this very framework. So, the effort of restricting identity in the object language to reflect the intended metaphysics is lost. If, on the other hand, the semantics is introduced by a non-reflexive framework as metalanguage, then it is the semantical understanding of that framework itself that comes to be questioned. How do we explain our semantic understanding of the metalanguage itself? Do we have any intuitively clear account of that? If the answer to that last question is negative, the resulting situation seems to put in danger our attempt at making precise logical sense of something having no identity. This would entail that our best efforts at a quantum ontology of non-individuals are incoherent, since no reasonable logic of non-individuals may be developed. Here we examine those difficulties, and argue that there is a reasonable formal semantics for non-reflexive logics with interesting features. We show that the formal semantics presented conforms to intuitive features expected of talk of non-individuals. We deal also with the problem of an intuitive semantics. That is, besides a formal interpretation for non-reflexive logics satisfying some desiderata, we argue that we may have an intuitive understanding of meaning for non-reflexive calculi from an intuitive level. The main difficulty in this case comes from natural language, which provides the framework in which such an intuitive understanding is furnished. It seems that natural language commits us with the thesis that identity always makes sense for everything (natural language seems to be reflexive). Of course, that would be the very opposite of what is desired when we design a non-reflexive logic. Do we allow identity to infiltrate in our understanding of non-reflexive logics from the simple fact that we use natural language in our intuitive understanding of such logics? We argue that no informal semantics should be seen as posing obstacles to our understanding of non-individuality; indeed, we argue that natural language does not impose on us a specific ontology. Philosophical debate is required to settle the issue of whether there is an ontology with which natural language is committed to and what kind of ontology it is. We suggest that metaphysical issues concerning the ontological commitments of our representational apparatus - natural languages and non-reflexive logics being examples of such apparatuses - rely more heavily on metaphysical assumptions then it is usually assumed.

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Quantum physics and the classical theory of identity
According to the classical theory of identity (CTI), every object is an individual in the sense that: (i) (at least in principle) it can always be distinguished from any other individual, and (ii) a situation involving one individual presents differences from a situation involving *another* individual, the other ones remaining the same. Standard assumptions about quantum objects seem to suggest that these entities would violate such a theory, independently either they are particles or fields. Superpositions would present us typical situations in which (i) is violated, while entangled states seems to impede a clear distinction be-

tween situations with one or with another quantum object. A metaphysics of *non-individuals* looks "consistent" with the standard formalisms of quantum physics, hence CTI can also be questioned. But, if this is so, would classical logic be violated in the quantum realm? In this paper we discuss this and other related questions, by suggesting a way to overcome the supposed violation of classical logic by quantum physics, namely, by distinguishing between *pragmatical* and *foundational* uses of the theory.

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Contextuality and indistinguishability in quantum mechanics: a modal ontology of properties

In previous papers (Lombardi & Castagnino 2008, Ardenghi & Lombardi 2011) a new member of the "modal family" of interpretations of quantum mechanics (see Dieks & Vermaas 1998, Dieks & Lombardi 2012) has been proposed. In that paper, the contextuality of quantum mechanics is addressed by means of a modal ontology of of type-properties, whose modal nature consists in the fact that they only set the possible case-properties of a quantum system, but not its actual case-properties: a quantum system is conceived as a bundle of type-properties. This view is immune to the challenge represented by contextuality, since the Kochen-Specker theorem imposes no constraint on type-properties but only on the assignment of case-properties. However, in that paper the problem of indistinguishability was only superficially treated. This question has been specifically addressed in a recent work (da Costa, Lombardi & Lastiri 2012), where "identical particles" are indistinguishable because bundles of instances of the same type-properties: to the extent that they are strictly bundles, there is no principle of individuality that permits them to be subsumed under the ontological category of individual.

By beginning from atomic bundles (corresponding to irreducible representations of the Galilean group, see Ardenghi, Castagnino & Lombardi 2009, Lombardi, Castagnino & Ardenghi 2010), that last work faces the question of indistinguishability from a bottom-up viewpoint. Nevertheless, from a top-down perspective, the problem of indistinguishability leads us to consider the features of the new "composite" bundle as a whole: it is a structure with an internal symmetry (see Narvaja, Córdoba & Lombardi 2012). Therefore, the resources supplied by logic and mathematics, in particular, the concepts of invariance and rigidity (Krause & Coelho 2005; da Costa & Rodrigues 2007), may be useful to discuss the problem. The purpose of this work is to apply these concepts to the picture of a modal ontology of properties.

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Heisenberg indeterminacy principle in the light of Galois-Grothendieck algebraic theory.

We shall consider Heisenberg indeterminacy principle in the light of the Galois-Grothendieck theory of polynomial equations. For that, we use the Pontryagin duality between the space of states defined by a (Schrodinger or polynomial) equation and the space of observables necessary to discern them. We apply this duality in the context of Galois-Grothendieck theory in the following sense:

the different Galoisian permutations of the solutions of a specific polynomial equation form a Galois group G of symmetries. The algebra C[G] can then be considered a sort of space of wave functions. We can thus define superpositions of several single states and distinguish between different levels of Galoisian indetermination. In the Galois-Grothendieck theory, the existence of these different levels results from the relativity of the Galoisian indetermination. Each level is defined by a subgroup H of Galoisian symmetries that can be progressively broken by increasing what we could call the resolving power of the corresponding observables. We argue that all these levels satisfy an indetermination relation analogous to the Heisenberg's relation.

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Quantum decoherence: a logical perspective
There are different perspectives to address the problem of the classical limit of quantum mechanics. The orthodox treatment introduces the phenomenon of decoherence as the key to solve this problem (Bub 1997). The mainstream approach of decoherence is the so-called "*environment induced decoherence*", developed by Zurek and his collaborators (see, e.g., Zurek 1981, 2003, Paz & Zurek 2002). In the context of this approach, the goal is to know whether the state becomes diagonal or not (Schlosshauer 2007). If the state becomes diagonal, then it acquires a structure to a mixed state of classical mechanics; this feature leads to the usual interpretation of the decohered state from a classical viewpoint.

In our group, we have developed a general theoretical framework for decoherence based on the study of the evolution of the expectation values of certain relevant observables of system (Castagnino, Fortin, Laura & Lombardi 2008, Castagnino, Fortin & Lombardi 2010). According to this framework, decoherence is a phenomenon relative to the relevant observables selected in each particular case (Lombardi, Fortin & Castagnino, 2012). This new approach and the orthodox treatment of decoherence are equivalent from a mathematical point of view. Nevertheless, there are good reasons to think that the treatment of decoherence by means of the behavior of the observables of the system instead that of its states may have conceptual advantages.

The purpose of this work is to argue that the main advantage of the study of decoherence in terms of the Heisenberg representation is that this approach allows us to analyze the logical aspects of the classical limit. On the one hand, we know that the lattices of classical properties are distributive or Boolean (Boole 1854): when operators are associated with those properties, they commute with each other. On the other hand, it is well-known that the lattices of quantum properties are non-distributive, a formal feature manifested in the existence of non-commuting observables (Cohen 1989, Bub 1997). In spite of this difference, there are certain quantum systems which, under certain particular conditions, evolve in a special way: although initially the commutator between two operators is not zero, due to the evolution it tends to become zero (Kiefer & Polarski 2009). Therefore, in these systems should be possible to show that, initially, they can be represented by a non-Boolean lattice, but after a definite time a Boolean lattice emerges: this process, that could be described from the perspective of the expectation values of the system's observables, deserves to be considered as a sort of decoherence that leads to the classical limit. In other words, from this perspective the classical limit can be addressed by studying the dynamical evolution of Boolean lattices toward Boolean lattices.

*This work is fully collaborative: the order of the names does not mean priority.

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Indivisibility, complementarity, ontology and logic

Indivisibility is one of the most significant intuitions of Bohr. Several related meanings of indivisibility are discussed. One is the limited divisibility of the action which implies the impossibility of applying the classical algorithms. Another is the impossibility of dividing in quantum phenomena what can be considered the object of observation and the instruments of observation, which implies that the quantum phenomena must be considered as an indivisible wholeness that includes the instrument of observation. A third meaning is that indivisibility does not signify only that we cannot conceptually divide the systems and the instruments of measurement, but also that we can neither do such division for the systems themselves. Bohr's indivisibility comprises then both the contextuality of the quantum variables and the entanglement of the quantum states. Some arguments are presented to show in which sense indivisibility's meanings as contextuality and entanglement are a consequence of the quantization of action.

Although Bohr did not express himself neither in detail, nor clearly enough, on issues of the ontological kind, arguments consistent with his general thought are presented, which go in the following direction: since an absolute distinction between the observer and the observed is not possible to make here, ontological questions have no sense because they are questions about observing an object defined in an absolute way, independent of the experimental context. Science then will not refer to a reality independent of the human, of the human observational context, of the human existence, but will only refer to the unambiguously communicable experience. In a nutshell: it will be argued that indivisibility impedes the attribution of properties in themselves to a quantum system, therefore the quantum formalism cannot be interpreted in terms of reality in itself . It is concluded that the conception implicit in Bohr's thesis is then that physical reality is just an epistemological reality.

If the quantum phenomena are an indivisible wholeness which includes the instrument of observation, the possibility arises that concepts valid in a given experimental context are not valid in a different experimental context. Within a conception in which the observed objects can be conceived completely separate from the observer, the observed properties can be attributed to the objects in a totally independent manner from the method of observation. They will be completely independent properties from the experimental context and there will not be any reason for not continue attributing them to the objects in totally different experimental contexts. The indivisibility opens then the possibility of mutually exclusive experiences and concepts: two concepts will be mutually excluding if the possibility does not exist to define their use by means of only one type of experience. In other words, in the case where the experimental contexts mutually exclude, the corresponding concepts will also be mutually exclusive. Two mutually excluding concepts cannot then be combined in the mind to have a conceptual single image. Thus by example, as the concepts of velocity and of position are mutually excluding, it is not permitted to us to combine them in the idea of path, in spite of the fact that from a logical point of view they are not excluding , as the concepts of wave and particle are.

It is the experience what defines the use of the concept. Now well, the meaning that is giving here to the word use is extremely restrictive: if the type of experience does not permit the use of a concept it signifies that we cannot utilize it in our reasoning. Thus, in an experience that permits us to use the concept of wave we cannot utilize in our reasoning the concept of particle and vice versa. It is for this reason that the duality wave-particle does not arise. Similarly in an experience that permits us to use the concept of position we cannot utilized in our reasoning the concepts of velocity and vice versa. That is to say, the employ of a concept in an experience which does not allows its use turns it into a meaningless concept. In other words relative to that experimental context the concept is meaningless. Because of this we can affirm then that the meaning of a concept is defined only by means of a precise type of experimental context. What is being proposes is not a mere operational definition of the concepts. What it is affirmed is that we are not authorized to utilize without restriction a concept whose use is sanctioned by a concrete type of experience, to account for any another type of arbitrary experience.

Complementarity affirms that although concepts like wave and particle, and position and velocity are mutually exclusive in the above mentioned sense, they are to be considered as complementary: as mutually exclusive, in the sense that we cannot combine them in the same reasoning in respect to an electron, but both being necessary to show everything that is possible about an electron. Given that the mutual exclusion of two complementary concepts is not defined by logic there is not any inconsistency implied by complementarity from the point of view of classical or ordinary logic. As a consequence it is argued that there is no need of any type of new logic where to insert complementarity.

4.2.14 Logic and Linguistics

This workshop is organized by

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Natural and artificial languages structured by grammar and logic are important tools of thinking, cognition of the world and knowledge acquisition which stand as foundations to our sense of existence. The aim of this workshop is to reveal the universal components functioning in logic, philosophy of the language and theoretical linguistics. It aims to constitute the inspiration for research on a general theory of logical linguistic structures, and also cognitive and computational linguistics.

From the universal perspective, logic and linguistics should answer the following example questions: What is language and its expressions and what are their ontological and conceptual counterparts, if any? What is the meaning of language expressions and how to calculate it? What is the difference between meaning and denotation of a language expression? What are the general principles of using language in communicative, cognitive and argumentative purposes?

Invited talks cover following or related topics:

a) Peirce's token-type distinction of language objects;

b) formal grammars, in particular categorial grammars;

c) universal syntax and semantics assumptions;

d) parts and constituents of language expressions;

e) intensional and extensional semantics;

f) meaning, object reference, denotation and interpretation of linguistic expressions;

g) principles of compositionality;

h) cognitive and computational linguistics issues.

The invited keynote speakers of this workshop are Jonathan Ginzburg (page 72) and Andrzej Wiśniewski (page 86).

Contributed Talks

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Inflected infinitive, control and anaphora in Portuguese

In this paper I will argue the so-called personal or inflected infinitive in Portuguese gives rise to resources deficit in a categorial analysis of control structures. I will suggest a solution to the tension between syntax of control and semantic resource sensitivity by analizing the categorial treatments of anaphora phenomena.

Formal grammars based in Lambek calculus are substructural logics; they reject the use of the structural rules of Contraction, Permutation and Weakening in the syntax of their Gentzen style sequent formulation of the calculus. So, these Categorial Grammars, that avoid counterparts of copy transformations and deletion rules, are conscious-resource logics.

The so-called control structures exhibit a potential resource conflict for the consciuous-resource logics: in control, there is a single syntactic element shared as the controller and control target that apparently realises two different semantic arguments, since it is an argument of both the matrix control verb and the subordinate predicate. Though, this resource deficit arises only if a clause and —consequently— a proposition are considered as controlled complement of the principal clause. On the contrary, if the controlled complement denotes a property, the only syntactic resource is not semantically consumed by the controlled complement and then, the resource deficit does not arise. So, the resource deficit can be avoid in categorial grammars, that consider an only —surface— level of syntax, admitting the property denotation.

Nevertheless, Portuguese has an infinitive with person endings, the so-called personal or inflected infinitive. In Portuguese the controlled complement in control can be finite complement with null pronominal subject or even lexical coreferencial subject. Then, the tension between syntax of control and semantic resource sensitivity come back in Portuguese and the categorial support to the property denotation has to be revised.

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Aspecto-temporal meanings analyzed by means of Combinatory Logics

What is the meaning of language expressions and how to compute or calculate it? We give an answer in analysing the meanings of aspects and tenses in natural languages inside the formal model of an Applicative and Cognitive Grammar and Enuntiative operations (GRACE) (Desclés, Ro, 2011; Ro, 2012), using the applicative formalism, functional types of Categorial Grammars and Combinatory Logic (Curry, 1958). In the enunciative theory (Culioli, 1999; Desclés, 2011) and following C. Bally (1932/1965), an utterance can be decomposed into two components: a *modus* and a *dictum* (or a proposition). In GRACE, the *modus* is a complex operator applied to a proposition, that is a predicative relation, and generated from more elementary operators of the categories of TAM (tense, aspect, and modality). In the temporal enunciative framework, the general scheme of enunciation is an applicative expression (presented with a prefixed notation):

$$(I) \quad \exists J_0 \ \exists \ \{(\mathrm{I})\ \mathrm{PROC}_{\mathrm{J}_0}(\text{I-SAY} \ (\ \& \ \ (\mathrm{ASP}_I(\text{proposition}))\ (\underline{\mathrm{rep}}\ \mathrm{J}_0 I)))\}$$

In this applicative expression, 'J_0' and 'I' are topological intervals of contiguous instants. The operator '$\mathrm{PROC}_{\mathrm{J}_0}$' designates a process actualized onto the interval of enunciation 'J_0', it is applied to the result of an application of a commitment operator 'I-SAY' on an aspectualized proposition coordinated, by ' & ', with a temporal condition. In the aspectualized proposition, the aspectual operator 'ASP_I' actualizes the proposition onto the interval 'I'. The temporal relation '$\underline{\mathrm{rep}}$ J_0 I' specifies how the interval 'I' is related to the interval 'J_0'; the value of the abstract locating operator $\underline{\mathrm{rep}}$ (in French "repérage") are: concomitance (=) or differentiation ($\neq$), by anteriority (¡) or posteriority (¿), between the right boundaries 'd(I)' and 'd(J_0)' of these intervals 'I' and 'J_0'. The operator 'ASP_I' is generic; its values are: 'STATE_O' (state), 'EVENT_F' (event), 'PROC_J' (process) and the generic interval 'I' is specified by an open interval 'O' (for a state), closed interval 'F' (for an event) and a left closed and right open interval J (for a process). These three operators are basic concepts for building a theory of aspects and temporal relations (Desclés, Guentchéva, 2012). The scheme (I) expresses the general meaning of aspects and temporal relations. For instance, for the two utterances:
(a) John is killing a bear
(b) John killed a bear
(c) John has killed a bear

the general scheme is specified by respectively :
(a') $\exists J_0\ \exists I$ {PROC_{J_0}(I-SAY(& (PROC_J(to-kill a-bear John))(= d(J_0)(d(O)))}
(b') $\exists J_0\ \exists I$ {PROC_{J_0} (I-SAY(& (EVENT_F(to-kill a-bear John))(¡ d(J_0)d(F)))}
(c') $\exists J_0\ \exists I$ {PROC_{J_0} (I-SAY(& (RESULT-STATE_O(to-kill a-bear John))(= d(J_0)d(0)))}

In the process (a), the meaning (a') expresses a concomitance between the process of killing with the process of enunciation; in the event (b), the meaning expresses an anteriority of the predicative event relative to the enunciative process; in the resulting state (c), the meaning expresses the result that follows immediately an occurrence of an event, this result is actualized onto an open interval that is concomitant with the interval 'J_0'. Our contribution explains how different grammatical meanings can be synthesized into complex operators (grammatical operators) applied to the predicate of the underlying predicative relation since the analyses of (a), (b) and (c) by a categorical grammar are:
(a") (((is ing) (to-kill) a-bear) John)
(b") (((-ed) (to-kill) a-bear) John)

(c") (((has ... ed) (to-kill) a-bear) John)

The Combinatory Logic gives formal tools to compose elementary operators and to build grammatical operators. We show, for instance, that the morphological operator ising in (a") and -ed in (b") are obtained by means of some (complex) combinator 'X' that combines together the semantic primitive operators 'I-SAY', ' & ', 'PROC$_{J_0}$', 'PROC$_J$' (or 'EVENT$_F$') and the temporal relations of concomitance or anteriority. The semantic definitions of these grammatical operators is given by relations between a definiendum and a definiens:

[is ing $=_{def}$ $\exists J_0$ $\exists I$ {X I-SAY & PROC$_{J_0}$ PROC$_J$ ((= d(J$_0$) (d(O))}]

[-ed = $_{def}$ $\exists J_0$ $\exists I$ {X I-SAY & PROC$_{J_0}$ EVENT$_F$ ((¡ d(J$_0$) (d(F))}]

The definition of has ... ed is more complex since we must specify the relation between the interval 'F' (of the occurrence of one event) and the interval 'O' of the resulting state of this event. By this way, we can give a semantic meaning associated to different grammatical aspecto-temporal operators. The applicative expressions of Combinatory Logic can be easily translated in a functional programming language as HAKELL or CAML (Ro, 2012).

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Complement Polyvalence and Polyadicity Early versions of phrase structure grammar (Chomsky 1957) were unable to formalize the fact that words may have different numbers of complements but belong to the same word class. Thus, such grammars can not, for example, treat intransitive, transitive and ditransitive verbs all as verbs. Let us call this *the subcategorization problem.* These grammars are equally unable to formalize generalization, well-known even then, that each phrase of a certain kind, say a noun phrase, contains a word of a related kind, a noun. Let us call this *the projection problem.* HPSG has a satisfactory solution to both of these problems, namely, the valence attribute in the lexical entry and the valence principle. As noted by Robert Levine and W. Detmar Meurers (2006), 'this principle essentially is the phrase-structural analogue to Categorial Grammar's treatment of valence satisfaction as combinatory cancellation'. One can formalize such a categorial rule as follows:

> $\mathrm{X}\colon \langle \mathrm{C}_1, \ldots, \mathrm{C}_n \rangle\ \mathrm{C}_1 \ldots \mathrm{C}_n \rightarrow \mathrm{X}\colon \langle\,\rangle$, where $\mathrm{X}\colon \langle \mathrm{C}_1, \ldots, \mathrm{C}_n \rangle$ is a category comprising a pos label X and a complement list $\langle \mathrm{C}_1, \ldots, \mathrm{C}_n \rangle$ and where C_i $(1 \leq i \leq n)$ is either a phrase (e.g., AP, NP, PP) or the label for some form of a clause.

However, as things stand, two other related problems are left outstanding. First, one and the same word may admit complements of different categories. This is true not only of copular verbs such as the verb *to be*,

> Alice is [AP smart] / [NP a genius] / [PP in Paris] / [VP walking]

but also of many other verbs as well. (Huddleston 2002 has a summary.) Let us call this *the complement polyvalence problem.* Second, the same word may permit the omission of one or more of its complements and the omission of the complement results in a predictable construal. Thus, the verb *to wash*, *to meet*, *to read* and *to arrive* take optional complements and the intransitive versions have a predictable meaning (reflexive, reciprocal, indefinite and contextual respectively). Let us call this *the complement polyadicity problem.* (See Fillmore 1986, Partee 1989 and Condoravdi and Gawron 1996 as well as Huddleston 2002.)

The question arises: is there a way to extend the solution of the subcategorization and projection problems to the problems of complement polyvalence and polyadicity? As the paper will show, this can be done by introducing two modifications to the cancellation rule above. To solve the complement polyvalence problem, one replaces the categories of the complement list with non-empty sets of categories, the resulting rule being formulated as follows:

> Let $\mathcal{C}_1, \ldots, \mathcal{C}_n$ be non-empty sets of complement categories. Let $\mathrm{C}_1, \ldots, \mathrm{C}_n$ be complement categories where $\mathrm{C}_i \in \mathcal{C}_i$, for each i, $1 \leq i \leq n$. Then $\mathrm{X}\colon \langle \mathcal{C}_1, \ldots, \mathcal{C}_n \rangle\ \mathrm{C}_1 \ldots \mathrm{C}_n \rightarrow \mathrm{X}\colon \langle\,\rangle$.

To solve the complement polyadicity problem, one permits the non-empty set of complement categories to include the diacritics ref, rec, ind and con. One also permits the number of slots in the complement list to outrun, as it were, the number of complements, subject to the proviso that any slot in the complement which does not have a correspondent among a word's complements has a diacritic in the slot's set. Here is the formalization:

> (1) Let $\mathcal{C}_1, \ldots, \mathcal{C}_n$ be non-empty subsets of complement categories and diacritics. (2) Let $\mathrm{C}_1, \ldots, \mathrm{C}_m$ be complement categories where $m \leq n$. (3) Let r be a monotonically increasing injection from $\mathbb{Z}_m^+$ to $\mathbb{Z}_n^+$ satisfying two conditions: (a) for each $i \in \mathbb{Z}_m^+$, $\mathrm{C}_i \in \mathcal{C}_{r(i)}$; and, (b) for each $j \in \mathbb{Z}_n^+$ which is not in the image set of r, $\mathcal{C}_j$ contains a diacritic. Then X: $\langle \mathcal{C}_1, \ldots, \mathcal{C}_n \rangle$ $\mathrm{C}_1 \ldots \mathrm{C}_m \rightarrow$ X: $\langle\,\rangle$.

The paper will present, in addition, the corresponding semantic rules.

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The Concept Formation Constraint

A restriction on natural concept formation and lexicalisation in natural language has hitherto gone unobserved. To bring it to light and show its relevance, three steps will be taken. First, it will be shown that there is a homology between colour percepts and quantifier concepts. This will be done by proving algebraically that definitions for traditional relations of opposition in predicate logic carry over to the mereological realm of colour percepts and capture the relations between primary (R, G, B) and secondary (yellow (Y), magenta (M), cyan (C)) colours. Secondly, we turn from this logic-colour parallel - to whose conceptual counterpart Wittgenstein devoted the last year of his life - to language and more specifically to lexicalisation. Here the homology is reflected in a parallel lexicalisation asymmetry. For quantifiers Horn (1989) observed the nonexistence of a single-item lexicalisation for the Boethian O-corner operator **nall* (= *not all*) and the artificial nature of the corresponding propositional operator *nand*. In the realm of primary and secondary colours, there is a parallel asymmetry between four natural words (RGBY) and two consciously crafted terms (MC). To prove that this parallel is not an accidental (hence simple-minded) analogy, it will be shown in a third section that a concept formation constraint makes sense of the asymmetry and crucially generalizes to loads of other lexical domains (demonstratives, spatial deixis, etc.).

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Logic, Language and Information – functional and structural integration without competence overstepping

Logic, language and information integration is one of areas broadly explored nowadays and at the same time promising. Authors use that approach in their 8 years long research into Structured Semantic Knowledge Base System (SKB). Natural language processing in the context of logics or information processing is ineffective, not to say pointless in the means of certainty. It is simply caused by the fact, that natural languages has been formed long time ago as the main communication channel, rather than for automatic information processing. Due to that logic has been founded as the universal language of mathematics and formalized communication in more extensive sense. Information processing however brought more sophisticated problems, that mathematics could not smoothly solve, what made a chance for computer science to appear. Studies over artificial intelligence revealed even more complex issues to solve. The unity of those three areas: logics, language and information is a necessity to acquire complementarity and synergy. This approach requires however to take into consideration natural, structural and functional limitations of each of them. As natural languages should not be used to information processing or its storage, logics will never become efficient in human-system communication. Each element has its own fields of implementation and the problem is in building adequate unity.

The solution proposed by the authors has three layers of data storage, named: conceptual, information and language layers. Each of them has its own functionalities. Some of those functionalities are internal operations, whereas others are interfaces between layers. Taking other points of view, the structure of the SKB is quite complex. There are specialized modules: Ontological Module (Ontological Core + Structural Object Module), Behavioral Module, Time and Space Module, Linguistic Module, Semantic Network Module. The last listed module is the most important in the aspect of logics implementation and for-malized information processing. This module is based on, extended by authors, semantic network idea, using roles, quantifiers, multiplicity, certainty, error margin over the structure of operators and operands. Taking the modules and the layers described authors created the system where each of the fields represented by logic, language and information is strictly imposed in the structure and functionalities.

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Ontology through Semantics in Donald Davidson

In this presentation I will address, firstly, Donald Davidson's argument that we can extract ontological consequences from an adequate semantic theory for ordinary languages. In the author's perspective, the linguistic structures that emerge from a rigorous and empirical semantics for natural languages may reflect a shared picture of the world by its speakers, evidencing thus their ontological commitments. For this reason, following the Quinean maxim that to be is to be the value of a variable, the notations used to explore the structure of natural languages - Davidson believes - must be tools that in systematically transforming the sentences of ordinary languages into sentences of a formal language, may bring to the surface covert quantificational structures, enabling that ontological commitments can be inferred from considerations on ordinary language. Secondly, I will regard Davidson's observation that even though a logical form that quantifies over variables for events accounts for a number of linguistic phenomena which until then had not received a satisfactory treatment - in particular action sentences - it is not sufficient to sustain the commitment of the users of a language with the existence of events as particulars. Quinean in spirit, Davidson believes that we should tolerate only entities to which some general criterion of identity can be applied, i.e., there can be no entity without identity. I will present the criterion proposed by Davidson, and I will speak about the reasons that later led the author to abandon it, without however giving up an ontology of events.

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A Generative Montagovian Lexicon for Polysemous Deverbal Nouns

We propose a computational formalization of some forms of polysemy. Here we focus on the resultative/processual polysemy of deverbal nouns like *assinatura* ("signing/signature") or *abertura* ("opening/aperture") in Portuguese — we also study similar constructs in French, Italian, and English. We follow the Montagovian Generative Lexicon (MGL) introduced in Bassac, Mery & Retor (2010) based on second-order Girard's F system with several entity types — including at least one type t for propositions and several entity types, as v (*event*), s (*state*) φ (*physical object*). Our formalization produces the readings involving one aspect of the polysemous noun, and it also handles properly co-predication phenomena. Indeed, co-predications on several aspects of a polysemous term can be correct or incorrect. For instance, one cannot assert a predicate of the resultative meaning and simultaneously a predicate of the processual meaning.

To do so, a lexical entry consists in the "usual" Montagovian λ-term expressing the argument structure (with fine-grained types) and optional modifiers turning the type of an object (e.g. v or s) into another type (e.g. φ). Consider the lexical entry to "assinatura" (whose type is v) as the following:

$$\langle \lambda x^v.(assinatura^{v\to t}x); \quad id = \lambda x^v.x, \quad f_r^{v\to s}, \quad f_\varphi^{v\to\varphi}\rangle$$

When there is a type mismatch, one is allowed to apply some optional modifier(s). We thus are able to derive "A assinatura atrasou três dias"[5] and "A assinatura estava ilegível"[6].

The definite article "a" ("the"), is handled by a typed choice function ι (a typed version of von Heusinger, 2007) whose type is $\Lambda\alpha.(\alpha\to t)\to\alpha$. When this polymorphic ι ($\Lambda\alpha...$) is specialised to the type v ($\alpha := v$) it becomes of type $(v\to t)\to v$ and when applied to $assinatura : (v\to t)$ it yields a term $\iota\{v\}assinatura$ of type v whose short hand in the examples is written $(sig)^v$. This term introduces a presupposition: $assinatura(\iota(assinatura))$, saying that the designed event is an "assinatura".

In the examples, let us denote by $atra3 : (v\to t)$ the predicate "atrasou três dias" (*took three days*) which applies to events and by $ilg : \phi\to t$ the predicate "estava ilegível" (*was illegible*) that applies to physical objects. Both predicates are computed from the lexicon, but we cannot include the details.

(1) "A assinatura atrasou três dias"[1]

$$\lambda y^v.(atras3^{v\to t}y)(sig^v)$$
$$atras3^{v\to t}(sig^v)$$

(2) "A assinatura estava ilegível"[2]

$$(\lambda y^\varphi.ilg^{\varphi\to t}y)((sig^v))$$
$$(\lambda y^\varphi.ilg^{\varphi\to t}y)(g^{v\to\varphi}(sig^v))$$
$$ilg^{\varphi\to t}(g\ sig^v)^\varphi$$

Now let us show that the co-predication between "took three days" and

[5] "The signing was delayed by three days." Example from http://noticias.uol.com.br/inter/efe/2004/03/05/ult1808u6970.jhtm.

[6] "The signature was illegible." Example from http://www.reclameaqui.com.br/3372739/dix-saude/cancelamento-do-plano-a-mais-de-um-mes-e-nada.

"was illegible" cannot be derived. Firstly, the conjunction of two predicates that apply to different types (different view of the same object) is depicted using the second order typing. The "and" formalisation is:

$$\Lambda\alpha\Lambda\beta\ \lambda P^{\alpha\to t}\lambda Q^{\beta\to t}\ \Lambda\xi\ \lambda x^{\xi}\lambda f^{\xi\to\alpha}\lambda g^{\xi\to\beta}\&(P(f\ x))(Q(g\ x))$$

The instantiations for our example should be as follows: $P = atras3, \alpha = v, f = Id_v, Q = ilg, \beta = \varphi, g = f$ and $\xi = v, x = sig^v$. This polymorphic "and" takes as arguments two properties P (here: $astras3$) and Q (here: ilg) which apply to entities of type α (here: v) and β (here: φ), returning a predicate that applies to a term x of type ξ. This predicates says that x of type ξ (here sig of type v) which via some f (here Id) can be viewed as an object of type α (here v) enjoying P (here $atras3(x)$) and that the same x can also be viewed via some g (here f_φ) as an object of type β (here ϕ) enjoying Q (here $ilg(f_\varphi(x))$). — hence x has both properties (here $atras3(x)\&ilg(f_\varphi(x))$), provided the proper meanings of x are considered.

The constraint that both the identity and the result are rigid modifiers, means that if one of the occurrences of the argument of the predicate is used via a modifier, so are the others. Here, if one occurrences is the process itself (through the identity) or the result (through the optional modifier) it ought to be used with the same meaning for each of the occurrences — the presence of the identity in the lexical entry allows us to express that the original type itself is incompatible with others that are derived from it. As expected, this flexible/rigid distinction properly blocks the above co-predication that effectively cannot be derived. A less strict rule is possible: such compound infringing the rigid rule are given bad marks.

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Intensional verbs, their intensional objects and axiomatic metaphysics

Intensional transitive verbs (ITVs) such as *look for*, *need* or *resemble* have attracted a lot of attention from formal semanticists in the past decades. That interest has been raised by the works of Quine [3] and Montague [2] (though Quine's interest was philosophical, not linguistic).

It is a well known fact that a sentence *John needs a horse* has two readings. According to the first, specific one there is a particular horse that John needs. According to the nonspecific reading, on the other hand, there is no particular horse that John is in need of. It is rather, as Quine would put it, *mere relief from* *horselessness* that John is after. It comes as no surprise that standard semantics for transitive verbs, taking them to denote relations between individuals, cannot handle the second reading.

Another well established viewpoint is that ITVs do not allow for substitution of coreferential terms in their object position. A sentence

John is looking for the Dean (I)

cannot be inferred from

John is looking for Mary's father (II)

even if the Dean happens to be Mary's father, because it might as well be that John is unaware of that fact.

In my previous work (cf. [4]) I challenged the two assumptions that have shaped the research on ITVs so far: first, that ITVs are evaluated with respect to the belief state of their agent, and second, that their nonspecific readings are derived from the specific ones. I show that in evaluating sentences like (1) or (2) the information state of the hearer is crucial. Considering the knowledge of the hearer rather than that of the agent leads to a conclusion that a novel, dynamic approach to semantics of ITVs is needed.

The formalism I designed was inspired by the work of Barwise and Cooper [1] on generalized quantifiers. I draw on their idea that all NPs should be given the same kind of denotation, namely a quantifier (i.e. a set of sets of individuals). The primary reading of an intensional transitive verb therefore relates two quantifiers, the first one specifying the individual that is the agent, the other one that provides a set of alternatives for filling the object position. Specific readings can be defined in purely algebraic terms as the ones where the object is given a denotation that is a filter generated by a singleton set from the domain. All of the above formal setup is done in a dynamic way (cf. e.g. [5]). Instead of a definition of truth of formulas I define an update relation between information states and sentences of my formal language.

However, taking a closer look at both the data I considered in my thesis and some further examples, I was led to a conclusion that even more fine tuning is needed to capture the intricacies of the use of intensional transitive verbs. While the data suggesting the need for a hearer-oriented approach are still convincing, I have found examples where the hearer's perspective does not seem to be a default one at all. It seems that we are in need of keeping track of three modes of reference to the object – i.e. by the speaker, the hearer and the agent of the sentence containing an intensional verb. And we seem to already have a formalism that with some further work could handle this much.

In his 1988 paper "A Comparison of Two Intensional Logics" [7] E. Zalta briefly sketches a proposal for representing intensional transitive verbs within the language of his axiomatic metaphysics, as developed in his earlier works [6]. However, his treatment fails when challenged with the data and analysis I have put forward. Moreover, Zalta's formalism is not ready for more elaborate natural language examples, e.g. there is no go-to way of translating *det NP* expressions that are supposed to refer to intensional objects.

Nonetheless, quite a lot of elements in Zalta's formalism look promising for our enterprise: the ease of interpreting intensional verbs as extensional relations on individuals (thanks to the more elaborate universe of individuals including intensional objects), intuitive way of spelling out the above-mentioned "three modes of reference" idea.

In this paper I propose to develop a more sophisticated semantics for intensional verbs, based on Zalta's Axiomatic Metaphysics. I put it to test against nontrivial examples of the use of ITVs, and end up expanding both original Zalta's proposal and my own update semantics for ITVs.

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4.2.15 Universal Logic and Artificial Intelligence

This workshop is organized by

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Artificial intelligence is different from traditional computer science and mathematics. Computer science and mathematics describes an ideal world in which all problems are abstracted to true/false clearly demarcated deterministic issues (up most uncertainty issues with a certain probability distribution), which is suitable for the description of binary mathematical logic. But artificial intelligence must be faced with the real world. Only a small part of the issues can be transformed into true/false clearly demarcated deterministic problems, and most of the issues are uncertainty problems in continuous development and evolution. The challenges are the following:

1. In the real world propositions should be described by multi-valued or even continuous values of truth degrees rather than binary state.

2. In the case of non-binary phenomena, the affect of the generalized correlation among propositions, measurement errors and partiality will gradually be displayed and the reasoning results will be affected directly and inevitably.

3. With the changing of time or environment, propositional truth will be continuously changing.

4. Incomplete inductive reasoning, the default inference in knowledge incomplete case and reasoning based on common sense are commonplace in the human intelligence activities, artificial intelligence needs therefore to simulate this kind of intelligent activity.

These factors determine the diversity of logic in artificial intelligence. And also the emergence and wide application of all kinds of non-standard logics proves this point. Do common logical laws exist in standard logic and non-standard logic or not? How to identify these common laws of logic? How to master and apply these common laws of logic to promote the development of artificial intelligence? This is the topic we need to discuss.

This topic of the UNILOG 2013's workshop will focus on the relationship in a variety of non-standard logics widely used in artificial intelligence, and the guiding significance of universal logic for the development of artificial intelligence.

Topics may include:

a) The ways and means to reveal the common law hidden in standard logic and a variety of non-standard logics;

b) The intrinsic link between the standard logic and a variety of non-standard logics;

c) The unified model accommodating standard logic and a variety of non-standard logical computing models;

d) The relationship between the logical and intelligent algorithms, how to extract the laws of logic from effective intelligent algorithms;

e) To explore the use of universal logic to describe the process of the imitation of anthropic activities in a realistic environment;

f) To explore the use of universal logic to describe the process of information-knowledge- intelligent, coordination in complex system.

The invited keynote speaker of this workshop is Zhitao He (page 75).

Contributed Talks

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Unified logics and artificial intelligence

Mutually-inversistic logic is unified logics. It unifies classical logic, Aristotelian logic, ancient Chinese logic, mutually-inversistic modal logic, relevance logic, inductive logic, many-valued logic, Boolean algebra, fuzzy logic, natural deduction, paraconsistent logic, non-monotonic logic. It is the unifications of mathematical logic and philosophical logic, of ancient logic and modern logic, of western logic and Chinese logic, of two-valued logic and many-valued logic, of inductive logic and deductive logic, of crisp logic and fuzzy logic, of extensional logic and intensional logic.

Mutually-inversistic logic is applied to such branches of artificial intelligence as automated theorem proving, logic programming, expert systems, planning, scheduling, semantic network, natural language understanding, machine learning, data mining, inference with uncertainty.

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Fixed Point and Undecidable Proposition in Logical Thinking – The Proof for Gödel Incomplete Theorem is Untenable
The fixed point theorem is one of the wild-ranging and profound theorems and it has penetrated into various fields of mathematics. This paper shall prove that Russel's paradox is the fixed point in set theory and Gödel undecidable proposition is the fixed point in natural number system N. It is further found that there is also fixed point in the logical thinking field. The fixed point is not included in the positive set or negative set. The logical natures of the fixed point are undecidable and are the system's inherent phenomenon. There is also fixed point in the natural number system N. The existence and undecidability of the fixed point do not have impact on recursiveness of the positive and negative set and completeness of the system. Therefore, the demonstration for Gödel incomplete theorem is untenable. Are the propositions related with Gödel incomplete theorem tenable? Is the system N complete? These are issues that must be reconsidered.

4.3 Contest – Scope of Logic Theorems

In view of the speedy and huge expansion of the universe of logics, the question of the scope of validity and the domain of application of fundamental logic theorems is more than ever crucial. What is true for classical logic and theories based on it, does not necessarily hold for non-classical logics.

But we may wonder if there is a logic deserving the name in which a theorem such as the incompleteness theorem does not hold. On the other hand a theorem such as cut-elimination does not hold for many interesting logical systems. Cut-elimination expresses the intrinsic analycity of a logic, the fact that a proof of a theorem depends only of its constituents, a not always welcome feature. Anyway, it is interesting to find necessary and/or sufficient conditions for cut-elimination to hold. And also for any important theorem of logic.

Any paper dealing with the scope of validity and domain of application of logic theorems is welcome, in particular those dealing with the following theorems:

a) Löwenheim-Skolem (1915-1920);

b) completeness (Post 1921 - Gödel 1930);

c) incompleteness (Gödel 1931);

d) cut-elimination (Gentzen 1934);

e) undefinability (Tarski 1936);

f) undecidability (Church-Turing, 1936);

g) Lindenbaum's extension lemma (1937);

h) compactness (Malcev 1938);

i) incompleteness for modal logic (Dugundji 1940);

j) Beth's definability theorem (1953);

k) Craig's interpolation theorem (1957);

l) completeness for modal logic (Kripke 1959);

m) independence of CH (Cohen 1963).

The best papers will be selected for presentation in a special session during the event and a jury will decide during the event who is the winner.

The members of the jury are Yuri Gurevich, Daniele Mundici and Hiroakira Ono.

Previous Winners of this contest are:

- Carlos Caleiro and Ricardo Gonalves
  UNILOG'2005 = How to define identity between logical structures

- Till Mossakowski, Razvan Diaconescu and Andrzej Tarlecki
  UNILOG'2007= How to translate a logic into another one?

- Vladimir Vasyukov
  UNILOG'2010 = How to combine logics?

The prize will be offered by Barbara Hellriegel, representative of Birkhäuser.

Papers selected for presentation at this contest

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Transferring model-theoretic results about $\mathcal{L}_{\infty,\omega}$ to a Grothendieck topos

One of the most significant discoveries of categorical logic is that for a first order language L the operations of $\mathcal{L}_{\infty,\omega}(L)$ can be described categorically. This observation allows us to study models of sentences of $\mathcal{L}_{\infty,\omega}(L)$ in categories other than the category of sets and functions. One class of categories which are especially well suited to interpret sentences of $\mathcal{L}_{\infty,\omega}(L)$ are Grothendieck toposes, i.e. categories which are equivalent to the category of sheaves on some site.

However, while it makes sense to study the model theory of $\mathcal{L}_{\infty,\omega}(L)$ in a Grothendieck topos, this model theory can behave very differently than model theory in the category of sets and functions. (For example, in general it will be intuitionistic and need not satisfy the law of excluded middle). A natural question to ask is: "If we fix a Grothendieck topos, which results about the model theory of $\mathcal{L}_{\infty,\omega}(L)$ in the category of sets and functions have analogs for the model theory of $\mathcal{L}_{\infty,\omega}(L)$ in our fixed Grothendieck topos?"

In this talk we will discuss a method of encoding models of a sentence of $\mathcal{L}_{\infty,\omega}(L)$ in a (fixed) Grothendieck topos by models of a sentence of $\mathcal{L}_{\infty,\omega}(L')$ in the category of sets and functions, (where L' is a language related to L). We will then discuss how to use this encoding to prove analogs in a fixed Grothendieck topos of the downward Löwenheim-Skolem theorem, the completeness theorem as well as Barwise compactness.

One of the most significant difficulties we will have to overcome is the fact that sheaves are fundamentally second order objects. We will discuss how our encoding deals with this fact and the tradeoffs which must be made in the final theorems to accommodate the non-first order nature of sheaves and the corresponding models in a Grothendieck topos.

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Aristotle's particularisation: The Achilles' heel of Hilbertian and Brouwerian perspectives of classical logic
We argue that the defining beliefs of both the Hilbertian and the Brouwerian perspectives of classical logic have fatal vulnerabilities—due to their uncritical acceptance in the first case, and their uncritical denial in the second, of Aristotle's particularisation. This is the postulation that an existentially quantified formula—such as '$[(\exists x)P(x)]$'—of a first order language S can be assumed to always interpret as the proposition: 'There exists some s in the domain D of the interpretation such that the interprertation $P^*(s)$ of the formula $[P(s)]$ holds in D', without inviting inconsistency. We show that: (a) If the first order Peano Arithmetic PA is consistent but not ω-consistent, then Aristotle's particularisation is false; (b) If PA is consistent and ω-consistent, then Aristotle's particularisation is false. We conclude that if PA is consistent, then the postulation is false; and that the standard Hilbertian interpretation of classical First Order Logic is not sound. However, we cannot conclude from this the Brouwerian thesis that the Law of the Excluded Middle is false.

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Is Hintikka's Independence-Friendly Logic a revolutionary non-classical first-order language?
This paper investigates Hintikka's Game-Theoretical Semantics (GTS), starting from its application to classical First-Order Logic (FOL), in preparation to its further transposition into Hintikka's Independence-Friendly Logic (IFL).

Hintikka has vigorously claimed that IFL together with GTS constitute the really fundamental first-order language, and should as such replace FOL in general practice —even for the purpose of teaching basic logic in universities.

Among the various advantages of IFL-GTS over FOL he points out, the most striking ones are related to the invalidity of central theorems of classical logic, especially Gdel's completeness and Tarski's undefinability of truth. Moreover, IFL-GTS is three-valued and, according to Sandu and Hintikka (*contra* Hodges), is not compositional in a suitable sense.

Now we ask: Does IFL really deserve the status of revolutionary non-classical logic?

I shall argue that the literal transferring of the clauses of GTS into IFL is, first, unjustified —Hintikka simply takes GTS for granted as the natural semantics for IFL with no further ado, as if the fact that GTS is suitable for FOL were the only reason for so proceeding—; and second, it is unnatural —specifically, GTS's clause for negation does not agree with the intended meaning of the independence slash operator introduced by IFL.

Finally, I will suggest that a natural semantics for IFL would be well-behaved, would not violate the central classical theorems which Hintikka claims

it violates, and would therefore constitute an extension of FOL with more expressive power, like any other extension of FOL with non-classical (e.g. generalized) quantifiers.

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On the scope of the completeness theorem for first-order predicate logic
This paper is a contribution to the study of first-order predicate logics. We consider Henkin's proof of completeness for classical first-order logic and discuss the scope of its adaptation to the realm of non-classical first-order logics. Such adaptation is based on works by Mostowski, Rasiowa, Sikorski, Hájek and others, where one builds the first-order system over a propositional logic with an implication defining an order that allows to interpret quantifiers as suprema and infima. We show that this is a very useful approach that can be stretched much further than in the previous systematic developments.

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Institution theoretic scope of logic theorems
In this talk we analyse and clarify the method to establish and clarify the scope of logic theorems offered within the theory of institutions. The method presented pervades a lot of abstract model theoretic developments carried out within institution theory. Our approach distinguishes two different levels: that of the scope of results themselves and another, more subtle, of the scope of methods to prove the results. We claim that the abstraction process involved in developments of logic theorems within the theory of institutions almost always implies a clarification and a significant expansion of the scope of the most important logic concepts involved, often correcting some common conceptual misunderstandings of rather subtle nature.

The power of the proposed general method is illustrated with the examples of (Craig) interpolation and (Beth) definability, as they appear in the literature of institutional model theory. Both case studies illustrate a considerable extension of the original scopes of the two classical theorems. The general results outlined in this talk easily instantiate to many other logical contexts as well, and determine the scopes of so generalised logic theorems in a variety of new logical situations at hand.

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Suszko's reduction in a topos

I study here Suszko's reduction in toposes. The originality of this paper comes from (1) recognizing the import of applying Suszko's reduction in a topos-theoretical setting, because the beautiful picture of logic in a topos rests on the ideas that (a) Ω is (or at least can be seen as) a truth-values object, (b) that the internal logic of a topos is in general many-valued and (c) that the internal logic of a topos is in general (with a few provisos) intuitionistic; (2) the construction used to give categorial content to the reduction, and (3) the extrapolation of the debate about Suszko's thesis to the topos-theoretical framework, which gives us some insight about the scope of another theorem, namely that stating the intuitionistic character of the internal logic of a topos. In the first section I expound Suszko's reduction. In section 2 I show that the internal logic of a topos can be described as algebraically many-valued but logically two-valued. I introduce there the notions of a *Suszkian object* and of *Suszkian bivaluation*, as opposed to a truth values object and a subobject classifier. I prove their existence and uniqueness (up to isomorphism) in a given topos. Even though the main result is the internalization of Suszkian bivalence in a topos, it is relatively straightforward once the hard part, the characterization of a Suszkian object and a bivaluation, has been done. Finally, in sections 3 to 6 I suggest how logical many-valuedness could be recovered, but at the price of letting the internal logic of a topos become variable.

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Forcing and the Omitting Type Theorem, institutionally

A type is a set of formulas in finite number of variables. A type Δ with free variables $x_1 \dots x_n$ is principal for a theory T if there exists a finite set of formulas p with free variables $x_1 \dots x_n$ such that $T \cup p \models \Delta$. The classical Omitting Types Theorem (OTT) states that if T is a complete theory and $(\Delta_n : n \in \mathbf{N})$ is a sequence of non-principal types of T, then there is a model of T omitting all the types Δ_n. In the context of proliferation of many logical systems in the area of mathematical logic and computer science, we present a generalization of forcing in institution-independent model theory which is used to prove an abstract OTT. We instantiate this general result to many first-order logics, which are, roughly speaking, logics whose sentences can be constructed from atomic formulae by means of Boolean connectives and classical first-order quantifiers. These include first-order logic (**FOL**), logic of order-sorted algebra (**OSA**), preorder algebra (**POA**), partial algebras (**PA**), as well as their infinitary variants $\mathbf{FOL}_{\omega_1,\omega}$, $\mathbf{OSA}_{\omega_1,\omega}$, $\mathbf{POA}_{\omega_1,\omega}$, $\mathbf{PA}_{\omega_1,\omega}$. However, there are examples of more refined institutions which cannot be cast in this abstract framework and for which we believe that the standard methods of proving OTT cannot be replicated. In addition to the first technique for proving the OTT, we develop another one, in the spirit of institution-independent model theory, which consists of borrowing the Omitting Types Property (OTP) from a simpler institution across an institution comorphism. More concretely, we prove a generic theorem for OTP along an institution comorphisms $\mathcal{I} \to \mathcal{I}'$ such that if $\mathcal{I}'$ has the OTP and the institution comorphism is *conservative*, then $\mathcal{I}$ can be established to have the OTP. As a result we export the OTP from **FOL** to first-order partial algebras (**FOPA**) and higher-order logic with Henkin semantics (**HNK**), and from the institution of $\mathbf{FOL}_{\omega_1,\omega}$ to $\mathbf{FOPA}_{\omega_1,\omega}$ and $\mathbf{HNK}_{\omega_1,\omega}$.

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Saturated Logic: Its syntax

In this paper we investigate the expressive power of Gentzen sequent calculi and the scope of Cut-elimination theorem. For this purpose, we introduce (propositional, non-modal, with Associative and Exchange) Saturated Logic, prove Cut-elimination and study the relationship between connectives and Weakening or Contraction rules. Compared to the most common logics, Saturated Logic maximizes, within the 'standard' sequent calculi frame, the number of connectives extending the language and expanding the stock of inference rules: hence its name, as it fills, *saturates* the space of the inference forms.

Saturated Logic can be seen as a refinement of one or more known logics, plausibly able to promote the development of new logics; moreover, it can be seen not only as a 'logic-to-use', but above all as an attempt to develop a 'toolbox', useful to reformulate the presentation of logical calculi and make comparisons among them, as well as able to shed new light on given logics or on the organization of the logic space. See in this perspective, Saturated Logic might help to further understand the Cut-elimination theorem, as well as the relationship between Structural Rules and meta-language on the one side and Operational Rules and object-language on the other side.

The structure of the paper is the following. In section 2 crucial notation and terminology is introduced: note that a list of connectives rules is not explicitly given in the paper, but they can be unambiguously reconstructed from conventions on the shape of the symbols given in this section. In section 3 some

conceptual links are highlighted between Structural and Operational rules: this discussion helps to assess the meaning of Saturated Logic in the context of the Universal Logic research program, but the reader who is only interested in the technical results can skip it. In section 4 the system *AEM.LSat* is introduced and the derivability of Weakening and Contraction as second rules (by use of some Operational rules) and the interderivability between Conjunctions connectives are studied. In section 5 the Cut-elimination for *AEM.LSat* is specified as Cut-tradeoff theorem and it is sketched.

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Completeness, Translation and Logicality

This article propounds a stroll along the winding roads of completeness, translations and logicality, looking for the places where they converge. We set ourselves within the forest dark of logic, and manage to pinpoint the significance of logicality and completeness theorem. Starting from the unifying framework of translation in Many Sorted Logic, introduced to cope with the extant proliferation of logics, we see how different methodologies converge to the framed of classical logic. Understandably one wonders where logicality of classical logic could reside. We look for an answer to this contest by searching through the branches of logical foundation. The upshot of this search revolves around the equivalence of several characterizations of logicality obtained from Model Theory, Proof Theory and other foundational areas. Completeness theorem is shown to be relevant for weighing some conjectures regarding each proposal.

Specifically Section 2 looks at the completeness theorem of MSL. In section 3 we give special consideration to the strategy of translation into MSL, distinguishing different levels and introducing general results concerning completeness of the logical calculuses. Later on we prove the reduction of MSL to unsorted FOL, while keeping our preference for MSL within the translation paradigm. In section 4 we turn to the problem of the logicality of classical logic, considered as an universal logical framework. By exploring different foundational alternatives we find out that completeness theorem plays an important role in order to articulate these approaches. Interestingly one of these foundational approaches coming from algebraic logic, focus on the purported relationship between completeness and representation theorems. In this case completeness theorem is considered as a mapping between two domains and entails to reject the construal of this theorem as founding a type of validity on the basis of the other.

It must be said that by looking at completeness theorem this way we are attempting a more philosphical view on an old-standing matter. We are not merely considering its signifcance from the metalogical point of view, but reinterpreting the role it plays into the logical theory by articulating different perspectives on the unifying logical framework.

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Completeness Theorems for Program-oriented Algebra-based Logics of Partial Quasiary Predicates

We provide motivation for developing and studying program-oriented logics of partial predicates. Such logics are algebra-based logics constructed in a semantic-syntactic style on the methodological basis that is common with programming; they can be considered as generalizations of traditional logics on classes of partial predicates that do not have fixed arity. Such predicates, called quasiary predicates, are defined over partial variable assignments (over partial data) and are sensitive to unassigned variables. These features complicate investigation of logics of quasiary predicates comparing with classical logic and violate some laws of classical logic. In particular, partiality of predicates violates Modus Ponens; unrestricted arity of quasiary predicates violates the property that a sentence (a closed formula) can be interpreted as a predicate with a constant value; sensitivity to unassigned variables violates the law $(\forall x \Phi) \rightarrow \Phi$, etc. To cope with such difficulties more powerful instruments should be introduced. As such instruments we additionally use an infinite set of unessential variables and variable unassignment predicates.

We describe the hierarchy of different logics which are based on algebras of quasiary predicates with compositions as operations. We construct sequent calculi for a number of defined logics and prove their soundness and completeness.

To preserve properties of classical first-order logic we restrict the class of quasiary predicates. Namely, we introduce a class of equitone predicates and its different variations. These predicates preserve their values under data extension. Logics based on such predicate classes are the "closest" generalization of classical first-order logic that preserves its main properties. These logics are called neoclassical logics. For these logics sequent calculi are constructed, their soundness and completeness are proved. We also develop algorithms for reduction of the satisfiability and validity problems in neoclassical logics to the same problems in classical logic.

The proposed methods can be useful for construction and investigation of logics for program reasoning.

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Actualizing Dugundji's Theorem

Although in the origin of modal logic some many-valued matrices was proposed as semantic of this new field, in 1940 Dugundji proved that no system between **S1** and **S5** can be characterized by finite matrices.

Dugundji's result forced the develop of alternative semantics, in particular Kripke's relation semantic. The success of this semantic allowed the creation of a huge family of modal systems called normal modal systems. With few adaptations, this semantic can characterize almost the totality of the modal systems developed in the last five decades.

This semantic however has some limits. Two results of incompleteness (for the systems **KH** and **KVB**) showed that modal logic is not equivalent to Kripke semantic. Besides, the creation of non-classical modal logics puts the problem of characterization of finite matrices very far away from the original scope of Dugundji's result.

In this sense, we will show how to actualize Dugundji's result in order to precise the scope and the limits of many-valued matrices as semantic of modal systems.

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Refutability and Post Completeness
When we deal with the set L of propositional formulas valid in some structures, we are usually interested in laws and inference rules generating valid formulas. However, we can also consider non-laws and refutation rules generating non-valid formulas. As a result, we have a pair $\mathbf{S} = (\mathcal{T}, \mathcal{F})$ of complementary inference systems ($\mathcal{T}$ for L and $\mathcal{F}$ for $-L$), which can be called a proof/refutation system. Such systems enable provability and refutability on the propositional level. In a manner of speaking, we have two engines rather than one.

We say that a logic (that is, a set of formulas closed under a consequence relation) is Post complete iff it is consistent and has no consistent proper extension. Everybody knows that Classical Logic is Post complete. However, it seems that there is no interesting standard non-classical logic that is Post complete. What happens if the term logic is construed more generally? There are two possibilities.

(1) A *logic* is a consequence relation $\vdash$ between finite sets of formulas and formulas. We now say that a logic $\vdash$ is Post complete iff $\vdash$ is consistent and has no consistent proper extension. However, it turns out that $\vdash$ is Post complete iff the set $\{A : \vdash A\}$ of its theorems is Post complete, so that in this case nothing happens.

(2) A *logic* is a consequence relation $\vdash\dashv$ between finite sets of formulas (or a multiple-conclusion consequence relation). Again, we say that $\vdash\dashv$ is Post complete iff $\vdash\dashv$ is consistent and has no consistent proper extension. Interestingly, the situation is now dramatically different. For example, consider any non-classical logic L that is not Post complete. Let

$\vdash\dashv^L = \{X/Y : \text{If } X \subseteq L \text{ then } Y \cap L \neq \emptyset\}$

(so $\vdash\dashv^L$ is the set of multiple-conclusion inferences preserving L). Then $\vdash\dashv^L$ is Post complete.

In this paper we give a necessary and sufficient condition (in terms of proof/refutation systems) for a multiple-conclusion consequence relation to be Post complete.

4.4 Call for Papers for Contributing Speakers

The deadline to submit a contribution (1 page abstract) to the congress is November 1st, 2012, (Notification: December 1st 2012). The abstract should be sent to: *rio2013@uni-log.org*. All talks dealing with general aspects of logic are welcome, in particular those falling into the categories below:

GENERAL TOOLS AND TECNHIQUES

- consequence operator
- diagrams
- multiple-conclusion logic
- labelled deductive systems
- Kripke structures
- logical matrices
- tableaux and trees
- universal algebra and categories
- abstract model theory
- combination of logics
- lambda calculus
- games

STUDY OF CLASSES OF LOGICS

- modal logics
- substructural logics
- linear logics
- relevant logics
- fuzzy logics
- non-monotonic logics
- paraconsistent logics
- intensional logics

- temporal logics
- many-valued logics
- high order logics
- free logics

SCOPE OF VALIDITY/DOMAIN OF APPLICATIONS OF FUNDAMENTAL THEOREMS

- completeness
- compactness
- cut-elimination
- deduction
- interpolation
- definability
- incompleteness
- decidability
- Lindenbaum lemma
- algebrization
- Dugundji's theorem

PHILOSOPHY AND HISTORY

- axioms and rules
- truth and fallacies
- identity
- lingua universalis vs calculus ratiocinator
- pluralism
- origin of logic
- reasoning and computing
- discovery and creativity

- nature of metalogic
- deduction and induction
- definition
- paradoxes

4.5 Sessions

4.5.1 Universal

The invited keynote speakers of this session are Gila Sher (page 82) and Sun-Joo Shin (page 82).

Contributed Talks

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HOL based Universal Reasoning
At UNILOG'2010 I have proposed classical higher-order logic HOL (Church's type theory [1,9) as a uniform framework for combining logics [2]. The main claim has been that many non-classical logics are just natural (embedded) fragments of classical HOL. The approach also supports combinations (e.g. fusions) of embedded logics; in particular, bridge rules can be postulated simply as axioms. In the UNILOG'2010 presentation the focus has been on combinations of quantified multimodal logics [7] (quantified wrt first-order and propositional variables) and I have claimed that the approach supports automation of both reasoning *within* and *about* embedded logics and their combinations with off-the-shelf higher-order automated theorem provers (cf. the experiments in [3]). Significant further progress has been made since the UNILOG'2010 event. For example, semantic embeddings for propositional and quantified conditional logics have been added [4,5]. This is particularly interesting since selection function semantics for conditional logics, which is what we have studied, can be seen as a higher-order extension of Kripke semantics for modal logics and cannot be naturally embedded into first-order logic. Moreover, important and timely application directions of the approach have been identified. Amongst these is the mechanization and automation of expressive ontologies such as SUMO (or Cyc), whose representation languages support combinations of first-order and even higher-order constructs with various modal operators [8]. Practical effectiveness of the HOL based universal reasoning approach can be evaluated by comparing it with implemented competitor approaches. To date, however, there are no such (implemented) systems available. Only for first-order monomodal logics, which are not in our primary interest since effective specialist reasoners for these logics can still be comparably easily achieved, some automated provers have meanwhile been implemented. A recent evaluation [6] confirms that the HOL based universal reasoning approach, which performed second best in this study, is competitive. At UNILOG'2013 I will present and discuss the project progress as sketched above. I will also demonstrate our HOL based universal reasoner, which calls various off-the-shelf HOL theorem provers and model finders remotely. Moreover, I will outline future directions of this research.

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The universalist conception and the justification of logic

According to a historiographical tradition started by Jean van Heijenoort and continued by authors such as Warren Goldfarb, Thomas Ricketts, Peter Hylton, among others, there is a universalist conception of logic within the philosophy of logic. In his classic "Logic as calculus and logic as language" van Heijenoort characterizes the universalist approach formulating a distinction between logic as language and logic as calculus. Briefly, while the algebraic tradition (Boole, Schröder, among others) conceives logic as a calculus, the universalist (paradigmatically Frege and Russell) thinks of it as a language (and not merely as a calculus). Even though there is no agreement on what would be for logic to be universal, several authors coincide on a minimal characterization, which has emerged from the notion of universal language.

In this paper, I would like to propose a brief characterization of the idea of logic as a universal language, pointing out some philosophical consequences that have been drawn from such conception. In particular, the association of the universalist point of view with an impossibility to adopt an external perspective about logic (the logocentric predicament), and the consequently blocking of any kind of metasystematic approach. Meaning by that, either the exclusion of any metalogical approach (as the kind illustrated by the familiar results of correctness, completeness, consistency, etc.) and/or the rejection of a theoretical discourse about logic, one with cognitive pretensions. The latter leads to the dissolution of a great number of major problems addressed by the philosophy of logic. The justification of logic is the one I'm interested in. I propose five arguments against the claim that the problem of the justification of logic is actually a pseudo problem.

Firstly, I consider and re-elaborate an argument proposed by Sullivan (2005), that points out that the interpretative tradition pioneered by van Heijenoort omits an important distinction, that between logic *per se* and a system of logic. Transferring the logocentric predicament from the first to the second. Secondly, I suggest that some controversial assumptions lie under the path starting in the logocentric predicament and ending at the impossibility of a theoretical discourse about logic and a justification of it. At least some of the following is being supposed: 1. that any candidate for the justification of logic should be inferential, 2. that all forms of circularity in a justification are vicious, and 3. that all metaperspective should be external.

Those considerations lead me to the suggestion that the dissolution of the problem only emerges when the universalist conception is combined with the logicist project (also adopted by the precursors of the universalist tradition). This project imposes a very strong notion of justification, as foundation. And only the problem of the justification of logic formulated in such stringent terms, if any, turn out to be a pseudo problem. In connection with this, the fourth argument is intended to show that, if the logical principles are at the bottom of the logicist project of founding mathematics, some questions regarding those principles stay in need of an answer. Particularly, what make them suitable to have such a function in the foundationalist enterprise. Finally, it's worth noting that even in those who paradigmatically illustrate the universalist conception we can find comments which seem to exemplify the apparently excluded possibility, those of a metaperspectival type. Of course, the central issue is how to interpret them. According to Dummett (1991) they are rhetoric, Goldbarb (1982) refers to them as "heuristic", Ricketts (1985), on the other hand, suggests that they work as "elucidations". What is common to all of them is the conviction that those comments only have a practical or pragmatic value, but not a theoretical one. I discuss some evidence that suggests that this last claim is, at least, suspicious.

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Predicate logic is not universal

We can call some system of logic *universal* if it is extremely wide so we can express and explain everything within it's scope. Well, modern *predicate logic* (PL) is not universal because it has the next limits: (a) PL uses the set-theoretic, non-logical notion of function; (b) equality is a logical relation but it is treated in PL as a concrete predicate, i. e., as a non-logical entity; so it generates the new type of the atomic formulas and requires separate axioms; (c) PL is grounded on the only one *standard semantics*, so one cannot built the predicate formal languages and calculi for an arbitrary semantics; (d) PL contains no general theory of descriptions; (e) PL is essentially one-sorted, so we can't formulate the predicate formal languages and calculi with irreducible sorts of variables; (f) there is no suitable explanation for the logical paradoxes within PL.

At the same time there is the truly universal system of logic; the author call it *function logic* (FL). (a') FL is based on the pure logical notion of function as a method to represent or to give it's values; this notion corresponds to set-theoretic notion of partial multimap; logical relation between value of a function in this sense and the very function (with arguments, if function has them) is no longer equality (=) but is *representation* ($\approx$). Predicates in FL are the representations by functions, so every predicate $F(x_0, x_1, \ldots, x_n)$ obtains the form $x_0 \approx f(x_1, \ldots, x_n)$ in FL. Every individual a can be represented as i-th value f^i or $f^i(a_1, \ldots, a_n)$ of the function $f^{(0)}$ or $f^{(n)}$ (with it's arguments $a_1, \ldots, a_n$, if it has them) [1]. (b') Equality is definable in every formal language within FL. (c') Every formal language within FL is build on the basis of the explicitly formulated semantics; moreover, one can build the formal languages and calculi for every semantics within FL. (d') General theory of descriptions can be easily build within FL; and it requires a non-standard semantics. (e') FL is principally many-sorted, so one can build within FL the one-sorted formal languages as well as languages with irreducible sorts. (f') Russel's and Cantor's paradoxes does not appear in FL and can be explained easily. The whole modern system of predicate logic can be replaced by the system of function logic.

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How universal can multi-dimensional logics be?

Multi-dimensional propositional logics are formal systems which we get if we extend the language of classical logic by ordered n-tuples of classical formulas and suitable operators having multi-dimensional expressions as their arguments. There are several kinds of motivation to deal with multi-dimensional logics in general and with multi-dimensional propositional logics in particular. One of them is connected with the program to reconstruct systems of non-classical logics within such a syntactically extended classical framework. Another kind of motivation is the possibility to show new basic aspects of formal systems which are of some important philosophical interest. Furthermore, it is possible to use the expressive power of such systems to translate expressions of natural languages (and also, e.g., structured elements of music) into more complex formal ones. The familiar one-dimensional classical language plays then the role of a theoretical language in showing the internal structures and the computing behavior of syntactically complex expressions.

(1) I will sketch the general form of multi-dimensional propositional systems with a fixed dimension n. It is possible to define several notions of validity/inconsistency for ordered n-tuples of classical formulas using only the classical vocabulary:

Let $< A_1, \ldots, A_n >$ be an ordered n-tuple of the classical formulas $A_1, \ldots, A_n$. Let $\models A$ indicating the classical validity of A. Then we can define a lot of n-dimensional notions of validity using only object language expressions:

$\|\!\models_i \; < A_1, \ldots, A_n >$ iff $\models \Phi_i^n A_1 \ldots A_n$, where Φ_i^n is an arbitrary n-ary classical connector (truth function) with $1 \leq i \leq 2^{(2^n)}$. Using quantifiers within such definitions gives us versions in the metalanguage; e.g.:

$\|\!\models_\exists \; < A_1, \ldots, A_n >$ iff $\exists A_i \models A_i$ $(1 \leq i \leq n)$. We can also define a variety of inconsistencies.

(2) With respect to more *formal applications* of our logics it will be shown how finite many-valued logics can be equivalently reconstructed. The atoms of, e.g., 2^n-valued logics can be represented by appropriate elementary expressions of the form $< A_1, \ldots, A_n >$. The simplest version – but not the only one – is

$< p_1, \ldots, p_n >$. The many-valued connectors will be understood as operators characterized by rules which allow to transform any complex formula to a formula of the form $< A_1, \ldots, A_n >$ which is $\|\!\models_i$-valid. The choice of i depends on the set of designated values.

(3) With respect to more *philosophical applications* it will be demonstrated that the distinctions atomic–molecular, atomic–complex depends on the underlying logic as well as the choice of the logical complexity of basic expressions. From our point of view the basic expressions of any propositional logic (including the classical one!) are not essentially atomic but can be explicated as structures already on a propositional level. But in well-defined sublanguages with n-dimensional formulas X_i each occurrence of well-chosen expressions of the form $< A_1, \ldots, A_n >$ can be replaced by any formula X_j of this sublanguage: $\|\!\models_i X_i \;\Rightarrow\; \|\!\models_i X_i[< A_1, \ldots, A_n > / X_j]$.

(4) With respect to *empirical applications* and leaving the strong analogy to finite many-valued logics (and other non-classical logics as well) we can consider other well-defined parts of the whole language. This allows us to compare several non-classical systems *within the same object language*. And it yields applications of the multi-dimensional framework *without the restrictions* other formal approaches usually have.

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Internal Logic of the Universe of Universal Logic

Universal Logic would be treated as a general theory of logical systems considered as a specific kind of mathematical structures the same manner Universal Algebra treats algebraic systems (cf.[1,p.6]). A category-theoretical approach where logical systems are combined in a category of the special sort provides us with some basis for inquiring the universe of Universal Logic. In the framework of such an approach we succeed in introducing categorical constructions which along with coproducts underlying fibring of logics describe the inner structure of the category of logical systems. It turned out that such universe posses the structure of a topos as well as the structure of a paraconsistent complement topos that was shown in [7].

The last circumstance is evidence of that for the structure of the universe of Universal Logic is characteristic a presence of both the intuitionistic and paraconsistent (Brouwerian) structure. The “internal logic” of a topos, as is well known, be a intuitionistic one while concerning the internal logic of the conplementary topos then to all appearance one would expect the presence of the anti-intuitionistic (Brouwerian) logic. Such metalogical systems would serve as a convenient tool for obtaining statements about logical systems and their translations. In itself such system of the Universe of Universal Logic is, in essence, a system of universal metalogic - a logic of logical systems and their translations.

In the paper an attempt of the exact description of this internal loguc of the Universe of Universal Logic is made. It is shown that such logic is based on the sequential formulation of H-B-logic.

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4.5.2 Combination

The invited keynote speakers of this session are Carlos Caleiro and Sérgio Marcelino (page 62).

Contributed Talks

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Homotopical Fibring

It has been as pointed out by many authors that (constrained) fibring of logics, i.e. the formation of colimits in categories of logics, can so far only be carried out in a very restricted setting: The maps along which one fibers have to be translations mapping the primitive connectives, which generate the domain language, to primitive connectives of the target language. It would be desirable to instead allow primitive connectives to be mapped to derived connectives, as it happens e.g. in the $\neg\neg$-translations from intuitionistic to classical logic. Applications abound, e.g. one could import modalities from a classical modal logic into intuitionistic modal logic. Unfortunately diagrams consisting of these more general translations do almost never have colimits.

In this talk we present a solution: Any category of logics known to the speaker comes with a natural definition of when a translation is an equivalence of logics. It is thus open to the methods of abstract homotopy theory, a toolkit that allows to transfer the homotopy theory of topological spaces to other situations. In particular the notion of homotopy colimit is defined, and this is what we call the *homotopical fibring*, or *hofibring*, of logics, and what overcomes the above mentioned problems of fibring.

As an example we present the concrete meaning of this in the simple setting of propositional Hilbert Calculi. We consider translations which fix the variables but are allowed to map generating connectives to derived connectives.

One can call a translation $f : (L, \vdash) \to (L', \vdash')$ a *weak equivalence*, if $\Gamma \vdash \varphi \Leftrightarrow f(\Gamma) \vdash' f(\varphi)$ (i.e. it is a "conservative translation") and if for every $\psi \in L'$

there exists a $\varphi \in L$ such that $\psi \dashv\vdash f(\varphi)$ (it has "dense image"). One can further call *cofibration* a morphism which maps the *generating connectives* of L injectively to *generating connectives* of L'.

This category of logics and translations, with the two given special classes of translations, now has the convenient structure of a so–called *ABC cofibration category*. The proof of this proceeds in close analogy to the original topological setting, e.g. by constructing mapping cylinders. The theory of ABC cofibration categories then yields a concrete and simple construction recipe for the homotopy colimit of a given diagram: It is the actual colimit of a different diagram which allows to

• express hofibring through fibring
• see that fibring is a special case of hofibring (which yields a new universal property of fibring)
• see that all homotopy colimits exist and
• transfer preservation results known from fibring to hofibring, for metaproperties which are invariant under equivalence

Among the preservation results obtained by the technique of the last point, those on the existence of implicit connectives are straightforward. Preservation of completeness and position in the Leibniz hierarchy require a homotopical view on semantics as well, which we will provide.

We will say how the above results extend to first order logics by admitting many-sorted signatures and to the fibring of institutions via the c-parchments of Caleiro/Mateus/Ramos/Sernadas. There are also homotopical versions of other variants of fibring, like modulated fibring, metafibring and fibring of non-truth functional logics.

To conclude, we point out a variety of approaches to abstract logic suggested by the homotopy theoretical view point.

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Classical and Paraclassical logics: A case of combination and analysis of different logics

The aim of this communication is to present an abstract approach to compare logical properties of different logics. In particular, the focus will be on the comparison between classical and paraclassical logic.

The presentation will be divided as follow. Firstly, it will be given an abstract definition of logic. Formally, a *logic* L is a structure $L = \langle F_L, \vdash_L \rangle$ such as:

(i) F_L is a non-empty set, whose elements are called *formulas* of L;

(ii) $\vdash_L$ is a relation in $\wp(F_L) \times \wp(F_L)$ called *consequence relation* of L

Secondly, it will be established, from the definition above, the classical and paraclassical logical system, the second being a particular paraconsistent logic. Formally, a *paraclassical propositional logic* P is a structure $P = \langle F_P, \vdash_P \rangle$ such as:

(i) $\langle F_P, \vdash_P \rangle$ follows the above definition of logic;

(ii) $F_P = F_{\mathbb{C}}$;

(iii) $T \vdash_P \alpha \Leftrightarrow$ there is $U \subseteq T$, $\mathbb{C}$-Consistent, such as $U \vdash_{\mathbb{C}} \alpha$[7].

This formal system was chosen for its simplicity, elegance and usefulness. Another interesting property of this system is that it can be defined without negation. This is important, for it shows that paraconsistency does not have a necessary relation to a specific kind of negation, nor does its negation need to be different from the classical one.

Bearing this conceptual framework in mind, we can analyze which properties - such as monotonicity, idempotency, inclusion, transitivity, among others - are invariant to both systems, and which are particular to one or another.

Hence, this presentation has two major contributions to the study of universal logic. It contributes to the discussion about the universality - or domain of validity - of logical proprieties. It gives a conceptual background to determine, for instance, if the deduction theorem holds to all logical systems. In addition to that, the techniques used to define logic and paralogic in a pure abstract way can be used as a method of paraconsistentization of logics, that is, we can define, for any given logic, its paralogical version.

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Safe Fibring or How to Revise Logic Systems

Fibring is a technique for combining logics. In short words, the fibring of two logics is the least conservative extention of both. There are situations, though, where fibring leads to a trivial logic. We propose the usage of belief revision techniques to define and characterize what we call a *safe fibring*. The safe fibring of two logics is a logic that agregates most of both without being trivial.

Belief revision is the field that studies the dynamics of rational belief change. It defines three main operations over theories: contraction, expansion and revision. The first consists in removing a sentence from a theory, the second consists in adding a sentence and the last in adding consistetly a sentence in a theory.

In previous works we presented an operation over logics analogous to contraction. It was presented a series of postulates for *contracting logics* that characterizes the bigest sub-logics $Ł - \xi$ of a given logic Ł that fails to derive certain rule ξ.

Safe fibring is the analogous of a revision operation in the context of combining logics. The operation $Ł_1 * Ł_2$ returns a logic that retains most of both $Ł_1$ and $Ł_2$ without trivializing it.

[7] $\mathbb{C}$ denotes classical logic. These definitions will be full explained throughout the presentation.

Safe fibring is presented via a series of postulates such as:

closure: $Ł_1 * Ł_2$ is a logic.

non-trivialization: $Ł_1 * Ł_2 \neq Ł_\perp$.

minimality: If $\xi \in Ł_1 \cup Ł_2$ and $\xi \notin Ł_1 * Ł_2$ then there is $Ł'$ such that $Ł_1 * Ł_2 \subseteq Ł'$, $Ł' \neq Ł_\perp$ and $Ł' + \{\xi\} = Ł_\perp$.

Our main result, the *representation theorem*, shows a construction for safe fibring that is fully characterized by this set of postulates.

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A sort-binding method of combining logics

The key levels in the definition of a logic are syntax, axiomatics, and semantics. When two logics are combined to produce a new one, ideally, the combination mechanism involves all the three levels. It is assumed that the definition of this mechanism specifies how one obtains the syntax of the new logic from the syntaxes of the original logics, what the relation between their axiomatics is, and what the semantics of the obtained logic is. From the practical point of view, the combination operation should be constructive allowing to effectively obtain the syntax and semantics of the new logic wrt given parameters of combining process. Besides, it should be adequate in the sense of relationship of the semantics and the syntactic consequence relation of the resulting logic.

An important question is a choice of combination parameters. Intuitively, combining means that logics in some way merge sharing a number of constructs from some levels in their definitions. In this work, the combination parameter is an extent to which the elements from the the syntax level are binded. Binding syntactic elements then directly influences the axiomatics and semantics of the obtained logic. This partially corresponds to the notion of constrained fibring from [1] closely related to merging signature symbols of logics. Note, however, that syntactic elements are not only signature symbols, but also syntactic sorts of logic, in other words, the sorts of expressions available in the underlying language. For instance, the classical propositional logic has only one sort, "formula", first–order logic and propositional dynamic logic have two sorts ("term", "formula" and "program", "formula", respectively), and some description logics have four (e.g. "concept", "role", "concept inclusion", "role inclusion") and even more. Sorts are implicitly present in axiomatics in inference rules, where they are directly related to the substitution operator. Each sort has its own interpretation in the scope of semantics. As demonstrated in Example 5 from [2] for da Costa's logic, introducing auxiliary sorts into a language can sometimes provide a basis for new results on the considered logic. We may note that syntax of new logics obtained from previously known ones is often defined with the help of *sort binding*. For example, the sort "formula" in the first–order modal logic is a join of sorts "formula of modal logic" and "first–order formula". The same can be said about temporalization of logics. From the syntactic point of view, in description logics the development of more expressive formalisms is often based on binding of sorts: the sorts "concept" and "role" of a base logic are extended with sorts of a supplementary language giving a more expressive formalism. Thus, in the context of combining logics, one may consider not only equality between sorts, but also a partial order on the union of the sets of sorts available in the input logics.

In the proposed approach, partial orders on sets of sorts are the first-class citizens in combining logics and are combination parameters defined at the level of syntax. Choosing a partial order on the union of sets of sorts of two given logics constructively determines axiomatics of their combination (the set of axioms and inference rules) on the basis of the original axiomatics. Essentially, this means taking full control over the substitution operator when combining two logics. We note that hints to the idea of sort binding are implicitly present in modulated fibring, where the notion of safe substitution is considered (Definition 8.3.1 in [1]), in graph-theoretic fibring (the notion of fibring of signatures in [3]), and in parameterisation of logics [4]. However, it is worth stressing that in our approach the have the key possibility of arbitrary ordering of sorts and besides, the same logical connective can be applicable to several different tuples of sorts. Though the latter option is not that important, sometimes it appears to be desirable. For instance, in description logics, the same symbol is often used for connecting expressions of sort "concept" and for connecting expressions of sort "role".

We propose an approach to combining logics which is aimed at implementation from its very basis. A logic is defined by means of a context–free grammar at the syntax level and by a set of axioms and inference rules respecting the grammar at the axiomatic level. The distinctive feature of our approach is that the notions of syntactic sort and order on sorts are defined completely in terms of formal grammar. Having two logics as input, the choice of a partial order on the union of their sets of sorts (and signatures, respectively) completely determines axiomatics of their combination. The consequence relation of the obtained logic is then completely defined and is recursively enumerable (provided the input logics are given effectively). For logics defined by finite context–free grammars and finite axiomatic systems, the sort–binding method of combining logics is implemented in the language of **mdl** automated prover [5]. The mdl prover provides features of automated proof verification and proof search in any logic specified effectively at the level of syntax and axiomatics. The source code of mdl is available at `http://russellmath.org`.

As algebraic semantics is a rather universal tool in combining logics and the notion of sort is central in our approach, we consider an algebraic semantics of many–sorted logics in the sense close to [2]. We show how the choice of partial orders on the sets of sorts and signatures of two logics influences semantics of their combination and formulate results on connection with axiomatics. Therefore, in our approach we assume that the input many–sorted logics are algebraizable wrt certain classes of many–sorted algebras. The subject of our study are the restrictions on the combination parameters (the mentioned partial orders) and the relationships between the class of algebras in the obtained semantics and the semantics of input logics which guarantee obtaining a many–sorted algebraizable combination. We provide numerous examples of combinations of well–known logics using the proposed method, as well as an example related to the collapsing problem and the problem of combining consistent logics into an inconsistent one.

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Classical and intuitionistic propositional logic
Gabbay has pointed out a difficulty with the fibring methodology for combining logics that became known as the collapsing problem. The argument given suggested that there would be no way of combining classical and intuitionistic propositional logics, *CPL* and *IPL* respectively, in a way that would not collapse the intuitionistic connectives into classical ones. Following this spirit, Andreas Herzig and Luis Fariñas del Cerro have proposed a combined logic $C + J$ that starts from the expected combined semantic setting, that could be axiomatized, not by adding the axiomatizations of *CPL* and *IPL* together with some interaction rules, but rather by modifying these axioms along with their scope of applicability.

We propose a new logic, inspired by the problems above, encompassing both classical and intuitionistic propositional logics. A preliminary step towards this logic was already taken by considering a combination of the implicative fragments of *CPL* and *IPL*, that was shown to be a conservative extension of both logics. The design of this logic is based on the key idea of keeping at the core of the deductive system the axioms and rules of both logics. As it would be expected, since we are aiming at a complete axiomatization, some extra axioms are needed to express the existing interaction between the two logics. For instance, intuitionistic implication is weaker than classical implication and this fact needs to be expressed in the logic. Still, these interaction axioms are carefully chosen to guarantee that the two logics do not collapse. The semantics for the logic was inspired by the semantic models obtained by cryptofibring. In this case, we adopt as models for the logic the usual Kripke model for intuitionistic logic. The semantic for the classical connectives over these models needs to be defined with care as to ensure that the usual semantics is preserved. The resulting logic is then proved to sound and complete and, furthermore, it is proved to extended conservatively both classical and intuitionistic propositional logics.

The logic is also shown to be decidable. To this end, we introduce a tableau system based on labeled formulas. We consider two kinds of labels. One of these labels concerns the truth value of the formula, that can be either true or false. The other one denotes the world (of the intended Kripke model) in which the formula is being evaluated. Each labeled formula will have one of each of these two labels. Due to the nature of the intuitionistic implication, the set of labels for the worlds cannot be arbitrary. In particular, we need it to be finite in order keep our tableaux finite. Hence, we consider a (finite) set of world labels, that is build up based on the formula that is being considered, where each label has a semantic flavor attached to it.

Another competing approach was recently proposed by Liang and Miller for intuitionistic and classical first order logics. Though different in spirit, it seems worth understanding its connection to our approach.

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Multi-Dimensional Products of Graphs and Hybrid Logics[8]
In this work, we address some issues related to products of graphs and products of modal logics. Our main contribution is the presentation of a necessary and sufficient condition for a countable and connected graph to be a product, using a property called intransitivity. We then proceed to describe this property in a logical language. First, we show that intransitivity is not modally definable and also that no necessary and sufficient condition for a graph to be a product can be modally definable. Then, we exhibit a formula in a hybrid language that describes intransitivity. With this, we get a logical characterization of products of graphs of arbitrary dimensions. We then use this characterization to obtain two other interesting results. First, we determine that it is possible to test in polynomial time, using a model-checking algorithm, whether a finite connected graph is a product. This test has cubic complexity in the size of the graph and quadratic complexity in its number of dimensions. Finally, we use this characterization of countable connected products to provide sound and complete axiomatic systems for a large class of products of modal logics. This class contains the logics defined by product frames obtained from Kripke frames that satisfy connectivity, transitivity and symmetry plus any additional property that can be defined by a pure hybrid formula. Most sound and complete axiomatic systems presented in the literature are for products of a pair of modal logics, while we are able, using hybrid logics, to provide sound and complete axiomatizations for many products of arbitrary dimensions.

4.5.3 Modal

The invited keynote speaker of this session is Tamar Lando (page 77).

Contributed Talks

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Individuals, Reference, and Possible Worlds : Sharpening Hintikka's Insights on Epistemic Logic and Propositional Attitudes
The aim of this talk is twofold, and grounded in two observations regarding Hintikka's epistemic logic as it is exposed in his landmark *Knowledge and Belief* (1962).

[8]The authors received financial support from the research agencies CNPq, FAPERJ and CAPES.

The first one is due to Chisholm, who in reviewing Hintikka's book in his "The logic of knowing" (1963) complains that "[f]or reasons that are not entirely clear, epistemic logic has been confined to a study of the analogies that hold between knowledge and necessity" – indeed as is well known, epistemic and doxastic notions are treated semantically by Hintikka in terms of quantification over possible (accessible) worlds, analogously to the approach of the notion of necessity in intensional logics.

The second one is due to Hintikka himself, in the opening lines of his 1969 "Semantics for Propositional Attitudes" (but the idea remains a central one throughout the paper): "In the philosophy of logic a distinction is often made between the theory of reference and the theory of meaning [...] this distinction, though not without substance, is profoundly misleading".

I shall argue that Hintikka's referential inclinations (centralized on his notion of "individuation") are compromised by his use of quantification over possible worlds; that we can treat epistemic notions without quantification; that this enables us to meet at the same time Chisholm's concerns and Hintikka's insights in a simpler (though still of a modal nature) logical framework semantically equivalent to Hintikka's. Moreover, this allows a straightforward solution to Frege's puzzle by simply making explicit the standard Principle of Substitutivity (rather than repairing it), without resorting to intensional entities as it has become common practice since the introduction by Frege of the notion of *Sinn* (especially through the works of Carnap and Kanger).

The essential idea consists in analyzing sentences like "a believes that p in world w" as "p belongs to what a believes in w", instead of Hintikka's "p belongs to every w' compatible with what a believes in w". The only thing we must add to the picture is a new class of points of evaluation related to what an agent believes in a given world.

Put it another way, the sentence p is to be evaluated in a possible world constituted by the beliefs of the relevant agent (in the relevant world). Or still more briefly, agents induce particular points of evaluation in the model, instead of compatibility relations.

A formal sketch of the reduction of Hintikka's system so that quantification is avoided and new points of evaluation enter the picture will be provided, as well as a proof of the equivalence with Hintikka's framework. As a consequence, the Kripke-style models for epistemic logic will no longer contain compatibility relations. On the other hand, they will single out a special subset of the set of worlds, namely the set of possible worlds related to the belief states of agents (with respect to the various possible worlds).

Since we may now directly speak of the evaluation of a sentence in a single world (and no longer in a totality of compatible worlds) built up from the beliefs of an agent (respective to the relevant world), it seems we have met Hintikka's referentialist (though in an overall underlying modal setting) point of view on belief sentences (and attitudes in general), through a simpler system semantically equivalent to his own.

Finally, an analysis of propositional attitudes will be outlined according to the ideas presented in this talk.

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Epistemic Entrenchment Contraction and the Minimal Change Criterion in the AGM Model of Belief Revision

Tennant in [1] and [2] criticizes the approach that the belief revision theory AGM adopts concerning the contraction operation. The AGM model [3] is based on the minimal change criterion, that is, when one changes a belief state, he must do so in a minimal way. In AGM, to block the derivation of $a\&b$ from the belief set K, at least one of the pair $\{a, b\}$ must be removed and, when there is no reason to choose one instead of the other, both must be removed. Tennant named this approach the "*en bloc* approach" and argued that it does not respect the minimal change criterion, since it removes from a belief set more than is necessary. He proposed another one, named "one-sentence-at-a-time", arguing as follows. To prevent that $a\&b$ be derivable from K (that is, to contract $a\&b$ from K), supposing that they are logically independent, one will not adopt the option of removing both a and b and even in the case in which there are no reasons to remove one instead of the other. We will argue that the "single-sentence-at-a-time" approach of Tennant is not adequate, since its adoption allows the inference of sentences one normally would not accept.

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Preference and Actions: A modal approach to Von Mises Time Preference

> "Things and actions are what they are, and their consequences will be what they will be; why then should we seek to be deceived?"
> Bishop Butler

A possible answer to Bishop Butler question could be: because we are humans, and we pervade our environment with intentions. That's the way human beings are, or the way evolution furnished us. However they are in fact things and consequences of actions, we have preference for certain things and actions to obtain and perform.

If we are to accept preference on states, describing how the things are, and judgments involving them we might as well ask ourselves if there are inferences that involve this last ones. In logic Von Wright's work is commonly taken as a first hand reference of the study of preference judgments and their logical relations. Von Wright (1963) develops a preference axiom system and an implicit semantic that has some flavor of a kripke semantic. Although it is clearly outline that the change of preference will be left out of the inquiry it is acknowledge that the concept has nevertheless an intrinsic dynamic dimension. Moreover, within the theory of computer science Dynamic Logic is used to prove correctness properties of computer programs. This is no more than saying that a computer program is in fact nothing but a sequence of actions of a certain kind.

As preferences are exerted over states, we can say that actions act over these same states. It seems natural to think of many examples involving some interaction between preferences and actions, that is, preferences over actions or actions over preferences. Asking about what kinds of restrictions govern these interactions it amounts to study the ways in which actions and preferences can be mixed up. From the point of view of logic this kind of inquiries are developed on what is known as combining logics and it extends to mixing different modal operators (each one governed by its own logic) and merging them into a new logic. In this article we propose a combination of logics of preference and actions to capture a qualitative concept of preference over actions taken from (Von Mises 1963). This concept of preference involves actions that give us a measure of time in order to rich some kind of satisfaction and according to which – in Von Mises words – "Satisfaction of a want in the nearer future is, other things being equal, preferred to that in the farther distant future." ([1], p.483.)

The article will be divided into five sections. The first one will be devoted to a short presentation of dynamic logic and the logic of preference. Once this is done a propositional modal language will be defined for combining preferences and actions. In the third section we will consider a semantic for this language. In fourth place Von Mises's time preference concept will be introduced and discussed. Finally what has been worked from section one to three will be applied to formalize the time preference concept introduced in section four.

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Semantics and Tableaus for Modal Syllogistic de re

Modal syllogistic deals with modal categorial propositions which are interpreted as *de re*. We have twelve kinds of propositions: *a*) four classical categorial propositions (without modalities), *b*) eight modal propositions such as:

- $Sa^{\Box}P$ — Every S is necessarily P.
- $Si^{\Diamond}P$ — Some S is possibly P.

As we know, some logicians (Aristotle, Jan Łukasiewicz, and others) formulated foundations for different systems of modal categorial syllogistic. On the other hand we start our research from semantical aspects of modal categorial propositions.

In our approach we do not use possible worlds' semantics. An initial model of modal categorial propositions language is any quadruple $\mathfrak{M} = \langle D, f^{\Box}, f, f^{\Diamond}\rangle$, where: D is a set and $f^{\Box}$, f, $f^{\Diamond}$ are functions defined on the set of terms into 2^D such that $f^{\Box}(X) \subseteq f(X) \subseteq f^{\Diamond}(X)$, for any term X. The models enable us to give few reasonable interpretations: one for general propositions and two for particular propositions. That is for all terms X, Y we put:

- (a) $\mathfrak{M} \models Xa^{\Box}Y$ iff $f(X) \subseteq f^{\Box}(Y)$
- (b1) $\mathfrak{M} \models Xi^{\Diamond}Y$ iff $f(X) \cap f^{\Diamond}(Y) \neq \emptyset$
- (b2) $\mathfrak{M} \models Xi^{\Diamond}Y$ iff $f^{\Diamond}(X) \cap f^{\Diamond}(Y) \neq \emptyset$

For each of interpretations we introduce a tableau system that corresponds to a given interpretation of modal propositions. The last part of our paper consists of a presentation of a generalized semantics for categorial propositions. We propose some possible world's semantics that allows to capture both interpretation: *de dicto*, as well as the former interpretation *de re*. Thanks to it we can compare propositions *de dicto* with propositions *de re*.

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Prefixed tableaux for logic of proofs and provability

Gödel-Löb logic (GL) has been the fundamental modal logic in the area of provability logic. On the other hand, the logic of proofs (LP, aka justification logic) was introduced (Artemov 2001) as an explicit modal logic to study the structure of 'proofs' at the level of propositional logic. Yet another logic has been studied to capture the notion of "being provable in PA and true" (called "strong provability"). The modal logic for strong provability is known as Grz (Boolos 1993).

Logics that combine GL (Grz) and LP have already been introduced, and their arithmetic interpretations (Yavorskaya 2001, Artemov and Nogina 2004, Nogina 2007) have been studied. One of the reasons why these logics are interesting is that, by combining GL and LP, we can observe how the notion of formal provability in PA and the notion of proofs in PA interact. A good illustration of this is one of the axioms in GLA, i.e., $\neg t : \varphi \to \Box\neg t : \varphi$ (we call this "mixed negative introspection"). This formula is of interest, not only because it is a kind of negative introspection (an analogue of S5 axiom), but also because this statement is valid in arithmetic interpretations. Also, complete axomatizations (via Hilbert-style systems) of these logics with respect to Fitting-style semantics are given under the name GLA and GrzA (Nogina 2007, Nogina 2008). However, no proof systems for these logics that have proof-theoretically interesting properties, say cut-free tableau or Gentzen-style systems, have been introduced so far.

In this paper, we introduce prefixed tableau systems for both GrzA and GLA and show cut-admissibility of these systems. We use prefixed tableau systems primarily because the mixed negative introspection in these logics makes it difficult to formulate cut-free destructive tableau systems for the logics.

Moreover, semantically proving cut-admissibility for the systems is not easy for the reasons summarized as follows.

(1) Unlike S5, an infinitary construction in proving completeness for cut-free proof systems for the logics is used, since we have to handle closure conditions of evidence functions. This is similar to the case of S4LPN (Kurokawa 2012).

(2) Unlike S4LPN, we finitize the height of the canonical model for GLA (GrzA) in a manner similar to the case of GL (Grz).

(3) In addition, for GLA, a new rule called "reflection rule" requires a special treatment.

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Degrees of Possibility

David Lewis's notion of comparative possibility has inspired a number of studies aiming to understand the logic of qualitative possibility in terms of a given ranking of the possible worlds. Recent studies, for instance that of Dick de Jongh and Sujata Ghosh, show that the link between the possibility comparisons at the object language and the similarity orderings at the semantic plane can be achieved at various levels of generality, as one weakens the relation between the two orderings. Lewis's treatment is based on an immediate link between the two orderings. I believe a generalization of Lewis's treatment promises to shed light on the nature of chance or natural necessity/possibility at a more general setting (than what can be found in probability theory) as qualitatively gradable. I illustrate the claim by using an extension of basic modal propositional logic, when we take as our language, to be called MPLO, a countable set of atomic propositions Γ and formulas φ defined inductively by:

$$\varphi \coloneqq \perp | p | \varphi \to \varphi | \Box\varphi | \varphi \succeq \psi | \text{ (where } p \in \Gamma).$$

The semantics of the "φ is at least as possible as ψ" relation, that is $\varphi \succeq \psi$, will be given by a model $\mathcal{M} = (W, \{\leq_w\}_{w \in W}, \{\sqsubseteq_w\}_{w \in W}, V)$ such that W is a non-empty set of worlds, V is a valuation of Γ on W; for each w in W, $\leq_w$ is a reflexive, transitive and linear order relation on W (a similarity/possibility ordering of the worlds at w), and $\sqsubseteq_w$ is a same type of order relation on the power set of W that is faithful to the subset relation (i.e., if $X \subseteq Y$, then $X \sqsubseteq_w Y$) and if $X \sqsubseteq_w \emptyset$ then $X = \emptyset$. Furthermore, it satisfies: If C_w is the set of worlds maximally similar to w, then whenever $C_w \subseteq X$ and $C_w \nsubseteq Y$ we have $Y \sqsubseteq_w X$. The crucial point here is that $\sqsubseteq_w$ is related to $\leq_w$ only to the extent that is satisfies the last condition. Given such a model $\mathcal{M}$, the truth conditions for formulas of MPLO are to be understood in the usual manner, where C_w's play the role of accessible worlds for each $w \in W$. In other words, $\mathcal{M}, w \models \Box\varphi$

iff $\mathcal{M}, w' \models \varphi$ for all $w' \in C_w$, and $\mathcal{M}, w \models \varphi \succeq \psi$ iff $\{u : \mathcal{M}, u \models \psi\} \sqsubseteq_w \{u : \mathcal{M}, u \models \varphi\}$.

In Lewi's system, $\Box\varphi \leftrightarrow (\varphi \succeq \neg\bot)$ is a validity. Within his framework, one cannot compare strengths of necessity/possibility of different formulas, as they all have the same strength. MPLO allows us to compare $\Box\varphi$ with $\Box\psi$ in a nontrivial manner. Sketching out a completeness result for this system, I further show in this talk that the system is less than a plausibility space (as defined by Joseph Y. Halpern), for the following is not a validity:

$$(\varphi \succeq \psi) \wedge (\bot \succeq \chi \wedge \varphi) \wedge (\bot \succeq \chi \wedge \psi) \rightarrow (\varphi \vee \chi \succeq \psi \vee \chi).$$

Being less than a plausibility space, yet enabling rudimentary comparisons of strengths of necessities and possibilities, MPLO and its various enrichments (from K to S5), come out as yet more abstract logical structures to project on chances.

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Belief Weakening

Models of belief revision provide a very abstract treatment of the correction of our beliefs before new information. But there is a special kind of "correction" where we are said to "restate" our convictions. In this restatement one necessarily abandons the aspect which was specifically contradicted, but does not necessarily abandon some weaker derivations from those original beliefs which may still be compatible with the new information. That is a particular and interesting kind of "correction" where one just weakens its original beliefs.

Belief revision on logically closed sets (like in the AGM model) is based on the principle of minimal change, which excludes only those sentences which may be "responsible" for the derivation of the conflicting sentence. This means that information originally derived from the excluded sentence may be kept (Maranhão calls this "conservatism" of the original compatible logical consequences). Minimality is transplanted without further reflection to the revision of bases (sets necessarily closed by logical consequences), but in this model any originally held information is lost whenever its support is excluded from the base, which means the loss of conservatism. In a first attempt Maranhão proposed a model of revision on bases which satisfies both minimality and conservatism. But the model was concerned only with the recovery of the consequences of the sentence which was specifically contradicted and deleted. It did not consider sentences originally derived from the deleted sentence in conjunction with other beliefs of the set. In the present paper a more general approach will be investigated, where the whole set (not only the contradicted sentence) is weakened. An interesting question is how to balance minimality in a conservative model, since minimality holds for the deletion but not for the selection of logical consequences of the original set. This selection of the logical consequences leads to a different consequence operator which is not Tarskian and whose properties are examined.

The paper then models the dynamics of weakening a set of sentences, studies the logical properties of such operation and investigates how one may keep track of its original convictions by taking care of original epistemic (but not explicit) commitments.

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Open Future and Relative Modalities

Counter to a long tradition of philosophers who offered ingenious semantics to explain the way we should properly evaluate future contingent statements - all the way from Prior's resetting of classical Peircean and Ockhamist solutions (Prior, 1967), to more recent Thin Red Line solutions (Belnap and Green, 1994) -, there still remains a special conundrum yielded by our ordinary experience of time, which still suggests we lack a better (or more elegant) way to analyze future contingents.

The problem may be stated in the following form: from one initial perspective, we have a strong intuition that our future is objectively open to a variety of distinct possible outcomes, which would mean that there are some statements about the future which are properly contingent, i.e., that it is possible for our world to be actualized in a way to satisfy the untensed statement, as much as it is possible for it to be actualized in a way to satisfy its negation.

Yet, if we now assert such a statement, and imagine ourselves in the future assessing this very statement, when the world has (or has not) been actualized in a way to satisfy it, we are strongly prompted to believe the statement is already true (or false); which would in turn mean that it is now indeed necessarily true (or false).

Both perspectives are regarded as incompatible. In fact, traditional solutions find no other way than to favor one of these perspectives, and fully reject the other, refraining us from retaining both intuitions. However, recent approaches (MacFarlane, 2003; Belnap, 2001) have focused on a solution to meet this challenge, exploring the uses of a new parameter of assessment contextuality.

Inspired by MacFarlane's initial proposal, I show how to construct a relational model which entertains a tool for accessibility shifting, built on a frame combining both tense and alethic modalities. This will show us how statements about the future, preceded by Necessity or Contingency operators, can be both satisfied by the model from a same point of utterance.

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Contextual Constructive Description Logics

Constructive modal logics come in several different flavours and constructive *description* logics, while much more recent and less studied, not surprisingly do the same. After all, it is a well-known result of Schild that description logics are simply variants of n-ary basic modal logic. There are several extensions of classical description logics, with modalities, temporal assertions, etc. As far as we know there are no such extensions for *constructive description logics*. Hence this note is a formal description of the extension of a constructive description logic in the style of Mendler and Scheele, with contexts as modalities, as described by de Paiva2003, following the blueprint for constructive modal logics set up by Wolter and Zakharyaschev.

We start by recalling the description logic $c\mathcal{ALC}$. We then consider one extra modality on top of $c\mathcal{ALC}$, following the blueprint of Wolter and Zakharyaschev, and prove decidability of the resulting system. In previous work one of us suggested the use of constructive modalities as MacCarthy-style contexts in AI. On that work, the application envisaged was constructive modalities for formalising natural language 'microtheories' and it was regretted that the system obtained only described constructive modalities over a *propositional basis*. The work in this paper gets closer to the desired ultimate system, as we can now talk about context as a constructive modality over a constructive description logic basis, as opposed to over a propositional logic basis. But most of the hard work is still to be done, as it concerns the interactions of the finite (but very large) collection of linguistics based contexts/modalities. Here we briefly discuss these intended applications as future work.

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Modal approach to region-based theories of space: definability and canonicity

The classical geometry and topology (as an abstract kind of geometry) are point-based in a sense that they take the notion of *point* as one of the basic primitive notions. At the beginning of 20th Century de Laguna [2] and Whitehead [6] by purely philosophical reasons take an alternative approach to the theories of space based on more realistic primitives: solid spatial bodies—*regions*—and the relation "a is connected with b"—*contact*. This approach is now known as *region-based* and in computer science and related areas Artificial Intelligence and Knowledge Representation it is considered as one of the basic qualitative approaches to the theories of space. In this setting, the full first-order theory of Euclidean spaces is non-axiomatizable, Grzegorczyk [3]. Recently, in order to obtain decidable fragments of the region-based theories of space, quantifier-free fragments viewed as propositional languages with topological semantics have been considered, [5, 4, 1]. A lot of the associated logics are axiomatized and completeness with respect to three kinds of semantics—algebraic, topological and relational Kripke-type—is demonstrated by slight modifications of techniques and tools from modal logic.

In the present talk definability by first-order and second-order conditions in comparison with classical modal logic will be discussed. Several cases of non-canonicity will be shown and a sufficiently general condition for canonicity based on first-order definability will be presented.

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Indeterministic Temporal Logic

The questions od determinism, causality and freedom are the main philosophical problems debated since the beginning of temporal logic. We consider indeterministic temporal logic based on the idea of temporal worlds and relation of accessibility between them.

To use a formal logic to solve a philosophical problem, we have to have:

1. a formal language in that the problem can be formulated in intuitively satisfactory way,
2. the logic should be neutral with respect to this problem, i.e. the formula that expresses the solution to the problem should not be a thesis of the logic (an analytical truth of the language).

The thesis of determinism as consisting of two theses, the thesis of predeterminism PRE-DET and the thesis of post-determinism $POST$-DET, can be formulated as follows:

DET. If α, then

PRE-DET. at any earlier moment it was true that there would be α,

and

$POST$-DET. at any later moment it will be true that there was α.

The principle of causality says:

PC. If α occurs at t, then at t_1, some moment earlier than t, and at any moment between t and t_1 it was true that there will be α.

The principle of effectivity, as symmetrical to PC, may be formulated as follows:

PE. If α occurs at t, then at t_1, some moment later than t, and at any moment between t and t_1 it will be true that there was α.

We are looking for a formal language such that:

- both the theses PRE-DET and $POST$-DET and both the principles PC and PE are expressible in it;
- the theses as formulated in the language are not truths of the logic of this language, i.e. they are not true in any model, especially irrespective of properties of time;
- the arguments from the principles PC and PE for PRE-DET and $POST$-DET, respectively, could be avoided even if both the principles are valid.

If all these conditions are fulfilled, we say that the logic is indeterministic.

Priorean temporal logic language does not satisfy the conditions that are imposed on interesting us language, namely the thesis of determinism DET is a theorem of of the minimal tense logic K_t.

In the new language temporal operators will be defined assuming that there could be more courses of events though branches do not differ in time (as a set of moments). A possible world consists of T — time (non-empty set of points), $<$ — a binary relation (earlier-later) defined on T and V — a function that to any element of T assigns a subset of propositional letters $V : T \to 2^{AP}$. Possible worlds will be indexed by elements of a non-empty set I. On the class of possible worlds $\lhd$, a relation of accessibility, is defined. Thus the frame $\langle T_i, <, \lhd \rangle, i \in I$ consists of T_i, the set T indexed by an element of I, a binary relation $<$ defined on T, and relation of accessability:

$$\lhd \subseteq \bigcup_{i,j \in I} T_i \times T_j.$$

In the language L_D the future and past tense operators will be defined with respect to a binary relation $\lhd$ of accessibility between possible worlds (possible courses of affairs):

Definition 1: $[G]$ $\langle T, <, V_i \rangle, t \models G\phi$ iff for any $t_1, t < t_1$, and any $j \in I$: if $(t,i) \lhd (t_1, j)$, then $\langle T, <, V_j \rangle, t_1 \models \phi$.

Definition 2: $[F]$ $\langle T, <, V_i \rangle, t \models F\phi$ iff there is $t_1,\ t < t_1$, such that for any $j \in I$: if for some $t_2, t < t_2, (t,i) \lhd (t_2, j)$, then $(t,i) \lhd (t_1, j)$ and $\langle T, <, V_j \rangle$, $t_1 \models \phi$.

Definition 3: $[H]$ $\langle T, <, V_i \rangle, t \models H\phi$ iff for any $t_1, t_1 < t$, and any $j \in I$: if $(t_1, j) \lhd (t, i)$, then $\langle T, <, V_j \rangle, t_1 \models \phi$.

Definition 4: $[P]$ $\langle T, <, V_i \rangle, t \models P\phi$ iff there is $t_1,\ t_1 < t$, such that for any $j \in I$: if for some $t_2, t_2 < t, (t_2, j) \lhd (t, i)$, then $(t_1, j) \lhd (t, i)$ and $\langle T, <, V_j \rangle$, $t_1 \models \phi$.

L_D satisfies the conditions imposed on indeterministic temporal language.

4.5.4 Empirical

The invited keynote speaker of this session is Newton C. A. da Costa (page 67).

Contributed Talks

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Models in constructions of scientific knowledge

In constructions of scientific knowledge always exist models. Often one of sciences is a model for all other sciences. For exemple, Galilei considered that God wrote book of nature with the help of mathematics and that who understand mathematics can read book of nature as God. A mathematical argumentation also was a model for all other sciences. If we compare the argumentation which is applied in different sciences we should recognize that the logical argumentation in its purest form is used in mathematics. Therefore there is such a problem in the history and theory of argumentation: Was the mathematical demonstration the sample for the theory of scientifical argumentation, that is syllogistics in Aristotle's logics, or not? Leibniz considered that argumentation more geometrico can be used in philosophy. Leibniz kept in mind those Spinoza's works in which Spinoza tried to use geometrical methods of proving in philosophical thinkings. Leibniz evaluated using geometrical methods of proving in philosophy by Spinoza positively. However in our times most of people consider using geometrical methods beyong geometry in other way because models of building scientifical knowledge changed. Also an attitude to mathematical demonstration changed. For example, in the 20th century Lakatos affirmed that mathematical demonstration didn't prove anything.

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Proof and Causation

In one of his notes Gödel briefly mentions a possible logical and philosophical program of a reduction of all "logical (set-theoretical)" axioms to the "axioms of causality", and, in philosophy, of all (Kantian) categories to the category of causality. From this standpoint, it seems natural to connect two Gödel's results: (1) a sketch of justification logic as a sort of a logic of proofs (Zilsel Lecture; further developed only recently by S.Artemov and M.Fitting), and (2) Gödel's second-order modal ontological system GO, devised for his ontological proof of the existence of the most positive being (a note from 1970). On the ground of this, we want to show that proofs can be formally conceived as a special (in a way, paradigmatic) case of causality, and causality as a central ontological concept.

We show that Gödel's ontological system can be transformed into a sort of justification logic (a modification and extension of FOLP by Artemov) in which the justification (proof) terms can be re-interpreted in a causal sense. To that end, we first analyze Mackie's INUS concept of causality, and relate it to historical concepts of causality in Aristotle's theory of proof (premises as causes of the conclusion), as well as to Leibniz' and Kant's concepts of causality (in connection with the principle of sufficient reason and the concept of consequence). After this conceptual-historical analysis, we apply the ontological concept of causality to the justificational transformation of GO, and transform it further into a causal ontological system CGO, where "essence" and "necessary existence" are reduced to causality, and in which it can be proved that every fact has a cause and that there is a first cause. We formally describe a first-order and a second-order version of CGO, and give an appropriate semantics with respect to which the soundness and completeness proofs for versions of CGO can be devised.

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Do we need yet another logic for the living?

Natural phenomena arises from a collection of interacting things which, by interacting, change the attributes we observe. Traditionally, things reside in space, attributes in phase spaces and time is a way of reckoning and registering changes. Moreover, except for simple constrains, every thing may freely interact with every other thing. Under this view, living things appear as protean open systems, barely stabilised by self-organisation, where interactions are a kaleidoscopic exchange of attributes presenting patterns difficult to discern due to their swiftness. Biologists handle these difficulties by totally abstracting from time and space and represent living things as static objects such as networks, organelles and architectures [2]. Although acknowledging their dynamic character and permanent reconstruction by bio-chemical processes, biological arguments seldom make use of this reconstruction.

Nevertheless, there are functionally relevant "things" in living phenomena, like the flagella motors [1] or the chaperone aggregates around ribosomes that can only be recognised along time. Changes in bio-molecules' conformation do change their interactions with other components and biologically relevant events are entailed to special "architectural" (con)formations. Moreover, architecture (henceforth organisation), and not mass nor energy, is what is ubiquitously preserved in living phenomena [2]. Organisation is also independent of any material elements that give them existence. Portions of DNA are frequently replaced by the SOS system to preserve its organisation.

An alternative view of natural phenomena, aimed at living phenomena, will be introduced where things may spread in time besides space, interactions are organised, and "things" may dynamically coalesce (organise) into new interacting "things". Thus, time will have two roles: to give things "existence" and to reckon changes in attributes. This view is grounded on a rich framework for representing organisations and their changes, which admits the definition of predicates and operators, such as equality, sameness, forgetting and detailing. It gives rise to organised volumes and to a concept of "in-formation" that accounts for changes in organisations. This organisation framework is based on hypergraphs and recursiveness, and relates to Rashevsky's and Rosen's ideas [5,6]. Notwithstanding, it is not centred on biological function and maps straightforwardly into living things and phenomena. Moreover, organisation and dynamics are connected through interaction graphs [4].

Inspired by biological examples, a concept for organisation [3] will be discussed, their symbolic framework introduced and information operationally defined. Time permitting, the anticipatory nature of this definition will be addressed [Kritz, in preparation].

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Logic Is an Empirical Science: All Knowledge Is Based on Our Experience and Epistemic Logic is the Cognitive Representation of Our Experiential Confrontation with Reality.

1. Introduction: Epistemological explanation that logic guides human conduct in reality. What is logic and what is its role in human affairs: this is the basic epistemological question. It is basic and universal science and the laws of logic represent the method of our self-control in Reality by proving that we actually represent this reality. This conception of Rationality in human affairs purposing

to guide our conduct in Reality which is our freedom in it and not from it. This is also Peircean conception of reasonableness explaining our logical cognitive confrontation with reality (Nesher, 2007a). 2. The epistemological deficiency of syntactic and semantic axiomatic formal systems. Formal systems cannot explain or generate human cognitive operations in proving our true representation of reality to guide human conduct. The difference between axiomatic formal systems and realist theories lies in their proof-conditions. Formal systems are by definition hermetically closed games; given their fixed formal proof-conditions, the axioms cannot be proved true and the formal rules of inference cannot evaluate the truth of their theorems. Hence, axiomatic formal systems are complete and thus isolated from external Reality (Gödel, 1930; Carnap, 1939; Tarski, 1941). In contrast, realistic theories are incomplete, a la Gödel, but true relative to their proof-conditions: the proved true facts of reality and methods of proving their hypotheses (Gödel, 1931; Nesher, 2002: X, 2011). 3. Can intuition compensate for the deficiency of formal inferences to represent reality? Thus, the axiomatic formal systems are artificially abstracted from human cognitive operations and cannot explain them; however, because of human confrontation with Reality, logicians have to accommodate their formal systems to Reality by intuiting new axiomatics and new modes of logics. The question is whether intuition can compensate for the deficiency of formal inferences to represent Reality and whether logicians can explicitly and rationally self-control these intuitive operations to ensure the truth of such proofs? Thus, if logicians do not know the logical Reality, they can infer Models from their intuition of axioms of formal systems, which by themselves are unfounded. Hence, the intuitive conceptions of Ontology and Models substitute only artificially for Reality in formalist epistemology and cannot explain cognitive representations of reality (Russell, 1914, 1919). 4. Pragmaticist epistemic logic is universal logic of complete proof relative to proof-conditions. Cognitive epistemic logic consists of the trio sequence of Abductive logic of discovery + Deductive logic of consistency + Inductive logic of evaluation (Peirce, 1903). Abduction and Induction are material logics while the meanings of our perceptual confrontation of Reality are their essential components. Thus, by means of epistemic logic, standing with its two material logics "legs" on Reality, we can prove the truth or the falsity of our cognitions as representing external reality, yet under our relative proof-conditions. In disregarding formalist epistemology, we do not have to assume the undefined meanings of primitive terms and the truths of axioms, but by epistemic logic we can quasi-prove the truth of ostensive definitions meanings (Nesher, 2005, 2007b). Hence, we can prove the truth of our basic perceptual facts of Reality, and from these facts Abductively discover our hypotheses, Deductively inferring their conclusions to evaluate them Inductively upon our true facts of reality (Peirce, 1903: MSS 448-449; Nesher, 2002: II, V, X). However, epistemic universal logic can comprehend different partial logics as its components. 5. We can prove the truth of universal epistemic logic as the epistemological basis of all sciences. Since all our knowledge is empirically proven, this can also hold in regard to our knowledge of epistemic logic. Such was Spinoza's endeavor to refute Cartesian formalism by proving the true logical method through self-reflecting on the first quasi-proved, true idea of our perceptual judgment and to formulate its structure. This is also Peirce's self-reflection on perceptual operation with the relation of the iconic sign of ego to the indexical sign of non-ego, whose consistency is the criterion for quasi-proving the truth of perceptual judgments representing external reality. Thus, we can be logically Realists in refuting a priorism and solipsism (Spinoza, 1662; Peirce, 1902; Nesher, 2002: Int.).

4.5.5 Paradox

Contributed Talks

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Affinity among some epistemic paradoxes
Epistemic paradoxes are paradoxes involving the notions of knowledge or belief. In this paper, we shall present five epistemic paradoxes: Moore's Paradox (sentences like "p, but I do not believe p" may be true, but sound/seem inconsistent when asserted), Fitch's Knowability Paradox (the thesis that all truths are knowable entails all truths are known), Believability Paradox (the thesis that all truths are believable entails all truths are believed), Preface Paradox (an author, despite believing every single proposition in his book, does not believe his book is free of error) and Surprise Test Paradox (an student, after concluding there will be no surprise test this week, is surprised as there is one). Using the formalism of modal epistemic-doxastic logic, we will show that these five paradoxes are related one to another, as they all rely on common or largely similar assumptions.

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Vague Desire: The Sorites and the Money Pump

The sorites paradox is usually treated as a problem about linguistic meaning. However, related phenomena occur for intransitivities in mental states, especially of perceptual indiscriminability and of indifferences in desire. Dummett [1] argues that those basic psychological phenomenona underlying vagueness make language incoherent. But Raffman [5] and Fara [2] have used the mental indeterminicies to try to explain linguistic vagueness and try to defuse the sorites paradox. Van Rooij [8,9] has adapted the formal models of economics and mathematical psychology, to develop formal semantics for vague language.

In this paper, we will invoke the Gricean [3] program of reducing linguistic meaning to mental states in acts of communication. A consequence of this model is that linguistic vagueness is based in vagueness in communicative intentions. By adopting a solution to the paradox of the money pump, we can solve the sorites paradox by showing that the problematic sorites premise corresponds to an appealing but mistaken principle of rational choice.

A typical version of the sorites paradox asks how many grains it takes to make a heap of sugar. It is very plausible to accept the sorites premise that if n grains of sugar make a heap, then so too do $n-1$ grains. But by repeated application of the sorites premise to any object which is a heap of sugar, we will keep maintaining that smaller collections of sugar are heaps, even after we eventually reach an object which is not a heap.

A similar phenomenon arises for desire, thanks to intransitivities of indifference. Consider these two examples (from Luce and Armstrong, respectively, cited by [4]). You prefer two lumps of sugar to one, and you prefer each to no sugar, yet you are indifferent between changes in a single grain of sugar. Hence your indifferences are intransitive, due to accumulating perceptual indiscriminabilities. Besides thresholds of detection, indifference may be intransitive due to multiple criteria of evaluation: a child prefers a bike with a bell to one without a bell, but is indifferent between each and a pony.

Preference is standardly connected with choice by the principle that an option is choiceworthy just in case undominated: there is nothing available which is preferred to it. Intransitivities of indifference lead to choiceworthiness being context-dependent. Especially, intransitive indifferences yield violations of this property of "expansion consistency" (as in [7]): if a pair of options are choiceworthy, then it isn't the case that only one remains so when the menu of options is enlarged.

Intransitive indifferences lead to a paradox of sequential pairwise choice similar to the sorites. If you are willing to trade up for more preferred options, and you are willing to exchange items that you are indifferent between, you will end up in a potentially endless cycle of choice (and an expensive one, if you must pay to upgrade). It is has been argued, originally by Schwartz [6], that the problem with the "money pump" is not the preferences but how choice is determined by them. We argue that the specific problem is that although an agent is required to choose something if it is preferred to all other options, she is not required to choose an item just because nothing is preferred to it. Indeed, depending on past choices, the agent may not even be permitted to choose an item despite no item being preferred to it.

In communication, the speaker intends for the audience to come to believe certain in- formation; according to Grice, meaning can be reduced to a complex intention of this kind. Communicative intentions involve preferences about the semantic interpretations which the audience will assign to the expressions uttered by the speaker. The speaker's objects of choice are the intended extensions and anti-extensions of predicates in a communicative con- text. So we can see how sorites predicates (like "heap") arise as in the sugar example, while multi-criteria words (like "vehicle") behave in a manner like the example of the child. The speaker will typically be indifferent between small changes in grains of sugar affecting the interpretation of heap", yet these indifferences will be intransitive. Following the incorrect choice principle – that an object is choiceworthy if undominated – will lead to the sorites paradox.

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The Logic of rational agent

The aim of the present article is to establish a new logical system called "the logic of rational agent". The logic of rational agent is supposed to be a new tool to resolve some logical and philosophical problems, for example, the knowability paradox (Fitch's paradox). I would like to present three possible formalizations of this logic, with different entailment relations. Each system will depend on what semantic insight we would like to use in a formal way. This fact is one of the aspects of "rationality" of the agent.

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What Achilles Did and the Tortoise Wouldn't

This paper offers an expressivist account of logical form. It argues that in order to fully understand logical form one must examine what valid arguments make us do (or: what Achilles does and the Tortoise doesn't, in Carroll's famed fable). It introduces Charles Peirce's distinction between symbols, indices and icons which signify (respectively) by arbitrary convention, by direct indication, and by resemblance. It is then argued that logical form is represented by the third, iconic, kind of sign. It is noted that because icons are unique in having parts which bear the same relationship to one another as the parts of the object

they represent, they uniquely enjoy partial identity between sign and object. This, it is argued, holds the key to Carroll's puzzle of why it seems that Achilles cannot explain to the Tortoise what he is failing to understand. Finally, from this new triadic account of signification original metaphysical morals are drawn: that (indexical) metaphysical realism and (symbolic) conventionalism - despite being traditional philosophical foes - constitute a false dichotomy. Although indices and symbols are vital aspects of signification, both our representations and our reality are also shot through with intriguingly inference-binding structures.

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Coping With Falakros Paradox in the Meaning of Special Linguistic Expressions

A special but very important class of linguistic expressions is formed by the, so called, *evaluative linguistic expressions* (cf. [2–3]). It includes several subclasses of expressions, namely: simple evaluative expressions (e.g., *very short, more or less strong, more or less medium, roughly big, extremely strong, silly, normal, extremely intelligent*), fuzzy numbers (e.g., *about twenty five thousand, roughly one hundred*), compound evaluative expressions (e.g., *roughly small or medium, small but not very (small)*), negative evaluative expressions (e.g., *not small, not strong*). Let us also mention *evaluative linguistic predications*

$$\langle \text{noun} \rangle \text{ is } \mathcal{A}$$

where $\mathcal{A}$ is an evaluative expression; for example *temperature is low, very intelligent man, more or less weak force, medium tension, extremely long bridge*, *short distance and pleasant walk*, *roughly small or medium speed*, etc. Note that evaluative expressions are permanently used, for example when describing processes, decision situations, procedures, characterize objects, etc. They are also considered in applications of fuzzy logic. Logical analysis of the semantics of evaluative expressions (and predications) reveals that it hides falakros (sorites) paradox. Moreover, the paradox extends even to real numbers in the sense that we may speak, e.g., about *(very) small amount of water, an extremely strong pressure*, etc. In this paper we present a formal theory of the semantics of evaluative expressions and demonstrate that the theory copes well with various manifestations of falakros paradox (and, of course, also of sorites one). The main assumption is that vagueness of the meaning of these expressions is a consequence of the *indiscernibility* relation between objects. Our main formal tool is the *fuzzy type theory* (FTT) (see [4], [6]), which is generalization of the classical type theory presented, e.g., in [1]. The semantics of evaluative expressions is developed within a special theory T^{Ev} of FTT (see [5]) which formalizes certain general characteristics of it. The theory also copes with the concept of context

(=possible world), i.e. it makes it possible to model semantics of *small, medium big*, etc. with respect to various specific situations. For example, a *small beetle* means size of an object very much different from that of a *small planet*. In the theory T^{Ev}, we can formally prove that semantics of evaluative expressions from the class ⟨*linguistic hedge*⟩ *small* has natural properties, for example that in each context, there is no *small* n such that $n+1$ is already *not small*, there are *big* n, etc. We also prove that in arbitrary context, there is no last surely ⟨*linguistic hedge*⟩ *small* x and no first *surely* ⟨*linguistic hedge*⟩ *big* x (e.g., no last x that would surely be roughly small, or no first x that would surely be more or less big). Of course, similar properties hold also for the class ⟨*linguistic hedge*⟩ *big*. In this contribution, we will discuss in detail the role of T^{Ev}. Note that the theory is syntactically formulated (we have formal syntactical proofs of its theorems) and so, they can have various kinds of interpretation. Our theory relates to wider paradigm of *fuzzy natural logic*.

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Fitch's Paradox of Knowability, Typing Knowledge and Ramified Theory of Types

It is already known that Fitch's knowability paradox can be solved by typing knowledge, cf. Paseau (2008), Linsky (2009), Giaretta (2009), or even Williamson (2000). (Recall that such approach blocks Fitch's paradox without a substantial change of classical logic.) Nevertheless, the method received an extensive criticism by Halbach (2008), Hart (2009), Florio and Murzi (2009), Jago (2010), and mainly by Carrara and Fasio (2011). It is not difficult to notice that most of the criticism pertains to Tarskian typing knowledge (i.e. stratification of K-predicate applicable to names of sentences), not to Russellian typing knowledge (i.e. stratification of K-operator applicable to propositions; the propositions in question are structured entities, not possible world propositions, thus they have intensional individuation). I will thus reject a part of this criticism as misdirected. (Note also that one should not ascribe to Russellian typing knowledge capability to protect verificationism, antirealism, because the framework is neutral as regards this philosophical position.) I will argue that within ramified theories of types (RTTs) such as that of Church (1976) or Tichý (1988) one naturally stratifies the knowledge operator because its very identity relies on the stratification of propositions on which it operates. The main justification of this kind of typing is thus provided by the Intensional Vicious Circle Principle (Gödel's alleged objections notwithstanding) which governs the very formations of propositions and thus also intensional operators. (There are other important and relevant features of the RTTs in question, e.g., cumulativity.) On the other hand, Russellian typing knowledge cannot block Fitch's knowability paradox only by its own principles (or the principles of the framework, i.e. RTT). One needs to accommodate also a special assumption concerning knowledge - namely, that some propositions can be known only in a higher-order way. This gives rise to a question for the character of the notion of knowledge provided by Russellian typing knowledge. Using two distinctions suggested by Paseau (2008), typing knowledge can be based on content / on epistemic access (stage of knowledge), while adopting a stratification for logical / philosophical reasons. I will argue that the best foundation of Russellian typing knowledge is on (stratification of) content for logical reasons. This enables to reject most of the criticism by Carrara and Fasio who concentrated largely on refutation of the philosophical aspect of Tarskian typing.

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On the Tenability of a Benacerraf-like Dilemma for Logic

The dilemma suggested by Paul Benacerraf (1973) is regarded by most philosophers to raise an hardly surmountable challenge for mathematical platonism. On the one hand, the platonist seems to offer a good semantic treatment of mathematical discourse through a referential (Tarskian) analysis of mathematical statements. On the other hand, the corresponding postulation of abstract referents for mathematical terms appears to rule out any feasible epistemology for mathematics, leaving our causal access to a causal mathematical entities wholly unexplained. This epistemological challenge appeared to many to be biased, if not question-begging, against the platonist, given its requirement for a causal connection between our true mathematical beliefs and facts involving abstract mathematical objects. Field (1988, 1989) revised the challenge, replacing the assumption of the causal theory of knowledge with the more general request that the reliability of our mathematical beliefs should be given some explanation (a request he thought the platonist could hardly satisfy). Benacerraf-style dilemmas can be thought of for different areas where knowledge has been traditionally conceived as being a priori. Field (2005) has for instance suggested that a Benacerraf- style dilemma can be formulated in connection to logic, and that a corresponding epistemological challenge can be raised to some accounts of our logical knowledge. Field suggests that, parallel to what happens in the mathematical case, there is a strong concern regarding the possibility of explaining the reliability of our logical beliefs. In both cases, what seems to be lacking, at least prima facie, is an explanation of the correlation between our logical (or mathematical) beliefs and the allegedly corresponding logical (or mathematical) facts. In the first half of the paper, after reviewing part of the recent literature on the problem at issue and the relevant epistemic notions, I will argue that Benacerraf-style dilemmas suffer from a general shortcoming, that can be expressed in the form of a (meta)dilemma. In particular, it does not seem possible to find ways of making such dilemmas both non-trivial (i.e. raising a genuine novel epistemological challenge to, e.g., platonism) and non-question-begging (i.e. not based on epistemological notions that by themselves rule out the opponent, e.g. platonist, account). I will then consider Field's way of dealing and defusing a Benacerraf-like dilemma concerning logic, and will finally apply the considerations above to the case of logic. I will point out that similar considerations apply in the case of mathematical and logical knowledge, and will thus suggests different reasons why a Benaceraf-like epistemological challenge to logical knowledge proves ineffective.

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Solving the Surprise Examination and Designated Student Paradoxes

Quine (1953) famously suggested that the error in the student's reasoning in the Surprise Examination Paradox is that she assumes that she will continue to know the teacher's announcement through the week; naturally, after the student completes her reasoning, this assumption turns out to be false. Formally, this suggestion amounts to the rejection of the *temporal retention principle* (R), which states the monotonicity of knowledge, as an axiom of epistemic logic. More recently, Sorensen (1982) introduced the Designated Student Paradox as an analogous case in an attempt to show that the paradox in the Surprise Examination Paradox is not the result of any temporal notion, which suggests that the culprit cannot be R. While both paradoxes have been widely discussed, they have not, to my satisfaction, received an adequate analysis that reconciles the two cases. In this paper, I will do two things: (1) I will offer a more technical presentation of Quine's analysis; and (2) I will suggest that the standard reliance on a principle of *synchronic logical omniscience* is what makes the Designated Student Paradox appear to eliminate all temporal qualities. Once we acknowledge that reasoning takes time and often changes the epistemic state of the agent (two features that are abstracted away by logical omniscience), we can see that the Designated Student Paradox also depends on an infelicitous use of R. As such, in my analysis, I will propose a weaker principle of *diachronic logical omniscience* that respects these features.

The intuitive idea behind my analysis is that when reasoning, the student first considers the state of affairs wherein she arrives on the penultimate day of the week without having received an exam; however, in order to reason about what she will then know, she supposes that she will know the teacher's announcement. She then reasons on this basis to deduce that the final day of the week is not a possible exam day; if it were it would contradict the teacher's announcement. She iterates this reasoning for the rest of the week and decides that the teacher's announcement is false. Naturally, this means that at the beginning of the week and for the remainder of the week, she does not believe the teacher's announcement. Since all of her reasoning was based on the presumption that she would retain knowledge through the week, the performance and result of her reasoning undermine one of its very assumptions. When formalized, R is the justification for the student's assumption that they will know the teacher's announcement on the penultimate day—of course, the fact that this fails to obtain is precisely what causes her to be surprised, so R is clearly suspect.

When considering the Designated Student Paradox, recognizing that the student's inference is not instantaneous allows us to identify the hidden use of R and see where the reasoning goes awry. There are numerous challenges to logical omniscience in the literature on epistemic logic; however, to my knowledge none have come from the literature on the Surprise Examination Paradox. In fact, all formal discussions of the Surprise Examination Paradox in static epistemic logic assume synchronic logical omniscience. To be fair, most Surprise Examination scholars explicitly recognize that the assumption of synchronic logical omniscience is problematic in general—nonetheless, such authors seem to think that in this specific case, the reliance of logical omniscience use is innocuous. So, it is interesting that the consideration of the Designated Student Paradox and its relation to the Surprise Examination Paradox gives us independent reason to doubt synchronic logical omniscience.

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Some Remarks on Moore's Paradox

Moore's 'paradox' arises from the fact that consistent propositions of the form of (1) and (2):
(1) It is raining but I believe it is not raining.
(2) It is raining but I don't believe it is raining.
strike us as being contradictory.

There are two main diagnoses concerning the adequate characterization of the absurdity Moore discovered the linguistic diagnosis and the doxastic diagnosis.

The linguistic diagnosis is based on a plausible analysis of the conversational constraints underlying the rules that define an interpersonal linguistic game of information transfer and persuasion. Within such a game, a move displaying an instantiation of a sentence of the forms Moore highlighted seems indeed to be defying the rules that constitute it.

However, a number of philosophers and logicians have voiced their dissatisfaction with the intrinsic limitation of this diagnosis to cases in which such linguistic games are actually being played. They claim that only a diagnosis produced at a deeper level of analysis will do justice to our intuition namely, a diagnosis produced at a doxastic rather than at a linguistic level. Among them, Shoemaker and Sorensen seem to me to hold the more interesting views.

Thus, Shoemaker explains the oddity characterizing the entertaining of Moore-like contents by producing what he takes to be a proof that belief in the sentences that instantiate them is either inconsistent or self-refuting.

Sorensen, in turn, puts forth a highly original view according to which a different number of propositional attitudes have scopes smaller than the class of consistent propositions. Thus, some consistent propositions are inaccessible to the exercise of those propositional attitudes. According to Sorensen's terminology, inaccessible consistent propositions are blindspots. In particular, Moore-like propositions are supposed to be the blindspots of belief.

Thus, either of them claims that Moore-like contents are unbelievable.

In opposition to them, I will contend that the doxastic diagnosis is not able to pin down a plausible constraint in terms of the reference to which belief in contents of the forms Moore identified is adequately criticized as violating some constitutive condition of meaningful thought. Thus, I will contend that there is no reason why such contents ought to be labelled as unbelievable.

4.5.6 Tools

The invited keynote speakers of this session are Sara Negri (page 79) and Giovanni Sambin (page 81).

Contributed Talks

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Multiset consequence relations for substructural logics

Consequence relations studied in logic are usually Tarskian, i.e., are relations between sets of formulae and single formulae, satisfying certain closure conditions. With the arrival of substructural logics, the option of more complex structures of premises has opened. In this talk we will focus on multiset consequence relations, i.e., such that have multisets of formulae for premises. Such non-Tarskian consequence relations have already been tackled by Avron [1]. However, most standard methods (e.g., those of abstract algebraic logic) are restricted to (substitution-invariant) Tarski consequence relations, and thus cannot be applied to multiset consequence relations; this opens a new area for research.

Arguably, multiset consequence relations are the natural consequence relations for contraction-free substructural logics, or the logics of commutative residuated lattices. In these logics, several meaningful consequence relations can be defined. The usually studied ("global") Tarski consequence relation for substructural logics transmits the full truth from the premises to the conclusion; however, this consequence relations does not correspond to the substructural implication connective, as it does not enjoy the classical deduction–detachment theorem. On the other hand, the ("local") consequence relation that does enjoy the classical deduction–detachment theorem (which is thus the one internalized by implication) is necessarily a multiset consequence relation due to the lack of contraction.

With this prime example in mind, we make first steps towards a general theory of substitution-invariant multiset consequence relations. Analogously to the matrix semantics of substitution-invariant Tarskian consequence relations we propose the semantics of ℓ-monoidal matrices for multiset consequence relations, and make an initial investigation of the correspondence between the logical and algebraic facets of multiset consequence relations.

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Bringing logic to the people. A diagrammatic proof system for classical logic extended with a pseudo-relevant implication

In this paper, I will present a diagrammatic proof system for classical logic, which can be extended with a pseudo-relevant implication via a strong paraconsistent logic. The diagrammatic proof system seems very promising for educational purposes. Due to its clarity and elegance, it can provide more insight in the structure of the proof than other more common proof styles. We have included an example of such a diagram at the bottom of the page. On top of its clarity, the presented diagrammatic system has the ability of making explicit the heuristic principles that the researcher uses when constructing proofs in a goal-directed way. This allows students to get an even better grip on how proof construction works and gives him/her a greater chance at successfully mastering this skill. This is achieved by restricting the use of composing rules: a formula preceded by a $\boxplus$ can be analyzed, but is never the result of a composing rule, a formula preceded by a $\diamondplus$ is the result of a composing rule and can therefore not be analyzed. The rules for the proof construction are exemplified at the bottom of the page. The diagrammatic system results in a very strong paraconsistent logic. This logic, devised by Diderik Batens, provides for all the consequences of classical logic, except Ex Contradictione sequitur Quod Libet (ECQL) i.e. $A, \neg A \vdash B$ for any A and any B. Consequently, one is never able to deduce a consequence which has no sentential letters in common with the premise set. This guarantees some non-trivial connection between the premises and the consequences.

Furthermore, we will show that we can easily extend this logic with a nice implication that solves most of the paradoxes of the material one; i.e. by demanding that, when a hypothesis is introduced in order to conclude an implication, this hypothesis must be used in the construction of the proof. This results in the fact that, given that one aims at concluding an implication, one is only able to introduce a hypothesis that is relevant for the deduction of the consequences. This contributes to the clarification of actual human reasoning on the one hand and the structure of the proof on the other. All this is obvious from the example in figure one. If our hypothesis (presented in the circle node) had not been necessary for the deduction of s, it would have been irrelevant for the inference of our goal - which leads to unwanted and ungrounded connections due to the equivalence of $A \supset B$ and $\neg A \vee B$. The hypothesis (p) was e.g. unnecessary for the inference of $q \vee r$. By demanding that it be used in the proof construction (as illustrated below), one guarantees a grounded connection between the implicandum and the implicans. We therefore lose most of the contra-intuitive consequences of material implications. Last, but not least, this system can be embedded in a fragment of the famous relevant logic **R**, by means of treating a disjunction in the premises as an *intensional disjunction* ($A + B =_{df} \neg A \rightarrow B$) and a conjunction in the conclusion as an *intensional conjunction* ($A \circ B =_{df} \neg(\neg A + \neg B)$). This system will turn out to be non-transitive and much richer than the fragment of **R** in a justified way i.e. without reintroducing most of the paradoxes of the material implication (only two justifiable paradoxes remain).

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Dialetheism and Galaxy Theory
The availability of multiple logics, although not a novelty, carries on provoking different kinds of puzzlement. From the point of view of those endeavoring to describe and understand parts of the world, it is a pressing issue to understand how different logics coexist and eventually how to choose between them. For metaphysicians, who often deal in necessity and make frequent use of modal reasoning, the appeal to a logic is also the appeal to a standard to decide what is possible typically in terms of which worlds are possible (see D. Lewis' *On the plurality of worlds*). The use of a single, fixed logic as a standard of possibility is clearly unsatisfactory as it biases all results. Clearly, what is impossible in classical logic is not necessarily so in paraconsistent or intuitionistic logics. Up till now, the use of classical logic as if it were there only logic available was defended on the basis of its entrenchment: in the absence of any reason to pick any other logic, classical logic is best retained once it is deemed sufficiently useful and intuitive in the past. Such a response, nevertheless, has been challenged by the development of tools for a universal logic. Universal logic engages with multiple logics simultaneously either by comparing them or by combining them. It made it possible to look at the plurality of logics not in order to choose one among them but rather to study relations between them. By considering the space of all logics, universal logic provides a general framework where features and capacities of a logic can be made evident. We have recently sketched a tool for universal logic called galaxy theory (see H. Bensusan and A. Costa-Leite, "Logic and their galaxies", forthcoming). Based on some developments in Kripke's semantics for modal logic, galaxy theory defines a logic (or rather, a relation of consequence) as a class of possible worlds. Such a class, called galaxy, is itself an element in a topology of galaxies. Typically, modal elements in a logic add to each corresponding galaxy some relations of access, but this can be taken not to affect the underlying galaxy. The emerging image is one where the plurality of logics can be studied as the plurality of galaxies. In this work we present the framework of galaxies and apply it to the debate about realism concerning different logics and related issues revolving around Priest's dialetheism. We consider galaxy theory together with some concepts developed by Kit Fine (mainly in papers collected in "Modality and Tense"), such as the notion of a inconsistent über-reality that brings together elements in a plurality. We then propose a realism about the different logics that is, at the same time, combined to a form of dialetheism. Galaxy theory paves the way to investigate such issues

because it takes each galaxy as a point in a topology. A side aim of this work, nevertheless important, is to show how fruitful the framework of galaxies can be.

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Towards Powerful and Decidable Formalizations Through Schematic Representation

Work done by Aravantinos et al.[1] on propositional schemata provides a way to give a finite formalization of an infinite set of propositional sentences by iterating a conjunction (disjunction) of the formula indexed by a free numeric variable. They also provide a decision procedure for a particular type of schemata, namely regular schemata. Regular schemata allow formalization of induction on propositional formulas up to the ordinal ω.

While regular propositional schemata are quite powerful tools, they are not expressive enough to formalize theorems such as the pigeonhole principle (PHP). While it is well known that an instance n of PHP (PHP_n) can be formalized in propositional logic [2], see Equation 9, a formalization of the set $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ requires induction over the ordinal $\omega\cdot\omega$. There are classes of schemata powerful enough to formalize $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$, however, all known classes of schemata which are powerful enough are undeciable for satisfiability [1].

$$\bigwedge_{i=1}^{n+1}\bigvee_{j=1}^{n} P_{i,j} \rightarrow \bigvee_{i=0}^{n+1}\bigvee_{m=i+1}^{n+1}\bigvee_{j=1}^{n}(P_{i,j}\wedge P_{m,j}) \tag{9}$$

Our goal is to study the relationship between induction and schemata, namely, finding classes of schemata powerful enough to represent induction up to a given countable ordinal α while remaining decidable for satisfiability. The set $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ is a perfect choice for studying this schemata to induction relationship, being that it is an elementary theorem which requires a larger than expected ordinal to formalize. Thus, we ask two main questions: what is the weakest class of schemata that can formalize $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$, and is this class decidable.

Many of the simple extensions of regular schemata have been proved undeciable. However, our prior work Cerna [3] lead to the discovery of a class of schemata (linkable schemata), which can express induction up to the ordinal $\omega\cdot m$, where m is a finite ordinal. Linkable schemata are created by changing the indexing terms used in regular schemata. Aravantinos et al. [1] used what they call linear expressions, essentially a form of Pressburger arithmetic. In Cerna [3] we instead use the following terms:

Definition 1. *Given the alphabet,* $\Sigma = \left\{0, S, \dot{0}, \mathbb{S}, \langle\cdot,\cdot\rangle\right\}$ *we construct the set of* $\mathbb{L}$*-terms,* $\mathbb{L} = \left\langle S^n(0), \mathbb{S}^m(\dot{0})\right\rangle$ *where* S *and* $\mathbb{S}$ *are successor functions,* 0 *and* $\dot{0}$ *are constants, and* n,m *are the number of nested successor functions. We also have two countably infinite distinct sets of variables* $\mathcal{V}_f$ *and* $\mathcal{V}_b$*. The set* $\mathcal{V}_f$ *ranges over the inductively constructed terms* $\mathbb{S}^n(\dot{0})$*, while the set* $\mathcal{V}_b$ *ranges over all the* $\mathbb{L}$*-terms. We will refer to the first part of a* $\mathbb{L}$*-term as the index* *numeral (α is the index numeral in $\langle\alpha,\beta\rangle$) and the second part of a $\mathbb{L}$-term as the numeric value (β is the numeric value in $\langle\alpha,\beta\rangle$).*

Definition 2. *Given a term $\langle\alpha,\beta\rangle \in \mathbb{L}$, we define its cardinality within $\mathbb{N}$ as $|\langle\alpha,\beta\rangle| = \{\ \#\ of\ \mathbb{S}\ in\ \beta\ \}$.*

Representing the $\mathbb{L}$-term $\langle\alpha,\beta\rangle$ as an ordinal is simply $\omega\cdot\alpha+\beta$. Thus, intuitively, it seems as if $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ can be represented if we create a new set of variables ($\mathcal{V}_n$) that ranges over the index numerals. This seems to allow a representation of the ordinal $\omega^2\cdot m$. However, the inner schema in the nesting is repeated, thus these new schemata are no more powerful than linkable schemata. To get around this repetition problem we extend the term language of linkable schemata. We add functions from the set $\mathbb{F} = \{f | f : \mathbb{L} \rightarrow \mathbb{L}\ \&\ \exists\lambda\forall\alpha|f(\alpha)| \leq |\lambda|\}$, e.g $f(\langle 3,10\rangle) = \langle 1,5\rangle$. We use the theory $(\mathbb{L},<)$ extended by functions from $\mathbb{F}$ in a schematic Π_0^0-predicate logic. We only allow the predicate $<$ and construct $\leq$ and $=$ from $<$. Aravantinos et al.[1] considered atoms of the form $a < b$ as iterations, e.g $\vee_{i=a}^{b}\top$. We instead consider them as binary predicate symbols.

We can formalize a weaker form of $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$ in this logic, where we check the numbers assigned to the pigeons in canonical order and see if any pigeons are neighbours (Equation 10), i.e. assigned the same number. This formalization requires induction over the ordinal $\omega + r$, where r is a finite ordinal.

$$\left(f(\langle 1, m+\overline{1}\rangle) \leq (\langle 0, m\rangle) \wedge \langle 0,\overline{1}\rangle \leq f(\langle 1,\overline{1}\rangle) \wedge \bigwedge_{i=\langle 1,\overline{1}\rangle}^{\langle 1,m\rangle} f(i) \leq f(i+\langle 1,\overline{1}\rangle) \right) \rightarrow \bigvee_{i=\langle 1,\overline{1}\rangle}^{\langle 1,m\rangle} f(\bar{i}) = f(i+\langle 1,\overline{1}\rangle) \tag{10}$$

It remains open whether or not this method can be extended to formalize $\{\mathrm{PHP}_i\}_{i=0}^{\infty}$. Also, it is not yet known if this method can formalize induction over the ordinal ω^2. Foreseeable extensions of this work include introduction of variables over index numerals and construction of a decision procedure for the new schematic logic, if it is found to be decidable or semi-decidable.

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On Valuations in Gödel and Nilpotent Minimum Logics

Some decades ago, V. Klee and G.-C. Rota [2,3] introduced a lattice-theoretic analogue of the Euler characteristic, the celebrated topological invariant of polyhedra. In [1], using the Klee-Rota definition, we introduce the Euler characteristic of a formula in Gödel logic, the extension of intuitionistic logic via the prelinearity axiom $(\varphi \to \psi) \vee (\psi \to \varphi)$. We then prove that the Euler characteristic of a formula φ over n propositional variables coincides with the number of Boolean assignments to these n variables that satisfy φ. Building on this, we generalise this notion to other invariants of φ that provide additional information about the satisfiability of φ in Gödel logic. Specifically, the Euler characteristic does not determine non-classical tautologies: the maximum value of the characteristic of $\varphi(X_1, \ldots, X_n)$ is 2^n, and this can be attained even when φ is not a tautology in Gödel logic. By contrast, we prove that these new invariants do.

In this talk, we present the aforementioned results and compare what has been obtained for Gödel logic with analogous results for a different many-valued logic, namely, the logic of *Nilpotent Minimum*. This logic can also be described as the extension of Nelson logic by the prelinearity axiom. The latter results are joint work with D. Valota.

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Normalizing N-graphs: why and how
De Oliveira's doctoral work introduced the system of N-Graphs, a multiple-conclusion Natural Deduction system where derivations are written as digraphs. Multiple conclusion versions of Natural Deduction are appealing, as they seem a direct formalization of the way humans reason, entertaining disjoint possibilities, in parallel, and this corresponds, directly, to the idea of multiple conclusions in a Natural Deduction proof. De Oliveira proved soundness and completeness of her graphical system with respect to Gentzen's (system for classical propositional logic) LK, using a soundness criterion similar to the one used for proof-nets in Linear Logic.

Were we only concerned with **provability** in classical propositional logic, the system N-Graph would seem a useless redundancy, as proofs in, e.g., the LK sequent calculus can tell us whether sequents are provable easier than the corresponding proofs in N-Graphs. But we are interested in proofs as abstract objects, and the hope is that N-Graph proofs might tell us about 'intrinsic' characteristics of these proofs, in a way similar to that which other geometric-inspired proofs systems (such as Buss' *logic flow graphs*) tell us about the complexity of proofs.

For this hope to be seen as plausible we need to show that the N-Graph system satisfies its own version of normalization/cut-elimination. Thus, we have introduced a normalization procedure for N-Graphs and prove its termination. Since cycles are an essential component of N-Graphs derivations, we develop a thorough treatment of these, including an algorithm for cycle normalization. Furthermore we discuss the usefulness of cycles in N-Graphs by classifying the different sorts of cycles that may occur in N-Graph multiple conclusion proofs.

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Dynamic Tableaux for Public Announcement Logic
Public Announcement Logic (PAL) is a formal system for reasoning about the knowledge of one or more agents and about the dynamics of their knowledge as facts are publicly announced to all agents. In this paper I present a new

'dynamic' proof system for PAL.

PAL adds one construct to the formal language of epistemic logic—viz. the form $[!\phi]\psi$, which can be read as 'It is the case that ψ after it is publicly revealed that ϕ'. In spite of having a more dynamic language, PAL is interpreted on the same $S5^n$ Kripke models that are often used for interpreting non-dynamic epistemic logics. The semantics for public announcements are nevertheless 'dynamic' in the sense that these announcements are modeled using a notion of 'update model'. Given a model $\mathcal{M}$, the update model $\mathcal{M}|_{!\phi}$ is the model that remains when all worlds (sometimes called states) in which ϕ does *not* hold are removed from $\mathcal{M}$. Specifically, $[!\phi]\psi$ is stipulated to hold in a world w in a model $\mathcal{M}$ if and only if ϕ being true in w in $\mathcal{M}$ (meaning ϕ is a fact) implies that ψ holds in w in $\mathcal{M}|_{!\phi}$.

The proof system presented in this paper is based on semantic tableaux. As such, given a formula ϕ, the proof strategy is to try to construct a model for ϕ. If successful then ϕ is said to be satisfiable; otherwise the negation of ϕ is a theorem of PAL.

One important selling point of semantic tableaux is intuitive clarity, which is afforded by basing the constituents of the proof system on corresponding elements of the semantics in a transparent manner. I have tried to adhere to this principle as I extended semantic tableaux to encompass public announcements. When a public announcement $[!\phi]\psi$ is encountered in a tableau this is interpreted as asserting that if ϕ is true then there is a tableau for ψ that stands in a $\xrightarrow{!\phi}$ relation to the original tableau. This creates a cascade of tableaux that have dynamic interdependencies.

Proof are provided for soundness and completeness. These proofs proceed by mutual induction, owing to the interdependencies of tableaux. It is also demonstrated that if a formula is satisfiable, a model can be found in a finite number of steps.

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The Logic of Determination of Objects (LDO) – a Paraconsistent Logic

The Logic of Determination of Objects (LDO) is a non-classical logic of construction of objects. It contains a theory of typicality. It is described in (Descles, Pascu, 2011). LDO is defined within the framework of Combinatory Logic (Curry, Feys, 1958) with functional types. LDO is inspired by the semantics of natural languages. It captures the following ideas: the mismatch between logic categories and linguistic categories (adjectives, intransitive verbs often represented by unary predicates); the determination as a logic operator (a book, a red book, a book which is on the table); the duality extension – intension; the lack of "typicality"(The French are arrogant). The LDO is an "typed applicative system"in the Curry's sense (Curry, Feys, 1958). It is a triple of: a network of concepts, a set of objects and a type theory. A concept is an operator, an object is an operand (in Curry's sense. With every concept f, the following are canonically associated (Descles,Pascu, 2011):

1. An object called "typical object", τf, which represents the concept f as an object. This object is completely (fully) indeterminate;
2. A determination operator δf, constructing an object more determinate than the object to which it is applied;
3. The intension of the concept f, Int$f$, conceived as the class of all concepts that the concept f "includes", that is, a semantic network of concepts structured by the relation "IS-A";
4. The essence of a concept f, Ess$f$; it is the class of concepts such that they are inherited by all objects falling under the concept f;
5. The expanse of the concept f, Expf, which contains all "more or less determinate objects" to whom the concept f can be applied;
6. A part of the expanse is the extension Ext$f$ of the concept f; it contains all fully (completely, totally) determinate objects such that the concept f applies to.

From the viewpoint of determination, in LDO, objects are of two kinds: "fully (completely, totally) determinate"objects and "more or less determinate"objects. From the viewpoint of some of their properties, LDO captures two kinds of objects: typical objects and atypical objects. The typical objects in Exp$f$ inherit all concepts of Int$f$. The atypical objects in Exp$f$ inherit only some concepts of Int$f$. The LDO contains axioms and rules of inference. Some of the rules decide of the "typicality" of an object as regard with some concept (Descles, Pascu, 2011). In this paper we analyse the nature of these rules issued from the "theory of typicality" of LDO versus the paraconsistence. More precisely, we show that the rule establishing that an object is an atypical object of a concept in the frame of the LDO, is a particular case of the RA1 rule of Da Costa (Da Costa, 1997). We arrive at the following interpretation of the weakening of the principle of contradiction ($\neg(B \wedge \neg B)$) contained by the RA1 rule inside the LDO: an atypical object of a concept can have in their Int-large both h and Nh and in their Int-caract Nh. From the point of vue of managing negation, we can conclude that LDO is a particular case of a paraconsistent logic. For its powerfull of description and especially for its basic notions (to emphasise the distinction between object and concept and between extention and intension), we can state that LDO is a description logic capturing at least one more cognitive feature: the typicality of objects.

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Plurivalent Logics
In this paper, I will describe a technique for generating a novel kind of semantics for a logic, and explore some of its consequences. It would be natural to call the semantics produced by the technique in question 'many-valued'; but that name is, of course, already taken. I call them, instead, 'plurivalent'. In standard logical semantics, formulas take exactly one of a bunch of semantic values. I call such semantics 'univalent'. In a plurivalent semantics, by contrast, formulas may take one or more such values (maybe even less than one). The construction I shall describe can be applied to any univalent semantics to produce a corresponding plurivalent one. In the paper I will be concerned with the application of the technique to propositional many-valued (including two-valued) logics. Sometimes going plurivalent does not change the consequence relation; sometimes it does. I investigate the possibilities in detail with respect to a small family of many-valued logics.

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Equivalence Under Determinacy Between Non-Selfdual Sets and Conciliatory Sets
We assume X, Y are sets of cardinalities at least 2. We write $X^{<\omega}, X^{<\omega^*}, X^{\omega}, X^{\leq\omega}$ for respectively, the sets of finite sequences, non empty finite sequences, infinite sequences, and finite or infinite sequences over X. We equip both X^{ω} and $X^{\leq\omega}$ with the initial segment topology . It is worth noticing that closed subsets of

$X^{\leq\omega}$ are of the form $T \cup [T]$ where T is a tree on X and $[T]$ stands for the set of its infinite branches; whereas the closed subsets of X^ω are those of the form $[T]$. Wadge designed the following game to characterize the continuous mappings as particular strategies [1].

Definition (Wadge) Given any mapping $f : X^\omega \longrightarrow Y^\omega$, the game $\mathbf{W}(f)$ is the two-player game where players take turn picking letters in X for I and Y for *II*, player I starting the game, and player *II* being allowed in addition to pass her turn, while player I is not. After ω-many moves, player I and player *II* have respectively constructed $x \in X^\omega$ and $y \in Y^{\leq^\omega}$. Player *II* wins the game if $y = f(x)$, otherwise player I wins.

Lemma (Wadge) *Given any* $f : X^\omega \longrightarrow Y^\omega$*, II has a winning strategy in* $\mathbf{W}(f) \iff f$ *is continuous.*

Definition Given any mapping $f : X^{\leq\omega} \longrightarrow Y^{\leq\omega}$, the game $\mathbf{C}(f)$ is the same as the game $\mathbf{W}(f)$ except that both players are allowed to pass their turns. Players take turn either picking letters in X for I and Y for *II*, or passing their turns. So that after ω-many moves, player I and player *II* have respectively constructed $x \in X^{\leq^\omega}$ and $y \in Y^{\leq^\omega}$. Player *II* wins the game if $y = f(x)$, otherwise player I wins.

Lemma *Given any mapping* $f : X^{\leq\omega} \longrightarrow Y^{\leq\omega}$*, II has a winning strategy in* $\mathbf{C}(f) \iff f$ *is continuous.*

Definition

1. For $A \subseteq X^\omega$ and $B \subseteq Y^\omega$, the Wadge game $\mathbf{W}(A, B)$ is the same as $\mathbf{W}(f)$, except that *II* wins iff $y \in Y^\omega$ and $(x \in A \iff y \in B)$ hold.

2. For $A \subseteq X^{\leq\omega}$ and $B \subseteq Y^{\leq\omega}$, the conciliatory game $\mathbf{C}(A, B)$ is the same as $\mathbf{C}(f)$, except that *II* wins iff $(x \in A \iff y \in B)$ holds.

We write $A \leq_w B$, resp. $A \leq_c B$ when II has a w.s. in $\mathbf{W}(A, B)$, resp. $\mathbf{C}(A, B)$. The subsets of the form $A \subseteq X^\omega$ that verifies $A \leq_w A^{\complement}$ are traditionally called *selfdudal*, while all the others are called *non-selfudal*.

Martin proved that determinacy implies that $\leq_w$ is a wqo [2], which also implies that $\leq_c$ is a wqo as well. As a consequence, one can define by induction, for $A \subseteq X^\omega$ and $A' \subseteq X^{\leq\omega}$, the following ranking functions:

1. $rk_w(A) = 0$ iff A or $A^{\complement} = \emptyset$, and otherwise $rk_w(A) = \sup\{rk_w(B) \mid B <_w A \wedge B \text{ non-selfdual } \subseteq X^\omega\}$.

2. $rk_c(A') = 0$ iff $A' = \emptyset$ or $A' = X^{\leq\omega}$, and otherwise $rk_c(A') = \sup\{rk_c(B) \mid B <_c A'\}$.

For any $A' \subseteq X^{\leq\omega}$, we write A'_s for the set $A'_s \subseteq (X \cup \{s\})^\omega$ that contains all $x \in (X \cup \{s\})^\omega$ which, once the letter s is removed, turn into some $y \in X^{\leq\omega}$ that verifies $y \in A'$.

Theorem *Assuming AD:*

1. *for any non-selfdual* $A \subseteq X^\omega$*, there exists* $F \subseteq X^{<\omega}$ *such that* $rk_w(A) = rk_c(A \cup F)$*.*

2. *For any* $A' \subseteq X^{\leq\omega}$ $rk_w(A'_s) = rk_c(A')$*.*

Corollary *The two hierarchies – the one induced by* $\leq_w$ *on non-selfdual subsets, and the one induced by* $\leq_c$ *on conciliatory sets – are isomorphic.*

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Proof theory for the theory of graded consequence

The term 'proof theory' refers to the syntactic presentation of the rules, altogether governing the way of drawing conclusion from a set of premises. There are different versions for presenting the proof theory of a logical system e.g. Hilbert type axiomatization, Gentzen's natural deduction and sequent calculus presentation etc. Existing literature on proof theoretic presentation has the following features which may raise uneasiness.

- Interpretations of object level connectives and meta level connectives are somehow blurred with each other. Like in LJ (sequent calculus for Intuitionistic logic) and LK (sequent calculus for Classical logic) with the help of the structural rules viz., contraction-left, weakening-left, cut, &-left and &-right one can obtain $\delta, \phi \vdash \psi$ if and only if $\delta \& \phi \vdash \psi$. This indicates that the usual practice in LJ, LK is to see ',' and object level conjunction '&' equivalently. Similar is the attitude towards the meta-implication $\vdash$ and object level implication $\supset$. In substructurals logics though some of the structural rules remain absent, with the help of a modified version of &-left this equivalence between ',' and '&' is retained.
- Meaning of ',' in the left hand side of a sequent in LK is different from the meaning of the same in the right hand side of a sequent.
- Interpretation of rules only focuses on the meaning of the object level connectives; meaning of the meta-linguistic connectives remains outside the realm of the concern of logic.

Keeping eye on these we would like to present the theory of graded consequence, which provides a general set up for logics dealing with uncertainties, generalizing the proof theory for classical as well as intutionistic logic. In the context of graded consequence, given a set of formulae X and a single formula α, $gr(X \mid\sim \alpha)$ denotes the degree/value of the meta level sentence 'α is a consequence of X' and $\mid\sim$ is a fuzzy relation between $P(F)$, the power set of formulae and F, the set of formulae, representing the notion of consequence in many-valued context. Generalizing Gentzen's notion of consequence relation Chakraborty axiomatized a graded consequence relation ($\mid\sim$) with the following axioms.

(GC1) if $\alpha \in X$ then $gr(X \mid\sim \alpha) = 1$ (Reflexivity/Overlap).

(GC2) if $X \subseteq Y$ then $gr(X \mid\sim \alpha) \leq gr(Y \mid\sim \alpha)$ (Monotonicity/Dilution).

(GC3) $\inf_{\beta \in Y} gr(X \mid\sim \beta) *_m gr(X \cup Y \mid\sim \alpha) \leq gr(X \mid\sim \alpha)$ (Cut).

These axioms are the generalized versions of the classical conditions viz., overlap, dilution and cut where '$*_m$' and 'inf' are the operators for computing the meta-linguistic 'and' and 'for all' present in the classical version of the above mentioned conditions.

The proof theory for the theory of graded consequence starts with an initial context $\{T_i\}_{i \in I}$ and a meta level algebraic structure $(L, *_m, \rightarrow_m, 0, 1)$. Now depending on the object language connectives and corresponding rules the object level algebraic structure, giving interpretation to the connectives, takes form.

In this presentation we shall try to show that, in one hand, the presence of a particular rule concerning a particular connective induces some interrelations between the algebraic interpretation of the object level and meta level of a logic, and on the other, algebraic structures presumed for both the levels of a logic determine the availability of rules in the logic. This study gives a new way of looking to the inherent meaning of the logical connectives overcoming the uneasiness pointed at the begining of this write-up.

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A plea for β-conversion by value

This paper solves, in a logically rigorous manner, a problem discussed in 2004 paper by Stephen Neale and originally advanced as a counterexample to Chomsky's theory of binding. The example we will focus on is the following. John loves his wife. So does Peter. Therefore, John and Peter share a property. Only which one? There are two options. (1) Loving John's wife. Then John and Peter love the same woman (and there is trouble on the horizon). (2) Loving one's own wife. Then, unless they are married to the same woman, both are exemplary husbands. On the *strict* reading of "John loves his wife, and so does Peter" property (1) is the one they share. On the *sloppy* reading, property (2) is the one they share.

The dialectics of this contribution is to move from linguistics through logic to semantics. An issue originally bearing on binding in linguistics is used to make a point about β-conversion in the typed λ-calculus. Since the properties (1) and (2) as attributed to John are distinct, there is room for oscillation between the sloppy and the strict reading. But once we feed the formal renditions of attribution of these two properties to John into the widespread λ-calculus for logical analysis, a *logical* problem arises. The problem is this. Their respective β-*redexes* are distinct, for sure, but they share the same β-*contractum*. This contractum corresponds to the strict reading. So β-conversion predicts, erroneously, that two properties applied to John β-reduce to one. The result is that the sloppy reading gets squeezed out. β-reduction blots out the anaphoric character of 'his wife', while the resulting contractum is itself β-expandable back into both the strict and the sloppy reading. Information is lost in transformation. The information lost when performing β-reduction on the formal counterparts of "John loves his wife" is whether the property that was applied was (1) or (2), since both can be reconstructed from the contractum, though neither in particular. The sentence "John loves his wife, and so does Peter" ostensibly shows that the λ-calculus is too crude an analytical tool for at least one kind of perfectly natural use of indexicals.

The problematic reduction and its solution will both be discussed within the framework of Tichý's Transparent Intensional Logic. Tichý's TIL was developed simultaneously with Montague's Intensional Logic. The technical tools of the two disambiguations of the analysandum will be familiar from Montague's IL, with two important exceptions. One is that we λ-bind separate variables $w_1, ..., w_n$ ranging over possible worlds and $t_1, ..., t_n$ ranging over times. This dual binding is tantamount to *explicit intensionalization* and *temporalization*. The other exception is that *functional application* is the logic both of extensionalization of intensions (functions from possible worlds) and of predication.

In the paper we demonstrate that, and how, the λ-calculus is up for the challenge, *provided* a rule of β-*conversion by value* is adopted. The *logical contribution* of the paper is a generally valid form of β-reduction *by value* rather than *by name*. The *philosophical application* of β-reduction by value to a context containing anaphora is another contribution of this paper. The standard approach to VP ellipsis based on λ-abstracts and variable binding can, thus, be safely upheld. Our solution has the following features. First, unambiguous terms and expressions with a pragmatically incomplete meaning, like 'his wife' or "So does Peter", are analyzed in *all* contexts as expressing an *open* construction containing at least one free variable with a fixed domain of quantification. Second, the solution uses β-conversion *by value*, rather than conversion *by name*. The generally valid rule of β-conversion *by value* exploits our *substitution method* and we show that the application of this rule does not yield a loss of analytic information. Third, the substitution method is applied to sentences containing anaphora, like 'so does' and 'his', in order to pre-process the meaning of the incomplete clause. Our declarative procedural semantics also makes it straightforward to infer that there is a property that John and Peter share.

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Computational interpretations of some substructural logics

Substructural logics are a wide family of logics obtained by restricting or rejecting some of Gentzen's structural rules [1]. Therefore, they are naturally connected with Gentzen's sequent calculus.

A computational interpretation of the intuitionistic implicative fragment of the sequent calculus with explicit structural rules, namely weakening and contraction was proposed in [2]. This formalism called *resource control lambda Gentzen calculus* contains operators for erasure and duplication, corresponding to explicit weakening and contraction, as well as a specific syntactical category of contexts, that enables right associativity of application, which is the distinctive feature of sequent-style lambda calculi. The operational semantics is based on cut-elimination together with an optimisation of resources, thus the reduction rules propagate duplication into expressions, while extracting erasure out of expressions. In a Curry-Howard correspondence, the simply typed resource control lambda Gentzen calculus yields the G1 variant of the implicative sequent LJ, with implicit permutation (i.e. exchange).

Here, we use the resource control lambda Gentzen calculus as a starting point for obtaining computational interpretations of implicative fragments of some substructural logics, namely relevant and BCK logic. The corresponding formal calculi are obtained by syntactic restrictions, along with modifications of the reduction rules and the type assignment system. The proposed approach is simpler than the one obtained via linear logic.

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A semantics for counterfactuals based on formal fuzzy logic

The logical analysis of counterfactual conditionals (conditionals with a false antecedent) has been for a long time one of the important issues of philosophical logic. Consider the sentences: *If Oswald didnt shoot Kennedy, someone else did* and *If Oswald had not shot Kennedy, someone else would have.* The first one is definitely true, but the second one can be true or false. However, the standard logical analysis makes both the sentences trivially true, contrary to our linguistic intuitions. The most influential logical analysis of counterfactuals was provided independently by David Lewis (1973) and Robert Stalnaker (1968). Their solution is based on the notion of similarity of possible worlds with respect to the actual world. According to Lewis, a counterfactual "A implies C" is true in the actual world (with respect to a given similarity ordering) iff the worlds in which both A and C hold are closer to the actual world than the worlds in which A and non C hold. The intuitive notion of closeness of worlds can be formally rendered in a number of ways, and the rendering impacts the resulting properties of the logic of counterfactuals. Lewis himself (1973) provides a variety of counterfactual logics differing in the properties of the similarity ordering. One of the frameworks in which the general concepts of closeness or similarity have been studied is that of formal fuzzy logic (Hájek 1998). Fuzzy logics, sometimes called logics of comparative truth, are designed to capture such notions as order on (more than two) truth degrees and can therefore be used for modeling closeness or similarity, which can in its framework be expressed by comparing the truth degrees of the proposition "x is close (or similar) to y". Thus in formal fuzzy logic, similarity becomes simply a binary predicate that satisfies some natural constraints (reflexivity, symmetry, transitivity, etc.). The notion of a fuzzy similarity relation was originally introduced in fuzzy set theory, however, use its axiomatization in first-order fuzzy logic (e.g., Gerla 2008, Běhounek et al. 2008), which is more suitable for formal considerations. By means of fuzzy similarity, Lewis' semantics of counterfactuals can be easily formalized in fuzzy logic. Besides a new perspective on the semantics of counterfactuals, the formalization within the framework of fuzzy logic has several further merits: (i) It automatically accommodates graded counterfactual propositions (such as If John were tall, then he would have played basketball), where tall is a graded predicate whose truth value reflects John's height in cm's or feet). Arguably, most propositions of natural language are gradual in this sense, rather than bivalent. (ii) The many-valuedness of fuzzy logics admits the gradual truth of counterfactuals, capturing the intuition that some counterfactuals seem truer than others. (iii) The structure of truth degrees makes it possible not only to compare, but also 'measure' the 'distance' between possible worlds using not only real numbers, but also abstract degrees of distance yielded by algebraic semantics of fuzzy logics. The aim of this article is to present an advanced version of the analysis of counterfactuals in the fuzzy framework provided in (Běhounek, Majer 2011), compare it to the original Lewis' system and discuss its relation to other systems based on many-valued semantics, in particular to probabilistic approaches to counterfactuals (e.g. Leitgeb 2012).

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Identity as a primitive in equational hybrid logic

The relation of identity is usually understood as the binary relation which holds between any object and itself and which fails to hold between any two distinct objects. Due to the central role this notion plays in logic, it is interesting in how to define it using other logical concepts and which kind of logic is required, or else in the opposite scheme; namely, in the definition of the other logical concepts (connectives and quantifiers) in terms of the identity relation, using also abstraction.

Respect to the question of equality being defined in terms of other logical symbols, it is known that in first order logic identity cannot be defined with the other logical concepts. In the case of second order logic identity can be defined by Leibniz's principle, and then, there is a formula to define the equality symbol for individuals using the rest of the logical symbols and the relation defined by this formula is 'genuine' identity in any standard second order structure. However in second order logic with non-standard structures, there is no guarantee that the equivalence relation defined by Leibniz's principle is identity [3].

In this work the reverse question is posed and affirmatively answered, that is: Can we define with only equality and abstraction the remaining logical symbols? It is known that the identity relation on the set of truth-values, *true* and *false*, serves as the denotation of the biconditional and is usually defined using other connectives, but our question here is how to use identity to obtain the rest. We know that in propositional logic we are not able to define connectives such as conjunction whose truth table shows a value *true* on an odd number of lines, not even with equality and negation. We can allow quantification over propositional variables of all types (including second order propositional variables) and then all connectives are defined with equality and quantifier. Theories of this kind were studied by Lésniewski and received the name of protothetic. But what about quantifiers? Can they be defined with equality? The idea of reducing the other concepts to identity is an old one which was tackled with some success in 1923 by Tarski [6], who solved the case for connectors; three years later Ramsey [5] raised the whole subject; it was Quine [4] who introduced quantifiers in 1937. It was finally answered in 1963 by Henkin [1], where he developed a system of propositional type theory. Later, in 1975, Henkin wrote a whole paper [2] on this subject in a volume completely devoted to identity.

This work is developed in the context of equational hybrid logic (i.e. a modal logic with equations as propositional atoms enlarged with the hybrid features: nominals and the hybrid @ operator). We will take equality, modal operator of possibility, @ operator and λ abstraction as primitive symbols and we will show that all of the remaining logical symbols can be defined.

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Rasiowan deductive systems

We deal with monotone structural deductive systems in the framework of an abstract propositional language $\mathcal{L}$. These systems fall into several overlapping classes, forming a hierarchy. Along with well-known classes of deductive systems such as *implicative*, [3] *Fregean* [1] and *equivalential* [2] systems, we consider new classes of Rasiowan and weakly implicative systems. The latter class is purely axillary, while the former is central in our discussion. Our analysis of the notion of a Rasiowan system leads to the concept of a Lindenbaum-Tarski algebra which, under some natural conditions, is a free algebra in a variety closely related to the deductive system in focus.

Let $\mathfrak{F}_{\mathcal{L}}$ be the algebra of the $\mathcal{L}$-formulas. A system $\mathcal{S}$ is called *Rasiowan* if for every $\mathcal{S}$-theory $\Sigma_{\mathcal{S}}$, the latter is a congruence class in $\mathfrak{F}_{\mathcal{L}}$ with respect to $\Theta(\Sigma_{\mathcal{S}})$, where $\Theta(\Sigma_{\mathcal{S}})$ is the congruence in $\mathfrak{F}_{\mathcal{L}}$ generated by $\Sigma_{\mathcal{S}}$. We show relationships between the class of Rasiowan systems and the classes of implicative, Fregean and equivalential systems in terms of inclusion and non-inclusion relations.

Then, given a Rasiowan system $\mathcal{S}$, we focus on $\mathfrak{F}_{\mathcal{L}}/\Theta(\boldsymbol{T}_{\mathcal{S}})$, where $\boldsymbol{T}_{\mathcal{S}}$ is the set of the theorems of $\mathcal{S}$. Denoting $\mathbf{1} = \boldsymbol{T}_{\mathcal{S}}$, we define the algebra $\mathfrak{F}_{\mathcal{S}} =< \mathfrak{F}_{\mathcal{L}}/\Theta(\boldsymbol{T}_{\mathcal{S}}), \mathbf{1} >$ and the following variety:

$$K_{\mathcal{S}} = \{\mathfrak{A} \mid \mathfrak{A} \models A = \mathbf{1}, A \in \boldsymbol{T}_{\mathcal{S}}\}.$$

A Rasiowan system is called *implicational* if there is a two-variable $\mathcal{L}$-formula, $I(p,q)$, such that for any algebra $\mathfrak{A} \in K_{\mathcal{S}}$, any $\mathcal{L}$-formulas A and B, and any valuation v in $\mathfrak{A}$,

$$v(A) = v(B) \Longleftrightarrow v(I(A,B)) = \mathbf{1}.$$

We prove that for any implicational Rasiowan system $\mathcal{S}$, the algebra $\mathfrak{F}_{\mathcal{S}}$ is a free algebra with the free generators $p/\Theta(\boldsymbol{T}_{\mathcal{S}})$, for every $\mathcal{L}$-variable p, in the variety $K_{\mathcal{S}}$.

Finally, we prove that, under some conditions on $I(p,q)$ related to an implicational Rasiowan system, this system is equivalential with respect to $\{I(p,q), I(q,p)\}$. However, even if a system is Rasiowan and equivalential, it is not necessarily imlicational; so it is unclear, whether $\mathfrak{F}_{\mathcal{S}}$ is free or even functionally free over $K_{\mathcal{S}}$.

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Constructing Logics oriented on Computing: Philosophical and Mathematical Aspects

Computing is an important and rapidly growing area that attracts careful attention of philosophers and logicians. Numerous results were obtained (see, for example, chapters relevant to the topic in [1]). Still, many problems call for further investigation. Among them are problems concerning *foundations of computing*, its *main notions*, and *logics* for reasoning in the area. Computing (as a science) is relatively young. As such, it borrows its foundations from those branches of sciences it is based on, primarily from mathematics, logic, linguistics, and philosophy. But coming into mature age, computing is searching for its own foundations, which should state its self-dependence and provide its self-development. The etymology of the term ‘computing’ indicates that the notion of *computability* should be among basic notions in the area. This notion was intensively studied within a Church–Turing approach. Still, many problems demonstrate that this approach is restricted and new notions of *generalized computability* that integrates various its aspects are required. Also, *logics* that support reasoning within this area should be developed. This talk aims to present our approach and results on constructing logics oriented on computing. We start with the notion of *generalized computable function* that is presented by a *program* in a certain *formal language* [2]. Based directly on such formal program models we develop *program logics* of various abstraction and generality levels. We distinguish three levels of development: 1) *philosophical*, 2) *scientific* (oriented on computing), and 3) *mathematical* levels. The *philosophical level* should provide us with general laws of development and with a system of categories that form a skeleton of such development. At this level we are based on Hegel’s Logic [3] because its main principle − *development from abstract to concrete* − nicely corresponds to methods of program development. Within Hegel’s Logic (subdivided into Being, Essence, and Notion) we identify categories that are important to program development: *subject and object; abstract and concrete; internal and external; quality, quantity, and measure; essence and phenomenon; whole and part; content and form; cause and effect; goal and tool; singular, universal, and particular;* etc. At the *scientific level* we follow the general development scheme and make particularization of categories obtaining computing notions (being finite notions in Hegel’s terminology) such as *user, problem, information, program, data, function, name, composition, description* etc. interconnected with such relations as *adequacy, pragmatics, computability, explicativity, origination, semantics, syntax, denotation,* etc. We use *triads* (*thesis – antithesis – synthesis*) to develop these notions later combined into development *pentads*. These notions are considered in integrity of their *intensional* and *extensional* aspects. These aspects are treated as particularization of Hegel's categories *universal–particular–singular*. At the *mathematical level* we formalize the above-mentioned notions in integrity of their intensional and extensional aspects paying the main attention to the notions of 1) *data*, 2) *function* (specified by its *applicative* properties), and 3) *composition* (considered as function combining mean). Thus, at this level we aim to develop the theory of *intensionalized program notions* and *intensionalized logics* based on this theory. The initial fragments of this theory are described in [4]. Let us admit that *conventional set theory* is considered as one component of this intensionalized theory. Though we aim to develop intensionalized logics, we also study their extensional components which are built according to mathematical traditions. Thus, a number of logics oriented on *partial* and *non-deterministic functions and predicates without fixed arity* were defined and investigated; corresponding *calculi* were constructed, their *soundness* and *completeness/incompleteness* were proved [5, 6]. We would like to note that *reasoning rules* for such logics *differ from the classical ones.* For example, *modus ponens fails* because of partiality of predicates, the *law* $(\forall x\Phi) \to \Phi$ *fails* because of partiality of data, etc; therefore reasoning within computing area indeed requires new logics. So, the proposed scheme of logic development at three levels (philosophical, scientific, and mathematical) seems to be fruitful and permits to construct a hierarchy of new logics that reflect the main aspects of computing.

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Evolutionary databases and quasi-truth: some relations via model theory

A relational database can be considered as a finite collection of finite relations in which the information is stored. If a database may be updated, in the sense to add, modify or remove some information, in such a way that contradictions are avoided; such databases are called evolutionary. However, two local databases can be mutually contradictory. From this paraconsistent environment arises naturally a relation with the concept of quasi-truth introduced by da Costa and collaborators (cf. [1]), in which the truth, in the field of Philosophy of Science, is given according to a context, i.e., the truth several times is restricted to certain circumstances, because this it could be partial. Thereby the Tarskian model theory of quasi-truth is paraconsistent. This talk describes some relations of the model theory of quasi-truth to the semantics of evolutionary databases introduced in [2].

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Automated Support for the Investigation of Paraconsistent and Other Logics

Non-classical logics are often introduced by adding Hilbert axioms to known systems. The usefulness of these logics, however, heavily depends on two essential components. The first is a corresponding *analytic calculus* where proof search proceeds by a stepwise decomposition of the formula to be proved. Such calculi are the key for developing automated reasoning methods. The second component is an intuitive *semantics*, which can provide insights into the logic, e.g., proving its decidability.

In this talk, I will present [3] where we provide a procedure for an *automatic generation* of analytic sequent calculi and effective semantics for a large class of Hilbert systems. The Hilbert systems are obtained (i) by extending the language of the positive fragment of classical logic with a finite set of new unary connectives, and (ii) by adding to a Hilbert axiomatization of classical logic axioms over the new language of a certain general form. The procedure then works in two steps: First, we introduce a sequent calculus equivalent to the Hilbert system. Second, we construct a semantics in the framework of partial non-deterministic finite-valued matrices [1] to reason about analyticity of the calculus.

Our method applies to infinitely many logics, which include a family of paraconsistent logics known as C-systems [4,2], as well as to other logics for which neither analytic calculi nor suitable semantics have so far been available. This approach is a concrete step towards a systematization of the vast variety of existing non-classical logics and the development of tools for designing new application-oriented logics.

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Hypothetical Logic of Proofs

The Logic of Proofs (LP) [1] is a refinement of modal logic introduced by Artemov in 1995 which has recently been proposed for explaining well-known paradoxes arising in the formalization of Epistemic Logic. Assertions of knowledge and belief are accompanied by justifications: the formula $[\![t]\!]A$ states that *proof witness* t is "reason" for knowing/believing A. Also, LP is capable of reflecting its proofs in the object-logic: $\vdash A$ implies $\vdash [\![t]\!]A$, for some t. This suggests that computational interpretations in terms of the Curry-de Bruijn-Howard isomorphism could yield programming languages supporting a uniform treatment of programs *and* type derivations. Although some avenues in this direction have been explored (eg. certifying mobile computation [3] and history-aware computation [2]), they have been centred on intuitionistic fragments of LP. This work proposes to extend this analysis to full LP, which is based on classical logic. To this effect, we define the Hypothetical Logic of Proofs (HLP).

The set of formulas and proof witnesses of HLP is defined by the following syntax:

Formulas: $A, B ::= P \mid \bot \mid A \supset B \mid [\![s]\!]A$
Proof witnesses: $r, s, t ::= x^A \mid v^A \mid \lambda x^A.s \mid s \cdot t \mid !s \mid \text{LETB}\, v^A \text{ BE}\, r, s \text{ IN}\, t \mid [\alpha^A]s \mid (\mu\alpha^A.s) \mid s + t$

Judgements take the form $\Theta; \Gamma; \Delta \vdash A \mid s$ where $\Theta = {v_1}^{B_1}, \ldots, {v_m}^{B_m}$, $\Gamma = {x_1}^{C_1}, \ldots, {x_n}^{C_n}$ and $\Delta = {\alpha_1}^{D_1}, \ldots, {\alpha_k}^{D_k}$. Hypotheses are represented by three kinds of variables: truth variables (x^A claims that A is *true*), validity variables (v^A claims that A is *valid*), and falsehood variables (α^A claims that A is *false*). The inference schemes of HLP are:

$$\frac{}{\Theta; \Gamma, x^A; \Delta \vdash A \mid x^A}\,\mathsf{Var} \qquad \frac{}{\Theta, v^A; \Gamma; \Delta \vdash A \mid v^A}\,\mathsf{VarM}$$

$$\frac{\Theta; \Gamma, x^A; \Delta \vdash B \mid s}{\Theta; \Gamma; \Delta \vdash A \supset B \mid \lambda x^A.s}\supset \mathsf{I} \qquad \frac{\Theta; \Gamma; \Delta \vdash A \supset B \mid s \quad \Theta; \Gamma; \Delta \vdash A \mid t}{\Theta; \Gamma; \Delta \vdash B \mid s \cdot t}\supset \mathsf{E}$$

$$\frac{\Theta; \cdot; \cdot \vdash B \mid s \quad \Theta; \cdot; \cdot \vdash s \equiv t \colon B}{\Theta; \Gamma; \Delta \vdash [\![t]\!]B \mid !t}\,\Box\mathsf{I} \qquad \frac{\Theta; \Gamma; \Delta \vdash [\![r]\!]A \mid s \quad \Theta, v^A; \Gamma; \Delta \vdash C \mid t}{\Theta; \Gamma; \Delta \vdash C\{v^A \leftarrow r\} \mid \text{LETB}\, v^A \text{ BE}\, r, s \text{ IN}\, t}\,\Box\mathsf{E}$$

$$\frac{\Theta; \Gamma; \Delta \vdash A \mid s}{\Theta; \Gamma; \Delta \vdash A \mid s + t}\,\mathsf{PlusL} \qquad \frac{\Theta; \Gamma; \Delta \vdash A \mid t}{\Theta; \Gamma; \Delta \vdash A \mid s + t}\,\mathsf{PlusR}$$

$$\frac{\Theta; \Gamma; \Delta, \alpha^A \vdash A \mid s}{\Theta; \Gamma; \Delta, \alpha^A \vdash \bot \mid [\alpha^A]s}\,\mathsf{Name} \qquad \frac{\Theta; \Gamma; \Delta, \alpha^A \vdash \bot \mid s}{\Theta; \Gamma; \Delta \vdash A \mid \mu\alpha^A.s}\,\mathsf{NAbs}$$

Note that each inference scheme updates the associated proof witness of the inferred judgement. Moreover, this witness is *reflected* in the logic via $\Box$I. The additional hypothesis $\Theta; \cdot; \cdot \vdash s \equiv t \colon B$ in $\Box$I is required for subject reduction, which would otherwise fail. This follows from the fact that proof witnesses are reflected in the logic and that normalisation equates them. The schemes PlusL and PlusR correspond to LP's multiple-conclusion nature (cf. axioms $[\![s]\!]A \supset [\![s + t]\!]A$ and $[\![t]\!]A \supset [\![s + t]\!]A$ of LP).

We show that HLP can prove all LP-theorems – hence, all classical tautologies – and we provide a translation from HLP to LP which preserves derivability. Also, we define a term assignment, where terms are proof witnesses with additional annotations, and define reduction rules to model proof normalization by means of term reduction. The resulting reduction system extends the λ-calculus, as well as Parigot's $\lambda\mu$-calculus [4]. Finally, we address some fundamental properties of the metatheory, including type preservation, normalisation and confluence.

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Update As Evidence
Like modal logics, *justification logics* are epistemic logics that provide means to formalize properties of knowledge and belief. Modal logics use formulas $\mathsf{K}A$ to state that A is known (or believed), where the modality K can be seen as an *implicit* knowledge operator since it does not provide any reason why A is known. Justification logics operate with *explicit* evidence for an agent's knowledge using formulas of the form $t : A$ to state that A is known for reason t. The evidence term t may represent a formal mathematical proof of A or an informal reason for believing A such as a public announcement or direct observation of A.

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A Game-Theoretic Decision Procedure for the Constructive Description Logic $c\mathcal{ALC}$
In recent years, several languages of non-classical description logics have been introduced to model knowledge and perform inference on it. The *constructive* description logic $c\mathcal{ALC}$ deals with uncertain and dynamic knowledge and is therefore more restrictive than intuitionistic $\mathcal{ALC}$. Specifically, the intuitionistic axioms $\neg\Diamond_r\bot$, $\Diamond_r(A \vee B) = \Diamond_r A \vee \Diamond_r B$ and $(\Diamond_r A \supset \Box_r B) \supset \Box_r(A \supset B)$ are not valid anymore in $c\mathcal{ALC}$. [4]

We make use of a game-theoretic dialogue-based proof technique that has its roots in philosophy to explain reasoning in $c\mathcal{ALC}$ and its modal-logical counterpart CK_n.

The game-theoretic approach we build on has been introduced by Kuno Lorenz and Paul Lorenzen [3] and later been extended for intuitionistic and classical modal logics by Shahid Rahman and Helge Rückert [5]. It contains a selection of *game rules* that specify the behaviour of the proof system. Other logics can be adapted or even constructed by changing the underlying game rules. For instance, there are rules introduced by Andreas Blass for Linear Logic [1].

The game-theoretic presentation can be considered as an alternative technique to tableau-based proofs, emphasising interaction semantics. We formalize the intuitive rules given by Laurent Keiff [2], making these rules more concrete from a mathematical perspective and thereby provide an adequate semantics for $c\mathcal{ALC}$. It turns out that the interaction semantics provides the right level of constructiveness to explain the absence of the axiom schemes stated above.

The game interpretation makes showing validity more complex but in return we have a philosophical approach that might make it possible to find out more about related constructive theories and that provides a rich playground for extending or altering the underlying semantics.

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On Granules and Granular Semantics for Multi-Valued Logics

The term "*granule*" is originated from Latin word *granum* that means grain, to denote a small particle in a real or imaginary world. The granules differ one from another by their nature, complexity, size, abstraction level.

Typical interpretations of granules are: part of the whole, sub-problem of the problem, cluster, piece of knowledge, variable constraints. There exists some hierarchy of granulation levels: for instance, decomposition may be seen as a transition to a lower level of granulation and composition inversely.

Conventional formal approaches aimed to construct granules are based on: subsets, intervals, relations, partitions, hierarchies, clusters, neighborhoods, bags, graphs, etc. Granulation is often induced by non-standard sets – relative mathematical objects with flexible boundaries (boundary zones) depending on observer's information level. Typical examples of non-standard sets are Zadeh's fuzzy sets, Pavlak's rough sets, Pedrycz's shadowed sets, Narin'yani's sub-definite sets, over-definite sets, and so on. A direct precursor of modern granulation theory was Lesniewski's mereology – the theory of parthood relations. In mereology instead of conventional membership relation between elements and sets the relation of "being a part" is taken. In short, mereology: 1) makes emphasis on the wholistic interpretation of set as a "collective class"; 2) employs mainly parthood (reflexive, anti-symmetric, transitive) relations, i.e. partial orderings; 3) rejects both membership relation and empty set. Here singular expressions are not used.

The procedure of information granulation is mainly reduced to specifying two regions: certainty region and uncertainty region, i.e. it can be based on set inclusion. By considering grades of uncertainty we obtain a flou set (ensemble flou by Gentilhomme) as a family of n nested sets A_i, $i = 1, ..., n$, $A_1 \subseteq ... \subseteq A_n$ in the universe U.

The generalization of this approach leads to an infinite family of mappings $[0, 1] \to 2^U$. Furthermore, if we take instead of $[0, 1]$ an arbitrary set of parameters Q and specify a function $\phi : Q \to 2^U$, then we have a pair (ϕ, Q) called soft set over the universe U. So the development of granular logical semantics in a wide sense supposes the use of subsets as truth values. It corresponds to Dunn-Belnap's strategy of constructing logical semantics, where both gaps and gluts were possible. Here the term "glut" stands for granular truth value "both true and false" (it means the generalization of singular truth values of classical logic) and the gap is viewed as none (neither true nor false – it extends bivalence principle of classical logic).

Let V be the set of logical truth values. Following Dunn, we shall take as truth values not only elements $v \in V$, but also any subsets of V including empty set $\emptyset$. In other words, a direct transition from the set V to its power set 2^V is performed. In his turn, L. Zadeh proposed further extension of this approach by introducing fuzzy truth values $v_f \in [0, 1]^V$ and linguistic truth labels.

Here we point out that any subset $g_\Lambda \in 2^V$ (or more generally $g^*_\Lambda \in [0, 1]^V$) viewed as an extended truth value is a granule. In this context the generalized matrix introduced by R. Wojcicki may be rewritten in the form $LM_G = \langle L, G \rangle$, where L is a lattice and $G \subseteq 2^L$ is a granular structure obtained as a family of sublattices of the lattice L. It is worth stressing that Vasiliev's and Da Costa's basic paraconsistent semantics are of special interest for granular logics.

Now let us apply Birkhoff-Jaskovski's approach to constructing product lattices and product logics as a way of generating logical granules. In the framework of algebraic logic various product logics are often expressed by direct product lattices. The applications of product semantics vary from Rescher's modalized truth to concerted truth for two experts (or sensors).

A natural way to produce complex truth/information granules consists in constructing Ginsburg's bilattices, in particular, logical bilattices. In the last section of this paper we introduce the concept of Belnap's sensor corresponding to the simplest "four" bilattice and investigate combinations of such sensors. Let us consider the sensor able to measure some parameter of inspected process and to interpret obtained information. Such a sensor equipped with four-valued data mining semantics will be referred as Belnap's sensor. The data obtained from this sensor are granulated according to four values:

- T – "measured truth" ("norm" – sensor data are located in a "green zone");
- F – "measured falsehood" ("out of norm" – sensor data are located in "red zone");
- B – "measured ambiguity" ("partial fault" – sensor data are located in "yellow zone");
- N – "total uncertainty" (sensor resources are exhausted or sensor is "sleeping").

Now let us interpret basic granular truth values for two communicating sensors:

- T1T2 – "measured concerted truth" (data issues from both sensors take norm values);
- F1F2 – "measured concerted falsehood" (both sensors data are out of norm that witnesses for failure stata);
- T1B2 $\sim$ B1T2 – "measured partial contradiction as the first-order fault" (the first sensor shows a norm value and the second sensor indicates partial fault, and vice versa);
- T1N2 $\sim$ N1T2 – "measured partial truth with uncertainty" (the first sensor indicates a norm value and the second sensor sleeps, and vice versa);
- T1F2 $\sim$ F1T2 – "measured full contradiction" (the first sensor indicates a norm value, and the second sensor informs about failure state, and vice versa);
- B1B2 – "measured concerted ambiguity" (the data from both sensors inform about first order fault);
- N1N2 – "total uncertainty" (the resources if both sensors are exhausted or both sensors sleep;
- F1B2 $\sim$ B1F2 – "measured partial contradiction as the second-order fault" (the first sensor indicates an out of norm value and the second sensor indicates a fault), and vice versa;
- F1N2 $\sim$ N1F2 – "measured partial falsehood with uncertainty" (the first sensor indicates a failure value and the second sensor sleeps), and vice versa.

As a result the map of extended anti-designated values for two Belnap's sensors combination is constructed. The procedures of solving partial contradictions in sensor networks are suggested.

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The Perspective of Non-Linear Multi-Valued Logics: Extended Logical Matrices

Current advances in multi-valued logics are closely related to the arrival of abstract algebraic approach rising to Tarski and Rasiowa. In the context of Beziau's universal logic viewed as an attempt to construct a general theory of logics based on mathematical structures, we reconsider such important algebra-related concepts as logical matrices and generalized matrices. A further development of multi-valued logics supposes the joint use of algebra, geometry and mathematical analysis tools.

Many existing generalizations of conventional Lukasiewicz-Tarski's logical matrix focus on the extensions of its components: transition from ordinary truth set V to power truth set 2^V (underlying Dunn-Belnap's semantics, $[0,1]^V$ (introduced by Zadeh) or L^V (suggested by Goguen, where L is a lattice); specification of basic operations over logical values by taking functional-axiomatic approach and using families of parameterized negations, implications, triangular norms and co-norms; rethinking of designated values with employing interval and fuzzy designated values. Here the most interesting proposals are generalized matrix by Wojcicki and non-deterministic matrix by Avron and Lev.

We introduce a natural complementation of standard logical matrix components by the concept of logical (or more generally, semiotic) space. It corresponds to current tendency of considering practical turn in logic and specifying the dependencies between logics and ontologies. The concept of logical space was introduced by Wittgenstein in his "Tractatus Logico-Philosophicus" and later on discussed by von Wright and Smirnov.

Our next proposal concerns the investigation of logical space geometry in the framework of Klein's Erlangen program. In this scientific manifesto Klein proclaimed that properties of any geometry are invariant under transformations natural for this geometry, i.e. symmetry expressed by the notion of transformation group determines the type of geometry. Nowadays Klein's idea to define geometry by using its symmetry group is extended to other mathematical branches. It is worth recalling that Tarski himself defined logical notions in the context of Erlangen program.

Thus, an extended logical matrix is given by the quadruple $ELM = \langle E, V, \Omega, D\rangle$, where E is a set of logical environments (logical spaces with appropriate geometries), V is a set of truth values, D is the set of designated truth values $D \subset V$, Ω is the set of operations over truth values.

In particular, we face this synergistic problem in constructing multi-valued logics with negations and implications expressed by non-linear functions. The pioneer of such an approach was Bochvar who introduced infinite-valued parabolic and hyperbolic logics. In these logics basic negation and implication operations are obtained from equations describing surfaces of parabolic/hyperbolic type.

For example, parabolic logic PL is given by the following logical matrix $LM_{\mathrm{PL}} = \langle p, [0,1], \{n_{\mathrm{p}}, I_{\mathrm{p}}\}, \{1\}\rangle$, where P are logical spaces with parabolic geometries, and two basic operations are: negation $n_p\,(x) = 1-x^2$ and implication $I_{\mathrm{p}}(x,y) = \min\{1, 1-x^2+y\}$.

Furthermore, a family of parameterized hyperbolic logics HL is specified by $LM_{\mathrm{HL}} = \langle H, [0,1], \{n_{\mathrm{h}}, I_{\mathrm{h}}\}, \{1\}\rangle$, where H are hyperbolic logical spaces, negation family is expressed by $n_{\mathrm{h}}(x) = k(1-x)/(1+x)$, $k = 1, 2, \ldots$, and implication family is given by

$$I_{\mathrm{h}}(x,y) = \begin{cases} 1, & \text{if } x \le y \\ k[(1-(x-y)/k+(x-y))], & \text{if } x > y \end{cases}.$$

Here for $k \to \infty$ we obtain as special cases $n_{\mathrm{L}}\,(x) = 1-x$ (linear Lukasiewicz's negation) and $I_{\mathrm{L}}(x,y) = \min\{1, 1-x+y\}$ (Lukasiewicz's implication).

Another family is expressed in the form $LM^*_{\mathrm{HL}} = \langle H, [0,1], \{n^*_{\mathrm{h}}, I^*_{\mathrm{h}}\}, \{1\}\rangle$, where $n^*_{\mathrm{h}}(x) = [(1-x)/(1+x)]^k$, $k = 1, 2, \ldots$ and

$$I^*_{\mathrm{h}}(x,y) = \begin{cases} 1, & \text{if } x \le y \\ (k+1)y/(k+x), & \text{if } x > y \end{cases}.$$

For $k \to \infty$ we obtain as special cases Gödel's negation and Gödel's implication respectively

$$n^*_G(x) = \begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{if } x \neq 0 \end{cases},$$

$$I^*_G(x) = \begin{cases} 1, & \text{if } x \le y \\ 0, & \text{if } x > y \end{cases}.$$

Apart logics with non-linear operations such extended matrices may be of special concern for applied dia-logics dealing with negotiation and argumentation processes.

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On Łoś Synthesis Theories

An analysis of the the celebrated *Łoś Ultraproduct Theorem* leads us to a natural generalization of the notion of sl-theory by Mostowski and Mycielski - from theories of sufficiently large finite models to arbitrary families of consistent theories. Such a syntactical integration operation may be compared with semantical fusion operators like reduced products or ultraproducts.

The finite models approach of mathematical analysis by J.Mycielski inspired M.Mostowski to study the family FIN of all finite initial segments of $\mathbb{N}$ under the operation sl which is a kind of fusion operator on theories (see e.g.[1]). Experiences show that computers may accede to finite parts of natural numbers only. Whence computer arithmetic is not the theory of the standard infinite model $\mathbb{N}$ of arithmetic. We have to replace the actual infinity of the standard model of arithmetic by the potentially infinite family of its finite submodels. Such an approach leads to useful applications in theoretical foundations of information systems, in the integration of data basis and in the information fusion theory(cf.[2]). Recall that $sl(\mathcal{A})$ denotes the set of sentences true in all sufficiently large initial finite segments $\{\mathcal{A}_n : n \in N\}$ of the model $\mathcal{A}$ i.e $sl(\mathcal{A}) = \{\varphi : \exists k \in N \forall n > k \ \mathcal{A}_n \models \varphi\}$.

Theorem[1] The following conditions hold:

1. $sl(A) = lim\ inf_{n\to\infty} Th(\mathcal{A}_n)$.
2. Sentence φ is consistent with the theory $sl(A)$ iff $\varphi \in lim\ sup_{n\to\infty} Th(\mathcal{A}_n)$.
3. $sl(\mathcal{A})$ is a complete theory iff the sequence $(Th(\mathcal{A}_n) : n \in N)$ is convergent.

We prove that one may use any filter and/or ultrafilter over a set I in the usual definition of sl-theory instead of the Frechet filter of cofinite subsets of N . Let $\mathcal{T}$ be a family of consistent first order theories expressed in the same relational language L i.e $\mathcal{T} = \{T_i : i \in I\}$. We consider a family of models of these theories $\mathcal{M} = \{\mathcal{M}_i : \mathcal{M}i \models T_i \quad and \quad i \in I\}$

Definition Let $\mathcal{F}$ be a non principal filter over the set I and let $\mathcal{T}$ be a family of first order sentences of a language L. Łoś synthesis theory Ł$(\mathcal{T}/\mathcal{F})$ is the set of sentences φ in the language L satisfying the following Łoś condition:

$$\varphi \in L(\mathcal{T}/\mathcal{F}) \quad iff \quad \{i \in I : \mathcal{M}_i \models \varphi\} \in \mathcal{F} \tag{11}$$

One may observe that in case when $\mathcal{F}$ is the Frechet filter over N the Łoś synthesis operator L is identical with the Mostowski's operator sl

Theorem Let $\mathcal{F}$ be the Frechet filter of cofinite sets over I=N. Let $\mathcal{T}$ be the family of theories of all finite initial segments of a standard model $\mathcal{A}$ of arithmetic. We have the equality $L(\mathcal{T}/\mathcal{F}) = sl(\mathcal{A})$.

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Some partial conservativity properties for Intuitionistic Set Theory with principle DCS

Let $ZFI2C$ be usual intuitionistic Zermelo-Fraenkel set theory in two-sorted language (where sort 0 is for natural numbers, and sort 1 is for sets). Axioms and rules of the system are: all usual axioms and rules of intuitionistic predicate logic, intuitionistic arithmetic, and all usual proper axioms and schemes of Zermelo-Fraenkel set theory for variables of sort 1, namely, axioms of Extensionality, Infinity, Pair, Union, Power set , Infinity, and schemes Separation, Transfinite Induction as Regularity, and Collection as Substitution. It is well-known that both ZFI2C and ZFI2C+DCS (where DCS is a well-known principle Double Complement of Sets) haves some important properties of effectivity: disjunction property (DP), numerical existence property (but not full existence property!) and also that the Markov Rule, the Church Rule, and the Uniformization Rule are admissible in it. Such collection of existence properties shows that these theories are sufficiently constructive theories. On the other hand, ZFI2C+DCS contain the classical theory ZF2 (i.e. ZFI2C+LEM) in the sense of Gödel's negative translation. Moreover, a lot of important mathematical reasons may be formalized in ZFI2C+DCS, so, we can formalize and decide in it a lot of informal problems about transformation of a classical reason into intuitionistical proof and extraction of a description of a mathematical object from some proof of its existence. In this talk we prove that ZFI2C+DCS+M+CT is conservative over the theory ZFI2C+DCS+M w.r.t. class of all formulae of kind $\forall a \exists \vartheta(a;b)$, where $\vartheta(a;b)$ is a arithmetical negative (in the usual sense) formula. Of course, we also prove that ZFI2C+M+CT is conservative over the theory ZFI2C+M w.r.t. class of all formulae of kind $\forall a \exists \vartheta(a;b)$, where CT is the usual schema of the Church Thesis.

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Labeled Natural Deduction for Peircean Branching Temporal Logics

In a labeled deductive system framework [2] for a modal logic, labels are typically used to denote worlds. E.g., the following introduction and elimination rules for the modal operators are defined in [5,6] for the logic K:

$$
\frac{\begin{array}{c}[x\mathcal{R}y]\\ \vdots\\ y{:}\alpha\end{array}}{x{:}\Box\alpha}\,\Box I
\qquad
\frac{x{:}\Box\alpha \quad x\mathcal{R}y}{y{:}\alpha}\,\Box E
\qquad
\frac{y{:}\alpha \quad x\mathcal{R}y}{x{:}\Diamond\alpha}\,\Diamond I
\qquad
\frac{x{:}\Diamond\alpha \qquad \begin{array}{c}[x\mathcal{R}y]\ [y{:}\alpha]\\ \vdots\\ z{:}\beta\end{array}}{z{:}\beta}\,\Diamond E
$$

where x, y are labels and $\mathcal{R}$ denotes the accessibility relation. Note that in $\Box I$, y is required to be *fresh*, i.e., it must be different from x and not occur in any assumption on which $y : \alpha$ depends other than the discharged assumption $x\mathcal{R}y$. Analogously, y must be fresh in $\Diamond E$.

If we consider a modal logic whose models are defined by using more than one accessibility relation and whose language allows for quantifying over the relations, then it seems convenient to introduce a second sort of labels for denoting such relations. This is the case, e.g., of branching temporal logics. In particular, for the branching-time logics defined over a Peircean language, where the only temporal operators admitted are those obtained as a combination of one single linear-time operator immediately preceded by one single path quantifier, having two sorts of labels allows for defining well-behaved rules for such composed operators. Namely, given the path quantifiers $\forall$ and $\exists$, and the linear-time operators $\Diamond$ and $\Box$, with the usual intended meanings, the Peircean language is defined by the grammar $\alpha ::= p, \neg\alpha, \alpha \supset \alpha, \forall\Box\alpha, \forall\Diamond\alpha, \exists\Box\alpha, \exists\Diamond\alpha$, where p ranges over a set of propositional symbols.

By using the labels $\mathcal{R}_1$, $\mathcal{R}_2$, ... for representing accessibility relations along the different paths and $x, y, \ldots$ for time instants, and by properly tuning the freshness conditions on such labels, one can, e.g., define the following rules for the operators $\forall\Box$ and $\exists\Box$, which follow the same patterns as the ones seen above for the logic K:

$$
\frac{\begin{array}{c}[x\mathcal{R}_1y]\\ \vdots\\ y{:}\alpha\end{array}}{x{:}\forall\Box\alpha}\,\forall\Box I
\qquad
\frac{x{:}\forall\Box\alpha \quad x\mathcal{R}_1y}{y{:}\alpha}\,\forall\Box E
\qquad
\frac{\begin{array}{c}[x\mathcal{R}_1y]\\ \vdots\\ y{:}\alpha\end{array} \qquad x\mathcal{R}_1z}{x{:}\exists\Box\alpha}\,\exists\Box I
\qquad
\frac{x{:}\exists\Box\alpha \quad x\,r(x,\Box\alpha)\,y}{y{:}\alpha}\,\exists\Box E
$$

where y and $\mathcal{R}_1$ are fresh in $\forall\Box I$, while only y is fresh in $\exists\Box I$.

In the rule $\exists\Box E$, the nesting of a "universal" operator inside an "existential" quantifier does not allow for designing a rule which follows the standard pattern of a $\Diamond E$. We face the problem by using a skolem function, i.e., we give the name $r(x, \Box\alpha)$ to the path starting from x and such that α holds in all the points along that path.

It is not difficult to add the *next-time* operator in this framework as well as to express the proper relational properties, e.g., seriality, transitivity, induction, thus obtaining a sound and complete system for the logic UB. Moreover, we believe that, by using a further skolemization over the time instants, it should be possible to define rules for the operator *until* and have full CTL. Past-time operators can also be easily introduced. Different labeled deduction approaches for the same class of logics have been proposed in [1,4]. We are currently working at studying normalization of the proofs derivable in our systems: we expect the standard subformula property to hold for the subsystems obtained by excluding the induction rules, as in the corresponding systems for Ockhamist branching-time logics [3].

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A shared framework for consequence operations and abstract model theory

In the present contribution we develop an abstract theory of adequacy for consequence operations on the syntactical side and abstract model theory. After presenting the basic axioms for consequence operations and some basic notions of logic, we state axioms for the connectives of classical propositional logic. The resulting concept of propositional consequence operation covers all systems of classical logic. We present an abstract semantics which has the same degree of generality as consequence operations. The set of structures on which the semantics is based on is not specified. As a consequence, our semantical framework covers many different systems, such as valuation semantics, semantics based on maximally consistent sets, and probability semantics. A model mapping Mod assigns to every formula the set of structures that verify it. The theory $Th(N)$ of a model N is the set of all sentences verified by N. Mod and Th form a Galois correspondence-a relation that is well-established within algebra. This observation is of main importance because many semantical facts derive immediately from the theory of Galois correspondences. The semantical consequence operation is given by the mapping $Th \circ Mod$. We study a special class of model mappings for propositional consequence operations. This class, 'propositional

model mappings', has the Negation property and the Conjunction property. A semantics is adequate for a consequence opeararation Cn iff Cn is identical with the semantical inference operation $Th \circ Mod$. After studying adequacy in its most general form, we investigate how properties of Mod reflect properties of Cn and vice versa. We treat the cases where Cn is a propositional consequence operation and where Mod is a propositional model mapping. Furthermore, we determine for every basic notion of the theory of consequence operations a semantical equivalent.

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4.5.7 Philosophy

The invited keynote speaker of this session is André Fuhrmann (page 71).

Contributed Talks

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Keeping metatheory in check: on the self-sufficiency of cCore Logic

This paper is concerned with the *self-sufficiency* of logics. Briefly put, a logic $\mathcal{L}$ is self-sufficient with respect to a property P iff in order to prove $P(\mathcal{L})$ we need not go beyond the *logical* resources provided by L itself. In the paper a number of steps are taken toward making this rough characterization more precise and amenable to mathematical treatment.

Self-sufficiency of a logic w.r.t. some property P is one thing and requiring that a logic be self-sufficient w.r.t. P in order to be correct is a further matter. Let's call a property P *endogenous* for $\mathcal{L}$ if it is required that $\mathcal{L}$ be self-sufficient w.r.t P.

Much metalogic is informal and the assumed background is often Classical first-order logic and standard set theory. Notably, this is often the case even when the target logic is weaker than Classical logic. This observation invites two questions: *To what extent are these logics self-sufficient with respect to their metatheory?* and *Which metatheoretic results are endogenous for these logics?* Apart from a few suggestions for how these issues can be addressed generally, this paper deals with these matters as they arise in the case of Neil Tennant's cCore logic. See (Tennant, 2011, 2005 and 1984) Tennant's main system, Core logic, is both constructive and relevant, but a non-constructive version – here called cCore logic – is singled out for separate study. The focus of this paper is cCore logic. The consequence relation of cCore logic is not unrestrictedly transitive, which in the favored Sequent Calculus presentation means that the CUT-rule does not hold. The crucial claim made on behalf of cCore logic is that it is adequate for non-constructive mathematics. The failure of CUT puts this claim into doubt. Tennant responds to this with a metaresult – a Cut-eschewal theorem – establishing that nearly all Classically provable sequents are matched by cCore provable sequents. The exceptions being the relevantistically offensive cases, in which case cCore logic proves epistemically more informative sequents. This result is crucial for for the claim that cCore logic could replace Classical logic. (The same goes, mutatis mutandis, for the case for replacing Intuitionistic logic with Core logic).

But is cCore logic self-sufficient with respect to this metaresult?

Tennant has provided an informal proof of this metatheorem, but no formal cCore (or Core) proof. It is at present unknown whether there is such a proof, and hence unknown whether cCore logic is self-sufficient w.r.t this theorem. Burgess has criticized Tennant on precisely this point. (Burgess, 2005 and 2009) This criticism is analyzed and sharpened here. Burgess's criticism relies on the assumption, rejected by Tennant, that cCore logic is normatively rather than descriptively motivated. I argue that in either case we can show that the metatheorem is endogenous for cCore logic. This argument is also generalized to a schematic endogenity-argument applicable in other cases.

Tennant has attempted to show that he can establish self-sufficiency of cCore logic even without a formal cCore proof of his metatheorem. The paper provides detailed analyses of the two arguments that are briefly sketched in (Tennant, 2011). The main result of this paper concerns the second of these arguments which seeks to give a method for converting a Classical logician to cCore logic; in short, if we accept Classical logic we will conclude that the Cut-eschewal theorem is true, and hence that cCore logic should replace Classical logic as 'the' logic of non-constructive mathematics. If successful this would provide a legitimate way of conducting metatheory of cCore logic in Classical logic.

I argue that this argument fails. In fact it leads to a paradox: We ought to accept Classical logic if and only if we ought to reject Classical logic. The most plausible way to avoid this paradox is to reject Tennant's idea that we can conduct the metalogic of cCore logic without heeding self-sufficiency. So, rather than avoiding the burden of self-sufficiency this argument serves to strengthen the case for requiring self-sufficiency.

Several aspects of this dialectical situation can be generalized. The final part of the paper is devoted to a general characterization of situations in which competing logics can lead to similar situations. This last is hopefully of interest in other cases where logics are pitted against each other.

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Synonymy and intra-theoretical pluralism
There are different ways to be a pluralist about logic. A first approach is rooted in Carnap's Principle of Tolerance [2], and ties the existence of different consequence relations to the existence of different languages. On that account, one can be a pluralist about logic because one is free to adopt the language that best suits the purpose at hand. A second approach is defended by Beall and Restall [1], and argues that even within a unique language there are multiple though equally good ways of making precise what it means for a conclusion to follow from a set or premises. On that account, the choice of one's language doesn't fully determine the extension of "follows from." More importantly, if logical pluralism can arise within a single language, we can come up with different logical systems that genuinely disagree. As implied by a famous remark of Quine [5], this type of disagreement presupposes that one can adopt a different logic without thereby also changing the subject; that is, without thereby changing the meaning of our logical vocabulary (meaning-variance).

Recently, Hjortland [3] has, following an earlier suggestion by Restall, argued for an alternative path to logical pluralism. On his proposal, different consequence relations are obtained within a single logical theory. Hence the name of *intra-theoretical* pluralism. Concretely, the idea is that a single calculus can be used to define different consequence-relations. Thus, we can use a single sequent-calculus to define the consequence-relations of intuitionistic and dual-intuitionistic logic (Restall's example), or we can use a single sequent-calculust to define the consequence-relations of classical logic, a Kleene-style paracomplete logic, and a paraconsistent logic (Hjortland's example). The main benefit of this type of pluralism is that the meaning of the logical vocabulary remains fixed. That is, if one believes that the meaning of the logical constants only depends on the proof-rules, then this meaning is fully determined by the calculus. Since this calculus is shared by different logics, the meaning of the connectives is common as well.

The upshot of this paper is to use the formal notion of synonymy to scrutinize the above argument. To begin with, if synonymy is defined relative to a consequence-relation [4, 6] by stipulating that two expressions A and B are synonymous ($A \equiv_{\vdash} B$) iff we have:

$$C_1(A), \ldots, C_n(A) \vdash C_{n+1}(A) \text{ iff } C_1(B), \ldots, C_n(B) \vdash C_{n+1}(B) \qquad \text{(Syn)}$$

Where $C_i(B)$ is obtained from $C_i(A)$ by replacing some (but not necessarily all) occurrences of A in $C_i(A)$ by B, we can only conclude that the meaning of the logical connectives (as defined by the calculus) isn't sufficient to decide whether two expressions are really synonymous. This suggests that intra-theoretical pluralism is committed to there being multiple synonymy-relations within one logical theory, which then puts some pressure on the idea that having a single calculus is sufficient to avoid meaning-variance. At this point, intra-theoretical pluralism can be further developed along two different lines. We can either object to the use of (Syn) within a context where there are different consequence-relations in a given theory, or we can simply accept that there are multiple synonymy-relations. In the former case, a new definition of synonymy is required; in the latter case, it needs to be argued that the meaning of the logical connectives can be fixed even though the synonymy-relation isn't. Throughout this paper we shall develop both these lines and compare their respective virtues.

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Can we omit Cantor's diagonalization argument in Gödel's incompleteness theorem?

We will show that there is no algorithm whose output list contains all valid (or true) propositions of first-order arithmetic and no false ones. This will be attained by using an iterative concept of formula generation, labeled natural numbers and our main argument – the forgotten number.

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What is the Internal Logic of Constructive Mathematics? The case of the Gel'fondSchneider Theorem

The question of an internal logic of mathematical practice is examined from a finitist point of view. The Gel'fond-Schneider theorem in transcendental number theory serves as an instance of a proof-theoretical investigation motivated and justified by a constructivist philosophy of logic and mathematics. Beyond the Gel'fond-Schneider theorem, transfinite induction is put to the test and is shown to be operating in most foundational programmes, from Voevodsky's univalent foundations and Martin-Löf's intuitionistic type theory to Mochizuki's inter-universal geometry for the abc conjecture. I argue finally that intuitionistic logic is not sufficient to handle constructive mathematics and a polynomial modular logic is proposed as the internal logic of Fermat-Kronecker "general arithmetic" (see[1]) for constructivist foundations of mathematics. The foundational perspective is briefly contrasted with a naturalistic philosophy defended by the philosopher of mathematics Penelope Maddy.

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On the Feynman's quantum art of solving difficult mathematical problems by drawing simple pictures

As it's well known by physicists ([5], p. 156) it's possible to analyze (sometimes, to solve) some physical problems formulated in some mathematical language by drawing a special kind of diagrams called Feynman diagrams. In this lecture we are going to try to explain why it's possible to study some of those advanced physical problems only by drawing some special kind of pictures. Our analysis will be guided by Feynmans views on non-relativistic quantum mechanics and on quantum electrodynamics [1,2,3]. Our studies were inspired by da Silva's structural approach [6] to the philosophy of mathematics we once applied [4] to analyze the applicability of mathematics to quantum mechanics.

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Elements of Special and General Logic – The crazy Hypothesis: Logic is about Truth, Goodness, Beauty and Utility

It is a common place to emphasize pluralism as an essential feature of contemporary logic as opposed to the more orthodox approaches following the Frege revolution. Yet, logic in its history, at least since Aristotle, has remained both aletheiotropic and epistemotropic (truth/knowledge-oriented), even after the analytic turn. The epistemic and deontic approaches of modal logic as well as that of pragmatics have certainly broadened the scope of logic. They add to the stakes of truth and knowledge that of duty, sometimes those of belief and desire, those of necessity, possibility, promise, command, etc. However, they all remain faithful to Frege's sentence: "The word 'true' is to logic what the word 'good' is to ethics and the word 'beautiful' is to aesthetics". I suggest that, given the pluralistic trends of contemporary logic, it is time to overcome the limits of classical logic due to its aletheiotropy and its epistemotropy by incorporating other types of rationality. This logic-philosophical reform leading to a kind of hetero-logic can be achieved at the basic level of semantics and syntax - and not only pragmatics, or modal logic - by including into the logical calculus some other values than truth, such as goodness, beauty, and utility. These other logical values are not the only ones possible, but they fit the main classical domains and stakes of judgement in philosophy: the epistemic (the true), the ethical (the good), the aesthetical (the beautiful) and the technical (the useful). For that, I suggest to make a difference between: (1) a special logic that is exclusively truth-centered (2) a general logic that is inclusively muti-values-oriented. This philosophical turn in logic has consequences in all its parts (semantics, syntax, pragmatics) for it requires modifying most of their existing basic tools of method as used in the various forms of logical calculus.

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Computer-Assisted Mathematics

Formal logic has for long been believed to be an impractical means for communicating mathematics. In particular proofs in mathematics are believed to be social processes rather than formal entities. We re-examine such beliefs in the backdrop provided by recent advances in computer science. In particular, we use the backdrop of research in the areas of 'Automated Reasoning' and 'Interactive Proof Checking', and the associated development of powerful tools known as 'Proof Assistants'. 'Proof Assistants' have given rise to an interesting consequence – viz. the practical feasibility of importing techniques developed in the computer science community and redeploying them to improve the main activity of the working mathematician, namely the process of proof development. At the core of such redeployed techniques lie the notions of formal systems, formal reasoning, and formal proofs. However the process of formalizing mathematics is a highly non-trivial task, and gives rise to a number of challenging and interesting issues which need to be addressed in order to make the discipline of computer assisted mathematics more prevalent in the future.

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Radical Interpretation, the Logical Constants and Gödel's Critique of Intuitionistic Logic

May the theory of radical interpretation developed by Davidson (e.g. in Davidson [1973] 2001) help fix the meaning of the logical constants? If so, how and with what results? In particular, may the theory promote ways of fixing their meaning which should be accepted as exclusively correct and force an interpreter to reject other ways as illegitimate? I examine the case of negation, disjunction and the existential quantifier, both in the context of a Davidsonian situation of radical translation and in the context of Gödel's objections to the constructive or intuitionistic definitions of "$\neg$", "$\vee$" and "$(\exists x)$" presented, e.g., in Gödel [1941] 1995, prior to the Dialectica paper of 1958 (Gödel [1958] [1972] 1990). I conclude first that Davidson's interpretative strategy fails to provide (i) a reason to believe that to change the meaning of the constants is, as the Quinean saying goes, "to change the subject," and (ii) a reason to prefer the classical reading to the intuitionistic one. Secondly, I conclude that despite Gödel's objection that intuitionistic logic "turns out to be rather a renaming and reinterpretation than a radical change of classical logic" (Gödel [1941] 1995: [3] 190), there remains a further disagreement over the meaning of the constants. Consider, e.g., negation. When one applies a version of the principle of charity according to which ascription of meaning to the ascribee's constant should be identical to that made by the ascriber to his own, the non-equivalence of the classical reductio or absurdity rule to the intuitionistic one may, in some very weak sense, be judged irrelevant to the debate over the meaning of the constant. But when one applies a stronger principle according to which any ascription of meaning to that constant should be grounded or justified, the non-equivalence of "$\sim$" (classical) and "$\neg$" (intuitionistic) is glaring. Of course, under Glivenko's translation of the classical constants into the intuitionistic ones, the classical calculus turns out to be a subsystem of the Heyting propositional calculus (Glivenko 1929). Adding a definition of existence such that $(\exists x)A(x) =_{DF} \neg(x)\neg A(x)$, non-constructive existence proofs become intuitionistically correct. I examine Gödel's critical reflexions regarding the extent to which classical existence proofs may be transformed into constructive ones for the formulas of Σ (an early version of the system T of the Dialectica paper of which Gödel op. cit. is an ancestor). I argue that in spite of Gödel's negative remarks on the notion of constructibility (to the point where the very notion of constructive proof provides a counterexample to its own admissibility), there is still ground for a disagreement over the meaningfulness, understanding or grasp of classical proofs. In the Davidsonian context, the translation of one idiom containing the classical "$\sim$", "$\vee$" and "$(\exists x)$" in another might perhaps maximize agreement, but the settlement thereby obtained takes for granted that there is no further disagreement over the independence of truth from the capacity that ascribers and ascribees alike may have of providing justifications for sentences containing occurrences of these constants. Such independence remains a sticking point despite the merits of the Gödel-Glivenko approach.

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On the Ontology of Logic

Acceptance of George Boole's judgment, "On the point of the principle of true classification, we ought no longer to associate Logic and Metaphysics, but Logic and Mathematics," [1] is why, "modern logic was designed with the language of mathematics largely in mind." [2] Affirming this identity, Bertrand Russell asserts, "Mathematics and logic . . . . are one. They differ as boy and man : logic is the youth of mathematics and mathematics is the manhood of logic." [3] Disassociating logic and metaphysics establishes logic as an autonomous being. Same experience is apparent as whole or parts, and if parts, different parts. Additionally, however apparent, constant experience can transition seamlessly in appearance. Logic determining experiential appearance, when experiential appearance is unchanging, logic is not constituent of experience. Not a transmutative experiential state, it is emergent, unbound by conservation of energy. Being thus, it is understandable as substantively autonomous of experience. This is manifest in the depiction of logic ontologically as abstract rather than material, and epistemologically as sensed rather than sensate. Of concern is the transformation of wholes, which occurs by integration or separation. As so, logic identifies qualitative states. Doing this, it is representable by qualitative states, if not in sensation, in imagination. Identifying qualitative states, then, logic does not compose an ontological environment autonomous of quality. Considering states of being, logic is not separate from ontology as Boole and others suppose. It is a subset of ontology, concerned with transformation of being. Presupposed by logic, thus, is the conservation energy. Emergence denying the conservation of energy, it is illogical, something coming from nothing. Logic identifies states of being intervening any two things constituting limits. Not necessarily observable, these states of being are imaginable. Infinite in extent, they are imperfectly representable. Incomplete, they are still identifiable by an abstract sense, if not a qualitative sensation. Thus, observationally indistinguishable, experience is distinguishable by an abstract sequential identity. Even an object is understandable in this way. Existing over time, if time is divided into moments, the object is understandable as a discrete set of distinguishable parts. This constitutes a process. Alternatively, if time is undivided into moments, it is understandable as a dense set of indistinguishable parts. This constitutes an event. More generally, distinguishing basic logical operators, implication is identity as fused whole, conjunction is identity as alternately fused whole and diffused parts, and disjunction is identity as diffused parts. Accepting this distinction, then presuming "Classes and concepts [exist] independently of our definitions and constructions" confuses epistemology with ontology. Epistemological identity of a constituted sequence is immediate, when ontological constitution of the sequence is mediate.

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Truth is pathological

In Kripke's classic paper on truth it is argued that by weakening classical logic it is possible to have a language with its own truth predicate. Usually it is claimed that a substantial problem with this approach is that it lacks the expressive resources to characterize those sentences which are neither true nor false. The goal of this paper is to offer a refinement of Kripke's approach in which this difficulty does not arise. I tackle the characterization problem by introducing a pathologicality or indeterminateness operator into the language.

I consider two different ways of doing this. The first one uses the framework of fuzzy logic. The presence of infinitely many semantic values allows us to have a really fine-grained characterization of the indeterminate sentences of the language. Also this approach is compatible with some solutions to the Sorites Paradox where a borderline operator is available in the language. The second one uses standard three-valued Strong Kleene logic, but the pathologicality operator is interpreted by using revision sequences. The main advantage with this strategy is that it affords a natural way of avoiding revenge worries that turn up in the standard Kripkean approach.

The central difference between the two ways of introducing the pathologicality operator has to do with self-referential sentences such as

(I) (I) is pathological

(II) (II) is not pathological

While the first approach handles these sentences by granting that there are true (false) pathological sentences, the second approach just declares the first one false and the second one true. Another important difference has to do with the iteration of the pathologicality operator.

I argue that the first approach is better in many respects. In particular, it not only provides a more intuitive and fine-grained characterization of the indeterminate sentences concerning truth and pathologicality itself, but also allows us to distinguish different kinds of indeterminate sentences containing vague predicates. This means that the first approach has more chances of providing a more attractive unified solution to the Liar and the Sorites Paradox.

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Set theoretical foundation of mathematics as unity of mathematical practice

I would like to present a reflection on the role that logic plays in the foundations of mathematics and, in particular, in the context of a set-theoretical foundation. First of all I propose a distinction between two different approaches, that parallel Benacerraf dichotomy outlined in the 1973 paper Mathematical truth. I will argue that the unity of mathematics is one of the main goals of any foundational attempt and I will distinguish – but I do not see this division as an alternative – between a philosophical foundation and a mathematical one. By a philosophical foundation I mean the attitude that sees in the foundation of mathematics the possibility of a reduction. This stance tries to answer a question on what there is in the mathematical world and how we can give a mathematical definition of our mathematical concepts. A reduction of this kind deals mostly with ontological or semantical problems. By a mathematical foundation I mean an attitude that aims to explain the unity of mathematics without proposing a reduction. It is, indeed, epistemological in character and the main question that it tries to answer is: why can we prove a theorem? why a proposition can be seen as a theorem of a theory? The main reason for calling it mathematical, in contrast with philosophical, is the attention that is devoted to mathematical practice. Logic is fundamental in both foundations. For example we can see Russell and Whitehead Principia Mathematica as a reductionist attempt, while acknowledging that Frege insists on the epistemological value of his analysis of arithmetic. Indeed this double role of logic depends on the bivalent nature of logic: a characteristica universalis and a calculus ratiocinator. On one hand the way an holistic reduction makes use of logical tools to reach a sufficient degree of generality, while the combinatorial aspects of logic are responsible for its use in the study of the main possibility of a proof. Then I propose set theory as the best mathematical foundation we have at disposal. The reason is twofold: on one hand it provides the analysis of necessary and sufficient conditions, on the other it allow a protocol of rules according to which our mathematical thinking proceeds. I will devote particular attention to the use of large cardinals in modern research in set theory as a way to codify second order principles in a first order setting; analyzing also the epistemological value of equiconsistency results. In the end some examples are given to support the primacy of set theory in finding a solution to any mathematical problem; thanks to methods that transcend the limits of first order logic.

4.5.8 Quantifiers

Contributed Talks

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Existence Axioms in Second-Order Arithmetic

The work presented in [2], [3] and [4], particularly the theory of set existence, is focused on axiomatic systems that are extensions of the usual system ZFC. The aim of that work was to analyze the axioms of set theory, including new axioms, with respect to their existential requirements. However it can be equally fruitful to consider aspects of this work on set existence in other systems.

Two of the most interesting systems that are not extensions of ZFC are the first-order arithmetic and second-order arithmetic. The first-order arithmetic can be obtained from ZFC by the elimination of the infinity axiom, the introduction of the negation of the infinity axiom and an axiom that asserts that every set has a transitive closure. The second-order arithmetic can be obtained from ZFC by restricting the power set axiom to finite set and inserting an axiom that asserts that every set is countable. In general, arithmetic of order n can be obtained in a analogous way, restricting the application of power set axiom.

These systems of first and second-order arithmetic are equivalents to usual presentations by means of the axiom-scheme of induction. This equivalence permits one to carry the main definitions in [2] [3] and [4] to the arithmetical context. The aim of our research project is to analyze the notion of existence in a general arithmetic context, (existence of numbers and existence of sets of numbers), in the light of the results presented in the set-theoretic context. This analysis will be successful if we obtain a stable classification of valid sentences in second-order arithmetic in terms of existencial requirements. In addition to its intrinsic interest, a classification of this type of arithmetic sentences will be relevant for foundations of mathematics. The foundational program named Reverse Mathematics is based on subsystems of second-order arithmetic, whose objective is finding the existence axioms required in fragments of mathematics. This program can be analyzed by the classification that we are looking for.

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A propositional logic for the term "few" presented in a natural deduction system

Historically, the work of Peirce and Frege gave rise to the classical first order logic, which deals with the quantifiers: universal "∀" and existential "∃". However, the classical first order logic is not enough to formalize any sentence of natural language. Mostowski (1957), in his scientific paper "On a Generalization of quantifiers", indicated the existence of many other quantifiers that are mathematically interesting, but that can not to be defined from the first order quantifiers: universal and existential. He called these new quantifiers, not defined in the classical first order logic, by generalized quantifiers. From that moment, several studies have been published on this topic. Sette, Carnielli and Veloso (1999), looking for logic formalization for a specific type of generalized quantifier, introduced a monotonic logical system, called ultrafilters logic. The name this system comes from the composition of its semantical structure: a universal set and an ultrafilter on its universe. The ultrafilters logic is an extension of first order logic, basically by adding a generalized quantifier to the classical first order language. The new quantifier "almost all" is interpreted by a structure called proper ultrafilter. Motivated by this work, Grácio (1999), in her doctorate thesis entitled "Lógicas moduladas e raciocínio sob incerteza", introduced a set of non-classical monotonic logic, called modulated logical. One of these modulated logic, the logic of many, motivated Feitosa, Nascimento and Grácio (2009) to write "Algebraic elements for the notions of 'many' ", where they introduced an algebra for "many" and a propositional logic for "many", which is a propositional modal logic with a modal operator to formalize the notion of "many" inside the propositional context. In a similar way, this work introduces a propositional logic for "few" (LPP) that, as the name suggests, intends to formalize the notion of "few". Although we recognize a duality between "many" and "few", the approach of the term "few", that will be made here, is not an adaptation to the dual approach given by Feitosa, Nascimento and Grácio (2009) for the term "many". The propositional logic to "few" will be displayed in a natural deduction system. As it is well known, a such system consists of deduction rules. The Hilbert method is, of course, a deductive method, and therefore we can develop some theorems of the theory in question. However it is not the only deductive system in the literature, there are many others as the sequent calculus, natural deduction and tableaux system. The main objective this paper is to present the propositional logic for "few" in a natural deduction system and to show the equivalence between the LPP in Hilbert version and in the LPP natural deduction version.

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A generalized time quantifier approach to similarity-based reasoning

The proposed paper is the further development of the ideas presented in the paper [1]. The new formal foundations for similarity-based reasoning in a temporal setting are introduced. The approach is based on a many-sorted monadic fuzzy first-order temporal logic where the underlying fuzzy logic uses Lukasiewicz implication. Here the monadic predicate symbols represent fuzzy sets. Concerning temporal quantifiers, crisp temporal specifications are considered as a special case of fuzzy temporal specifications, so that in a crisp specification a temporal operator quantifiers over a crisp time set and in a fuzzy specification over a fuzzy time set.

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Analysis of generalized square of opposition

We continue development of the formal theory of intermediate quantifiers (see [2]). The latter are expressions of natural language such as "most", "many", "few", etc. We introduce an analysis of the generalized Aristotelian square of opposition which, besides the classical quantifiers, is extended also by several selected intermediate quantifiers. Our analysis is again based on Peterson's (see [3]) analysis in his book and our goal is to demonstrate that our formal theory fits well also in this respect. The main problem we faced was proper formalization of the corresponding relations occurring in the square since fuzzy logic offers many various kinds of possibilities. It became clear that the studied relations are close to the proved generalized syllogism and so, our formalization follows the way of proving of the latter. Of course, our definitions also work when confining to classical square only.

Furthermore, we introduced general principle for introduction new intermediate quantifiers and proved that the generalized the square of opposition works with them accordingly. Because our results are proved syntactically, they hold in arbitrary model and so, our theory has great potential for various kinds of applications.

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A Logic of Improbable: Semantics and Soundness

Andrzej Mostowski, in 1957, was one of the first researchers to publish papers on quantifiers different from usual, which can't be expressed through the classical quantifiers, denominating generalized quantifiers. From this, several other authors began to publish articles on this subject, studying different aspects of this new range of quantifiers. Jon Barwise and Robin Cooper, in 1981, studied quantifiers "vague" or subjective as "most", "many", among others, in an approach associated with natural language, even giving a demonstration that the quantifier "many" can't be expressed from the usual first-order classical logic quantifiers. In an approach to formalize mathematical structures, we can observe several different studies, including the formalization of the quantifier "there exist uncountably many" and quantifiers that serves to formalize the notion of topology. Antonio M. Sette, Walter Carnielli e Paulo A. S. Veloso, in 1999, an approach focused on the environment of formalization of first-order classical logic, showed the logic of ultrafilters, for the formalization of the quantifier "almost all". In continuing this perspective, Maria C. C. Grácio in 1999, presented the modulated logics, a family of logical systems in order to formalize the intuitive notions of "many", "most" and "a 'good' part". Within the framework of modulated logic, in previous works, we presented a formalization for a quantifier with dual aspects of the quantifier "many" of Maria C. C. Grácio, the quantifier "few". Thinking about these quantifiers, in continuation of the dualization of modulated logical from Grácio, this paper presents a new generalized quantifier to formalize the notion of "a 'small' part". It presents an initial conceptualization to this notion. Next, we propose an axiomatic system of first order extended by adding a new quantifier, the quantifier of "improbable" to formalize the notion of "a 'small' part". We define a topological structure underlying the quantifier "improbable" called parareduced topology that is presents in the logical system created for this quantifier, its semantics and at the end we give a proof that this system is correct.

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A system of natural deduction for the "almost always" propositional logic

R. Reiter, in 1980, introduced the Default Logic. The concern of Reiter (1980) was with default arguments of the type "in the absence of any contrary information, it is assumed that ...", that is, arguments that "almost always" are true, but with some possible exceptions. A relevant critical on the Reiter's Default Logic is that this system is not monotonic. In all deductions made on a non-monotonic system we need to analyze all the obtained rules, the initial assumptions and the deduced theorems for to be sure that no inconsistency appeared in the system. For this and other disadvantages of the non-monotonic systems, Sette, Carnielli and Veloso (1999) developed a monotonic system, based on the concept of ultrafilter, to be an alternative for the Default Logic. The Logic of Ultrafilter is formalized in a language which is an extension of the classical first order language, obtained by the inclusion of the generalized quantifier. In Rodrigues (2012) it was introduced a new logic for to deal with concept of "almost always" in a propositional environment. The "almost always" propositional logic is an extension of classical propositional logic and was presented in an axiomatic style. The objective of this work is to present the "almost always" propositional logic in a system of natural deduction and show the equivalence between this system and the original one.

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Naive restricted quantification

A naive theory of truth is a formal theory of truth that upholds the unrestricted equivalence between ‘P’ and ‘‘P’ is true’. As shown by the semantic paradoxes, under minimal assumptions naive theories are trivial if their underlying logic is full classical logic, and hence they are committed to rejecting some classical patterns of reasoning. The paper investigates how the non-classical logics proposed on behalf of naive theories fare with respect to a set of extremely plausible principles of restricted quantification, which prominently include the logical relationships codified by the traditional square of opposition (under natural existential assumptions). Firstly, it is argued that standard naive theories that reject either the law of excluded middle or the law of non-contradiction cannot validate the relationships of contradictoriness between universal and particular statements figuring in the square of opposition. Secondly, it is argued for the more general claim that any naive theory whose logic is non-substructural (i.e. such as to be representable as a Tarski-Scott closure operation)—and even *prima facie* viable substructural approaches rejecting the structural rules of monotonicity or transitivity—cannot validate all of the proposed extremely plausible principles of restricted quantification. Thirdly, pursuing further an approach to the semantic paradoxes that the author has inaugurated and developed elsewhere, the theory of restricted quantification obtainable from a naive theory which rejects the structural rule of contraction is explored. It is proved that such non-contractive theory validates all of the proposed extremely plausible principles of restricted quantification and also all the principles codified in the square of opposition (under natural existential assumptions). It is then shown that the non-contractive theory fails to validate some other *prima facie* plausible principles of restricted quantification, but such principles are argued to be spurious and to belie a misunderstanding of the workings of quantification.

4.5.9 Class

The invited keynote speakers of this session are Daniele Mundici (page 78) and Hiroakira Ono (page 80).

Contributed Talks

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A quantitative-informational perspective of logical consequence
We present an analysis of logical consequence from a quantitative perspective of information, as developed by thinkers such as Shannon and adopted in large part by Mathematical Theory of Communication. In this perspective, the definition of the quantity of information of an event, and also of a formula of a language, depends on the concept of probability. For this reason, initially, we developed a semantics for a sentential logic classical (SLC) based on usual Probability Theory, built from the Zermelo-Fraenkel Set Theory. We define some usual concepts like random experiment, sample space of a random experiment and event. A situation for the language of SLC consists of an association between the formulas well-formed of this language and the events of a random experiment by means of a function. Then we present the definitions of probability value of a formula and formula probabilistically valid. We have listed and comment some results of this semantics. From there we set the informational value of a sentence of SLC. Finally, we defined the notion of informational logical consequence and presented some results of this definition.

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A Generalization of the Principles of Weak Induction, Complete Induction and Well Order and Their Equivalence Conditions
The principles of weak induction, complete induction and well order are all well known for well ordered collections. We propose generalizations of these principles for a kind of collections which are not necessarily well ordered, and also answer for the question of for what kind of structure these three new principles are equivalent, in the sense that all are true or all are false. It is intended here that these structure establishes a minimum set of conditions in order that this equivalence occurs. We also formulate and prove weak and complete double induction laws for any structure in which these three principles are valid.

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Belief Revision in Description Logics with Tipicality

Description Logics (DLs) [1] are a family of knowledge representation formalisms more expressive than propositional logics, but generally decidable. They are well-suited to represent concepts, objects and relationships between objects.

There have been some proposals in the literature [2,3] on how to extend DLs to deal with information about typicality, which is information about properties a concept or object usually has, for example: "Tigers are usually big". On the other hand, there is work on nonmonononic reasoning about concepts, through the ideas of stereotypical or prototypical objects [4,5]. The idea is that besides the concept description, some extra properties define the typical objects (for example, "being big" for a typical tiger), and that, on the absence of other information, an unknown tiger is assumed to be typical.

When the definition of a concept changes, due to new incoming information, the information about typicality may change as well. To address the nature of these changes, in this work, we explore the application of belief revision techniques such as [6] to DLs with typicality.

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Paraconsistency and a complementation operator

Classically, the notion of consequence and inconsistency are interwoven. Could any such connection be found in case of paraconsistent consequence? This has been the main motivation behind this study. Paraconsistent notion of consequence, by definition, negates the explosiveness condition. That is, in paraconsistent logics a set of formulae yielding a formula α and its negation $\neg\alpha$, does not necessarily yield every β. So what should be the proper way of addressing such a set X which yields a pair of formulae $\alpha, \neg\alpha$ and does not yield at least one of $\beta, \neg\beta$. In [1], we proposed sets of axioms characterizing the notion of consequence as well as the notion of inconsistency ($ParaINCONS$) in the context of paraconsistent logics. It has also been observed [1] that this axiomatization for consequence-inconsistency connection in paraconsistent logics captures some important paraconsistent logical systems like, $D2$ (Discussive logic of Jaskowski), J_n, $1 \leq n \leq 5$, of Arruda, da Costa, J_3 of da Costa, D'Ottavino, C_{ie} systems of Carnielli, Coniglio, Marcos, Logic of Paradox of Priest, Pac as well as $RM3$ of Avron.

In this axiomatization for the notion of consequence as well as the notion of inconsistency of paraconsistent logics, we found that the property of double negation play a crucial role. But there are paraconsistent logics which have nothing to do with the double negation property. This leads us towards a new axiomatization for the duo consequence-inconsistency in the context of paraconsistent logics.

In this new context, the basic intention behind the notion of inconsistency with respect to a formula is such that a set of formulae X is said to be inconsistent with respect to a formula if the formula along with its complement formula cause direct contradiction. In the earlier case [1], complement of a formula is obtained only by adding one negation before it. Here, complement of a formula $\neg\beta$ is not necessarily $\neg\neg\beta$; rather it is β. That is if β, $\neg\beta$ both follow from X then X is inconsistent with respect to both β and $\neg\beta$, as each of these formulae is a complement of the other. In order to capture this intention an unary

operator $'$ over formulae has been introduced in the following way.
$\alpha' = \beta$ if $\alpha = \neg\beta$ for some β and $= \neg\alpha$, otherwise.

The notion viz., 'inconsistency with respect to a formula' is now defined as - $(X, \alpha) \in ParaINCONS'$ if both α, α' follow from X. It can be shown that the above problem is resolved in the context of $ParaINCONS'$ which is defined in terms of an operator which maps every formula to its complement by adding or deleting the connective negation in front of a formula. In this paper we propose two sets of axioms characterizing the notion of inconsistency ($ParaINCONS'$) and its corresponding notion of consequence, and show that these two sets of axioms are equivalent. Apart from this, we shall explore the interrelation between $ParaINCONS$ and $ParaINCONS'$, and show that this new axiomatization is no more restricted to those paraconsistent logics which have double negation property.

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Bounded-Degree Second-Order and Transitive Closure Logics

We investigate the descriptive complexity of the logic obtained by restricting second-order quantification to relations of bounded degree. Based on previous work from Schwentick et al. [1] and Grandjean and Olive [2], we introduce the Bounded-Degree Second-Order Logic and show that it captures the class ALIN of classes of unary structures accepted by an alternating random access machine in linear time and bounded number of alternations. We also extend this logic with the transitive closure operator on high-order relations on bounded-degree relations. We show that the Bounded-Degree Second-Order Logic with Transitive Closure Operator captures linear number of registers in a nondeterministic random access machine provided that registers store values bounded by a linear function in the cardinality of the input structure.

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A logic with almost no properties

The logic is defined on a language with just implication $\to$ by the axiomatic system that has the law of Identity "$\phi \to \phi$" as its only axiom, and the rule of *Modus Ponens* "from ϕ and $\phi \to \psi$ to infer ψ" as its only rule of inference. The theorems of this logic are exactly all formulas of the form $\phi \to \phi$. In this talk I will summarize the results of studying this logic with the techniques and from the point of view of abstract algebraic logic. It is argued that this is the simplest protoalgebraic logic, and that in it every set of assumptions encodes in itself not only all its consequences but also their proofs. Besides this, it appears that this logic has almost no properties: It is neither equivalential nor weakly algebraizable, it does not have an algebraic semantics, it does not satisfy any special form of the Deduction Theorem (besides the most general Parameterized and Local Deduction-Detachment Theorem that all protoalgebraic logics satisfy), it is not filter-distributive, and so on. It satisfies some forms of the interpolation property but in a rather trivial way. The peculiar atomic structure of its lattice of theories is determined. Very few things are known about its algebraic counterpart, except that its "intrinsic variety" is the class of all algebras of the similarity type.

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A New Theory of Conditionals (Almost Completely Made Up of Old Ideas)

Conditionals are sentences that may have the form *If A, then B*. Conditional assertions (henceforth simply 'conditionals') have antecedents *(A)* and consequents *(B)* that are themselves assertions. Asserting a conditional, however, does not usually involve asserting its antecedent and consequent. Rather, what a conditional asserts is a relation between them. The truth (or the acceptability) of a conditional cannot be derived from the truth (or the acceptability) of its constituents, but its falsity (or its unacceptability) can, when A is true (or acceptable) and B is false (or unacceptable). Genuine conditional sentences (or implicative conditionals) must be distinguished from at least three classes of similar sentences: (1) concessive conditionals; (2) relevance conditionals; (3) whether-or-not conditionals.

Concessive conditionals typically have the form *Even if A, B*, but they may also have the form *If A, B*. Exceptionally, they may even have the form *If A, then B*. Moreover, not all conditionals of the form *Even if A, B* are concessive: some are implicative. Thus, the presence of even is neither necessary nor sufficient for a conditional to be concessive. Paraphrasability, however, gives a suitable test for identifying a concessive conditional. Concessive conditionals can always be paraphrased as *Even if A, still B*. Relevance conditionals are those that can be paraphrased as *If A, then it is relevant to know that B*. Whether-or-not conditionals are conjunctions (or the conjuncts of such conjunctions) of two conditionals with opposite antecedents and same consequent *(If A, B and if not A, B)* meaning *Whether or not A, B.*

Implicative conditionals are modal sentences. The meaning of an implicative conditional *(If A, then B)* is: *possibly (A and B) and possibly (not A and not B) and not possibly (A and not B).* Modus ponens, modus tollens and contraposition are valid for implicative conditionals. Hypothetical syllogism is also valid, counterexamples in natural language being explained by contextual shifts. Some implicative conditionals can only be explained when it is recognized that the meaning of their consequent, in the context of the whole sentence, includes the content of their antecedent. I call them AIC-conditionals (for 'antecedent included in consequent'). Contraposition seems to fail for Jackson's famous example If he doesn't live in Boston, he lives somewhere in New England because it is an AIC-conditional. In the context of the supposition that he does not live in Boston, somewhere in New England excludes Boston. This exclusion must be made explicit in a suitable contrapositive.

Conditionals may reflect one of three possible epistemic stances of the speaker toward the truth of their antecedents and consequents: neutral (in so-called 'indicative' conditionals); negative (in so-called 'subjunctive' conditionals); and positive (in conditionals sometimes called 'factual', in which if can be paraphrased by since). The contrapositive of a counterfactual subjunctive conditional is normally a since-conditional.

The meaning of a concessive conditional (*Even) if A, B* is: *B and not(If A, then not B).* The meaning of a whether-or-not conditional is *B and not (If A, then not B) and not (If not A, then not B).*

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Trans-arithmetic and principle of non-contradiction

The aim of this conference is presenting a criterion for distinguishing what propositions obey the principle of non-contradiction and what propositions do not obey such principle. This criterion is entirely defined on trans-arithmetic, whose creator is the English computer scientist James Anderson. Basically, the idea is the following: within a trans-metric space (an extension of metric space), we can map the propositions of some logical language. We can evaluate the "distance" between propositions. If the distance between a proposition and its contradictory is "infinity" (a precise and definite transreal), then such proposition obeys the principle of non-contradiction; otherwise, the proposition does not obey.

The idea behind such criterion is treating propositions as vectors within a system of reference whose coordinates are transreal numbers; and these transreal numbers are interpreted as positions in possible worlds. So, in a possible world where a proposition is absolutely true (what is represented by the transreal "infinity"), its contradictory is located at transreal number "minus infinity" and the distance between them is evaluated as infinite (here we use the concept of transmetric for evaluating the distance between the vectors p and not p).

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Revising Formal Program Specifications Using KMTS

Belief revision [1] deals with the problem of changing a knowledge base in view of new information, preserving consistence. This theory has been used in [4] to solve a practical problem called *Model Repair*: how to automatically fix errors on a model based system specifications. Several works on model repair use Kripke structures to represent the system model. We aim to explore belief revision on a more general model representation, adequate to deal with systems with partial information, called Kripke Model Transition System (KMTS) [5].

In our paper, we analise the implications of building a theory of belief revision to temporal logics through KMTS models. Our interest goes beyond its practical application, since we can address several theoretical issues of classical belief revision theory. Many of the existing results, even those from applications of belief revision to non-classical logics, as description logics [2,6] and modal logics [3], relies on logical properties that may not hold on most of temporal logics, such as the compactness assumption.

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Geometric Modal Logic

Modal iteration is the superposition of modal clauses, as when some proposition is said to be necessarily necessarily true. Modal iteration has a very strong meaning: saying that a proposition is necessarily necessarily true amounts, intuitively, to saying that the proposition is necessarily true whatever the actual range of the possible may be. On that score, modal iteration (including the limit case of a single modal clause) corresponds semantically to a "change of scale," namely to a shift from a possible world to a range of worlds, and then to a range of ranges of possible worlds, and so forth. Such a progression ultimately refers to an open-ended collection of nested ranges of possible worlds of higher and higher order.

The standard Kripkean semantics does not allow one to really account for the change of scale prompted by modal iteration. An alternative semantical framework is put forward to that end, that stands in sharp contrast to the metaphysical absoluteness that Leibniz bestowed on his possible worlds and that Kripke inherited in some way, despite fully allowing for modal iteration.

Tools coming from modern differential geometry are used. The basic idea is to represent a possible world x as a point of a geometric manifold M, and the higher level possible worlds which are relative to x as points of the tangent space to M at x. The ensuing modal semantics, owing to the complexity induced by the existence of worlds of different levels, is worked out in an enriched, Riemannian setting. It aims at endowing modal logic with a deepened geometric meaning, and gives rise to a completeness result concerning a variant of the modal system S4.

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Freedom from the Belief in Ungrounded Probability Functions

Although partial belief [Hall 2004, Ismael 2011, Lewis 1980, 1994, Meacham 2010] is a degree of belief in a probability of an event, reasons will be provided to challenge the idea of its grounding and object argument being universal. Rather than the outcome of an event being effected by a persons subjective belief in the particular distribution of real properties[Shoemaker 1975, 1980], I will consider what it means for properties to be interactive. Inequalities and equalities between grounded probability functions Pr_g and ungrounded ones Pr_u will be presented as an alternative within Intuitionistic Speculative Logic *ISL* that sublates any empirical and or formal implications.

To do this I will take van Fraassen's [1995] proposal for rational superiority which leans on the relation of two propositions; *A is a priori for P and A* $P >$ B given K is normal and disjoint with B, and argue that the valid propositions are in a determinate relation; B is posterior for P. That is, if A is comparative to K and B is abnormal by being equivalent to the empty set then the relation will be false.

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Explosiveness and Model Existence

In [3] we studied the classical model existence property CME (every consistent set has a classical model) and constructed several proof systems satisfying CME. While constructing such proof systems, the choice of contradiction or inconsistency does make differences on forming proof systems. In order to get a better understanding on this, in [4] a study of inconsistency based on classical semantics was developed. In general, the theory of inconsistency can be described as follows: given a semantics $\models$ (assuming that contradiction exists in $\models$), and consider the collection of all $\models$-unsatisfiable set $U_{\models}$. Any nonempty sub-collection of $U_{\models}$ forms an inconsistency. Given an inconsistency $\Phi(\subseteq U_{\models})$, the explosiveness with respect to Φ means the equivalence of Φ inconsistency and the absolute inconsistency (and anything inbetween). Similarly, the model existence property for Φ means the equivalence of Φ inconsistency and $U_{\models}$ inconsistency (and anything inbetween). Note that explosiveness with respect to Φ and model existences with respect to Φ are collapses from Φ inconsistency to two extreme cases: the absolute inconsistency and the $U_{\models}$ inconsistency. Also note that any proof system which satisfies both Φ explosiveness and model existence property w.r.t. Φ inconsistency has all inconsistencies equivalent.

Traditionally, paraconsistent logics are defined as logics with non-explosive contradictory theory, where a theory Γ is *contradictory* in logic L iff from Γ one can derive α and $\neg\alpha$ in L for some α. This definition of paraconsistency seems to be too limited from the general theory of inconsistency given above, for it not only relies on the existence of $\neg$, but also relies on the specific form of contradiction $\alpha, \neg\alpha$.

In this talk we will generalize the concept of paraconsistency according to the general theory of inconsistencies. We will study inconsistencies of $\{\bot\}$, $\{\alpha, \neg\alpha\}$, $\{\alpha, \alpha \to \beta, \neg\beta\}$ (where $\bot, \neg$ are both primitive symbols) according to explosiveness and model existence property. We are interested in the following question: Is there a logic which has Φ_1 explosiveness and model existence w.r.t. Φ_2 but Φ_1 and Φ_2 are not equivalent?)

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Traveling Around the Moebius Band: The Logic of Scientific Discovery
The main goal of this article is to use the Cog-Con+FBST framework, that is, the Cognitive Constructivism epistemological framework equipped with the Full Bayesian Significance Test for accessing the epistemic value of sharp statistical hypotheses, to address Piaget's central problem of knowledge construction, namely, the re-equilibration of cognitive structures.

The Cog-Con+FBST perspective is illustrated using some episodes in the history of chemistry concerning the definition or identification of chemical elements. Some of Heinz von Foerter's epistemological imperatives provide general guidelines of development and argumentation.

Keywords: Bayesian statistics; Chemical elements; Eigen-Solutions; Inductive inference; Scientific ontologies and evolution.

A slide presentation is available at http://www.ime.usp.br/ jstern/ miscellanea/jmsslide/moebs.pdf

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Many-valued (Fuzzy) Type Theories
Mathematical fuzzy logic is a well established formal tool for modeling of human reasoning affected by the vagueness phenomenon. The latter is captured via degree theoretical approach. Besides various kinds of propositional and first-order calculi, also higher-order fuzzy logic calculi have been developed that are in analogy with classical logic called *fuzzy type theories* (FTT).

FTT is a generalization of classical type theory presented, e.g., in [1]. The generalization consists especially in replacement of the axiom stating "there are just two truth values" by a sequence of axioms characterizing structure of the algebra of truth values. The truth values should form an ordered structure belonging to a class of lattices having several more specific properties. The fundamental subclass of them is formed by *MTL-algebras* which are prelinear *residuated lattices* $\mathcal{L} = \langle L, \vee, \wedge, \otimes, \rightarrow, \mathbf{0}, \mathbf{1} \rangle$ in which $\rightarrow$ is tied with $\otimes$ by the adjunction. The first FTT has been developed for IMTL$_\Delta$-algebra of truth values, i.e. the MTL-algebra keeping the law of double negation and extended by a unary Δ-operation (in case of linearly ordered $\mathcal{L}$, Δ keeps $\mathbf{1}$ and assigns $\mathbf{0}$ to all the other truth values). The most distinguished algebra of truth values for FTT is the standard Łukasiewicz$_\Delta$ MV-algebra $\mathcal{L} = \langle [0, 1], \vee, \wedge, \otimes, \rightarrow, 0, 1, \Delta \rangle$.

A very general class of algebras especially convenient as the algebras of truth values for FTT is formed by EQ-algebras $\mathcal{E} = \langle E, \wedge, \otimes, \sim, \mathbf{1} \rangle$ where $\wedge$ is the meet, $\otimes$ is a monoidal operation (possibly non-commutative), and $\sim$ is a fuzzy equality (equivalence). Unlike residuated lattices, implication in EQ-algebras is a derived operation $a \to b = (a \wedge b) \sim a$ which is not tied with $\otimes$. The syntax of FTT is a generalization of the lambda-calculus constructed in a classical way, but differing from the classical one by definition of additional special connectives, and by logical axioms. The fundamental connective in FTT is that of a *fuzzy equality* $\equiv$, which is interpreted by a reflexive, symmetric and $\otimes$-transitive binary fuzzy relation.

This paper provides an overview of the main calculi of FTT based on IMTL$_\Delta$-algebra, EQ$_\Delta$-algebra, and some of their extensions, especially Łukasiewicz$_\Delta$-algebra (see [6–8]). The generalized completeness theorem has been proved for all kinds of FTT.

We also mention one of the motivations for the development of FTT — the program of *fuzzy natural logic* (FNL). Similar program under the name "fuzzy logic in broader sense" was announced already in [5]. Note that the concept of natural logic was announced by several authors (cf. [3–4]). The paradigm of FNL is to develop a formal theory of human reasoning that would include mathematical models of the meaning of special expressions of natural language (including generalized quantifiers) with regard to presence of vagueness. Note also that the paradigm of FNL overlaps with paradigms of *commonsense reasoning* (cf. [2]) and *precisiated natural language* [9].

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Alternatives to the law of double negation and modal logic of actions of semigroups

We study extensions of the fragment of classical logic in which double negation introduction $\varphi \to \neg^2\varphi$ and double negation elimination $\neg^2\varphi \to \varphi$ are not derivable, by formulas $\neg^m\varphi \to \neg^n\varphi$ where m and n are arbitrary natural numbers. Similarly generalizing the law of excluded middle and its dual, we extend it by the formulas $\neg^m\varphi \vee \neg^n\varphi$ and $\neg^m\varphi \wedge \neg^n\varphi$. Besides obvious connections to classical and intuitionistic logics, the construction connects to paraconsistent logics. We observe that resulting logics relate closely to modal logic of actions of semigroups, recently introduced in (1). This observation provides a natural way to construct models of the logics and, moreover, throws a new light upon the modal nature of intuitionism and paraconsistency.

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Composition-Nominative Logics of Partial Quasiary Functions and Predicates

Mathematical logic is a powerful instrument for investigation of software systems [1]. Still, analysis of application of existing or modified logics to software systems (later simply referred to as programs) demonstrates certain *discrepancies* between problems to be solved and a logic in use. For example, programs are represented by *partial functions* whereas in traditional logic *total functions and predicates* are usually considered; programming languages have a *developed system of data types* whereas traditional logic prefers to operate with *simple types* (sorts); *semantic aspects* of programs prevail over *syntactic ones* whereas in traditional logic we have an inverse situation. To avoid discrepancies of the above types we propose to construct *logics based directly on program models*.

To realize this idea we should first construct adequate models of programs. To tackle this problem we use *composition-nominative approach* to program formalization [2]. Principles of the approach (*development* of program notions *from abstract to concrete*, *priority of semantics*, *compositionality* of programs, and *nominativity* of program data) form a methodological base of program models construction. Such models can be represented by algebras of *partial functions and predicates.* Various classes of algebras form a semantics base for corresponding program logics, called *composition-nominative program logics* (CNPL). *Predicate logics* can be treated as special cases of CNPL. We investigate here *first-order composition-nominative logics* (FOCNL) *of partial quasiary functions and predicates.*

These logics are defined in the following way. Let V be a set of *names* (variables) and A be a set of *basic values* (urelements). Partial mappings from V to A are called *nominative sets*; their class is denoted ${}^{V}\!A$ (in traditional terms nominative sets are *partial variable assignments*). Partial predicates and functions over ${}^{V}\!A$ are called *quasiary*, their classes are denoted $Pr^{V,A}$ and $Fn^{V,A}$ respectively. Then we define a two-sorted algebra $< Pr^{V,A}$, $Fn^{V,A}$; $\vee$, $\neg$, $R^{v_1,\dots,v_n}_{x_1,\dots,x_n}$, $S_F^{v_1,\dots,v_n}$, $S_P^{v_1,\dots,v_n}$, $'x$, $\exists^x$, $= >$ with such operations (called compositions) as *disjunction* $\vee$, *negation* $\neg$, *renomination* $R^{v_1,\dots,v_n}_{x_1,\dots,x_n}$, *superpositions* $S_F^{v_1,\dots,v_n}$ and $S_P^{v_1,\dots,v_n}$ of quasiary functions into quasiary function and predicate respectively, *denaming* function $'x$, existential *quantification* $\exists^x$, and *equality* $=$. These compositions are defined in the style of Kleene's strong connectives; parameters $x, x_1, \dots, x_n, v_1, \dots, v_n$ belong to V, $n \geq 0$. A class of algebras of the above type (for various A) forms a *semantic base* for FOCNL. The *language* of FOCNL is specified by the terms of such algebras constructed over sets Ps and Fs of predicate and function symbols respectively. Formulas are interpreted in the class of such algebras; a formula Φ is *valid* if it is *not refutable.* Obtained logics can be treated *generalizations* of classical first-order logics (FOL) on classes of partial predicates and functions that *do not have fixed arity.* In such generalizations *some reasoning rules of classical logic fail*, for example modus ponens is not valid, the law $(\forall x\Phi) \to \Phi$ is not valid either, etc.

The new main results that generalize [3, 4] are the following:

- a calculus of sequent type for FOCNL is constructed, its soundness and completeness is proved;
- a comparison of FOCNL with classical FOL is made, subclasses of FOCNL having properties of classical FOL are identified;
- a (non-trivial) algorithm for reduction of FOCNL formulas to formulas of classical FOL that preserves their validity and satisfiability is constructed, its correctness is proved.

The logics constructed are *more adequate to software domain* because they are semantics-based logics of partial predicates and functions defined over partial data (over nominative data). Such predicate logics can be used as a base for further construction of more expressive and powerful program logics.

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4.5.10 History

The invited keynote speaker of this session is Itala Maria Loffredo D'Ottaviano (page 68).

Contributed Talks

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Peirce's Hint on the Inclusive Disjunction according to His Mathematics of Discrete Series

The concept and the doctrine of continuity Synechism (Peirce, 1935, Vol. 6, pp. 168-173, 1902) is Peirce's attempt to bring his huge philosophical architectonic together into a full and comprehensive understanding. Consequently, I will emphasize through this paper the link between the mathematical-logical treatment and the metaphysical point of view This thematic cut will be supported by some appropriate chronological marks, which have become familiar to Peirce scholars, particularly to those dedicated to continuity. Chronologically, my argument will focus on the period around the passage from the 19th to the 20th century (Parker, 1998), particularly, between 1895 and 1911, when Peirce developed his concept of continuity leaving behind his previous Cantorian reminder (Potter, Shields, 1977). According to Zalamea, the interest for the continuum becomes central to Peirce's thought around 1895 and is mature through 1900 (Zalamea, 2003, pp. 137138). This period approximately corresponds to the fourth stage (out of five) on the development of Peirce's continuity according to Havenel, who called this period the supermultitudinous period (18971907), during which the approach to continuity concerns on one hand Mathematics and Logics, while on theother it deals with the metaphysical continuity in Kant, Leibniz and Hegel (Havenel, 2006, pp. 3739). The supermultitudinous period major character lies at the heart of Peirce's previous attempt to overcome Cantor's continuity, proving that it is not continuous but a discrete collection. In order to achieve this aim he had to study discrete collections and continuous multiplicities in order to understand their mathematical organization. Discrete collections are formed by units that could be individually assigned, while the unities of continuous multiplicities could not. Indeed, the line between discrete collections and continuous multiplicities is harder to draw than it would appear at a first glance, because there are discrete collections that include individually or/and generally designated units. In the long run, the mature philosophical achievement of continuity through Peirce's writings, according to Havenel (2006) and Sfendoni-Metazou (1997), is scrutinized through the Mathematics of Logics, the Mathematics of Discrete Series and the Mathematics of Continua (Peirce, 1932, Vol. 1, p. 185). I look forward to inspect a specific passage on Peirce's Mathematics of Discrete Series around the relationship between two kinds of discrete series: the denumerable and the postnumeral collections. They are diverse kinds of collections because the first unit in a postnumeral collection is the "denumerable point", it means, the latter part of the denumerable in which is concentrated all the points of the collection. Postnumeral collections are as infinite as the denumerable collections are, so that they merge in a scale of quantity adapted to multitudes, as "the abnumeral multitudes are not quantities, because they do not represent places upon a scale" (Peirce, 1976, Vol. 3, p. 57), although they are capable of mathematical discrimination. The postnumeral collections hold the coexistence of individually and generally assigned units, so that it is worth scrutinizing their generating relations in order to observe that continuous phenomena appear in mathematical defined discrete series under the logical inclusive disjunction behavior (and/or).

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The Master Argument and the Truth Criterion for Conditional Sentences

Diodorus Cronus' renowned Master Argument (Epictetus, *Arrian's Discourses*, II 19, 1) has been long debated by scholars. From the premisses "Everything true as an event in the past is necessary" and "The impossible does not follow from the possible", Diodorus infers that "Nothing is possible which is neither true now nor ever will be". An adequate evaluation of this argument has to face the following difficulties:

1. The modal notions that form the backdrop of Diodorus' argument are not the ones to be found in Aristotle, but are directly borrowed from the Megarian tradition; for example, Aristotle (*Metaphysics*, 1047a 21-23), claims that a possible event (or sentence) is a sort of a *bilateral contingent* (*ἐνδεχόμενον*), while Diodorus maintains that a possible event is what is now or will be at some given moment of time in the future; it is or will be as *ἐνέργεια*. There exists no possible event that will never occur.

2. Understanding the word *ἀκολουθεῖν* is crucial to make sense of the second premiss. In this case *ἀκολουθεῖν* refers neither to a logical inference, nor

is to be understood in a chronological sense. We suggest that this premise is to be interpreted as meaning that the possible is not compatible with the impossible (*simul formatis*).

In this talk, we argue that further light can be shed on the Master Argument by taking into account the Diodorean truth criterion for conditional sentences. According to Sextus Empiricus (*Against the Logicians*, II 115), Diodorus' hypothetical proposition is true *when neither admitted nor admits of beginning with truth and ending in falsehood.* This condition must be constantly checked and it must generate a permanent denial of both the truth of the antecedent and the falsehood of the consequent. If the possible is what is or will be true at a given moment of time, and the impossible is what, being false, will not be true at a given moment of time, then from the second premise of the Master Argument it follows that: [the impossible =] *what is not* (in time), does not follow from [the possible =] *what is* (in time). This principle is universally valid from the point of view of Diodorus and it entails that *"being does not imply not-being"*.

The bearing of the truth criterion for conditional sentences for the evaluation of the Master argument will be advocated by referring to an example concerning the existence of atoms.

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What Kind of Logic is Hegel's Logic?

Is it possible to consider Hegel's logic from the perspective of contemporary logic? What is (could be) the role of Hegel's reflections on logic within contemporary debates about logic, and basic logical notions such as logical consequence, truth, and validity? In my paper, I try to react to two classical objections against the attempt to consider Hegel's dialectical logic from the point of view of contemporary formal logic. The first (1.) is that Hegel's logic, being a conceptual logic, is incompatible with contemporary logic, which is propositional logic. The second (2.) is that Hegel criticised formal logic, and this seems to compromise every attempt to consider his logic from a formal point of view.

As to 1., I argue that dialectical logic is a conceptual logic, which deals with the properties of some concepts or predicates, and their relations, while modern logic is a propositional logic. This seems to suggest a major difference. However, the two perspectives are not incompatible. The attempts of formalising dialectical logic since the 1960s show that dialectical relations between concepts can be adequately expressed using propositional logic. Point 2. can be retraced to the question whether Hegel's logic is properly a "logic", in the contemporary meaning of the term. It is a complex issue, which would require a historical consideration about the misfortunes of logic in the European tradition, as well as an inquiry into the meaning of "logic" in contemporary philosophy. I will limit myself here to fix some basic points concerning the meaning of "formal" in both Hegel and contemporary logic, hinting at the importance of Hegel's view for recent debates in the philosophy of logic.

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Kazimierz Ajdukiewicz on the Scope and Nature of Logic: A Case from the History of the Polish School of Logic

Kazimierz Ajdukiewicz (1890-1963) was a disciple of Kazimierz Twardowski. In 1908, he began attending lectures given by, among others, Wacław Sierpiński, Marian Smoluchowski and Jan Łukasiewicz. Four years later, he took part in seminars led by David Hilbert and Edmund Husserl in Göttingen. Among his most famous disciples in the field of logic are Roman Suszko, Maria Kokoszyńska-Lutmanowa, Seweryna Romahnowa-Łuszczewska and Ludwik Borkowski. He is the godfather of a number of concepts, including Roman Suszko's Diachronic Logic – the first model-theoretic study on the dynamics of scientific knowledge, erotetic logic and semantic epistemology. The first structural grammar drawn up in a precise and complete way was Ajdukiewicz's grammar presented in 'Die syntaktische Konnexitaet', Studia Philosophica 1 (1936), pp. 1–27.

Our paper is aimed at presenting the scope and nature of logic, but also its social value according to Ajdukiewicz. Because he was focused on the application of formal logic, it is worth considering if he is also a precursor of the Practical Turn in Logic in the sense of Gabbay & Woods (2005).

In the context of the history of western rationality, Ajdukiewicz was an outstanding organizer of the institutional framework (academic and non-academic) of the teaching of logic. He was the first Editor-in-Chief of Studia Logica and created a true "Logical Empire", comprising 100 researchers and, at the time, only the United States compared with Poland in terms of the academic environment associated with logic. He was an author of a wide-ranging reform of the system of administration by means of education in logic, and always emphasized the role of education in logic for the proper functioning of a society.

He is also the godfather of the concept of idealization developed in the Poznan Methodological School (Giedymin, Kmita, Nowak). This approach resulted in the original idealizational theory of science, the socio-pragmatic theory of culture and the program of historical epistemology.

Within the conception of radical conventionalism, Ajdukiewicz developed a version of theory of language based on meaning directives. The reconstruction of language competence requires a formulation of three kinds of rules: axiomatic, deductive and empirical. Ajdukiewicz's projects seems most original in the context of the opposition between analytical and continental traditions.

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Fixed-Domain Interpretation and Logical Consequence in Aristotle

Etchemendy's conception of logical consequence as a conditional which always has a model in a non-fixed domain, has been criticised by Mancosu (2006; 2010) and Williamson (2011a; 2011b; 2012). Non-fixed domains, the criticism goes, were not presupposed in Tarski's (1936) analysis of logical consequence (this is Mancosu's objection to Etchemendy) and they reflect the idea of logic's being uninformative, which is characteristic of folk logic, not of scientific logic (this is what Williamson argues for). Williamson takes traditional logic to be the main influence of what he calls "folk logic". I argue that these criticisms, however relevant for Etchemendy's understanding of Tarski, for David Kaplan's view on the uninformativeness of logic and for Michael Dummett's contempt for abductive methods in logic, leave a great part of traditional logic intangible. Indeed the most distinctive part of traditional logic: Aristotelian syllogistic, if properly understood, does not encourage views akin to what Williamson calls folk logic. For one thing, Aristotle's view of necessity as a feature of valid inference does not involve what contemporary logicians call "logical necessity". It rather involves necessity as a feature of sentences which are true at all times. Valid inferences according to Aristotle have to fulfil some prerequisites which are informative, substantial or (in Etchemendy's sense) extra-logical. I will show that some of these prerequisites involve the fixity of the domain of discourse. Further, I will argue that such prerequisites justify two well-known features of Aristotelian logic which are that it does not consider every true implication to be a valid inference and that it even considers some false implications to be valid inferences. Particularly I shall deal with the following topics of Aristotelian logic: quaternio terminorum due to vagueness, the fallacy a dicto simpliciter ad dictum secundum quid, the subalternation of sentences at the right-hand side of the square of opposition, modal syllogisms with de re modalities and others.

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Completeness and continuity in Hilbert's Foundations of Geometry: on the Vollständigkeitsaxiom

One of the most interesting historical issues regarding David Hilbert's axiomatic system for elementary Euclidean geometry was the introduction, in the first

French edition of *Foundations of geometry* (Hilbert 1900a), of his famous "axiom of completeness" [*Vollständigkeitsaxiom*]. The axiom was originally conceived for the axiomatic system for the real numbers (Hilbert 1900b) and its adaptation and incorporation into the geometrical context had dramatic consequences for the geometrical theory, *i.e.* by means of the axiom of completeness the usual analytic geometry over the real numbers became the unique model (up to isomorphism) of his axioms for elementary Euclidean geometry. At the same time, with Hilbert's formulation of the axiom of completeness the notion of a *maximal model* was first conceived. The aim of this talk is to report and analyze the vicissitudes around Hilbert's inclusion of the axiom of completeness into his axiomatic system for Euclidean geometry. This task will be undertaken on the basis of his unpublished notes for lecture courses, corresponding to the period 1894–1905. I will argue that this historical and conceptual analysis not only sheds new light on how Hilbert originally conceived the *nature* of his geometrical axiom of completeness, but also it allows to clarify some misunderstandings concerning the relation between the axiom and the metalogical property of completeness of an axiomatic system, as it was understood by Hilbert in this initial stage. Finally, I will claim that the material included in his notes for lecture courses brings new insights into how Hilbert regarded the importance of metalogical properties like completeness and independence for his axiomatic investigations on the foundations of geometry.

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Logical meaning and linguistic sign at the turn of the 20th century

My talk will focus, with a historical perspective, on the divergence of two conceptions of signification. The first is the lexical one. It therefore concerns units of signification as inserted into the language conceived as a system. The second is the one of the sentence, and in this case, it deals with the meaning as it can be determined by correspondence to a model. In their modern forms, both can be rooted at the turn of the 20$^{\text{th}}$ century – *i.e.*, in the seminal works of Saussure and Frege, which respectively lead to the structuralist theory of the linguistic sign and the model-theoretic semantics. My point is to fix their divergence as the result of an alternative between two possibilities of binarizing the aristotelian semiotic triangle (linguistic symbols, thought, reality).

The first part of my talk will concern Frege's ideographic projects. I will focus on three of its steps linking the first Ideography (1879) to the second (Frege, 1893). That is: [Frege, 1879], where the computation of proofs (*i.e.*, the *calculus*) is not distinguished from its meaning (in terms of *lingua characterica*); [Frege, 1884], where an extentionalist theory of concept and definition of number is developed; and [Frege 1892], where the theory of meaning is reduced to the binarity *Sinn* / *Bedeutung* by merging thought and reality as semantic components. Concerning these three steps, I will explain how the binarized theory of meaning 1) results of Frege's extensionalization of logic, 2) prefigures the dichotomy between syntax and semantics, 3) presupposes the lost of the intensional meaning (still operative in 1879).

These are the three features that anticipate the model-theoretic conception of meaning, especially regarding the dualisation syntax / semantics. Such a conception is indeed adapted to some formal languages, but not to natural ones. To contrast, the second part of my talk will concern Saussure's semiology project.

In this case, I will first recall why the binarization of the sign (into a *signifier* and a *signified*) is explicitly done by rejecting the referent out of the meaning processes of natural language, and what such a rejection implicates in terms of sign arbitrariness. Then, I will explain why the dichotomy syntax / semantics should be traced here at the level of the articulation between the two principles of semiology (*linearity* and *arbitrary*). And finally, I will show how such a localization of that dichotomy implies an interactive and internalist conception of its inner relation – conception that revokes for instance the principle of compositionality, but which is much more adequate to describe meaning processes in natural languages.

The result of this confrontation between the formal language theory of meaning and the natural languages theory of the linguistic sign is to anchor their divergence in their ways of setting up the dichotomy syntax / semantics – the first being dualistic, external and static, whilst the second is monistic, internal and dynamic. From this, I will end my talk by briefly presenting Husserl's project of a pure logical grammar (Husserl, 1901) – as it is the last possibility of binarizing the semiotic triangle (into the peer concept / referent throughout the notion of intentionality), and because with its transcendental conception of intensionality, it merges the dichotomy between syntax and semantics. At last, comparing all those binarization processes of meaning or sign, it should be possible to figure out how the positive points of each – the dualisation for Frege's binarity, its internalization in Saussure's one, and for Husserl, its merger are actually handled by Ludics (Girard, 2007). And that, as a way of concealing the two divergent conceptions of signification.

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Existential Graphs as Universal Logic

In an influential paper Jean van Heijenoort proposed, more than forty years ago, the distinction between Logic as Calculus and Logic as Universal Language as two basic directions in the development of symbolic logic at the turn of the 20th Century. The logic contributions of Charles Sanders Peirce (1839 - 1914) do not fall into the second direction, the universalistic one. In fact, the idea of universal language, in spite of its importance, played no role in his conception of logic. Now, he developed in the last period of his thinking a diagrammatic account of logic, with the implicit idea that diagram construction should be a general tool for analyzing deductive reasoning in general. Peirce was undoubtedly one of the "grounding fathers" of mathematical logic and developed in an algebraic framework a logic system for quantification and relational reasoning that counts as a prototype of what is now called First Order Logic. However, due to philosophical reasons (the same reasons that led him to his semiotic theory), Peirce pursued finally a diagrammatic approach to logic, leading to the formulation of his Existential Graphs. He regarded them as his chef d'oeuvre in logic and as the logic of the future. Peirce developed three diagrammatic systems, called Alpha, Beta and Gamma, as the diagrammatic counterparts to propositional logic, first-order logic with identity and modal logic, respectively. The aim of this contribution is to show that Peirce's idea of his Existential Graphs as a general methodology for the mathematical treatment of deduction can be properly considered as a kind of universal logic framework (in a broad sense, not restricted to algebraic structures). In this sense, the diagrammatic approach can be interpreted as an account of what features are common to all logical structures: The properties of logical concepts are expressed in terms of diagrams. Besides, it will be suggested that the diagrammatic framework can be understood as a tool in order to analyze the properties of deduction in general. In fact, according to Peirce all mathematics is diagrammatic, including mathematical proof. As he stressed in volume IV of his work New Elements of Mathematics, mathematical proof is characterized as the construction of a diagram; it consists in a process of transformation of diagrams showing its logical structure. Since diagrams are icons, a proof has an iconic function with respect to deduction. In Peirce's semiotics, icons are characterized not only as being similar to their objects, but also as being manipulated in order to obtain information concerning what they represent. This characterization implies performing actions on signs. Thus, deduction is the construction of an icon or diagram, whose relations correspond to some extent to the relations in the 'object of thinking'. Diverse perspectives concerning logic arise from Peirce's ideas: (a) a non linguistic conception of logic and logical form; (b) a semiotic view on formal systems; (c) a diagrammatic tool for metalogic ("topological semantics", "topological proof theory"), and (d) a diagrammatic conception of logical constants.

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Leibniz's Conceptions of Modal Necessity
An inconsistency between two notions of necessity in Leibniz is explored: necessity as true in all cases, as mooted in his logic, and necessity as finite having a finite proof, as found in the Theodicy. Suppose, for the sake of argument, that the two concepts of necessity are coextensive. The logical sense suggests an S5 modal necessity, and entails that for all p, $\Diamond\neg p \models \Box\Diamond\neg p$. The finite proof concept allows three possibilities: a proposition has a proof and is necessary, its negation has a proof and is impossible, or neither. It follows that for some p, $\Diamond\neg p$ and $\neg\Box\Diamond\neg p$. The contradiction is resolved by proposing that the intended notion is provability rather than having a proof, and that such a notion coincides with the concept of completeness in an S5 modal system. The paper concludes by showing that the S5 notion of necessity coincides with provability as completeness in an extension of Chris Swoyer's intensional logic for Leibniz to an S5 modal system.

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The Path of Italian Logic from Peano to the Second World War

In the first half of the last century, logical studies in Italy have been dominated by the figure of the great logician Giuseppe Peano, who deeply influenced many logicians worldwide. In the summer of 1900, in conjunction with the Paris International Exposition, the French capital hosted two congresses: the second International Congress of Mathematics and the rst International Congress of Philosophy. The two meetings were attended by the best scholars who, through their lectures and the ensuing debates, set up the state of the art of the two disciplines, as well as the programmatic manifestos for their future. All this, along with the suggestiveness of the date, made the two congresses a major cultural turning point. Among the participants was a young Bertrand Russell, who remembers his Paris experience with these words:

> The Congress was a turning point in my intellectual life, because I there met Peano. I already knew him by name and had seen some of his work, but had not taken the trouble to master his notation. In discussion at the Congress I observed that he was always more precise than anyone else, and that he invariably got the better of any argument upon which he embarked. [...] It became clear to me that his notation afforded an instrument of logical analysis such as I had been seeking for years, and that by studying him I was acquiring a new and powerful technique for the work that I had long wanted to do. By the end of August I had become completely familiar with all the work of his school. I spent September in extending his methods to the logic of relations. It seems to me in retrospect that, through that month, everyday was warm and sunny.

The words of the great British logician leave no doubts: the Italian group headed by the Turinese logician and mathematician Giuseppe Peano imposed itself as one of the strongest and most promising schools on the international scene. Unfortunately, the school born from this circle never did become a leading one. Just a few years after these enthusiastic declarations by Russell, the state of logic in Italy was in huge crisis and decadence - a state that was not to change until fty years later, after World War II. Among the reasons of this decline was the deleterious role played by the Italian neo-idealist philosophy. The aim of this paper is to identify and clarify the cultural, methodological and technical causes in the first half of 20th century which brought about the discontinuity of the research in logic in Italy and to chart the evolution of this path.

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What Are the Inference Steps Really Stepping on? – Brief History of Two-Dimensional Inference
Our main aim is to examine various shapes that inference steps took in the not so distant past. In particular, we focus on similarities that can be found in the works of Frege (1879), Tichý (1988) and de Queiroz, de Oliveira and Gabbay (2011). Their general approach to logical inference constitutes quite distinct school of thought which may be roughly dubbed as two-dimensional conception of inference (deduction). Frege himself, however, never explicitly used the term two-dimensional inference, unlike Tichý and de Queiroz, de Oliveira and Gabbay, who view him as their main source of inspiration.

Although Tichý and de Queiroz, de Oliveira and Gabbay slightly differ in their motivations for the introduction of the so called two-dimensional inference and despite the fact that they rely on different proof theories (Tichý prefers as a basis for his deduction system Gentzen's sequent calculus, while de Queiroz, de Oliveira and Gabbay favour natural deduction), deep conceptual similarities can be found between their notions of two-dimensional inference, which can be indeed traced back to the father of modern logic, Frege. In other words, all of them seem to share common goal: do better "book-keeping" of proof steps, so that no unacknowledged assumptions get in the inferential sequence. And by better book-keeping methods for inference we mean more rigorous, elaborate and extended way of recording inference steps, i.e., explicit tracking of all the assumptions, which have been previously made, withdrawn or are still in force. As Frege put it himself in the preface of his *Begriffsschrift*, we need to keep the chain of inferences free of gaps.

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Logic, Kantian Intuition and Quantification Dependence in Russell's Principles of Mathematics
Russell's criticism of Kant has had an enormous influence on the subsequent study of Kant's philosophy of mathematics. Though largely responsible for the pervasive disrepute in which Kant's views on mathematics were held for over six

20th Century decades, Russell's analysis is also the primary source of what has become a dominant school of interpretation of Kant's philosophy of mathematics – the so called Logical Interpretation.

The school is often identified by two of its principal claims:

1. Kant appeals to intuition in an account for mathematical reasoning. (Whether or not intuition also plays a role in establishing mathematical truths, on which questions proponents may disagree).
2. The role that by Kant's lights intuition fulfills is logical, and by our lights is filled by essentially logical means.

Even interpreters of Kant who reject every premise of Russell's conception of Kant's do not tend to question this: that Russell's views of the inadequacy of Kant's incursion of intuition into his philosophy of mathematics has its sources in (classical) logic's genesis with the discovery of nested polyadic quantification. Thus that Russell holds that the resources of modern polyadic quantification theory are adequate for addressing what Kant perceived correctly to be inadequate logical resources available to his 18th Century self. But, as is well known, the most elaborate statement of Russell's diagnosis of the role of intuition in Kants philosophy of mathematics is 1903 Principles of Mathematics. A neglected benefit of the extensive work in Kant studies carried in the framework of the Logical Interpretation lies in its elaborating the implications of Russell's original insights, arising as they do well before Russell himself has articulated his major logical discoveries, and of course well before we have arrived at an articulated conception of quantification.

The evolution of the Logical School from its Russellian sources presents distinguishable approaches to one of the two sets of apparently unsatisfiable descriptions comprising Kant's introduction of his notion of pure intuition as a factor in what he describes as mathematical construction: the construction of concepts in intuition. The logical approach can be contrasted with its alternatives in its focus on the introduction of pure intuition in the context of mathematical reasoning, as that which, as an intuition, is singular, but as the construction of a concept must underwrite universal validity for anything that falls under the concept constructed. In the tradition of Russell, the aim of a Logical interpretation is to provide an account of intuition in Kantian mathematical construction that tell us a. How a representation can be singular and carry or "codify" information that, when rigorously represented, is sufficient to support conclusions that only something general would be enough to support, and b. How a singular representation can carry with it the cognitive excess that is encapsulated by its construction.

Michael Friedman who adapts Russell's diagnosis undertakes to address both questions in its development. Jaakko Hintikka's account discriminates between aspects of the diagnosis but adopts Russellian insights about logic more fully in those aspects of the Russellian diagnosis that he accepts. I argue that this is specifically in evidence in Hintikka's association of intuition with the analysis of quantification and, that the view can naturally and suggestively be extended by deploying the contextual analysis of definite descriptions.

Jaakko Hintikka's contribution is subtle, for it isolates a logical discovery in

Russell and the singularity criterion of intuition, and shows how the one can be used to clarify the other in such a way as to give a coherent logical account of the role for intuition in mathematical reasoning that meets Kant's description of intuition in mathematical construction, is faithful to the contrast between intuitions and concepts, and appeals to no other characteristic of intuition – specifically, not to immediacy, and not to sensibility, pure forms, space time or ideality.

The logical approach as developed following the work of Michael Friedman assumes that the logical replacement for intuition will accommodate a poverty of expressive powers. By articulating the extended view of Hintikka we can reconsider what the precise nature is of the expressive powers assumed unavailable to Kant for which intuition is supposed to compensate.

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Lawvere and Hegel

Throughout his works in Categorical logic Lawvere refers to Hegel's Dialectical logic as a major source of his intellectual inspiration. At several instances Lawvere also suggests to consider his newly invented Categorical logic as a version of Dialectical logic put into a precise mathematical form. The aim of this talk is to explore the impact of Hegel's philosophical logic onto Lawvere's pioneering works in Categorical logic.

Among several Hegelian themes referred to in Lawvere's writings the most frequent and, in my view, the most important is Hegel's distinction between the *subjective* and the *objective* logic. Hegel describes these two kinds of logic not as two parts of the same static whole but rather as two stages of the same dialectical reasoning, which begins with the most general ontological categories of Being, Non-Being and Becoming, then proceeds through some further categories like Thisness, Quality, Finiteness, Infinity, One, Many, Quantity, Measure, and some other, and only at a late stage involves the notion of subjectivity. Hegel's criticizes the traditional logic (descending from Aristotle) as well as Kant's Transcendental logic as merely subjective (and thus having no objective significance) by pointing the fact that these types of logic assume some notion of thinking subject from the outset without grounding this notion in the objective dialectics of categories.

Lawvere turns this Hegelian critique of logical subjectivism against the received notion of formal (symbolic) calculus by arguing that conventional symbolic presentations of formal logical calculi are chosen arbitrarily and fail to capture the objective invariant algebraic structures behind these calculi. As a remedy Lawvere puts forward his notion of Cartesian Closed Category (CCC), which captures a large class of relevant structures; then Lawvere upgrades the concept of CCC to that of *hyperdoctrine*, which captures the invariant structure of logical quantification with the notion of adjoint functor.

On a parallel development Lawvere discovers what he describes as the "unity of opposites [..] between logic and geometry" in the then-new field of topos theory pioneered by Grothendieck and his collaborators. Lawvere's basic observation is that all main features of a given topos have not only geometrical but also logical significance. This allows Lawvere to conceive of Topos logic as the *objective* logic in Hegel's sense, where usual (*subjective*) logical operation are grounded in spatial and other fundamental categories, which are commonly called non-logical.

Lawvere's work in Categorical logic paves the gap between between Hegel's Dialectical logic and the modern mathematical logic and thus makes it necessary to take Hegel's views on logic seriously in today's debates.

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Criticism to psychologism, logical truths and language in Gottlob Frege's "The Thought"

Anti-psychologism is a pervasive topic in Frege's works. In his Grundlagen, already in 1884 indeed, he sought to show that arithmetic specifically and mathematics in general have nothing to do with "psychological methods of reflexion" that in his view would abound in philosophy and even in logic. In this work, Frege distinguishes the objective from the subjective and claims that in mathematics we deal only with the objective. And also in the Preface to his Grundgesetze, the first volume of which was released in 1893 and the second in 1903, we find his criticism to psychologism in the distinction between the laws of logic and the laws of psychology. The laws of logic are laws of being true (Gesetze des Wahrseins), he says, aiming at absolutely distancing logic from any psychological consideration about our mental acts, contents or states, our Vorstellungen, as he says, which term is usually translated as representations or ideas. The definitive step away from psychologism is taken very clearly in his article "The Thought", which is the first of his Logical Investigations, and appeared in 1918. In this article, Frege passes from the level of the objects of discourse to the level of discursive objects, to the level we now call semantic: to investigate truth, we should investigate language. This claim is in line with the project of his concept-language, which he thought of as an exact, unambiguous medium to express thought. In doing this, Frege opens roads to be taken later on by such authors as Austin and Wittgenstein in philosophy of language. It is this step we mainly aim at clarifying in this communication.

4.5.11 Algebra and Category Theory

The invited keynote speaker of this session is Jonathan Seldin (page 81).

Contributed Talks

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Algebraic Resolution Methods for Propositional Logics Characterized by Boolean Algebras with Operators

The *polynomial ring calculus (PRC)*, introduced in [2], is as an algebraic resolution method for classical and non-classical logics. This method basically consists in translating formulas of a logical system L into polynomials over a finite (Galois) field, in such a way that the deduction problem for L (i.e. determining whether a propositional formula can be deduced from a set of formulas in L) becomes equivalent to reduce polynomials by performing algebraic operations (defined in accordance with the ring properties of addition and multiplication and the reduction rules of polynomials over finite fields). As it is shown in [2], PRC is apt for many propositional logics, including finitely-many-valued logics and some non-truth-functional logics. Moreover, in [1], it is shown that PRC is also apt for some modal logics, by defining two different but equivalent PRCs for $S5$, one of them based on the fact that $S5$ is characterized by a class of *boolean algebras with operators (BAOs)*.

On the other hand, Gröbner Basis had also been used to define a resolution method for many-valued propositional logics (see, for instance, [3]). This method is also based on translating formulas of a logical system into polynomials over a field, but in this case the deduction problem for the respective logic is transformed into the algebraic one of determining whether a polynomial vanishes on an algebraic variety, which can be solved by calculating Gröbner Basis. The author do not know works using Gröbner Basis to define resolution methods for non-truth-functional logics like normal modal logics and intuitionistic logic.

In this work, it is presented a general framework for defining PRCs and resolution methods base on the calculation of Gröbner Basis to propositional logics characterized by BAOs. Taking into account that many normal modal logics are characterized by BAOs, and the well-known conservative translation of intuitionistic logic into $S4$, the algebraic methods here presented are particularly apt for these logics.

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Gödel-Dummett logic, the category of forests, and topoi

Joint work with Pietro Codara and Vincenzo Marra, Università degli Studi di Milano.

Gödel-Dummett Logic $\mathcal{G}$ is the schematic extension of Intuitionistic Logic by the prelinearity axiom $(\varphi \to \psi) \vee (\psi \to \varphi)$; equivalently, it is the extension of Hájek's Basic Fuzzy Logic by the idempotency axiom $\varphi \to (\varphi \& \varphi)$. This is an algebraisable logic in the sense of Lindenbaum-Tarski and of Blok-Pigozzi, and its equivalent algebraic semantics is given by the variety $\mathbb{G}$ of *Gödel algebras*, which are precisely idempotent, divisible, prelinear, integral, bounded, commutative, residuated lattices. Equivalently, Gödel algebras are exactly the Heyting algebras satisfying the prelinearity law $(x \to y) \vee (y \to x) = \top$, where $\top$ denotes the top element of the Heyting algebra. In this talk we offer three perspectives on the formal semantics of Gödel-Dummet logic, different from Hájek's approach based on degrees of truth. We shall see that the third, topos-theoretic perspective, has a twist to offer—which embodies the main point of our talk. **Stone-type dualities.** Seen as a category, $\mathbb{G}$ is dually equivalent to the pro-finite completion of the category F of finite forests and open maps. Here, a (finite) *forest* is a (finite) poset such that the lower set of each element is totally ordered, and an *open map* between posets is an order-preserving map carrying lower sets to lower sets. The duality $\mathbb{G} \equiv (\mathrm{Pro}\,\mathsf{F})^{\mathrm{op}}$ yields a sound and complete semantics for propositional Gödel logic, as each finitely generated Gödel algebra arises as the algebra of subforests of a suitably chosen finite forest, exactly in the same way as each finitely generated Boolean algebra is the algebra of subsets of a suitably chosen finite set. This construction of $\mathbb{G}^{\mathrm{op}}$ is intimately related to Kripke semantics for Intuitionistic Logic, and is an example of Stone-type duality. **Lawvere theories.** Following F. W. Lawvere's category-theoretic approach to universal algebra, $\mathbb{G}$ is equivalent to the category of finite-product preserving functors $\mathsf{F}_0 \to \mathsf{Set}$, and natural transformations between them. Here, F_0 is the full subcategory of F whose objects are dual to finitely generated free Gödel algebras. This can be used to show that F_0 (as opposed to the larger F) provides a sound and complete semantics for $\mathcal{G}$. **Topoi.** It is well known that the category Set^{ω} of *sets through time* is a topos that provides a sound and complete semantics for $\mathcal{G}$. Does F, too, provide a topos-theoretic semantics for Gödel-Dummett Logic? We recall that an elementary topos is a finitely complete category with exponentiation and subobject classifier. We shall prove that

F is finitely complete and has the subobject classifier, but it lacks exponentiation. We shall explore the import of this fact with respect to first-order Gödel logic, and offer some tentative interpretations. By contrast, we prove that the subcategory of F dual to three-valued Gödel-Dummett logic is indeed a topos, while this is not the case for any n-valued Gödel-Dummett logic with $n > 3$.

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Partial Algebraic Semantics
Let K be a class of algebras that is an algebraic semantics for some propositional logic L. That is, a formula φ is a theorem of L if, and only if, some related equation, say $\varphi = 1$, holds in K. Consequently, φ is not a theorem of L if, and only if, the inequation $\varphi \neq 1$ is satisfiable in K. Recall that $\varphi \neq 1$ is satisfiable in K if there exists an algebra A in K and an assignment of elements $a_1, \ldots, a_n$ of A to the variables in φ such that $\varphi^A(a_1, \ldots, a_n) \neq 1^A$. Observe that in evaluating $\varphi^A(a_1, \ldots, a_n)$ only a finite subset of the elements of A and finitely many instances of the operations in A are used. Such a subset with partially defined operations constitutes a 'partial subalgebra' of A. Satisfiability in such a partial subalgebra implies satisfiability in A and hence in K. Conversely, if φ is a theorem of L, then $\varphi \neq 1$ is not satisfiable in K and hence not in any partial subalgebra of any algebra in K. Thus, the class of all partial subalgebras of algebras in K is a class of models for the logic L, which we call a 'partial algebraic semantics'.

By the 'size' of φ we mean the number of constants, operation symbols and distinct variables occurring in φ. If $\varphi \neq 1$ is satisfiable in K then it is satisfiable in a partial subalgebra of size at most that of φ. Thus, to determine satisfiability of $\varphi \neq 1$ in K we need only search through all partial subalgebras of members of K up to the size of φ. So we have an immediate finite model property with known bounds. The problem of satisfiability is now reduced to that of recognizing, amongst the set of all partially defined algebras on the same language as K, those that are partial subalgebras of members of K. That is, we require an internal characterization of the partial subalgebras. With such a characterization, we may infer complexity results for the class K and logic L. In fact, this partial subalgebra approach is more generally applicable to the satisfiability problem of any quantifier-free first order formula in the language of K, so we obtain complexity results for this more general satisfiability problem and hence for the universal theory of the class K.

In this talk, we shall discuss the above notions in more detail and illustrate them in the case of Intuitionistic propositional logic. In particular, we characterize the partial subalgebras of Heyting algebras, which form an algebraic semantics for Intuitionistic propositional logic, and explore the consequences for the complexity of satisfiability and the universal theory of Heyting algebras.

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Anti-associative groupoids
Given a groupoid $\langle G, \star\rangle$, and $k \geq 3$, we say that G is *antiassociative* iff for all $x_1, x_2, x_3 \in G$, $(x1 \star x2) \star x3$ and $x1 \star (x2 \star x3)$ are never equal. Generalizing this, $\langle G, \star\rangle$ is k-antiassociative iff for all $x_1, x_2, \ldots, x_k \in G$, any two distinct expressions made by putting parentheses in $x_1 \star x_2 \star x_3 \star \ldots \star x_k$ are never equal.

We prove that for every $k \geq 3$, there exist finite groupoids that are k-antiassociative. We then obtain similar results for algebras with operations of any finite arity.

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A note on the hierarchy of algebraizable logics
A logic L (a structural consequence relation) in a language $\mathcal{L}$ is *algebraizable* [1, 2] w.r.t. a class of L-algebras $\mathbf{L}$ with translations $\rho\colon \mathrm{Eq}_{\mathcal{L}} \to \mathcal{P}(\mathrm{Fm}_{\mathcal{L}})$ and $\tau\colon \mathrm{Fm}_{\mathcal{L}} \to \mathcal{P}(\mathrm{Eq}_{\mathcal{L}})$ if

1. $\Pi \models_{\mathbf{L}} \varphi \approx \psi$ iff $\rho[\Pi] \vdash_{\mathrm{L}} \rho(\varphi \approx \psi)$

2. $p \dashv\vdash_{\mathrm{L}} \rho[\tau(p)]$

There are numerous strengthenings of this notion in the literature, which are often confused, the usually mistakes being that finitary of L implies that $\mathbf{L}$ is an elementary class (a counterexample is given in [3]) or vice versa (a counterexample is given in [4]). Moreover, the relation of these two notions with the finiteness of ρ (called *finite algebraizability*) is another usual source of confusions.

The goal of this talk is to clarify these confusions by considering the overlooked condition of finiteness of τ. We show that by combining these four properties we obtain 7 distinct classes of logics (the smallest class coinciding with that of *B–P algebraizable logics* [1]). Then we add two more well-studied properties: regularity of algebraization (a special requirement for τ) and algebraic implicativeness of L (a special requirement for ρ). We eventually obtain a hierarchy of 17 classes logics of algebraizable logics and show their separation examples.

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Interpolation in First Order Logics with Constructors

We prove an interpolation result for first-order logics with constructors in the signatures. The framework used is that of the so-called *institution theory* invented by Goguen and Burstall which is a categorical-based formulation of the informal concept of *logical system* sufficiently abstract to capture many examples of logics used in computer science and mathematical logic, and expressive enough to elaborate our general results. *Constructor-based logics* are obtained from a base logic, for example classical first-order logic, by enhancing the syntax with a sub-signature of constructors and by restricting the semantics to models with elements that are *reachable by constructors*. The sentences and satisfaction condition are preserved from the base institution, while the signature morphisms are restricted such that the reducts of models that are reachable in the target signature are again reachable in the source signature. The interpolation property is very difficult to obtain, in general. In constructor-based institutions interpolation holds under certain extra conditions which are added on top of the hypothesis under which it holds for the base institution. In this paper we provide a general method of borrowing interpolation from a base institution for its constructor-based variant across institution morphisms. This result depends on *sufficient completeness*. Intuitively, a specification (Σ, Γ), where Γ is a set of formulas over the signature Σ, is sufficient complete if every term can be reduced to a term formed with constructors and operators of loose sorts using the equations in Γ. As far as we know, the interpolation problem in logics with constructors is still open and therefore, the conditions under which we prove interpolation for constructor-based first order institutions are new. We instantiate our general interpolation results to concrete institutions such as constructor-based variants of first-order logic and preorder algebra.

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Polynomial Ring Calculus and Gröbner Basis, Two Algebraic Resolution Methods for Classical and Non-classical Logics

On one hand, the *polynomial ring calculus (PRC)* introduced in [2] is as an algebraic resolution method for classical and non-classical logics. This method basically consists in translating formulas of a logical system L into polynomials over a finite (Galois) field, in such a way that the deduction problem for L (i.e. determining whether a propositional formula can be deduced from a set of formulas in L) becomes equivalent to reduce polynomials by performing algebraic operations (defined in accordance with the ring properties of addition and multiplication and the reduction rules of polynomials over finite fields). As it is shown in [2], PRC is apt for many propositional logics, including finitely-many-valued logics and some non-truth-functional logics. In [1], it is shown that PRC is also apt for some modal logics, particularly for $S5$. Moreover, in [3], the PRC is extended to deal with the monadic fragment of first-order logic.

On the other hand, Gröbner Basis had also been used to define resolution methods for many-valued propositional logics (see, for instance, [4]). These method is also based on translating formulas of a logical system into polynomials over a field, but in this case the deduction problem for the respective logic is transformed into the algebraic one of determining whether a polynomial vanishes on an algebraic variety, which can be solved by calculating Gröbner Basis.

In this work, the two algebraic resolution methods mentioned above are characterized (considering only the case of propositional logics), pointing out some of their connections and differences, and providing a general algebraic framework for determining deductibility on many propositional logics.

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Lattice tolerances and their blocks

Tolerances are in the focus of current interest as an important tool in Lattice Theory. A *tolerance relation* of a lattice L is a reflexive and symmetric relation compatible with operations of the lattice. Equivalently, a tolerance of L is the image of a congruence by a surjective lattice homomorphism onto L [3]. All tolerances of a lattice L form an algebraic lattice (with respect to inclusion) [1].

Let R be a tolerance of L. If $X \subseteq L$ is a maximal subset with respect to the property $X \times X \subseteq R$, then X is called a *block* of R. All blocks of the tolerance R form a new lattice called a *factor lattice* [2]. The skeleton tolerance of a lattice and its factor lattice called a *skeleton* play a special role. It is known [5] that every finite lattice is a skeleton of a finite distributive lattice. However, we are going to show that there are lattices which cannot be blocks of the skeleton tolerance of any finite lattice. It is clear for distributive or modular lattices, especially, as their blocks of the skeleton tolerance are maximal boolean or, respectively, complemented intervals of such lattices [4]. Our goal is to characterize lattices which can be blocks of the skeleton tolerance in the general case.

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New Involutive FL_e-algebra Constructions FL_e-algebras are algebraic counterparts of substructural logics with exchange. A structural description and also a classification of certain subclasses of involutive FL_e-algebras have been obtained in 4 and 3, respectively, as follows: If an involutive FL_e-monoid $\langle X, \circledast, \leq, t, f\rangle$ is conic then $\circledast$ is the twin-rotation of the two cone operations $\otimes$ and $\oplus$, that is,

$$x \circledast y = \begin{cases} x \otimes y & \text{if } x, y \in X^- \\ x \oplus y & \text{if } x, y \in X^+ \\ \neg(x \to_\oplus \neg y) & \text{if } x \in X^+,\ y \in X^-, \text{ and } x \leq \neg y \\ \neg(y \to_\oplus \neg x) & \text{if } x \in X^-,\ y \in X^+, \text{ and } x \leq \neg y \\ \neg(y \to_\otimes (\neg x \wedge t)) & \text{if } x \in X^+,\ y \in X^-, \text{ and } x \not\leq \neg y \\ \neg(x \to_\otimes (\neg y \wedge t)) & \text{if } x \in X^-,\ y \in X^+, \text{ and } x \not\leq \neg y \end{cases}.$$

If U is an absorbent-continuous sharp FL_e-algebra on a subreal chain then its negative cone is a BL-algebra with components (see 1.) which are either cancellative or MV-algebras with two elements, and with no two consecutive cancellative components, its positive cone is the dual of its negative cone with respect to $\neg$, and its monoidal operation is given by the twin-rotation of its cones.

In this talk we introduce new construction methods resulting in involutive (but not sharp) FL_e-algebras along with some related characterizations.

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A Fibred and Indexed Semantics for a Hoare-like Logic

Indexed and fibred category theory have a long tradition in computer science as a tool for the semantics of logic and computation, as for instance, in the theory of Institutions and General Logics, and as categorical models for a wide range of logics and type theories as well. In particular, pointwise indexing $\langle A_i \mid i \in I\rangle$ and display maps $d : \biguplus\langle A_i \mid i \in I\rangle \to I$ of sets can be considered as the motivating set-theoretical constructions for their categorical counterparts, indexed and fibred categories respectively. In fact, the interplay between these two categorical constructions are well-known. For instance, we have the equivalences between indexed categories and cloven fibrations, as well as between strict indexed categories and split fibrations.

In this work we present a detailed discussion of a Hoare-like logic using a small imperative language as an example. The main point is to present a categorical semantics using both indexed and fibred categories, with a special focus on the adequacy of both techniques for the specific task mentioned here.

The general ideas of our approach are as follows. Consider an imperative programming language $\mathcal{L}$. Typically it has at least one data-type, and commands for assignment, selection and iteration. In providing the semantics of imperative programming languages, such $\mathcal{L}$, it is necessary to assume some kind of a *State* concept. Concretely, mappings from locations (or variable names) to values could be taken into account. The point is that, anything that serves as a model to *States* can be taken as objects in a category B, and, by choosing an adequate morphism notion, this category can be viewed as providing a denotational model for the semantics of $\mathcal{L}$. This semantics is formally provided by means of a B-valued functor from the syntactic category (the term-algebra of the language).

Let us consider B as having integer-valued finite mappings as objects (maps from finite sets of $\mathcal{L}$-variables into Nat). A morphism between m_1 and m_2 is a computable function that turns m_1 into m_2, by means of a piece of code of $\mathcal{L}$. On top of B one can consider a logic model, namely, to each *State* m one consider a Heyting Category for m (H_m). This will be the logic over the *State* m. The most usual H_m is the cartesian closed category of propositions (objects) and morphisms (proofs) on m. For instance, consider the semantics of the following assignment statement $x := x + y$ and the corresponding denotation of $\lambda m.\ update(m, x, m(x) + m(y))$. It is a morphism in B and naturally induces a functor from H_m into $H_{update(m,x,m(x)+m(y))}$ that maps propositions $p[m(x \mapsto (x + y))]$ into $p[m]$. This mapping, whenever is functorial, induces a substitution functor $(\_)^\star$ part if we take either the indexed or fibred way. On top of this initial construction then it is possible to define a Hoare logic for $\mathcal{L}$ and a sound categorical model for the calculus as well.

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Nuclei over (Quasi–ordered) Monoids and Symmetric Constructive FL–algebras

In [3] we have developed an algebraic semantics for symmetric constructive logic of I.D. Zaslavsky [5] devoid of structural rules and have shown how it is related to cyclic involutive FL–algebras and Nelson $\mathrm{FL}_{\mathrm{ew}}$–algebras. Because of this analogy we called the obtained calculus *symmetric constructive full Lambek calculus* (**SymConFL**) and its algebraic models *symmetric constructive FL–algebras*.

In this talk we focus on the construction of symmetric constructive FL–algebras and its basic subclasses (also distributive, i.e. in which the operations of join and meet distribute over each other) from monoids. A way of completing a monoid, by means of a nucleus, in order to obtain a FL–algebra is commonly known and used in the field of substructural logic [2]. Here we present a way of completing a monoid, by means of a nucleus and its dual (interior operator), in order to obtain a symmetric constructive FL–algebra. The so–called *twist–structure* construction (independently developed by M.M. Fidel [1] and D. Vakarelov [4] for Nelson algebras) is adapted.

We show not only how to construct a symmetric constructive FL–algebra by means of a nucleus and its dual, but we also define several nuclei over a quasi–ordered monoid. For some of them the obtained symmetric constructive FL–algebras are distributive. Particularly interesting are nuclei yielding complete embedding of some quasi–ordered monoids into their nuclear completions. As a consequence we obtain an algebraic proof of the cut elimination theorem for basic variants of **SymConFL** enriched with the axiom $\Gamma \Rightarrow$, and the finite model property for most of them.

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Hybridizing logics for quantitative reasoning

It is introduced in [1] a method that extends an arbitrary institution with hybrid logic features [2], concretely by adding a Kripke semantics, for multi-modalities with arbitrary arities, as well as nominals and local satisfaction operators. This lead to the characterization of an *institution-independent method* [3] to specify *reconfigurable systems*, i.e., computational systems that evolve along different modes of execution in response of external stimulus. The relevance of such a generalization step is in line with a basic engineering concern which recommends that the choice of a specification framework should depend on the nature of the system's own requirements. Depending on them one may, for example, equip each local state with a partial algebra, a hidden algebra, a propositional valuation, a first-order structure or even a hybrid logic model (since the method recurs).

These techniques and results are, until now, exclusively focussed on dealing with *qualitative* and *discrete* properties, whose verification is expressed as a Boolean outcome. Recently, however, *quantitative* reasoning, dealing with weighted or probabilistic system's behaviour and evolution, and the underlying mathematical techniques, emerged as a main challenge for Computer Science. This witnesses a shift from classical models of computation, such as labeled transition systems, to more elaborate ones where quantities can be modelled, such as weighted, hybrid or probabilistic automata. On the logic side, one may also mention *multi-valued*, *probabilistic*, *probabilistic hybrid*, *fuzzy* and *possibilistic* logics.

We discuss on this talk that 'quantitative' dimension on the context of the hybridisation method [1]. This is done in two distinct, but interrelated, directions: via the direct application of the method to logics for *quantities*, typically to multi-valued and fuzzy logics e.g. [4]; and via the adaptation of the method, generalising the underlying semantic structures, replacing the relational component of models (upon which modalities are interpreted), typically regarded as coalgebras [5] for *Set*-functors, by coalgebras over suitable categories of probability distributions or metric, or topological spaces (e.g. [6]).

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Product Models on Positive Process Algebras A Preliminary

Positive Process Fragment algebras are defined and applied to obtain models by defining a language category over the signatures. Fragment consistent model techniques (author 1996-2005) are applied to generate Kleene models. Generic diagrams from the author's 1980's allow us to define canonical models with specific functions and primitive extensions. String ISL algebras (2005) is an example Σ-algebra with an additional property that the signature Σ's has a subsignature Λ with only 1-1 functions. Specific applications to Kleene algebras with ISL are presented. Positive categories and Horn models are further technical areas presented and applied to the above. Specific filters, generic products, positive and Horn categories are presented and applied to the areas developing new model theoretic products that provide further foundations to computing and parallel language classes.

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Representation Theory of Logics: a categorial approach

In the 1990’s rise many methods of combinations of logics (**[2]**). They appear in dual aspects: as processes of analysis of logics (e.g., the “Possible Translation Semantics” of W. Carnielli) or as a processes of synthesis of logics (e.g., the “Fibrings” of D. Gabbay). This was the main motivation of categories of logics. The major concern in the study of categories of logics (CLE-UNICAMP, IST-Lisboa) is to describe condition for preservation, under the combination method, of meta-logical properties. Our complementary approach to this field is study the “global” aspects of categories of logics (**[1]**, **[3]**). In **[1]** it is considered a simple notion of category of (finitary, propositional) signatures $\mathcal{S}$ and a corresponding category of logics $\mathcal{L}$: a logic is a signature with a Tarskian consequence relation and a morphism between logics is a “translation”, i.e., a signature morphism whose induced function of formula algebras preserves the consequence relation. It is considered a category $\mathcal{A}$ of Blok-Pigozzi algebraizable logics and translations that preserves algebraizable pairs; $U : \mathcal{A} \to \mathcal{L}$ denote the obvious functor. The categories $\mathcal{S}, \mathcal{L}, \mathcal{A}$ have good categorial properties – they are finitely-accessible categories – with many small limits and colimits (only the last category is not complete/cocomplete), but they does not allow a good treatment of the “identity problem” for logics: for instance, the presentations of “classical logics” (e.g., in the signature $\{\neg, \vee\}$ and $\{\neg', \to'\}$) are not $\mathcal{L}$-isomorphic. In this work, we sketch a possible way to overcome this “defect”, by a mathematical device.

Recall that in the theory of Blok-Pigozzi, to each algebraizable logic $a = (\Theta, \vdash)$ is canonically associated a *unique* quasivariety $QV(a)$ in the same signature Θ (its “algebraic codification”), thus the inclusion functor has a left adjoint $(L, I) : QV(a) \rightleftarrows \Theta - Str$. If $f \in \mathcal{A}(a_0, a_1)$ and we consider $(L_j, I_j) : QV(a_j) \rightleftarrows \Theta_j - Str$, then the induced functor $f^\star : \Theta_1 - Str \to \Theta_0 - Str$ $(M_1 \mapsto (M_1)^f)$ restricts to $f^\star\restriction: QV(a_1) \to QV(a_0)$, thus $I_1 \circ f^\star\restriction = f^\star \circ I_0$ and $L_1 \circ f^\star \cong f^\star\restriction \circ L_0$.

To each logic $l = (\Sigma, \vdash)$, are associated three pairs (left and right) of data: **(I)** two comma categories (over $\mathcal{A}$ or $\mathcal{A}_{fin.pres.}$): $(l \to U)$, the “left algebrizable spectrum of l” (analysis process); $(U \to l)$, “right algebrizable spectrum of l”

(synthesis process); **(II)** two canonical arrow: $l\text{-}can : l \to lim(l\text{-}D)$, obtained from the canonical cone associated to the diagram $l\text{-}D : (l \to U) \to \mathcal{L}$; $can\text{-}l : colimD\text{-}l \to l$ is obtained from the co-cone encoded by $D\text{-}l : (U \to l) \to \mathcal{L}$; **(III)** two categories of categories (left and right "representation category"): $l\text{-}Mod$, whose objects are functors from the quasivarieties associated to objects of $(l \to U)$ into $\Sigma - Str$, arrows associated (contravariantly) to arrows of $(l \to U)$; $Mod\text{-}l$ is obtained from $(U \to l)$.

Morphisms between logics $f : l \to l'$ induce three pairs (left and right) of data: **(I)** a left "spectral" functor: $(l \to \mathcal{U}) \stackrel{-\circ f}{\leftarrow} (l' \to \mathcal{U})$; **(II)** a pair of morphism of arrows canonical arrows (= commutative squares): $(lim(-\circ f), f) : l\text{-}can \to l'\text{-}can$; **(III)** a left "representation" functor: $(l\text{-}Mod) \stackrel{f^{\star}\circ -}{\to} (l'\text{-}Mod)$.

In this setting we propose the study of (left and right) "Morita equivalence" of logics and variants. We introduce the concepts of logics (left/right)-stably-Morita-equivalent and show that the presentations of classical logics are stably Morita equivalent but classical logics and intuitionist logics are not stably-Morita-equivalent: they are only stably-Morita-adjointly related (on left and on right). In this work in progress, the following themes will be developed: categories of fractions of categories of logics; necessary/sufficient conditions for Morita equivalency of logics; new (functorial) morphisms between logics from the categories $Mod\text{-}l$ and $l\text{-}Mod$; new notions of identity of logics; similar construction on alternative base categories (**[3]**).

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An axiomatic approach to weak lattice-valued orderings

We present an axiom system whose models are ordering structures and their lattice valued generalizations in general.

For a given poset (P, ρ), we deal with weakly reflexive, antisymmetric and transitive subrelations of ρ, so called *weak orderings* contained in ρ. The collection of all these relations is an algebraic lattice under inclusion and we investigate its structure. Particular retracts in this lattice coincide with the lattices of all suborders of ρ and the boolean lattice of all its subposets. In addition, some convex sublattices are lattices of orders on these subposets. Some lattice identities satisfied on this structure are presented.

We also connect these with more general lattices containing special relations and we present their common properties.

In the second part, we deal with analogue notions and properties in the framework of lattice valued orderings. Namely, for a given domain P, we consider all mappings from P into a complete lattice L, fulfilling analog properties as above, called weak *lattice valued orderings* on P. We investigate the collection of all such mappings contained in a given lattice valued order on P, which is a complete lattice under the lattice valued inclusion. Similarly to the classical case, this lattice has particular convex sublattices connected to lattice valued suborders and subposets. We also prove the representation theorem for such mappings.

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The logic of equality and the logic of order

Roughly speaking, *equational reasoning* is the underlying reasoning used by mathematicians within algebraic practice, particulary, when equalities must be handled and one equality should be inferred from others. The *logic of equality*, sometimes called *equational logic*, is a formalization of equational reasoning by presenting both a formal language and a set of inference rules adequate in the manipulation of equations to produce equational proofs. Equational logic has been extensively investigated and important results, both model theoretical and proof theoretical, have been established.

However, equalities are not the only objects usually dealt with when some form of algebraic reasoning is at stake. More specifically, when the subjacent algebraic structures one is working with are equipped with some kind of ordering relation, it may be more useful or easy to manipulate inequalities instead of equalities. It is convenient then to have a *logic of order* within which such reasoning may be formalized.

The manipulation of inequalities seems to be more convenient and basic than that of equalities due to two main reasons. First, once a partial order is at disposal, a congruence relation may be defined, which sometimes turns out to be equality itself. That is, apparently, the logic of order is not more complex than the logic of equality. Nevertheless, the converse of this fact seems not to be true in general, that is, the underlying reasoning to manipulate inequalities seems not to be formalizable on top of equational logic.

In this work we propose a formalization for the logic of order, proving soundness and completeness, besides a comparison with the logic of equality, as far as proof theory is concerned, as a first step in formally supporting the claim that order is more basic than equality.

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General formulas for compatible relations

In a first order language with one binary relational symbol, we present a set of particular universal formulas whose model are binary relations forming special algebraic lattices. We list some lattice identities which are fulfilled, provided that the diagonal relation of the domain satisfies mentioned formulas. We prove some structure properties of these lattices in terms of lattice endomorphisms, retracts and convex sublattices. Then we switch to compatible relations on algebras and show that these are also modeling starting formulas. In this context we analyze properties of lattices consisting of congruences, weak congruences, compatible preorderings and of some other compatible relations. Some known algebraic properties are generalized and all are formulated in this new language. Namely, we deal with relational extension and intersection property (CEP and CIP for congruences, respectively) and characterize these in lattice theoretic framework. We prove that all these algebraic lattices have analogue structural properties.

After adding some new type of formulas, we analyze compatible orderings, in particular ordered groupoids and other similar algebras. We characterize all these within the same structure using, among other properties, special weak reflexivity.

As a generalization, we prove representation theorems for fuzzy ordered algebras.

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5 Book Exhibition

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23. AFHIC – Associação de Filosofia e História da Ciência do Cone Sul

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